



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

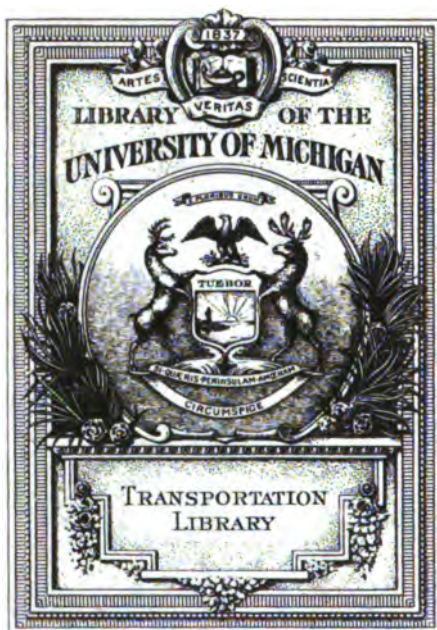
Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>



THE GIFT OF
R.B. Rifenberick

Transpo
Libr

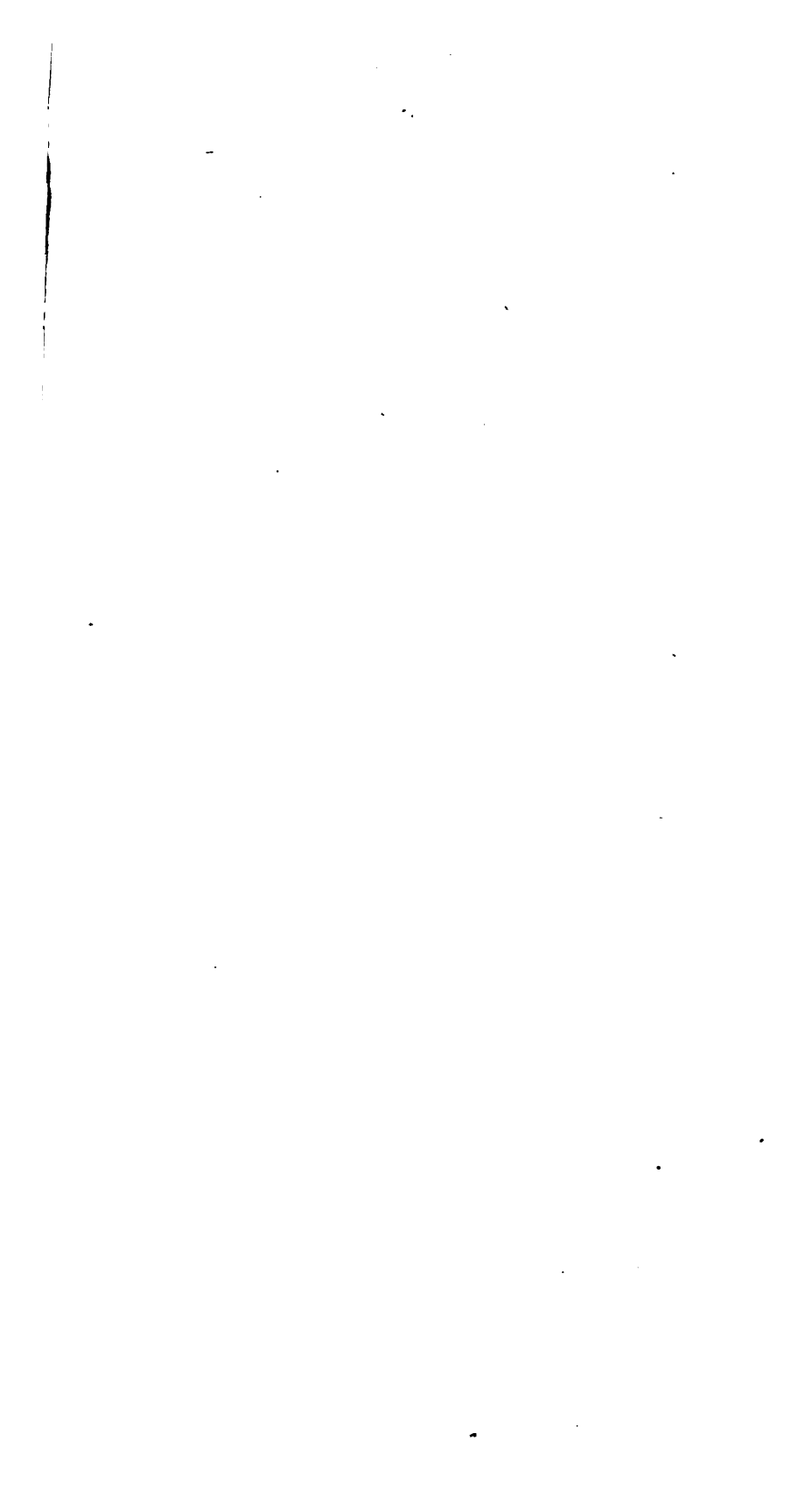
T/

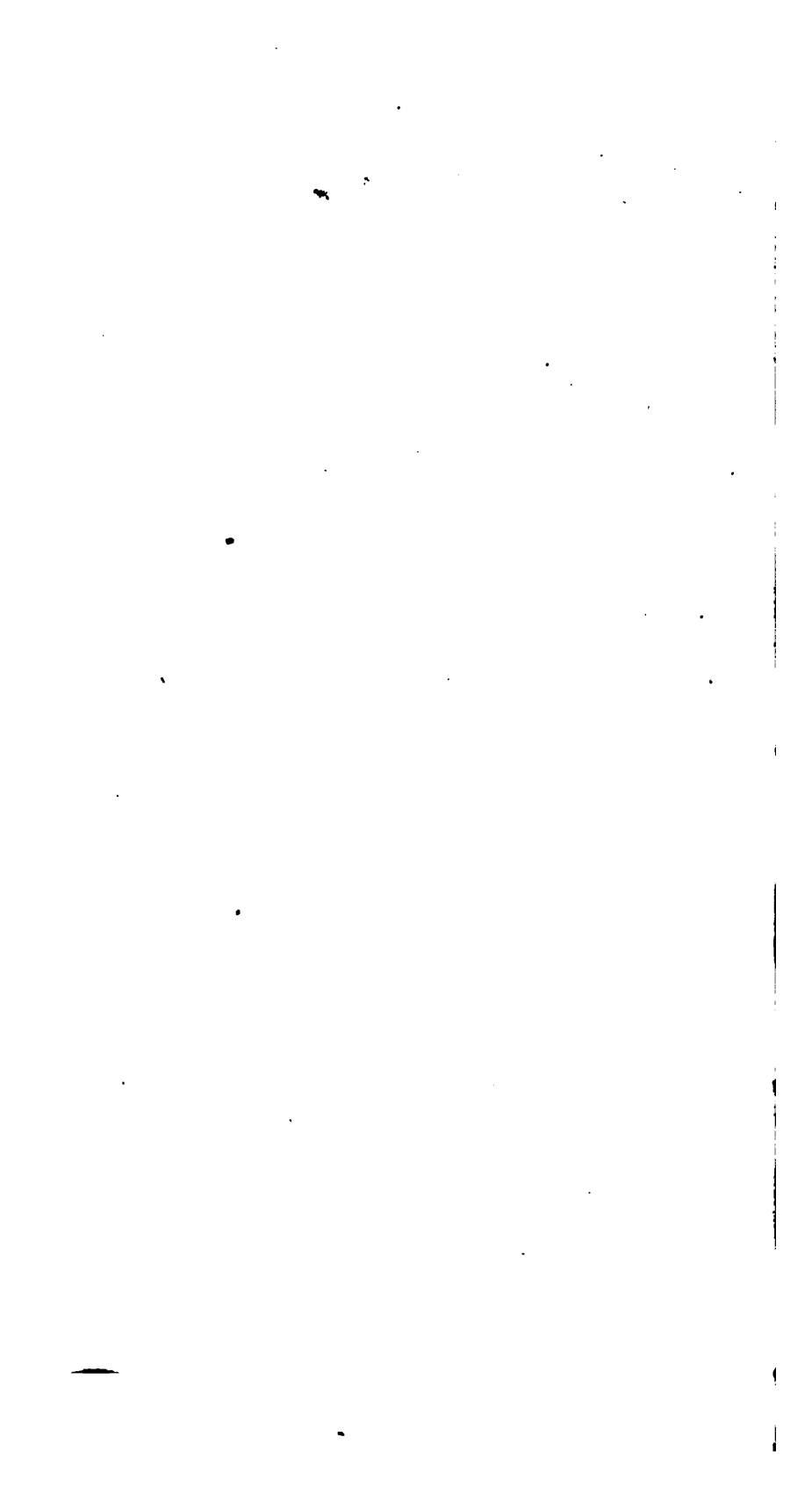
15

.M1

R. B. Rifenberg,
Cincinnati, O.







THE
CIVIL-ENGINEER & SURVEYOR'S
MANUAL:

COMPRISING
Surveying, Engineering, Practical Astronomy,
GEODETICAL JURISPRUDENCE,
ANALYSES OF
MINERALS, SOILS, GRAINS, VEGETABLES,
VALUATION OF
LANDS, BUILDINGS, PERMANENT STRUCTURES, ETC.

BY
MICHAEL McDERMOTT. C.E.,
CERTIFIED LAND SURVEYOR FOR GREAT BRITAIN AND IRELAND; PROVINCIAL LAND
SURVEYOR FOR THE CANADAS; FORMERLY CIVILIAN ON THE ORDNANCE
SURVEY OF IRELAND, PAROCHIAL SURVEYOR IN ENGLAND,
CITY SURVEYOR OF MILWAUKEE AND CHICAGO;
MEMBER OF THE ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE, CHICAGO
COLLEGE OF PHARMACY, AND THE CHICAGO CHEMICAL ASSOCIATION.

R. B. Riftenberich,
Cincinnati, O.

CHICAGO:
FERGUS PRINTING COMPANY,
244-8 ILLINOIS STREET.
1879.

Entered according to Act of Congress, in the year 1879, by
MICHAEL McDERMOTT,
In the Office of the Librarian of Congress, at Washington.



Trans. Lib.

AUTOBIOGRAPHY.

I have been born on the 10th day of Sep., 1810, in the village of Kilmore, near Castlekelly, in the County of Galway, Ireland. My mother, Ellen Nolan, daughter of Doctor Nolan, was of that place, and my father Michael McDermott was from Flaskagh, near Dunmore, in the same County, where I spent my early years at a village school kept by Mr. James Rogers, for whom I have an undying love through life. Of him I learned arithmetic and some book-keeping. He read arithmetic of Cronan and Roach, in the County of Limerick. They excelled in that branch. John Gregory, Esq., formerly Professor of Engineering and Surveying in Dublin; but now of Milwaukee, read of Cronan, which enabled him to publish his "Philosophy of Arithmetic," a work never equalled by another. By it one can solve quadratic and cubic equations, the diophantine problems, and summation of series.

After having been long enough under my friend Mr. Rogers, I went to the Clarenbridge school, kept by the brothers of St. Patrick, under the patronage of the good lady Reddington. I lived with a family named Neyland, at the Weir, about two miles from the school, where I had a happy home on the sea-side. There I read algebra, grammar, and book-keeping. After being nearly a year in that abode of piety and learning, I went to Mathew Collin's Mathematical school, in Limerick. He was considered then, and at the time of his death, the best mathematician in Europe. His correspondence in the English and Irish diaries on mathematics proves that he stood first. I left him after eight months studying geometry, etc., and went to Castleircan, near Cahirconlish, seven miles from Limerick, where I entered the mathematical school, kept by Mr. Thomas McNamara, familiarly known as Tom Mac, and Father of X, on account of his superior knowledge of algebra, he was generally known by the name of "Father of X." Of him I read algebra and surveying; lived with a gentleman farmer—named William Keys, Esq., at Drimkeen, about one and one-half miles south-east of the school. Mr. Mac had a large school, exclusively mathematical, and was considered the best teacher of surveying. After being with him nearly a year, I left and went to Bansha, four miles east of the town of Tipperary. Here Mr. Simon Cox, an unassuming little man, had the largest mathematical class in Ireland, and probably in the world, having 157 students, gathered from every County in Ireland, and some from England. Like Mr. McNamara, he had special branches in which he excelled; these were the use of the globes, spherical astronomy, analytical geometry, and fluxions. The differential and integral calculus were then slowly getting into the schools. I lived

69-26-46 9.H.P.

with Dairyman Peters, near the bridge of Aughahall, about three miles east of Bansha. I remained two years with Mr. Cox, and then bade farewell to hospitable and learned Munster, where, with a few exceptions, all the great mathematical and classical schools were kept, until the famine plague of 1848 broke them up. I next found myself in Athleague County, Roscommon, with Mr. Mathew Cuniff, who was an excellent constructor of equations, and showed the application to the various arts.

I received my diploma as certified Land Surveyor on the sixth of September, 1836, after a *rough* examination by Mr. Fowler, in the theoretical, and William Longfield, Esq., in the practice of surveying. I soon got excellent practice, but wishing for a wider field of operation, for further information, I joined the Ordnance Survey of Ireland. Worked on almost every department of it, such as plotting, calculating, registering, surveying, levelling, examining and translating Irish names into English. Having got a remunerative employment from S. W. Parks, Esq., land surveyor and civil engineer, in Ipswich, County of Suffolk, England. I left my native Isle in April, 1838. Surveyed with Mr. Parks in the counties of Suffolk, Norfolk, and Essex, for two years, then took the field on my own account. I left happy, hospitable, and friendly England in April, 1842, and sailed for Canada. Landed in Quebec, where I soon learned that I could not survey until I would serve an apprenticeship, be examined, and receive a diploma.

I sailed up the St. Lawrence and Ottawa Rivers to Bytown,—then a growing town in the woods,—but now called Ottawa, the seat of the Government of British America. I engaged as teacher in a school in Aylmer, nine miles from Bytown (now Ottawa). At the end of my term of three months, I joined John McNaughton, Esq., land surveyor, and justice of the peace, until I got my diploma as Provincial Land Surveyor for Upper Canada, dated December 16, 1843, and my diploma or commission for Lower Canada, dated September 12, 1844.

I spent my time about equally divided between making surveys for the Home (British) Government four years, and the Provincial Government, and private citizens, until I left Bytown in September, 1849, having thrown up an excellent situation on the Ordnance Department. I never can forget the happy days I have been employed on ordnance surveys in Ireland, under Lieutenants Broughton and Lancy. In Canada, under the supervision of Lieutenants White and King, and Colonel Thompson, of the Royal Engineers. In my surveys for the Provincial Government of Canada, I always found Hon. Andrew Russell and Joseph Bouchette, Surveyor-Generals, and Thomas Devine, Esq., Head of Surveys, my warmest friends. They are now—October 7, 1878—living at the head of their respective old Departments, having lived a long life of usefulness, which I hope will be prolonged. To Sir William Logan, Provincial Geologist, I am indebted for much information. I lived nearly eight years in Ottawa, Canada, where my friends were very numerous. The dearest of all to me was Alphonso Wells, Provincial Land Surveyor, who was the best surveyor I ever met. He had been so badly frost-bitten on a Government survey that it was the remote cause of his death.

On one of my surveys, far North, I and one of my men were badly frost-bitten. He died shortly after getting home. I lost all the toes of my left foot and seven fingers, leaving two thumbs and the small finger on the

right hand. After the amputation, I soon healed, which I attribute to my strictly temperate habits, for I never drank spirituous liquor nor used that narcotic weed—tobacco.

In Sept., 1849, I left the Ordnance Survey, near Kingston. Having surveyed about 120 miles of the Rideau Canal, in detail, with all the Government lands belonging to it. On this service I was four years employed. I came to the City of Milwaukee, September, 1849; could find no surveying to do. I opened a school, October 1. Soon gathered a good class, which rewarded me very well for my time and labor. Here I made the acquaintance of many of the learned and noble-hearted citizens of the Cream City—Milwaukee, amongst whom I have found the popular Doctors Johnson and Hubeschman; I. A. LAPHAM; Professor Buck; Peters, the celebrated clock-maker; Byron Kilbourne, Esq.; Aldermen Edward McGarry, Moses Neyland, James Rogers, Rosebach, Furlong, Dr. Lake; John Furlong, etc., etc. I found extraordinary friendship from all Americans and Germans, as well as Irishmen. I was appointed or elected by the City Council, in the following April, as City Engineer, for 1850 and part of 1851. I was reappointed in April, 1851, and needed but one vote of being again elected in 1852. I made every exertion not to have my name brought up for a third term, because, in Milwaukee the correct rule, "Rotation in office is true democracy," was adhered to. In accordance with a previous engagement, made with Wm. Clogher, Esq., many years City Surveyor of Chicago, I left Milwaukee with regret, and joined Mr. Clogher, as partner, in April, 1852, immediately after the Milwaukee election. Worked together for one year, and then pitched my tent here since, where I have been elected City Surveyor, City Supervisor, and had a hand in almost if not all the disputed surveys that took place here since that time.

I have attended one course of lectures on chemistry, in Ipswich, England, in 1840, and two courses at Rush Medical College, under the late Prof. J. V. Z. Blaney, and two under Dr. Mahla, on chemistry and pharmacy. By these means, I believe that I have given as much on the subject of analysis as will enable the surveyor or engineer, after a few days application, to determine the quality and approximate quantity of metal in any ore. To the late Sir Richard Griffith, I am indebted for his "Manual of Instructions," which he had the kindness to send me, May 23, 1861. He died Sept. 22, 1878, at the advanced age of 94 years; being the last Irishman who held office under the Irish Government, before the Union with England. He was in active service as surveyor, civil engineer, and land valuator almost to the day of his death.

The principles of geometry and trigonometry are well selected for useful applications. The sections on railroads, canals, railway curves, and tables for earthwork are numerous.

The Canada and United States methods of surveying are given in detail, and illustrated with diagrams. Sir Richard Griffith's system of valuation on the British Ordnance Survey, and the various decisions of the Supreme Courts of the United States are very numerous, and have been sometimes used in the Chicago Courts as authority in surveys. Hydraulics, and the sections on building walls, dams, roofs, etc., are extensive, original, and comprehensive. The sections and drawings of many bridges and tunnels are well selected, and their properties examined and defined. The tables of sines and tangents are in a new form, with guide lines at every five min-

minutes. The traverse table is original, and contains 88 pages, giving latitude and departures for every minute of four places, and decimals, and for every number of chains and links. The North and South polar tables are the results of great labor and time. The table of contents is full and explicit. I believe the surveyors, engineers, valuers, architects, lawyers, miners, navigators, and astronomers will find the work instructive.

I commenced my traverse table, the first of my Manual, on the 15th of October, 1833, and completed my work on the 8th of October, 1878.

The oldest traverse table I have seen was published by D'Burgh, Surveyor General, in Ireland, in 1723, but only to quarter degrees and one chain distance. The next is that by Benjamin Noble, of Ballinakill, Ireland, entitled "Geodesia Hibernica," printed in 1768, were to $\frac{1}{4}$ degrees and 50 chains. The next, by Harding, were to $\frac{1}{4}$ degrees and 100 chains. In my early days, these were scarce and expensive; that by Harding, sold at two pounds two shillings Sterling, (about \$10.50).

Gibson's tables, so well known, are but to $\frac{1}{4}$ degrees and one chain distance.

Those by the late lamented Gillespie, were but to $\frac{1}{4}$ degrees, three places of decimals, and for 1 to 9 chains. Hence appears the value of my new traverse table, which is to every minute, and can be used for any required distances.

Noble gave the following on his title-page: "Ye shall do no unrighteousness in meteyard, in weight, or in measure." Leviticus, chap. xix, 35; "Cursed be he that removeth his neighbor's landmark." Deuteronomy, chap. xxvii, 17.

I lost thirty-two pages of the present edition of 1000 copies in the great Chicago fire, Oct. 9th and 10th, 1871, with my type and engravings; this caused some expense and delay.

The Manual has 524 pages, strongly bound, leather back and corners.

MICH'D McDERMOTT.

GENERAL INDEX.

	Section.
<i>Square.</i> Area, diagonal, radius of inscribed circle, radius of the circumscribing circle, and other properties,	14
<i>Rectangle</i> or parallelogram, its area, diameter, radius of circumscribing circle. The greatest rectangle that can be inscribed in a semi-circle. The greatest area when $a = 2b$. Hydraulic mean depth. Stiffest and strongest beams, out of —	
OF THE TRIANGLE.	
Areas and properties by various methods,	25
To cut off a given area from a given point,	38
To cut off from P, the least triangle possible,	41
To bisect the triangle by the shortest line possible,	43
The greatest rectangle that can be inscribed in a triangle,	44
The centre of the inscribed and circumscribed circles,	55
Various properties of,	52
Strongest form of a retaining wall,	58
OF THE CIRCLE.	
Areas of circles, circular rings, segments, sectors, zones, and lunes, ..	60
Hydraulic mean depth,	77
Inscribed and circumscribed figures,	78
To draw a tangent to any point in the circumference,	87
To find the height and chord of any segment,	137
To find the diameter of a circle whose area, A , is given,	141
Important properties of the circle in railway curves and arches,	78
OF THE ELLIPSE.	
How to construct an ellipse and find its area,	88, 115
Various practical properties of,	89
Segment of. Circumference of,	116
PARABOLA.	
Construction of, 123. Properties, 124. Tangent to, 128. Area, 129. Length of curve, 130. Parabolic sewer, 133. Example, 133. Remarks on its use in preference to other forms, 134. Egg-shaped, 140. Hydraulic mean depth, 136. Perimeter,	139
Artificers' works, measurement of,	310x9
PLAIN TRIGONOMETRY—HEIGHTS AND DISTANCES.	
Right angled triangles, properties of,	148
The necessary formulas in surveying in finding any side and angle, ..	171b
Properties of lines and angles compared with one another,	194
Given two sides and contained angle to find the remaining parts,	203
Given three sides to find the angles,	20
Heights and distances, chaining, locating lots, villages, or towns, ...	311
How to take angles and repeat them for greater accuracy,	212
How to prove that all the interior angles of the survey are correct, ..	213
To reduce interior angles to quarter compass bearings,	204
To reduce circumferentor or compass bearings to those of the quarter compass,	214
How to take a traverse survey by the English Ordnance Survey method,	216
De Burgh's method known in America as the Pennsylvanian,	217
Table to change circumferentor to quarter compass bearings,	218
To find the Northings and Southings, Eastings and Westings, by commencing at any point,	219

	Section.
Inaccessible distances where the line partly or entirely is inaccessible,	221
This embraces fourteen cases, or all that can possibly be met in practice.	
From a given point P to find the distances P A, P B, P C,.....	
in the triangle A B C, whose sides A B, B C, and C D are given,	
this embraces three possible positions of the observer at P,	238

SPHERICAL TRIGONOMETRY.

Properties of spherical triangles. Page 72H*9,	345
Solution of right angled spherical triangles,	362
Napier's rules for circular parts, with a table and examples,	363
Quadrantal spherical triangles,	364
Oblique angled spherical triangles,	365
Fundamental formula applicable to all spherical triangles,	366
Formulas for finding sides and angles in every case,	367

SPHERICAL ASTRONOMY.

Definitions and general properties of refraction, parallax dip, greatest azimuth, refraction in altitude, etc., etc.,	375
Find when a heavenly body will pass the meridian,	376
Find when it will be at its greatest azimuth,	384
Find the altitude at this time,	384
Find the variation of the compass by an azimuth of a star,	383
Find latitude by an observation of the sun,	377
Find latitude when the celestial object is off the meridian,	378
Find latitude by a double altitude of the sun,	379
Find latitude by a meridian alt. of polaris or any circumpolar star, ..	380
Find latitude when the star is above the pole,	381
Find latitude by the pole star at any hour,	382
Errors respecting polaris and alioth in Ursamajoris when on the same vertical plane. (Note.)	389
Letters to the British and American Nautical Ephemeris offices,	389
Application and examples for Observatory House, corner of Twenty-sixth and Halsted streets, Chicago. Lat. $41^{\circ} 50', 30''$. Long. $87^{\circ} 34', 7''$, W.,	389
Remarkable proof of a Supreme Being. Page 72H*24,	386
True time; how determined; example,	387
True time by equal altitudes; example. Page 72H*23,	390
True time by a horizontal sundial, showing how to construct one, ...	390*
Longitude, difference of,	392
Longitude by the electric telegraph,	391
Longitude; how determined for Quebec and Chicago, by Col. Graham, U. S. Engineer.	393
Longitude by the heliostat. Page 72H*30,	393a
Longitude by the Drummond light and moon culminating stars,	394
Longitude by lunar distances; Young's method and example,	395
Reduction to the centre, that is reducing the angle taken near the point of a spire or corner of a public building, to that if taken from the centre of these points; by two methods,	244
Inaccessible heights. When the line A B is horizontal,	246
When the ground is sloping or inclined, three methods,	249

TRAVERSE SURVEYING.

Methods of. Sec. 213 to 217 and	255
To find meridian distances,	237
Method I. Begin with the sum of all the East departures,	258
Method II. First meridian pass through the most Westerly station, ..	239
Method III. First meridian pass through the most Northerly station, ..	260
Offsets and inlets, calculation of,	261
Ordnance method of keeping field-books,	62
Supplying lost lines and bearings. (Four cases.)	263
To find the most Westerly station,	264
To calculate an extensive survey where the first meridian is made a base line, at each end of which a station is made, and calculated by the third method,	264

CANADA SURVEYING.

Who are entitled to survey,	301
-----------------------------------	-----

Section.

Maps of towns, how made to be of evidence,	304
How side lines are to be ran. Page 72w, in townships,	302
How side lines in seignories. Page 72w, in townships,	306
Where the original posts or stakes are lost, law to establish,	306
<i>Compass</i> .—Variation of examples. 264h and	264a
Find at what time polaris or any other star will be at its greatest azimuth or elongation,	264b
Find its greatest azimuth or elongation,	264c
Find its altitude at the above time,	264d
Find when polaris or any other star culminate or pass the meridian,	264e
Example for altitude and azimuth in the above,	264f
How to know when polaris is above, below, East or West of the true pole,	264g
How to establish a meridian line. Page 71,	264h
To light or illumine the cross hairs,	265

UNITED STATES METHOD OF SURVEYING.

System of rectangular surveying,	266
What the United States law requires to be done,	267
Measurements, chaining, and marking,	269
Base lines, principal meridians, correction or standard lines,	270
North and south section lines, how to be surveyed,	272
East and west section lines, random and true lines,	273
East and west intersecting navigable streams,	274
Insurmountable obstacles, witness points,	275
Limits in closing on navigable waters and township lines,	276
Meandering of navigable streams,	277
Trees are marked for line, and bearing trees,	278
Township section corners, witness mounds, etc.,	279
Courses and distances to witness points,	285
Method of keeping field notes,	288
Lines crossing a navigable river, how determined,	292
Meandering notes,	293
Lost corners, how to restore,	294
Present subdivision of sections,	297
Government plats or maps,	299
Surveys of villages, towns, and cities,	300
Establishing lost corners in the above,	300

TRIGONOMETRICAL SURVEYING. Page 72H*35.

Base line and primary triangles, secondary triangles. How triangles are best subdivided for detail and checked. Method of keeping field-books. When there are wood traverse surveying. To protract the angles, ordnance method,	396
Method of protraction by a table of tangents, etc.	401
Plotting, McDermott's method, using two scales,	412
Finishing the plan or map, and coloring for various States of cultivation,	413
Registered sheets for contents,	402
Computation by scale,	403
Contouring, field-work, final examination,	411

DIVISION OF LAND. 403A.

Area cut off by a line drawn from a given point, 405. By a line parallel to one of its sides, 401. By a line at a given angle to one of its sides, 406,	403
From a given point P within a given figure to draw a line cutting off a given area,	420
From a given triangle to cut off a given area by a line drawn through a given point,	420a
To divide any quadrilateral figure into any number of equal parts, 409, continued in 419a,	409

LEVELLING.

Form of field-book used by the English and Irish Boards of Public Works,	414
--	-----

	Section.
By McDermott's method,	415
By barometrical observations,	416
Table for barometrical. Tables 416 and 417,	417
Example by Colonel Frome,	418
By boiling water. Tables A and B,	419

CORRECTIONS.

Additional, and corrections, geodetical jurisprudence, laying out curves, canals, corrections of D'Arcy's formula,	421
--	-----

GEODETICAL JURISPRUDENCE.

United States laws respecting the surveying of the public lands,	306a
Supreme court decisions of land cases of the State of Alabama,	307
Supreme court decisions of land cases of the States of Kentucky and Illinois,	303
Various supreme court decisions of several States on boundary lines, highways, water courses, accretion and alluvion, 309c, highways, 309d, backwater, Page 72B5, 309d, up to date,	309a
Ponds and lakes,	309b
New streets (continued 421). Page 72B10,	309d

SIR RICHARD GRIFFITH'S SYSTEM OF VALUATION.

Act of Parliament in reference to,	309f
Average prices of farm produce, and price of live weights,	309f
Lands and buildings for scientific, charitable, or public purposes, how valued,	309g
Field-book, nature and qualification of soils. 309g and 309h,	309h
Calcareous and peaty soils. 309k and 309l,	309l
Von Thae'r's classification of soils, table of,	309m
Classification of soils with reference to their value,	309n
Tables of produce, and scale for arable land and pasture. 309c, 309p, 309q	
Fattening, superior finishing land, dairy pasture, store pasture, land in medium situation and local circumstances, ...	309r
Manure, market, condition of land in reference to trees and plants, 309c	
Mines, Tolls, Fisheries, Railway waste,	310
Valuation of buildings, classification of same, measurement of,	310a
Modifying circumstances,	310c
Valuation in cities and towns,	310f
Comparative value,	310g
Scale of increase,	310i

WATER POWER.

Horse power, modulus of, for overshot wheels,	310j
Form of field-book for water wheels, head of water, etc.,	310k to 310l
Overshot, undershot, and turbine wheels,	310k
Valuation of water power, modifying circumstances,	310m to 310n
Horse power determined from the machinery driven,	310o
Beetling and flour mills. Mills in Chicago, note on,	310p
Valuator's field-book, form of, used on the Ordnance valuation of Ireland,	310x to 310w
Valuation of slated houses, thatched houses, country and towns. Tables I to V,	310v to 310A
Geological formation of the earth. Table, 72B52,	310B
Rocks, quartz, silica, sand, alumina, potash, lime, soda, magnesia, felspar, albite, labradorite, mica, porphyritic, hornblende, augite, gneiss, porphyritic, gneiss, protogine, serpentine, syenite, porphyritic granitoid, talc, steatite or soapstone, limestones, impure carbonate of lime, Fontainebleau do., tufa, malacelite satin spar, carbonate of magnesia or dolomite,	310C
Sir William Logan's report on six specimens of dolomite,	310C
Magnesian mortars. Page 72B56,	310C
Limestones, cements used in Paris, artificial cements, plaster of Paris, water lime, water cement, building stones. Page 72B56,	310C
Sands (various), Fuller's earth, clay for brick, potter's, pipe, fire brick, marl, chalk marl, shelly and slaty marl. Page 72B57,	310C
Table of rocks, composition of 310C, composition of grasses,	310D
Table of rocks, composition of trees, weeds, and plants,	310E

Section.

Composition of grains, straws, vegetables, and legumes,	310F
Analysis and composition of the ashes of miscellaneous articles,	310G
Analysis and percentage of water, nitrogen, phosphoric acid in manures,	310I
Sewage manure. Opposition to draining into rivers,	310J

DESCRIPTION OF MINERALS,

Including antimony, arsenic, bismuth, cobalt, copper, nickel, zinc, manganese, platinum, gold, silver, mercury, lead, and iron, with all the varieties of each metal, where found, its lustre, fracture, specific gravity, etc.,	310K
<i>Solid bodies</i> , examination of 310L. By Blow-pipe,	310M
Metallic substances. Qualitative analysis of,	310N
Metallic substances. Quantitative analysis,	310Q
Table—Of symbols, equivalents, and compounds,	310P
Table—Action of reagents on metallic oxides,	310Q
Table—Analysis of various soils,	310K
Analysis of soils, how made,	310S
Analysis of magnesian limestone,	310T
Analysis of iron pyrites,	310U
Analysis of copper pyrites, 310N, zinc, 310W,	310U to 310W
To separate gold, silver, copper, lead, antimony,	310X
To separate lead, and bismuth. Page 72B94,	310X
To determine mercury, 310V, tin, 310. Page 72B35,	310Y

HYDRAULICS.

Hydraulic mean depth of a rectangular water course of a circle,	79
Parabolic sewer, 134. Table showing hydraulic mean depths of parabolic and circular sewers, each having the same sectional areas, . . .	135
Egg-shaped sewer, its construction and properties,	140
Rectilinear water courses, 144. Best form of conduits, including circular, rectangular, triangular, parabolic, and rectilinear,	146
Table of rectilinear channels, where a given sectional area is enclosed by the least perimeter, or surface in contact,	167
A table of natural slopes and formulas,	147
Estimating the density of water, mineral, saline, sulphurous, chalybeate,	310Z
BOUSINGAULT's remarks on potable water. Page 72B93,	310Z
Supply of towns with water,	310Z
Solid matter in some of the principal places. Page 72B97,	310Z
Annual rain fall in various places and countries,	310A*
Daily supply in various cities,	310
Conduits, or supply mains,	310B*
Discharge throw pipes, and orifices under pressure,	310C*
VENA CONTRACTA and coefficient <i>m</i> of contraction. P. 72B100,	310C*
Adjutages, experiments by Michellotti Weisbach. P. 72B101,	310C*
Orifices with cylindrical and conical adjutages,	310D*
Table—Angles of convergence, discharge, and velocity,	310D*
Table—Blackwill's coefficient for overfall weirs. First and second Experiments,	310E*
Experiments by Poncelet and Lebros. DuBuats, Smeaton, Brinley, Rennie, with Poncelet and Lebros' table,	310E* and 310F*
Example from Neville's hydraulics. Page 72B105,	310F*
Formula of discharge by Boileau,	310G
Formula of discharge <i>Q</i> for orifices variously placed,	310H
Formula of time and velocity for the above,	310I
Formula by D'Arcy incorrect, page 264, but here corrected,	310I
Formula, value of coefft., by Frances of Lowell, Thompson of Belfast and Girard, of France,	310I
Spouting fluids,	310I
Water as a motive power. Available horse-power,	310K
High pressure turbines for every ten horse-power. P. 72B106, . . .	310J*
Artesian wells, and reservoirs. Page 72B108,	310X
Jetties,	310X9

LAND AND CITY DRAINAGE AND IRRIGATION.

Hilly districts, tile and pipe drains,	310P
Draining cities and towns, sewers,	310R
Sanitary hints,	310x10
Irrigation of lands,	310Q
Rawlinson's plan,	310Q
Supply of guano will soon be exhausted,	310B
On the steam engine, horse-power. Admiralty rule, work done by expansion,	310S

PRESSURE OF FLUIDS ON RETAINING WALLS.

Centre of pressure against a rectangular wall, cylindrical vessel, dams in masonry, foundations of basins and dams, waste weir, thickness of rectangular walls, cascades, 72B111,	310R
Retaining walls, Ancient, and Hindoo reservoirs,	310t
To find the thickness of the rectangular wall A B to resist its being turned over on the point D. Page 72B112,	310U

REVTMENT WALLS.

Wall having an external batter,	310U, 310U*
Table for surcharges, by Poncelet,	310w2
Walls in masonry, by Morin, 310w3, dry walls,	310w4
The greatest height to which a pier can be built,	310w6a
Piers and abutments,	310x10
Vauban, Rondelet, English engineers, and Colonel Wurmb. P. 72B115.	
Pressure on the key and foundations, by Rankine, Fox, Brunlee, Blyth, Hawkshaw, General Morin, Vicat,	310w4
Outlines of some important walls of docks and dams, including India docks, London, Liverpool Seawall, dams at Poona and Toolsee, near Bombay, East Indies, Dublin quay wall, Sunderland docks, Bristol do. Revetment wall on the Dublin and Kingston Railway, Chicago street revetment walls, dam at Blue Island, near Tunnels,	310w3
Blasting rock,	310w7
Chicago, dam at Jones' Falls, Canada,	310w11
PILE DRIVING, COFFER-DAMS, AND FOUNDATIONS. P. 72B116,	310v
The power of a pile, screw pile, hollow pile,	310v1
Examples—French standard, Nasmyth steamhammer. When men are used as power. 72B117,	310v1
MR. MCALPINE'S formula derived from facts,	310v
Cast-iron cylinders, when and where first used,	310v1
Foundations of timber. Pile driving engine,	310v2
Coffer-dams of earth, Thames tunnel, Victoria bridge in Montreal, Canada,	310v3

WOOD AND IRON PRESERVING.

When trees should be cut, natural seasoning, artificial do., Napier's process, 310v4, Kyan's process, corrosive sublimate, Brnnett's method, 310v4c, Bethell's method, Payne's do., Boucher's do., Hyett's do., Lege and Perenot, Harvey's by exhausted steam, ..	72B119
---	--------

MORTAR, CONCRETE, AND CEMENT.

At Woolwich, Croton Water Works, Forts Warren and Richmond. Page 72B121.	
--	--

VICAT'S METHOD. GROUTING, BY SMEATON.—Iron Cement.

STONE'S experiments on cement. Page 72B121,	310v6
Cement for moist climates. Page 72B122,	310v6
Concrete in London and United States,	310v7
Beton—Mole at Algiers, Africa,	310v7
PRESERVATION OF IRON,	31v8
VICTORIA ARTIFICIAL STONE. Page 72B123,	310v4
RANSOM'S method to make blocks of artificial stone,	310v10
Silicates of potash, of soda,	310v10

WALLS, BEAMS, AND PILLARS.

To test building stone,	310x4
Chimneys,	310w9

	Section.
Walls and foundations,	310v11
Table—Kind of wood, spec. grav., both ends fixed and loaded in the middle. Breaking weight. Transverse strain,	310v12
Formula for beams. Page 72b125,	310v12
Timber pillars, by RONDELET,	310v13
HODGKINSON'S FORMULA for long square pillars,	310v14
BRERETON'S experiments on pine timber,	310v15
Safe load in structures,	310v15
Strength of cast-iron beams,	310v16
Strongest form, Fairbairn's form,	310v16
Calculate the strength of a truss-beam,	310v17
To calculate a common roof,	310v17
Angles of roofs,	310v18
Beams, wrought-iron, —box,	310v18
GORDON'S RULES for cast-iron pillars,	310v20
Depth of foundations,	310v24
Walls of buildings,	310v28

FORCE AND MOTION.

Parallelogram of forces. Polygon of do.,	311
Falling bodies. Theoretical and actual mean velocities of. Virtual velocities,	312 and 319n
Composition of motions. Page 72E. When motion is retarded, ...	312
Centre of gravity in a circle, square, triangle, trapezoid,	313
In a trapezium, cone or pyramid, frustum of a, circular, sector, semicircle, quadrant, circular ring,	313
Of Solids.—Of triangular pyramid, a cone, conic frustum, in any polyhedron. Paraboloid, frustum of a, prismoid, or ungula. Spherical segment,	314
SPECIFIC GRAVITY, and density. Page 72H. Various methods, ...	315
Of a liquid, 316, body lighter than water, 318, of a powder soluble in water,	316
Table—Specific gravities of bodies. Weight one cubic foot in pounds,	319a
Table—Average bulk in cubic feet of one ton, 2240 pounds, of various materials,	319a
Table—Shrinkage or increase per cent. of materials,	319
Mechanical powers, levers, pulleys, wheels, axles, inclined planes, screws, with examples,	319a to 319n
Virtual velocity,	319n
FRICITION. COULOMB AND MORRINS' experiments coefficient of the angle of repose,	319n
Table—Friction of plane surfaces sometime in contact,	319v
Table—Friction of bodies in motion,	319v
Friction of axles in motion,	319v
Table—Motive power. Work done by man and horse moving horizontally,	319v
Table—Motive power. Work done by man and horse vertically, ...	319v
Motive power. Actions on machines,	319

ROADS AND STREETS.

Roman roads, Appian way, Roman military roads, Carthaginian, Greek, and French roads,	319n
German, Belgium, Sweden, English, Irish, and Scotch roads,	319n
Presentment for making and repairing roads,	319n
Making or repairing MCADAMIZED ROADS,	319v
Shrinkage allowance for,	
How the railroad was built over the Menominee marsh near Milwaukee, Wisconsin,	319v
RETAINING WALLS for roads. Page 72j11,	319v
Parapet walls, drainage, drain holes, materials, sandstone, limestone. Table—WALKER'S EXPERIMENTS on the durability of paving,	319v
Stones in London, England, in A.D. 1830 and 1831. Seventeen months,	319v
Table of compression of materials in road making, etc. Page 72j13,	319v

	Section.
Table. Uniform draught on roads. Page 72j13,.....	319r
Table and formulas of friction on roads. Page 72j14,.....	319r
McNeil's improved dynamometer. Page 72j14,.....	319r
PONCELET'S VALUE of draught to overcome friction,.....	319r
Table—Showing the lengths of horizontal lines, equal to ascending and descending planes. Pressure of a load on an inclined plane. Page 72j13.	
Table—MORIN'S EXPERIMENTS. With examples. Page 72j16.	
Table C—Laying out curves. Radius 700 feet to 10,560 feet radius, by chords and their versed sines in feet, showing how to use them in laying out curves of less radius than 700. Page 72j17.	

CANALS AND EXCAVATIONS.

Page 72k. (See Sec. 421),.....	320
To set out a section of a canal on a level surface,.....	321
To set out a section when the surface is inclined,.....	321a
To find the embankment, and to set off the boundary of,.....	321b
Area of section of excavation or embankment,.....	321b
When the slope cuts the bottom of the canal,.....	332
Mean height of a given section whose area = A, base = B V, ratio of slopes = r,.....	323
When the slopes are the same on both sides,.....	323
When the slopes are unequal,.....	323
How the mean heights are erroneously taken,.....	326
Erroneous or common method, of calculation,.....	326
To find the content of an excavation or embankment. Page 72r,...	327
Prism, prismoid, cylinder, frustrum of a cone, pyramid frustrum of a pyramid, prismoid, 334,.....	327
BAKER'S method of laying out curves, and calculating, earth works, do. modified. Page 72v,.....	339
Tables for calculating earthwork deduced from BAKER, KELLY, and SIR JOHN MCNEIL'S tables. Page 72y to 72h*.	

TABLES.

Comparative values of circular and parabolic sewers,.....	135
Rectilinear channels and slopes of materials,.....	167
Sines in plane trigonometry,.....	171r
To change circumferentor to quarter compass bearings,.....	218
Classification of land by Sir Richard Griffith,.....	309m
Indigenous plants,.....	309
Classification of soils,.....	309n
Scale for arable land,.....	309o
Table of produce,.....	309p
Scale of prices for pasture,.....	309q
One hundred statute acres under a five years' rotation. Page 72u21.	
Superior finishing land,.....	309r
Increase in valuation for its vicinity to towns,.....	310
Classification of buildings,.....	310c
Modifying circumstances,.....	310e
Valuation of water-power, 310m, 310w,.....	310k
Valuation of horse-power,.....	310o
Flour mills. Page 72b40, 72b41, 72b42,.....	310p, 310y
Form of field-book,.....	310t
Form of town-book,.....	310w
Annual valuation of houses in the country, slated,.....	310z
Annual valuation of houses in the country, thatched,.....	310w
Basement, stories, offices thatched,.....	310z, 310y
Prices of houses. Page 72b51.	
Geological formation of the earth,.....	310r
Composition of rocks,.....	310c
Composition of grasses and trees,.....	311d
Analysis of trees and weeds or plants,.....	310f
Analysis of grains and straws, vegetables and legumes,.....	310f
Analysis of ashes of miscellaneous articles,.....	310g
Per centage value of manures for nitrogen and phosphoric acid,	310i

Section.

Table of symbols, and equivalents,	310r
Action of reagents on metallic substances,	310q
Analysis of various soils,	310r
Supply of towns with water,	310z
Value of the <i>Vena contracta</i> from various writers on hydraulics, ...	310c*
Angles of convergence. Page 72b102.	
Coefficients of discharge over weirs,	310e*
Coefficients of Blackwell's experiments,	310e*
Poncelet and Lebros' experiments. 72b104,	310f*
Value of discharge <i>Q</i> through various orifices,	310h
Available power of water,	310j
Retaining walls, by Poncelet,	310w2
Specific gravities, breaking weight and traverse strains of beams supported at both ends, and loaded in the middle,	310v12
Specific gravities of bodies,	319a
Average bulk in cubic feet per ton of 2240 pounds. Page 72j1, ...	319a
Shrinkage or increase per cent. of materials. Page 72j1,	319a
Friction of plane surfaces,	319o
Friction of bodies in motion, one upon another,	319p
Work done by man and horse moving horizontally,	319r
Work done by man moving vertically,	319s
Action on machines,	319t
Walker's experiments on paving stones in a street in London,	319v
Compression pounds <i>avoirdupois</i> required to crush a cube of one and one-half inches. Page 72j13.	
Table of uniform draught on given inclinations. Page 72j13.	
Lengths of horizontal lines equal to ascending planes. Page 72j15.	
Morin's experiments with vehicles on roads. Page 72j16.	
Table c.—For laying out curves, chord <i>AB</i> = 200 feet, or links or any multiple of either giving radius of the curve. Half the angle of deflection the versed sine at one-half, the chord, or the versed sine of the angle, also versed sine of one-half, one-fourth, and one- eighth the angle. Page 72j17.	
Table a—Calculating earthwork prismoids. Page 72j.	
Table b—Calculating earthwork prismoids. Page 72A*.	
Table c—Calculating earthwork prismoids. Page 72E*.	
<i>Sundial</i> . Table for latitudes 41°, 49°, 54°, 36' 12", 30'. Page 211*27, 390*	
Levelling books, English and Irish Board of Works, method,	414
M. McDermott's method,	415
Levelling by barometrical observation. Table A,	416
Levelling by barometrical observation. Table B,	417
Table A and table B,	419
Natural sines to every minute, five places of decimals from 1° to 90°. Page 72i* to 72r*.	
Natural cosines as above. A guide line is at every five minutes.	
Natural tangents and cotangents, same as for the sines. 72s* to 72b**.	
The sines are separate from the cosine and tangents to avoid errors.	
Both tables occupy twenty pages nicely arranged for use.	
Traverse table, by McDermott, entirely original, calculated to the nearest four places of decimals, and to every minute of degree in the left hand column numbered from 1 at the top to 60 at the bot- tom, at the top are 1 to 9 to answer for say 9 chains 90 chains, 90 links or 9 links. The latitudes on the left hand page, and de- partures on the right hand page for 45 degrees, then 45 to 90 are found at the bottom, contains 88 pages.	
Solids, expansion of,	165
To reduce links to feet,	166
To reduce feet to links,	168
Lengths of circular arcs to radius one,	170
Lengths of circular arcs obtained by having the chord and versed sine,	171
Areas of segments of circles whose diameter is unity,	173
To reduce square feet to acres and <i>vice versa</i> ,	175
Table VIIIa. Properties of polygons whose sides are unity,	176
Table IX. Properties of the five regular bodies,	176

	Section
Table X. To reduce square links to acres,	176
Table XI. To reduce hypotenuse to base, or horizontal measurement,	177
Table XII. To reduce sidereal time to mean solar time,	178
Table XIII. To reduce mean solar time to sidereal time,	178
Table XIV. To reduce sidereal time to degrees of longitude,	179
Table XV. To reduce longitude to sidereal time,	179
Table XVI. Dip or depression of the horizon, and the distance at sea in miles corresponding to given heights,	179
Table XVII. Correction of the apparent altitude for refraction,	180
Table XVIII. Sun's parallax in altitude,	181
Table XIX. Parallax in altitude of the planets,	181
Table XX. Reduction of the time of the moon's passage over the meridian of Greenwich to that over any other meridian,	181
Table XXI. Best time for obtaining apparent time,	182
Table XXII. Best altitude for obtaining true time,	183
Table XXIII. Polar tables, azimuths or bearings of stars in the Northern and Southern hemispheres when at their greatest elongations from the meridian for every one-half a degree of latitude, and from one degree to latitude 70°, and for polar distances 0°, 40', 0 45, 0 50', 0 55', 1° 0', 1° 5', 1° 10', 1° 15', 1° 20', 1° 25', 1° 30', 3° 20', 3° 25', 7° 45', 7° 50', 7° 55', 8° 0', 11° 30', 11° 35', 11° 40', 11° 45', 11° 50', 11° 55', 12° 0', 12° 5', 12° 40', 12° 45', 12° 50', 12° 55', 13-0-13-5-15° 20', 15° 25', 15° 30', 15° 35', 15° 40', 15° 45', 15° 50',	184
[These will enable the Surveyor, at nearly any hour of the night, to run a meridian line in any place until A. D. 2000.]	
Azimuth of Kochab (Beta Ursaminoris), when at its greatest elongations or azimuths for 1875 and every ten years to 1995,	193
Table XXIV. Azimuths of Polaris when on the same vertical plane with gamma in Cassiopea at its under transit in latitudes 2° to 70° from 1870 until 1940,	194
Table XXV. Azimuths of Polaris when vertical with Alioth in Ursa majoris at its under transit, same as for table XXIV,	195
Table XXVI. Mean places of gamma (cassiope), and epsilon (alieth), in ursa majoris at Greenwich from A. D. 1870 until 1930,	196
Table XXVII. Azimuth, or bearings of alpha, in the foot of the Southern cross (Crucis), when on the same vertical plane with beta in Hydri, or in the tail of the serpent from A. D. 1850 until 2150, and for latitude 12° to 6°,	197
Table XXVIII. Altitudes and greatest azimuths for January 1, 1867. For Chicago latitude 41°, 50', 30" N., longitude 87°, 34', 7" W., and Buenos Ayres 34°, 36', 40" S., longitude 58°, 24', 3" W., for thirteen circumpolar stars in the Northern hemisphere, and ten circumpolar stars in the Southern hemisphere, giving the magnitude, polar distance, right ascension, upper meridian passage, time to greatest azimuth, time of greatest E azimuth, time of greatest W azimuth, greatest azimuth, altitude at its greatest azimuth of each,	198
Table XXVIII. A. Table of equal altitudes,	199
Table XXVIII. B. To change metres into statute miles,	200
Table XXVIII. C. Length of a degree of latitude and longitude in miles and metres,	200
Table XXIX. Reduce French litres into cubic feet and imperial gallons,	201
Table XXX. Weights and measures.	
Table XXXI. Discharge of water through new pipes compiled from D'Arcy's official French tables for 0.01 to 1.00 metres in diameter, and ten centimetres high in 100 metres to 200 centimetres in 100 metres high,	201
D'Arcy's formula and example,	264

THE SURVEYOR AND CIVIL ENGINEER'S MANUAL.

STRAIGHT-LINED AND CURVILINEAL FIGURES.

OF THE SQUARE.

1. Let $A B C D$ (Fig. 1) be a square. Let $A B = a$, and $A D = d$, or diagonal.
2. Then $a \times a = a^2 =$ the area of the square.
3. And $\sqrt{2} a^2 = a \sqrt{2} = a \times 1,4142136 =$ diagonal.
4. Radius of the inscribed circle $= O E = \frac{a}{2}$
5. Radius of the circumscribing circle $= O D = \frac{a \times 1,4142136}{2} = a \times 0,707168$.
6. Perimeter of the square $= A B + B D + D C + C A = 4 a$.
7. Side of the inscribed octagon $F G = a \sqrt{2} - a = a \times 1,4142136 - a = a \times 0,414214$, *i. e.*, the side of the inscribed octagon is equal to the difference between the diagonal $A D$ and the side $A B$ of a square.
8. Area of the inscribed circle $= a^2 \times 0,7854$.
9. Area of the circumscribed circle $0,7854 \times 2 a^2$.
10. Area of a square circumscribing a circle is double the square inscribed in that circle.

RHOMBUS.

11. (Fig. 3.) In a rhombus the four sides are equal to one another, but the angles not right-angled.
12. The area $=$ the product of the side $\times$ perpendicular breadth $= A B \times C E$.
13. Or, area $= a^2 \times$ natural sine of the acute angle $C A B$; *i. e.*, $A B \times A C \times \text{nat. sine of the angle } C A B =$ the area.

OF THE RECTANGLE OR PARALLELOGRAM.

14. (Fig. 2.) Let $A. B = a$, $B D = b$, and $A D = d$.
15. $A D = d = \sqrt{a^2 + b^2}$.
16. $\frac{d}{2} =$ radius of the circumscribing circle.
17. Area $= a b$ or the length $\times$ by the breadth.
18. When $a = 2 b$, the rectangle is the greatest in a semi-circle.
19. When $a = 2 b$, the perimeter, $A C + C D + D B$ contains the greatest area.

20. *Hydraulic mean depth* of a rectangular water-course is found by dividing the area by the wetted perimeter; i. e., = area divided by the sum of $2 AC + CB$.

21. When the breadth is to the depth as $1 : \sqrt{2}$, i. e., as $1 : 1.4152$, the rectangular beam will be the strongest in a circular tree.

22. When the breadth is to the depth as $1 : \sqrt{3}$, i. e., as $1 : 1.732$, the beam will be the stiffest that can be cut out of a round tree.

23. *Rhomboid.* (Fig 4.) In a rhomboid the four sides are parallel. Area = longest side $\times$ by the perpendicular height = $AB \times CE = AB \times AC \times \text{nat. sine } \angle CAB$.

24. *Trapezoid.* In a trapezoid only two of its sides are parallel to one another. Let $ADEB$ (fig. 4) be a trapezoid.

Area = $\frac{1}{2} (CD + AB) \times$ by the perpendicular width CE .

OF THE TRIANGLE.

25. Let ABC (Fig. 5) be a triangle.

26. If one of its angles, as B , is right-angled, the area = $\frac{AB}{2} \times BC = \frac{BC}{2} \times AB = \frac{1}{2} (AB \times BC)$

27. Or, area = $\frac{1}{2} AB \times$ tangent of the angle BAC .

28. When the triangle is not right-angled, measure any side; AC as a base, and take the perpendicular to the opposite angle, B ; then the area = $\frac{1}{2} (AC \times EB)$.

In measuring the line AC , note the distance from A to E and from E to C , E being where the perpendicular was erected.

29. Or, area = $\frac{AC \times AB}{2} \times \text{nat. sine of the angle } CAB$.

When the perpendicular EB would much exceed 100 links, and that the surveyor has not an instrument by which he could take the perpendicular EB , or angle CAB , his best plan would be to measure the three sides, $AB = a$, $BC = b$, and $AC = c$. Then the area will be found as follows:

30. Add the three sides together, take half their sum; from that half sum take each side separately; multiply the half sum by the three differences. The square root of the last product will be the area.

$$31. \text{ Area} = \left\{ \left(\frac{a+b+c}{2} \right) \cdot \left(\frac{a+b+c}{2} - a \right) \cdot \left(\frac{a+b+c}{2} - b \right) \cdot \left(\frac{a+b+c}{2} - c \right) \right\}^{\frac{1}{2}}$$

32. Let s equal half the sum of the three sides then

$$\text{Area} = \sqrt{s \cdot (s-a) \cdot (s-b) \cdot (s-c)}$$

$$33. \text{ Or, area} = \frac{1}{2} \left\{ \log s + \log (s-a) + \log (s-b) + \log (s-c) \right\} \text{ i. e.,}$$

to the logarithm of half the sum add the logs of the three differences, divide the sum by 2, and the quotient will be the log of the required area.

34. Or, to the log of A C add the log of A B and the log sine of the contained angle C A B. The number corresponding to the sum of these three logs will be double the area, i. e.,

$\text{Log } a + \text{log } c + \text{log sine angle C A B} = \text{double the area.}$

35. Or, by adding the arithmetical complement of 2, which is $\bar{1},698970$, we have a very concise formula.

$\text{Area} = \text{log } a + \text{log } c + \text{log sine angle C A B} + \bar{1},698970.$

Example. Let A B = $a = 18,74$, and A C = $c = 1695$ and the contained angle C A B = $29^\circ 48'$

Log 18,47 chains,	-	-	-	-	-	-	-	1,2664669
Log 16,95 chains,	-	-	-	-	-	-	-	1,2291697
Log sine $29^\circ 48'$	-	-	-	-	-	-	-	9,6952288
Constant log,	-	-	-	-	-	-	-	<u>1,6989700</u>
								11,8898354
Reject the index 10,	-	-	-	-	-	-	-	<u>10</u>
								1,8898354

The natural number corresponding to this log will be the required area = 77,5953 square chains, which, divided by 10, will give the area = 7,75953 acres.

35a. In Fig. 5, let the sides A C and B C be inaccessible. Measure A B = a ; take the angles A and B, then the area = $\frac{a^2 \text{ sine A} \times \text{sine B}}{2 \text{ sine C}}$

which, in words, is as follows:

Multiply together the square of the side, the natural sines of the angles A and B; divide the contained product by twice the sine of the angle C. The quotient will be the required area.

Or thus: Add together twice the log of a , the log sine A, and the log sine B; from the sum subtract $\text{log } 2 + \text{log sine C}$. The difference will be the log of the area.

Example. Let the $\angle A = 50^\circ$, angle B = 60° , and by Euclid I. 32, the $\angle$ at C = 70° ; and let A B = $a = 20$ chains to find the area of the triangle:

Log 20,	-	-	-	-	-	-	-	1,8010200
								<u>2</u>
								2,6020400
Angle A = 50° , log sine,	-	-	-	-	-	-	-	9,8842540
Angle B = 60° , log sine,	-	-	-	-	-	-	-	9,9875206
								<u>(A) = 22,4238146</u>
Constant log of 2 =	-	-	-	-	-	-	-	0,3010300
Angle C = 70° , log sine,	-	-	-	-	-	-	-	9,9729858
								<u>(B) = 10,2740158</u>
								2,1498288

From the sum A subtract the sum B, the difference, having rejected 10 from the index will be the log of the natural number corresponding to the area 141,198 square chains, which divided by 10 gives the area = $14\frac{119}{1000}$ acres.

Or thus: By using the table of natural sines. Having used Hutton's logs, we will also use his nat. sines.

$$\begin{array}{rcl} \text{See the formula (34) } a^2 = 20 \times 20, & - & - & - & 400 \\ \text{Nat. sine } 50^\circ = \text{nat. sin. } < A = & - & - & - & ,7660444 \\ \text{Product,} & - & - & - & 306,4177600 \end{array}$$

$$\begin{array}{rcl} \text{Let us take this} = & - & - & - & 306,418 \\ \text{Nat. sine } 60^\circ = \text{nat. sin. } < B = & - & - & - & ,86603+ \\ \text{Product,} & - & - & - & 265,367007334 \end{array}$$

$$\text{Nat. sine of } 70^\circ = ,989693$$

2

$$\text{Divisor, } = 1,879386 \text{) } 265,367007334$$

Quotient, = 141,198 square chains, which, divided by 10, gives $14\frac{1198}{10000}$ acres. Q. E. F.

35b. If on the line AB the triangles ACB , ADB , AEB , etc., be described such that the difference of the sides AC and CB , of AD and DB , and of AE and EB is each equal to a given quantity, the curve passing through the points C , D and E is a hyperbola.

36. If the sum of each of the above sides $AC + CB$, $AD + DB$, $AE + EB$ is equal to a given quantity, the curve is an ellipsis.

37. In the $\triangle ACB$, (Fig. 5,) if the base CE is $\frac{1}{2}$ of the line AC , the $\triangle CEB$ will be $\frac{1}{4}$ of the $\triangle ACB$, and if the base AC be n times the base CE , the $\triangle ACB$ will be n^2 times the area of the $\triangle CEB$.

38. From the point P in the $\triangle ACB$, (Fig. 11,) it is required to draw a line PE , so that the $\triangle APE$ will be $\frac{1}{4}$ the area of the $\triangle ACB$.

Divide the line AB into 4 equal parts, let $AD =$ one of these parts, join D and C and P and C , draw DE parallel to PC , then the $\triangle AEP$ will be $= \frac{1}{4}$ of the $\triangle ACB$; for by Euclid I. 37, we find that the $\triangle OEC = \triangle ODP$. $\therefore$ the $\triangle AEP = \triangle ACD = \frac{1}{4}$ the $\triangle ACB$, Q. E. F.

39. From the $\triangle ACB$, required to cut off a $\triangle ADE =$ to $\frac{1}{4}$ of the $\triangle ACB$ by a line DE parallel to BC .

By Euclid VI. 20, $\triangle ADE : \triangle ACB :: AD^2 : AB^2$; therefore, in this case, divide AB into two such parts, so that $AD^2 = \frac{1}{4}$ the square of AB . Let D be the required point, from which draw the line DE parallel to BC , and the work is done.

40. In the last case we have $\triangle ADE : \triangle ACB :: AD^2 : AB^2$; i. e., $1 : 5 :: AD^2 : AB^2$. Generally, $1 : n :: AD^2 : AB^2$; and by Euclid VI. 16, $n \times AD^2 = AB^2$; therefore, $AD = \frac{AB}{\sqrt{n}}$, which is a general formula.

Example. Let $AB = 60$ and $n = 5$; then $AD = \frac{60}{\sqrt{5}} = 26,7$.

41. If D be a point in the $\triangle ACB$, (Fig. 13,) through which the line PE is drawn parallel to CB , make $CE = EF$, join FD , and produce it to meet CB in G , then the line FDG will cut off the least possible triangle.

42. By Euclid VI. 2, $FD = DG$, because $FE = EC$.

43. To bisect the $\triangle A C B$ (Fig. 16,) by the shortest line $P D$. Let $A C = b$, $B C = a$, $C P = x$, and $C D = y$, $\triangle C P D = \frac{1}{2} \triangle A C B$, conditions which will be fulfilled when $x = C P = \sqrt{\frac{a b}{2}}$ and $y = C D = \sqrt{\frac{a b}{2}}$. Hence it follows that $C P = C D$. (See Tate's Differential Calculus, p. 65.)

44. The greatest rectangle that can be inscribed in any $\triangle A C B$, is that whose height $n m$, is $= \frac{1}{2}$ the height $n C$ of the given triangle (see Fig 14,) $A B C$. Hence the construction is evident. Bisect $A C$ in K , draw $K L$ parallel to $A B$, let fall the perpendiculars $K D$ and $L I$, and the figure $K L I D$ will be the required rectangle.

45. The centre of the circumscribing circle $A C B$, (Fig. 7,) is found by bisecting the sides $A B$, $A C$, and $C B$, and erecting perpendiculars from the points of bisection; the point of their bisection will be the required centre. (See Euclid IV. 5.)

46. The centre of the inscribed circle (Fig. 6,) is found by bisecting the angles A , B and C , the intersection of these lines will be the required centre, O , from which let fall the perpendicular $O E$ or $O D$, each equal to the perpendicular $O F =$ to the required radius.

47. Let $R =$ radius of circumscribing circle and $r =$ radius of the inscribed circle, and the sides $A B = a$, $B C = b$, and $A C = c$ of the

$$\triangle A B C; \text{ then } R = \frac{a b c}{2 r (a+b+c)}$$

$$\text{and } r = \frac{a b c}{2 R (a+b+c)}$$

48. To find r , the radius of the inscribed circle in (Fig. 6,)

$$\frac{r}{2} (a+b+c) = \text{area of the } \triangle A B C = A,$$

$$\therefore r = \frac{2 A}{a+b+c} = \frac{A}{\frac{1}{2} (a+b+c)} = \text{area divided by half of the sum of the sides of the } \triangle,$$

$$R = \frac{a b c}{2 r \cdot (a+b+c)} = \frac{a b c}{(a+b+c)} \div \frac{4 A}{(a+b+c)} \text{ i. e.,}$$

$$R = \frac{a b c \cdot (a+b+c)}{4 A \cdot (a+b+c)} = \frac{a b c}{4 A} \text{ i. e.,}$$

49. Radius of the circumscribing circle is equal to the product of the three sides divided by 4 times the area of the triangle, and substituting the formula in § 81 for the area of the triangle, we have

$$R = \frac{a b c}{4 A} = \frac{a b c}{2 r (a+b+c)} \text{ i. e.,}$$

$$R = \left\{ \frac{a b c}{s \cdot (s-a) \cdot (s-b) \cdot (s-c)} \right\}^{\frac{1}{2}} \text{ where } s \text{ is } \frac{1}{2} \text{ the sum of the sides,}$$

$$\text{but } (a+b+c) \frac{r}{2} = A; \text{ therefore,}$$

$$50. \quad r = \frac{2 A}{a+b+c}$$

51. The area of any $\triangle CKL$ (Fig. 14,) will be subtended by the least line KL , when $CK = CL$. Let $x = CK = CL$, and A = the required area, then $x = \frac{2A}{\text{nat. sine } < C}$.

52. Of all the triangles on the same base and in the same segment of a circle, the isosceles $\triangle$ contains the greatest area.

53. The greatest isosceles $\triangle$ in a circle will be also equi-lateral and will have each side $= r\sqrt{3}$ where r = radius of the given circle.

54. In a right-angled $\triangle$, when the hypotenuse is given, the area will be a maximum when the $\triangle$ is isosceles; that is, by putting h for the hypotenuse the base and perpendicular will be each $= \frac{h}{\sqrt{2}} = \frac{h}{1.414213}$

55. The greatest rectangle in an isosceles right-angled $\triangle$ will be a square.

56. In every triangle whose base and perpendicular are equal to one another, the perimeter will be a maximum when the triangle is isosceles.

57. Of all triangles having the same perimeter, the equi-lateral $\triangle$ contains the greatest area.

58. In all retaining walls (*walls built to support any pressure acting laterally*) whose base equals its perpendicular, or whose hypotenuse makes an angle of 45° with the horizon, will be the strongest possible.

OF THE CIRCLE.

Let $\log$ of 8,1416 = 0,4971509, of 0,7854 = $\overline{1}$,8950909, and of 0,07958 = 2,9008039.

59. Let a = area, d = diameter and c = circumference. $n = 3,1416$ and $m = 0,7854$. Const. $\log$ 8,1416 = 0,4971509. $d \times 3,1416$ = circumference, or $\log d + \log 0,4971509 = \log$ circumference.

60. $d^2 \times 0,7854$ = area = twice $\log d$ + constant $\log$ of 0,7854 = $\overline{1}$,8950909, and $c^2 \times 0,07959$ = area = $\frac{d}{2} \times \frac{c}{2} = \frac{dc}{4}$,
 $\log$ of area = $2 \log c$ + constant $\log$ $\overline{2}$,9008039.

61. *Example.* Let $d = 46$, then $46 \times 3,1416 = 144,5186$ = circumference; or, by logarithms,

$$\begin{array}{r} 46, \log = 1,6627578 \\ 3,1416 \text{ constant } \log 0,4971509 \\ \hline 2,1599087 = 144,5186 \\ 8979 \text{ circumference.} \end{array}$$

$$\begin{array}{r} 108 \\ 90 \\ \hline 18 \end{array}$$

$$62. \quad d = \frac{c}{3,1416} \text{ or } c = 144,5186 \quad \text{Log} = 2,1599087$$

$$3,1416 \quad \text{Log } 0,4971509$$

$$\text{Difference, } 1,6627578$$

$$d = 46$$

$$63. \text{ Area} = d^2 \times 0,7854 = \frac{cd}{4} = \frac{c}{4} \cdot d = \frac{d}{4} \cdot c = c^2 \cdot 0,07958.$$

Log area = twice log d + log 1,8950909, the nat. number of which will give the required area.

$$\begin{array}{rcl} \text{Example. Let } d = 45, \text{ its log} & = & \begin{cases} 1,6627578 \\ 1,6627578 \end{cases} \\ \text{Constant log of } 0,7854, & & 1,8950909 \\ \hline \text{Area} = 1661,909 & = & 3,2206065 \end{array}$$

$$64. = c^2 \times 0,07958 = \text{twice log } c + \text{log of } 0,07958 = \text{log area.}$$

Example. Let c = 154.

$$\begin{array}{rcl} \text{Log } c & = & 2,1875207 \\ & & 2 \\ \hline \text{Log } c^2 & = & 4,3750414 \\ \text{Constant log of } 0,07958 & = & 2,9008039 \\ \hline \text{Log area} & = & 8,2758453 \\ \text{Area} & = & 1887,3191 \end{array}$$

$$65. d = \left(\frac{a}{0,7854} \right)^{\frac{1}{2}} \text{ and } c = \left(\frac{a}{0,07958} \right)^{\frac{1}{2}}$$

66. Area of a Circular Ring = $(D^2 - d^2) \times 0,7854$. Here D = diameter of greater circumference, and d, that of the lesser circumference.

67. Area of a Sector of a Circle. (See Fig. 8.) Arc EGF is the arc of the given sector OEGF, area = $\frac{r}{2} \cdot \text{arc EGF}$ or area = $r \cdot \frac{\text{EGF}}{2}$; but arc EGF = 8 times the arc EG, less the chord EF, the difference divided by three = arc EGF (i. e.,)

$$\text{Arc EGF} = \frac{8 \text{ EG} - \text{EF}}{3}, \therefore \text{area of sector} = \frac{r}{2} \times \frac{8 \text{ EG} - \text{EF}}{3},$$

68. i. e., Area = $\frac{r}{6} (8 \text{ EG} - \text{EF})$. EG, the chord of $\frac{1}{2}$ the arc, may be found by Euclid I. 47. For we have OE = to the hypotenuse, given, also $\frac{1}{2}$ the chord EF = EH, $\therefore \sqrt{(OE^2 - EH^2)} = OH$, and OE - OH = HG, then $\sqrt{(EH^2 + HG^2)} = EG$.

69. Area = degrees of the $\angle EOF \times \text{diameter} \times \text{by the constant number, or factor } 0,008727$, i. e., area = $d \times 0,008727$ where a $\angle = EOF$ in degrees and decimals of a degree.

70. Segment of a Ring. NKMFG E, the area of this segment may be found by adding the arcs NKM and EGF of the sector ONKM and multiplying $\frac{1}{2}$ their sum by EN, the height of the segment of the ring, i. e., area = $\frac{\text{arc NKM} + \text{arc EGF}}{2} \times G K$.

71. Segment of a Circle. Let EGF be the given segment whose area is required. By § 67 find the area of the sector OEF, from which take the area of the $\triangle OEF$, the difference will be the required area.

$$72. \text{ Or, area} = \frac{2 \text{ EF} \times \text{GH}}{3} + \frac{\text{GH}^3}{2 \text{ EF}}; \text{ i. e., to } \frac{2}{3} \text{ of the product of}$$

the chord by the height, add the cube of the height divided by twice the chord of the segment, the sum will be the required area.

78. Or, divide the height GH by the diameter GL of the circle to three places of decimals. Find the quotient in the column Tabular Heights of Table VII., take out the corresponding area segment; which, when multiplied by the square of the diameter, will give the required area.

74. When GH, divided by the diameter GL, is greater than .5, take the quotient from 0,7854, and multiply the difference by the square of the diameter as above, when GH divided by GL does not terminate in three places of decimals, take out the quotient to five places of decimals, take out the areas less and greater than the required, multiply their difference by the last two decimals of the quotient, reject two places of decimals, add the remainder of the product to the lesser area, the sum will be the required tabular area.

Example. Let GH = 4, and $\frac{1}{2}$ the chord = EH = 9 = $\frac{1}{2}$ EF. By Euclid III. 35, $\text{HG} \times \text{HL} = \text{EH} \cdot \text{HF} = \text{EH}^2 = 81$; $\therefore \frac{81}{4} = 20,25 = \text{HL}$; consequently, by addition, $20,25 + 4 = 24,25 = \text{GL} = \text{diameter}$. And 4 divided by 24,25 = 0,16494 = tabular number.

Area corresponding to 0,164 = ,084059

“ “ “ 0,165 = ,084801

,000742

94

,000697,48

Lesser area for ,164 ,084059

Correction to be added for 00094 = 697

Corrected tabular area, ,084756; which, multiplied by the square of the diameters will give the required area.

OF A CIRCULAR ZONE.

75. Let EFVS (Fig. 8.) be a circular zone, in which EF is parallel to SV, and the perpendicular distance Et is given; consequently ES = tv may be found by Euclid I. 47, $st = \frac{1}{2} (S^2 v - EF) = d$, and $SV - d = tv$, and by Euclid III. 35, $\frac{st \times tv}{Et} = tW$, $\therefore Et + tU = EU$ is given.

And by Euclid I. 47, the diameter UF is $= \sqrt{EU^2 + EF^2}$

And by Euclid III. 3, by bisecting the line, OZ is at right angles to FV; and by Euclid III. 81, the $\angle UVF$ is a right angle; and by Euclid VI. 2 and 4, $UV = 2ox$.

And $Et : ES :: vt : VU$, by substitution we have

$$Et : ES :: vt : 2ox.$$

$$\text{By Euclid VI. 16, } ox = \frac{1}{2} (ES \times vt) \div Et = \frac{ES \times vt}{2Et}$$

Now having ox and oy = radius, we can find the height of the segment xy ; $\therefore$ having the height of the segment xy , and diameter WF of the segment FYV , we can find its area as follows:

The area of the trapezium $EFVS = \frac{1}{2} (EF + VS) \times Et$, to which add twice the segment FYV , the sum will be the required area of the zone $EFVS$.

In fig. 8, let $EF = a$, $SV = b$, $Et = p$, $St = d = \frac{1}{2} (SV - EF)$,

and $Tv = e$, $EW = p + \frac{ed}{p} = \frac{p^2 + ed}{p}$, and by Euclid L. 47.

$$WF = \left\{ \left(\frac{p^2 + ed}{p} \right)^2 + a^2 \right\}^{\frac{1}{2}} \text{ i. e.,}$$

$$WF = \sqrt{\frac{p^4 + 2p^2ed + e^2d^2 + p^2a^2}{p}}$$

$$ES = (p^2 + d^2)^{\frac{1}{2}}$$

Because $Et : ES :: Vt : VW$

$$Et : ES :: Vt : 2ox$$

$$\therefore ox = \frac{ES \cdot Vt}{2Et}$$

And by substituting the values of ES , Vt and $2Et$, we have

$$ox = \frac{e(p^2 + d^2)^{\frac{1}{2}}}{2p}$$

$$xy = \frac{WF}{2} - ox$$

$$WF = 2xy + 2ox$$

Example. Let $EF = a = 20$, and $sv = b = 30$, $Et = p = 25$, $St = d$, and $tv = e$, to find the diameter WF and height xy . Here $d = 5$ and $tv = e = 25$.

$$ES = \sqrt{650} = 25,494.$$

$$ox = \frac{25 \sqrt{625 + 25}}{50} = \frac{25 \sqrt{650}}{50} = \frac{25 \times 25,495}{50}, \text{ i. e.,}$$

$$ox = 12,747,$$

$$WF = \frac{1}{p} \sqrt{390625 + 156250 + 15625 + 390625} = \frac{908,05}{25}$$

therefore $WF = 36,12 =$ required diameter.

$WF = 36,07 =$ diameter; and having the diameter WF and height xy , the area of the segment, subtended by the chords Fv and ES , can be found by Table VII., and the trapezium $EFvt$ by section 24.

OF A CIRCULAR LUNE.

76. Let $ACBD$, fig. 10, represent a lune. Find the difference between the segment ACB and ADB , which will be the required area.

b

77. Hydraulic mean depth of a segment of a circle is found by dividing the area of the segment by the length of the arc of that segment. Of all segments of a circle, the semi-circular sewer or drain, when filled, has the greatest hydraulic mean depth.

78. The greatest isosceles $\triangle$ that can circumscribe a circle will be that whose height or perpendicular CF is equal to 3 times the radius OE .

79. Areas of circles are to one another as the squares of their diameters; i. e., in fig. 8, circle $AKBI$ is to the area of the circle $CGVL$ as the square of AB is to the square of CD .

80. In any circle (fig. 9), if two lines intersect one another, the rectangle contained by the segments of one is = to the rectangle contained by the segments of the other; i. e., $OM \times MC = FM \times MH$, or $OA \times AC = FA \times AH$.

81. In fig. 8, $aT \times bT = IT \times KT = \text{square of the tangent } TM$.

82. In a circle (fig. 9), the angle at the centre is double the angle at the circumference; i. e., $\angle CAB = 2 \angle COB$. Euclid III. 20.

83. By Euclid III. 21, equal angles stand upon equal circumferences; i. e., $\angle COB = \angle CLB$.

84. By Euclid III. 26, the $\angle BCL = \angle BLC = \angle COB$.

85. By Euclid III. 32, the angle contained by a tangent to a circle, and a chord drawn from the point of contact, is equal to the angle in the alternate segment of the circle; i. e., in fig. 9, the $\angle TBC = \angle BOC = \frac{1}{2} \angle CAB$. This theorem is much used in railway engineering.

86. The angle TBC is termed by railroad engineers the tangential angle, or angle of half deflection.

87. To draw a tangent to a circle from the point T without the circle. (See fig. 9.) Join the centre A and the point T , on the line AT describe a semi-circle, where A cuts the circle, in B . Join T and B , the line TB will be the required tangent or the square root of any line $QTH = TB$; i. e., $\sqrt{(QTH)} = TM$.

Then from the point T with the distance TB , describe a circle, cutting the circle in the point B , the line TB is the required tangent.

In Section 81, we have $Ta \cdot TB = TM^2$, $\therefore \sqrt{(Ta \cdot TB)} = TM$, and a circle describe with T as centre and TM as radius will determine the point M .

OF THE ELLIPSE.

88. An ellipse is the section of a cone, made by a plane cutting the cone obliquely from one side to the other.

Let fig. 89 represent an ellipse, where AB = the transverse axis, and DE = the conjugate axis. F and G the foci, and C the centre.

Construction.—An ellipse may be described as follows: Bisect the transverse axis in C , erect the perpendicular CD equal to the semi-conjugate, from the point D , as centre with AC as distance describe arcs cutting the transverse axis in the foci F and G . Take a fine cord, so that when knotted and doubled, will be equal to the distance AG or FB . At

the points or foci F and G put small nails or pins, over which put the line, and with a fine-pointed pencil describe the curve by keeping the line tight on the nails and pencil at every point in the curve.

89. *Ordinates* are lines at right angles to the axis, as $O I$ is an ordinate to the transverse axis $A B$.

90. *Double ordinates* are those which meet the curve on both sides of the axis, as $H V$ is a double ordinate to the transverse axis.

91. *Abscissa* is that part of the axis between the ordinate and vertex, as $A O$ and $O B$ are the abscissas to the ordinate $O I$; and $A G$ and $G B$ are abscissas to the ordinate $G H$.

92. *Parameter* or *Latus rectum* is that ordinate passing through the focus, and meeting the curve at both sides, as $H . V$.

93. *Diameter* is any line passing through the centre and terminated by the curve, as $Q X$ or $R I$.

94. *Ordinate to a diameter* is a line parallel to the tangent at the vertex of that diameter, as $Z T$ is the ordinate being parallel to the tangent $X Y$ drawn to the vertex X of the diameter $X Q$.

95. *Conjugate to a diameter* is that line drawn through the centre, terminated by the curve, and parallel to the tangent at the vertex of that diameter, as $C b$ is the semi-conjugate to the diameter $Q X$.

96. *Tangent to any point H' in the curve*, join $H F$ and $G H$, bisect the angle $L H G$ by the line $H K$, then $H K$ will be the required tangent.

97. *Tangent from a point without*, let P be the given point, (see fig. 40) join $P F$; on $P F$ and $A B$ describe circles cutting one another in X , join $P X$ and produce it to meet the ellipse in T , then $P T$ will be the required tangent, and $H K' =$ tangent to the point h .

98. *Focal tangents*, are the tangents drawn through the points where the *latus rectum* meets the curve, $K H$ is the focal tangent to the point H .

99. *Normal* is that line drawn from the point of contact of the tangent with the curve, and at right angles to the tangent, $H N$ is normal to $K H$.

100. *Subnormal* is the intercepted distance between the point where the normal meets the axis, and that point where an ordinate from the point of tangents contact with the curve meets the axis, as $N O'$ is the subnormal to the point H .

101. *Eccentricity* is the distance from the focus to the centre, as $C G$.

102. All diameters bisect one another in the centre C ; that is, $C X = C Q$ and $C I = C R$.

103. *To find the centre of an ellipse*. Draw any two cords parallel to one another, bisect them, join the points of bisection and produce the line both ways to the curve, bisect this last line drawn, and the point of bisection will be the centre of the ellipse.

104. $A B = F D + G B = F I + G I = F H + G H$, etc.; that is, the sum of any two lines drawn from the foci to any point in the curve, is equal to the transverse axis.

105. The square of half the transverse, is to the square of half the conjugate, as the rectangle of any two abscissas is to the square of the ordinate to these abscissas; i. e.,

$A C^2 : C D^2 :: A O \cdot O B : O I^2$; therefore,

$$O I = \sqrt{\frac{A O \cdot O B \cdot C D^2}{A C^2}} = \frac{C D}{A C} \cdot \sqrt{A O \cdot O B}$$

Let us assume $\frac{C D}{A C}$ equal to n , then

$$G H' = \sqrt{A G \cdot G B} \cdot n.$$

106. Rectangles of the abscissas are to one another as the squares of their ordinates; i. e.,

$$A O \cdot O B : A G \cdot G B :: O I^2 : G H'^2$$

107. The square of any diameter is to the square of its conjugate, as the rectangle of the abscissas to that, diameter, is to the square of the ordinate to these abscissas; i. e.,

$$Q X^2 : H' b^2 :: Q T \cdot T X : T Z^2; \text{ i. e.,}$$

$$C X^2 : C b^2 :: Q T \cdot T X : T Z^2.$$

108. To find where the tangent to the point H will meet the transverse axis produced:

$C O' : A C :: A C : C K'$. Substituting x for $C O'$, and a for $A C$

$$x : a :: a : C K'; \therefore C K' = \frac{a^2}{x}; \text{ therefore,}$$

$$O K' = \frac{(a+x) \cdot (a-x)}{x} = \frac{a^2 - x^2}{x}. \text{ Now, by having the ordinate } O I = y, \text{ we have}$$

$$H K' = \left\{ y^2 + \left(\frac{a^2 - x^2}{x} \right)^2 \right\}^{\frac{1}{2}} = \left\{ y^2 + \frac{a^4 - 2 a^2 x^2 + x^4}{x^2} \right\}^{\frac{1}{2}}$$

$$109. \text{ Tangent } H K' = \sqrt{\frac{x^2 y^2 + a^4 - 2 a^2 x^2 + x^4}{x^2}}, \text{ here } x = C O.$$

$$110. \text{ Equation to the ellipse } \frac{y^2}{b^2} + \frac{x^2}{a^2} = 1;$$

$$\text{or, } y = \left\{ \frac{b^2}{a^2} \cdot (a^2 - x^2) \right\}^{\frac{1}{2}} \text{ here } y = \text{any ordinate } O H.$$

Having the semi-transverse axis = a , the semi-conjugate = b
 $O = O H = \text{any ordinate, } x = C O = \text{co-ordinate of } y.$ Let

$A O = S = \text{greater abscissa, and } O B = s = \text{lesser abscissa. We will from the above deduce formulas for finding either } a, b, S, s, O \text{ or } x.$

$$111. O H = o = \left(\frac{S \cdot s \cdot b^2}{a^2} \right)^{\frac{1}{2}} = \text{ordinate} = \frac{b}{a} \sqrt{S \cdot s}.$$

$$112. A C = a = \frac{b \cdot S}{o^2} \left\{ b + \sqrt{(b^2 - o^2)} \right\} = \text{semi-transverse.}$$

$$113. \quad CD = b = \sqrt{\left(\frac{a^2 - o^2}{S \cdot s}\right)} = a \cdot o \sqrt{\frac{1}{S \cdot s}} = \text{semi-conjugate.}$$

$$114. \quad AO = S = a + \frac{a}{b} (b^2 - o^2)^{\frac{1}{2}} = \text{greater abscissa.}$$

$$115. \quad \text{Area of an ellipse} = AB \times DE \times ,7854 = 4 a b \cdot 7854 = 3,1416 \times a b.$$

116. *Area of an elliptical segment.*—Let h = height of the segment. Divide the height h , by the diameter of which it is a part; find the tabular area corresponding to the quotient taken from tab. VII; this area multiplied by the two axes will give the required area, i. e.,

$$\text{Tab. area } \frac{h}{2a} \cdot 4 a b, \text{ when the base is parallel to the conjugate axis;}$$

$$\text{or, tab. area} = \frac{h}{2b} \cdot 4 a b, \text{ when the base is parallel to the transverse axis.}$$

$$117. \quad \text{Circumference of an ellipse} = \sqrt{\left(\frac{4a^2 + 4b^2}{2}\right)} \cdot 3,1416; \text{ i. e.,}$$

$$\text{Circumference} = \sqrt{(2a^2 + 2b^2)} \cdot 3,1416.$$

$$118. \quad \text{Application.}—\text{Let the transverse} = 85, \text{ and conjugate} = 25. \\ \text{Area} = 35 \times 25 \times ,7854 = 875 \times ,7854 = 687,225.$$

$$\text{Circumference} = \sqrt{\left(\frac{35^2 + 25^2}{2}\right)} \cdot 3,1416 = 22,09 \times 3,1416 = 69,3979.$$

Let $AO = 28$ = greater abscissa, then 7 = the lesser abscissa, to find the ordinate OH .

$$OH = \left(\frac{28 \times 7 \times 25^2}{85^2}\right)^{\frac{1}{2}} = \sqrt{100} = 10.$$

$$\text{or, } OH = \frac{25}{85} \sqrt{28 \times 7} = 10. \quad (\text{See section 111.})$$

$$\text{Abcissa } AO = 17,5 + \frac{17,5}{12,5} \sqrt{625 - 100} = 17,5 + 1,4 \times 7,5 = 28.$$

OF THE PARABOLA.

122. A parabola is the section of a cone made by a plane cutting it parallel to one of its sides (see fig. 41).

123. *To describe a parabola.*—Let DC = directrix and F = focus; bisect AF in V ; then V = vertex; apply one side of a square to the directrix CD ; attach a fine line or cord to the side HI ; make it fast to the end I and focus F ; slide one side of the square along the edge of a ruler laid on the directrix; keep the line by a fine pencil or blunt needle close to the side of the square, and trace the curve on one side of the axis.

Otherwise, Assume in the axis the points $F B B' B'' B''' B''''$ etc., at equal distances from F ; from these points erect perpendicular ordinates to the axis, as $F Q, B P, B' O, B'' N, B''' M$; from the focus F , with the distances $A F, A B, A B', A B'',$ describe arcs cutting the above ordinates in the points Q, P, O, N, M , etc., which points will be in the curve of the required parabola; by marking the distances $F B = B B' = B' B'',$ etc., each distance equal about two inches, the curve can be drawn near enough; but where strict accuracy is required, that method given in sec. 122 is the best.

124. *Definitions*.— $C D$ is the directrix, $F =$ focus, $V =$ vertex, $A B =$ axis. The lines at right angles to the axis are called ordinates. The double ordinate $Q R$ through the focus is equal to four times $F V$, and is called *parameter*, or *latus rectum*.

Diameter to a parabola is a line drawn from any point in the curve parallel to the axis, as $S Y$.

Ordinate to a diameter is the line terminated by the curve and bisected by the diameter.

Abscissa is the distance from the vertex of any diameter to the intersection of an ordinate to that diameter, as $V B$ is the abscissa to the ordinate $P B$.

124a. Every ordinate to the axis is a mean proportional between its abscissa and the latus rectum; that is $4 V F \times B'' V = B'' N^2$, consequently having the abscissa and ordinate given, we find the latus rectum

$$= 4 V F = \frac{B'' N^2}{B'' V}; \text{ also the distance of the focus } F \text{ from the vertex}$$

$$= F V = \frac{B'' N^2}{4 B'' V}.$$

125. Squares of the ordinates are to one another as their abscissas; i. e., $B P^2 : B' O^2 :: V B : V B'$.

$$126. F Q = 2 F V \therefore Q R = 4 F V.$$

127. The ordinate $B S^2 = V B \cdot 4 V F$; hence, the equation to the curve is $y^2 = p x$, where $y =$ ordinate $= B S$, and $x =$ abscissa $V B$, and $p =$ parameter or latus rectum.

128. To draw a *tangent* to any point S in the curve, join $S F$; draw $Y S L$ parallel to the axis $A B$; bisect the angle $F S L$ by the line $X S$, which will be the required tangent.

Otherwise, Draw the line from the focus to the directrix, as $F L$; bisect $F L$ in m ; draw $m X$ at right angles to $F L$; then $m X S$ will be the tangent required, because $S L = S F$.

Otherwise, Let S be the point from which it is required to draw a tangent to the curve; draw the ordinate $S B$, produce $B'' V$ to G , making $V G = V B$; then the line $G S$ will be the required tangent.

129. *Area* of a parabola is found by multiplying the height by the base, and taking two-thirds of the product for the area; i. e., the area of the parabola $N V U = \frac{2}{3} (B'' V \cdot N W)$.

180. To find the length of the curve N V B of a parabola :

Rule.—To the square of the ordinate N B'' add four thirds of the square of the abscissa V B''; the square root of the product multiplied by 2 will be the required length. Or, by putting $a = \text{abscissa} = V B''$, and $d = \text{ordinate} = N B''$; length of the curve N V U $= \sqrt{\left(\frac{a^2 d^2 + 4 a^3}{8}\right)} \cdot 2$, i. e.,
Length of the curve N V U $= \sqrt{(8 d^3 + 4 a^3)} \times 1,155$.

Rule II.—The following is more accurate than the above rule, but is more difficult.

Let $q = \frac{2 d}{4 V F}$ = to the quotient obtained by dividing the double ordinate by the parameter.

$$\text{Length of the curve} = 2 d \cdot \left(1 + \frac{q^2}{2 \cdot 3} - \frac{q^4}{2 \cdot 4 \cdot 5} + \frac{8 q^6}{2 \cdot 4 \cdot 6 \cdot 7}\right) \text{ etc.}$$

181. By sec. 57, of all triangles the equilateral contains the greatest area enclosed by the same perimeter; therefore, in sewerage, the sewer having its double ordinate, at the spring of the arch, equal to d ; then its depth or abscissa will be ,866 d ; i. e., multiply the width of the sewer at the spring of the arch by the decimal ,866. The product will be the depth of that sewer, approximately for parabolic sewer.

182. The great object in sewerage is to obtain the form of a sewer, such that it will have the greatest hydraulic mean depth with the least possible surface in contact.

OF THE PARABOLIC SEWER.

183. Given the area of the parabolic sewer, N V U = a to find its abscissa V B'' and ordinate B'' N such that the hydraulic mean depth of the sewer will be the greatest possible.

Let $x = \text{abscissa} = V B''$

and $y = \text{ordinate} = N B''$; then $N U = 2 y$.

By section 129, $\frac{4 y x}{8} = a$; i. e., $4 y x = 8 a$

$$y = \frac{8 a}{4 x} = ,75 \frac{a}{x} = ,75 \frac{a}{x}$$

To find the length of the curve N V U.

$$2 \sqrt{\frac{5,625 a^2}{x^2} + \frac{4 x^3}{8}} = \text{perimeter.}$$

$$2 \sqrt{\left(\frac{1,6875 a^2 + 4 x^4}{8 x^2}\right)} = \frac{2 \sqrt{1,6875 a^2 + 4 x^4}}{1,732 x} = \text{perimeter.}$$

i. e., $\frac{1,155\sqrt{1,6875 a^2 + 4 x^4}}{1,782 x} = \text{perimeter, which, divided into the given area, (a) will give}$

$$\frac{a x}{1,155\sqrt{1,6875 a^2 + 4 x^4}} = \text{hydraulic mean depth.}$$

$$\text{i. e., } \frac{a x}{1,155\sqrt{1,6875 a^2 + 4 x^4}} = \text{maximum.}$$

And by differentiating this expression, we have

$$\text{Differential } u = a dx \cdot \frac{1,155\sqrt{1,6875 a^2 + 4 x^4} - a x \left(\frac{1,155 \cdot 8 x^3 dx}{\sqrt{1,6875 a^2 + 4 x^4}} \right)}{1,155\sqrt{1,6875 a^2 + 4 x^4}}$$

rejecting the denominator and bringing to the same common denominator.

$$\frac{du}{dx} = a \cdot 1,155 (1,6875 a^2 + 4 x^4) - a x (9,24 x^3) = 0.$$

$$\text{i. e., } 1,949 a^2 + 4,62 a x^4 - 9,24 a x^4 = 0.$$

$$1,949 a^2 = 4,62 a x^4$$

$$x^4 = ,4218 a^2$$

$$x^2 = ,6494 a$$

$$x = ,806\sqrt{a} = \sqrt{,649 a} = \text{required abscissa.}$$

$$y = \frac{8 a}{4 x} = \frac{0,75 a}{x} = \text{required ordinate.}$$

Example.—Let the area = 4 feet = a ;

$$\text{then } ,806\sqrt{a} = ,806 \cdot 2 = 1,612 = \text{abscissa} = x;$$

$$\text{and } y = \text{ordinate} = \frac{8 a}{4 x} = \frac{12}{6,448} = 1,868.$$

Now we have the abscissa $x = 1,612$, and ordinate = 1,868.

By Sec. 130, we find the length of the curve $N V U = 5,26$; and by dividing the perimeter, 5,26, into the area of the sewer, we will have the

$$\text{hydraulic mean depth} = \frac{4}{5,16} = 0,76 \text{ feet.}$$

134. The circular sewer, when running half full, has a greater hydraulic mean depth than any other segment; but as the water falls in the sewer, the difference between the circular and parabolic hydraulic mean depths, decreases until in the lower segments, where the debris is more concentrated in the parabolic, than in the circular, the parabolic sewer with the same sectional area will give the greatest hydraulic mean depth. This will appear from the following calculations: Where the segment of a circle is assumed equal to a segment of a parabola, which parabola is equal to one-half of the given circle. The method of finding the length of the curve, area and hydraulic mean depth, will also appear.

That the parabolic sewer, whose abscissa $= 0,806\sqrt{a}$ and ordinate $= \frac{\sqrt{a}}{1,075}$ (where $a =$ given area), is better than either the circular or egg-shaped sewer, will appear from the following table and calculations.

135. TABLE, SHOWING THE HYDRAULIC MEAN DEPTH IN SEGMENTS OF PARABOLIC AND CIRCULAR SEWERS, EACH HAVING THE SAME SECTIONAL AREA. THE DIMENSIONS OF THE PRIMITIVE PARABOLA AND CIRCULAR ARE AT THE TOP.

PARABOLA, *Latus Rectum* 2,7. SEMICIRCLE, *Diameter* = 4 feet.

Abcissa or depth filled.	Ordinate or half chord filled.	Area of segment filled.	Perimeter of segment filled.	Hydraulic mean depth of segment filled.	Abcissa or depth filled.	Ordinate or half chord filled.	Area of segment filled.	Perimeter of segment filled.	Hydraulic mean depth of segment filled.
Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
2.019	2.335	6.286	6.737	0.933	2.00	2.00	6.286	6.283	1.0
2.0	2.324	6.197	6.553	0.946	1.98	1.999	6.197	6.241	0.993
1.9	2.265	5.738	6.307	0.909	1.86	1.995	5.738	6.002	0.956
1.8	2.205	5.292	6.060	0.873	1.75	1.984	5.292	5.781	0.912
1.7	2.142	4.855	5.811	0.835	1.64	1.967	4.855	5.560	0.873
1.6	2.079	4.435	5.562	0.797	1.53	1.944	4.435	5.334	0.831
1.5	2.013	4.026	5.311	0.758	1.43	1.917	4.026	5.121	0.786
1.4	1.944	3.629	5.056	0.719	1.32	1.881	3.629	4.900	0.741
1.3	1.874	3.248	4.802	0.676	1.22	1.842	3.248	4.680	0.694
1.2	1.800	2.880	4.543	0.634	1.12	1.796	2.880	4.462	0.645
1.1	1.723	2.527	4.281	0.590	1.02	1.744	2.527	4.224	0.598
1.0	1.643	2.191	4.016	0.545	0.92	1.683	2.191	4.001	0.547
0.9	1.559	1.871	3.747	0.499	0.83	1.622	1.871	3.784	0.494
0.8	1.470	1.568	3.472	0.451	0.73	1.544	1.568	3.530	0.444
0.7	1.375	1.283	3.190	0.402	0.64	1.466	1.283	3.291	0.389
0.6	1.273	1.018	2.898	0.351	0.54	1.367	1.018	3.010	0.338
0.5	1.162	0.775	2.595	0.299	0.45	1.264	0.775	2.737	0.283
0.4	1.039	0.559	2.274	0.246					

Because the hydrostatic or scouring force in a sewer is found by multiplying the sectional area by the depth and $62\frac{1}{2}$ pounds, and that the depths of the segments of a parabola are greater than in the segments of the semicircle, each being equal to the same given area; therefore, from inspecting the above table, it will appear that the parabolic sewers have greater hydrostatic depths and pressure than the circular segments. It also appears that in the lower half depth of the semicircle, and in all other depths lower than half the radius, the hydraulic mean depth is greater than in circular segments of the same areas.

Calculation of the foregoing Table.

Example. Required, the ordinate at abscissa 1,2 of the given parabola, whose abscissa = 2,019, and ordinate 2,335, and *latus rectum* 2,7.

Rule. Multiply the *latus rectum* by the abscissa of the parabolic segment. The square root of product will be the required ordinate.

Or by logarithms, let log of 2,7 = 0,431364
 log of the given abscissa = 0,041893
 log of the product of abscissa and *latus rectum* = 0,472757
 which divide by 2 will give the log of the square root of the product = 0,236378
 the natural number corresponding to which gives the ordinate = 1,800

To Find the Area.

The given ordinate = 1,800.

The chord or double ordinate = 3,600.

$$\begin{array}{r} \text{abscissa} \quad 1,2 \\ \hline 4,82 \end{array}$$

This product multiplied by 2 and divided by 3, gives the area = 2,88.

That is, two-thirds of the product of the abscissa and double the ordinate is equal to the required area.

To Find the Perimeter of the given Segment.

186. *Rule.* To one and one-third times the square of the abscissa, add the square of the given ordinate. The square root of the sum, if multiplied by 2, will give the perimeter.

In the example, abscissa = 1,2, and ordinate = 1,80.

$$\begin{array}{r} \text{Abscissa squared} = (1,2)^2 = 1,44 \\ \text{one-third of } (1,2)^2 = 48 \\ \text{square of the ordinate} = (1,8)^2 = 3,24 \\ \hline 5,16 \\ \text{the square root of } 5,16 = 2,2715 \\ \hline 2 \\ \text{Required perimeter} = 4,5430 \end{array}$$

To Find the Hydraulic Mean Depth.

Rule. Divide the area of the segment by the wetted perimeter. The quotient will be the hydraulic mean depth.

$$\text{That is, } \frac{2,880}{4,648} = \text{hydraulic mean depth} = 0,634.$$

To Find the Height and Chord of a Circular Segment.

187. To find the chord corresponding to a circular segment whose area = that of the parabolic segment (see segment No. 10 in table), where area = $a = 1,880, \frac{a}{d^2} = \text{tabular segment area, opposite tab. ver. sine.}$ This multiplied by the diameter will be the height of the segment.

Here we have $a = 2,880$.

$$d^2 = 4 \times 4 = 16, \text{ and the quotient } \frac{a}{d^2} = 0,18000.$$

Tab. area segment = ,18000.

Corresponding ver. sine = ,280 (by TAB. VII).

$$\begin{array}{r} 4 \\ \hline 1,120 = \text{depth or abscissa.} \end{array}$$

To Find the Chord or Ordinate to this depth.

188. Diameter of the circle, 4 feet,
given height or depth of wet segment = $a = 1,12$

$$\begin{array}{r} \text{remaining or dry segment} = b = 2,88 \\ \hline 1,12 \end{array}$$

$$\text{product} = a b \quad 3,2256$$

the square root of this product will (Euclid III, prop. 35) give the ordinate or half chord = 1,796, and the chord of the segment = $c = 3,592$.

To Find the Perimeter.

139. We have the height of the segment = $a = 1,12$,
 the chord or double ordinate, $c = 8,592$.
 Then by TAB. VI, find the tabular length corresponding to the quotient
 in column tabular length. The tabular number thus found, multiplied by
 the chord, will be the required length.

$$\begin{array}{r} 8,592 \overline{) 1,12} \\ \text{quotient, } 1,118, \end{array}$$

whose tabular length = $1,2419$,
 which multiplied by the chord $c = 8,592$,
 will give the product = the required perimeter = $4,461$, and the perimeter
 divided into the given area will give the hydraulic mean depth, $0,646$.

EGG-SHAPED SEWER.

140. The egg-shaped sewer, in appearance, resembles a parabola, and
 is that now generally adopted in the new sewerage of London and Paris
 since 1857.

Let AB (fig 41) = width of sewer at the top. Bisect AB in O , erect
 the perpendicular $OC = AB$.

On AB describe the circle $EADB$, and on DC describe the circle
 $DICK$. Produce AB both ways. Making $AG = BH =$ the total height
 CE , join GF and HF . Produce them to the points I and K . From G
 as centre describe the arc AI , and from H as centre describe the arc BK .

Let $AB = 4$ feet, then $DC = 2$, and $CE = 6$, and $OC = 4$, and $OF = 3$.
 Also $HB = AG = GI = HK = 6$, and $HA = BG = 2$.
 $HG = 8$. Because $GQ = AG \therefore GQ^2 - GO^2 = OQ^2$.

In this example, $GQ^2 = 6^2 = 36$,

$$OQ^2 = 4^2 = 16.$$

The square root of $20 = 4,472 = OQ$.

To Find the $\angle O G Q$ by Trigonometry.

$4,472$ divided by radius $6 = 0,745383$, which is the natural cosine of
 $41^\circ 49' 2''$, and OF divided by $GO = 0,75 =$ nat. tangent of $\angle AGF = 36^\circ 52'$.
 (By sec. 69) $d^2 \times n \times ,00218175 = 12^2 \times 36^\circ, 86667 \times ,00218175 =$
 $\text{area } GAI = 11,5825$. Here $d^2 =$ diameter = 12 , and $n = 36^\circ 52' = 36,86967$.

$$\text{Area of the } \triangle GOF = \frac{GO \times OF}{2} = 2 \times 3 = 6$$

$$\text{Sector } GAI - \triangle GOF = 5,5825.$$

To Find the Sector IFC.

Because the angle $GOF = 90^\circ$, and the angle $OGF 36^\circ 52'$, their
 sum $126^\circ 52'$ taken from 180° will give $\angle GFO = 53^\circ 8'$; but Euclid I,
 prop. 15, the angle $GFO = \angle IFC = 53^\circ 8'$, and $FC =$ radius = 1 ,
 consequently $d^2 = 4$;

And by section 69, $d^2 \times n \times ,00218175 = 0,4636$, etc.;

Or by TAB. V, length of the arc corresponding to the angle $IFC 53^\circ 8' = 53^\circ, 18333 = 0,927851$. This multiplied by $\frac{1}{2} = \frac{1}{2}$ the radius, will
 give the area $ICF = 0,4636$, etc.

And from above we have the area $A I G = 11,5825$.

The sum of these two areas = area of the figure $G O A I C F G = 12,0461$

From this area deduct the $\triangle G O F$ found above, = 6

There remains the area of half the sewer below the spring of the arch, 6,0461

This multiplied by 2 gives the area of sewer to the spring of the arch; that is, area of $A O B K C I =$ 12,0922

Length of the curve $A I$ may be found by TAB. V.

$\angle O G F = 36^\circ 52' = 36^\circ, 86$, length of arc to radius 1 = ,658444
radius $G Q$ = 6

arc $A I =$ 8,920664

arc $I C$ from above = 0,927351

length of arc $A I C =$ 4,848

2

do. $A I C K B =$ perimeter = 9,696

This perimeter, 9,696, if divided into the area, 12,0922, will give the *hydraulic mean depth* of the sewer below the spring of the arch = 1,247 feet.

141. *To Find the Diameter of a Circle whose Semicircular Area = 12,0922.*

12,0922

2

Area of required circle = 24,1844

This divided by 0,7854, will give the square of the required diameter = 30,792462, square root = diameter = 5,550. Half of the diameter multiplied by 3,1416 = perimeter of semicircle = 8,718. This perimeter divided into the area 12,0922 = hydraulic mean depth 1,387.

Let us Find a Parabolic Sewer equal in area to 12,0922.

142. Abscissa = $0,806 \sqrt{a} = 0,806 \sqrt{12,092} = 2,803$. By sec. 138.

Ordinate = $\frac{\sqrt{a}}{1,075} = \frac{3,4774}{1,075} = 3,2344$.

Double ordinate, 6,4688.

Area corresponding to double ordinate 6,4688, and abscissa 2,803 = 12,088.

To Find Perimeter of this Parabolic Sewer.

143. Abscissa squared = $(2,803)^2 =$ 7,856809

one-third of do. = 2,618936

Ordinate squared = $(3,2344)^2 =$ 10,461343

20,937098

The square root of the sum = 4,575

2

Perimeter of wetted parabola = 9,15

This perimeter divided into 12,088, gives H. M. D. = 1,321.

Now we have the following summary :

	Circular Sewer.	Parabolic Sewer.	Egg-shaped Sewer.
Area filled in sewer,	12,0922	12,088	12,0922
Depth of water in sewer,	2,775	2,803	4,000
Hydraulic mean depth of part filled,	1,387	1,321	1,247
Hydrostatic pressure on bottom of sewer = depth of water $\times$ by $62\frac{1}{2}$ lbs. $\times$ sectional area,	2097 lbs.	2271 lbs.	3241 lbs.

Hence it appears that the scouring force, or hydrostatic pressure, is greater in a parabola than in the semicircle, and greater in the egg-shaped sewer than in the parabolic sewer.

And that the hydraulic mean depth, and consequently the discharge, is greater in the parabolic than in the egg-shaped, and greater in the circular than in the parabolic.

The great depths required by the egg-shaped, renders them impracticable excepting where sufficient inclinations can be obtained.

The parabolic segments will give greater hydraulic mean depths than circular or egg-shaped segments, and are as easily constructed as the egg-shaped sewers; therefore, ought to be preferred.

Having so far discussed curvilinear water courses or sewers, we will now proceed to the discussion of

RECTILINEAL WATER COURSES.

144. Let the nature of the soil require that the best slope to be given to the sides be that which makes the $\angle DCA = Q$. Let the required area of the section $ABDC$ be a , and h the given depth, to find the width $AB = x$.

Let $x = AB = EF$, and having the $\angle DCA$, we have its complement $\angle CAE$. By Trigonometry, $h \times \cotangent Q = CE = FD$, and $h \times \cot. Q \times h = h^2 \times \cot. Q = \text{area of the triangles } CEA + \triangle BFD$, and $h \times x = \text{area of the figure } AEFB$; therefore,

$$hx + h^2 \cot. Q = a,$$

$$x + h \cot. Q = \frac{a}{h},$$

$$x = \frac{a}{h} - h \cot. Q. \quad \text{A general formula. (1.)}$$

$$\text{Or, } x = \frac{a}{h} - h \tan. \text{ comp. } Q. \quad (2.)$$

When the $\angle CAE = 0$ then AC , coincides with AE , and $-h \cot. Q$ vanishes; then

$x = \frac{a}{h} = \text{value for rectangular figures, where } h \text{ the depth is limited, as in the case of canals; but if it were required to enclose the area } a \text{ in a rectangular figure, open at top, so that the surface will be a minimum.}$

Here we have $AB = x$, and $AC = BD = \frac{a}{x}$. $\therefore$ perimeter $CABD = x + \frac{2a}{x} = \frac{x^2 + 2a}{x}$;

that is, $y = \frac{x^2 + 2a}{x}$, and by differentiating this expression,

$$dy = \frac{2x^2 dx - x^2 dx - 2a dx}{x^2} = \frac{x^2 dx - 2a dx}{x^2}$$

$$\frac{dy}{dx} = \frac{x^2 - 2a}{x^2} = 0,$$

$$x^2 - 2a = 0,$$

$$x = \sqrt{2a} = AB,$$

and $\frac{a}{\sqrt{2a}} = AC = \frac{\sqrt{a} \cdot \sqrt{a}}{\sqrt{a} \cdot \sqrt{2}} = \frac{\sqrt{a}}{\sqrt{2}}$. Multiply this by $\sqrt{2}$;

then $= \frac{\sqrt{2} \cdot \sqrt{a}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2a}}{\sqrt{2}} = \frac{1}{2} \sqrt{2a} = AC$.

But $\sqrt{2a} = AB$.

Consequently, $AB = \text{twice } AC$, as stated in sec. 19.

Having determined the natural slope from observing that of the adjacent hills—and if no such hills are near, it is to be determined from the nature of the soil,—

Let $AC =$ required slope, making angle n degrees with the perpendicular AE ; then $CE =$ tangent of angle n to radius AE .

Let $s =$ secant of the angle CAE ; then $AC =$ secant to radius AE and angle n degrees. See fig. 42.

Let $x =$ height of the required section, and $a =$ area of the required section $CABD$, to find the height x and base AB , $nx^2 =$ area of the two triangles $ACE + BFD$, because $CE = nx$, and $AE = x$, $\therefore nx^2 =$ double area of triangle ACE .

Now, we have $a - nx^2 =$ area of the rectangle $ABEF$. $\therefore$

$$\frac{a - nx^2}{x} = AB. \text{ But } sx = AC, \text{ and } 2sx = CA + BD;$$

therefore, $\frac{a - nx^2}{x} + 2sx =$ perimeter $CABD = a$ minimum;

$$\text{i. e., } \frac{a - nx^2 + 2sx^2}{x} = \frac{2sx^2 - nx^2 + a}{x} = \frac{x^2 \cdot (2s - n) + a}{x}$$

and by differentiating the last expression,

$$\text{we have } dy = \frac{4sx^2 dx - 2nx^2 dx + nx^2 dx - a dx}{x^2},$$

$$\text{and } \frac{dy}{dx} = 2sx^2 - nx^2 - a = 0,$$

$$\text{and } x^2 = \frac{a}{2s - n},$$

$$\text{and } x = \left(\frac{a}{2s - n} \right)^{\frac{1}{2}} = AE = \text{height, or required depth.} \quad (3.)$$

When there is no slope, AC coincides with AE , and $S = 1$, and $n = 0$;

$$\text{then for rectangular conduits } x = \left(\frac{a}{2} \right)^{\frac{1}{2}} \quad (4.)$$

Example. What dimensions must be given to the transverse profile (or section) of a canal, whose banks are to have 40° slope, and which is to conduct a quantity of water Q , of 75 cubic feet, with a mean velocity of 3 feet per minute?—*Weisbach's Mechanics*, vol. 1, p. 444.

Here we have the $\angle DCA = 40^\circ$, consequently $\angle CAE = 50^\circ$, and the sectional area of figure $CABD = a = 25$ feet.

By formula 3, $x = \left(\frac{a}{2s-n}\right)^{\frac{1}{2}}$ where $s = \secant\ of\ 50^\circ = 1,555724$,

and $n = \text{tangent of } 50^\circ, \quad 1,191754.$

$$2s = 3,111448$$

$$n \quad 1,191754$$

1.919694 divided into 25, gives 13,022868,

the square root of which $= x = \text{depth } AE = 3,6087 = 3,609$ nearly, and tangent $= 1,191754$ if multiplied by $3,609 \times 3,609 = \text{area of the triangles } ACE + BFD = 15,522809$, which taken from 25, will leave the rectangle $AEFB =$

$$9,477691$$

This divided by the height, 3,609, gives $AB =$

$$2,626$$

But $3,609 \times 1,191754 = CE =$

$$4,301$$

and $FD,$

$$4,301$$

$$\text{Upper breadth } CD = 11,228$$

Bottom AB

$$2,6260$$

$$1,555724 \times 3,609 = AC =$$

$$5,6146$$

and $BD =$

$$5,6146$$

$$p = \text{perimeter} = AC + AB + BD =$$

$$13,8552$$

which is the least surface with the given slopes, and containing the given area $= 25$ feet.

The results here found are the same as those found by Weisbach's formula, which appears to me to be too abstruse.

145. From the above, the following equations are deduced:

$$AE = BF = x = \left(\frac{a}{2s-n}\right)^{\frac{1}{2}}$$

$$AC = BF = \left(\frac{a}{2s-n}\right)^{\frac{1}{2}} \cdot s = \left(\frac{as^2}{2s-n}\right)^{\frac{1}{2}}$$

$$AB = \frac{a-nx^2}{1} \times \frac{\sqrt{a}}{\sqrt{2s-n}} = (a-nx^2) \frac{\sqrt{(2s-n)}}{\sqrt{a}}$$

146. Hence it appears that the best form of *Conduits* are as follows:

Circular, when it is always filled.

Rectangular, that whose depth is half its breadth.

Triangular, when the triangle is equilateral.

Parabolic, when the depth of water is variable and conduit covered, and in accordance with section 133.

Rectilineal, when opened, and in accordance with section 144.

For the velocity and discharge through conduits, also for the laying out of canals, and calculating the necessary excavation and embankment, see *Sequel*.

147. TABLE, SHOWING THE VALUE OF THE HEIGHT $A E = x$, in the equation $x = \left(\frac{a}{2s - n}\right)^{\frac{1}{2}}$, where a = area of the given section, having given slopes, and such that the area a is inclosed by the least surface or perimeter in contact. s = secant and n = tangent of the angle $D B F$, or complement of the angle of repose (see fig. 42).

Ratio of base B G to perpendicular B F.	Angle of repose or angle D B G.	Angle Q or $\angle D B F$.	Value of $x = \left(\frac{a}{2s - n}\right)^{\frac{1}{2}}$ or A E.
Perpendicular 0 to 1	90° 00'	00° 00'	$x = \sqrt{\left(\frac{a}{2}\right)}$
1 to 1	45° 00'	45° 00'	$x = \sqrt{\left(\frac{a}{1,828427}\right)}$
1,5 to 1	23° 41'	66° 49'	$x = \sqrt{\left(\frac{a}{2,745287}\right)}$
2 to 1	26° 34'	63° 36'	$x = \sqrt{\left(\frac{a}{2,472025}\right)}$
2,5 to 1	21° 48'	68° 12'	$x = \sqrt{\left(\frac{a}{2,885318}\right)}$
3 to 1	18° 26'	81° 34'	$x = \sqrt{\left(\frac{a}{6,892288}\right)}$
3,5 to 1	15° 56'	74° 04'	$x = \sqrt{\left(\frac{a}{3,782686}\right)}$
4 to 1	14° 02'	75° 58'	$x = \sqrt{\left(\frac{a}{4,247024}\right)}$
5 to 1	11° 19'	48° 41'	$x = \sqrt{\left(\frac{a}{1,891684}\right)}$
Perfectly dry soil,	38° 49'	51° 11'	$x = \sqrt{\left(\frac{a}{1,947647}\right)}$
Moist soil,	42° 48'	47° 17'	$x = \sqrt{\left(\frac{a}{1,865171}\right)}$
Very dry sand,	30° 58'	59° 02'	$x = \sqrt{\left(\frac{a}{2,220497}\right)}$
Rye seed,	30° 00'	60° 00'	$x = \sqrt{\left(\frac{a}{2,267949}\right)}$
Fine shot,	25° 00'	65° 00'	$x = \sqrt{\left(\frac{a}{2,587897}\right)}$
Finest shot,	22° 30'	67° 30'	$x = \sqrt{\left(\frac{a}{2,812088}\right)}$

Slopes for the sides of canals, in very compact soils, have $1\frac{1}{2}$ base to 1 perpendicular; but generally they are 2 base to 1 perpendicular, as in the Illinois and Michigan Canal.

Sea banks, along sea shores, have slopes whose base is 5 to 1 perpendicular for the height of ordinary tides; base 4 to 1 perpendicular for that part between ordinary and spring tides; and slopes 3 to 1 for the upper part. By this means the surface next the sea is made hollow, so as to offer the least resistance to the waves of the sea. The lower part is faced with gravel. The centre, or that part between ordinary and spring tides, is faced with stone. The upper part, called the swash bank, is faced with clay, having to sustain but that part of the waves which dashes over the spring tide line. (See *Embankments*.)

PLANE TRIGONOMETRY.

RIGHT ANGLED TRIANGLES.

148. Let the given angle be CAB' (fig. 9). Let $AB = c$, $CB = a$, and $AC = b$, be the given parts in the right angled triangle ACB .

149. Radius $= AB' = AC$.

150. Sine $\angle CAB' = CB = \text{cosine of the complement} = \cos. \angle ACD$.

151. Cos. $\angle CAB = AB = \text{sine of the comp. of } \angle CAB = \text{sine } \angle ACB$.

152. Tangent $\angle CAB' = BT = \text{cot. of its complement} = \text{cot. } \angle HAC$.

153. Cotangent $CAB' = HK = \text{tan. of its complement} = \text{tan. } \angle HAC$.

154. Secant $\angle CAB' = AT = \text{cosec. of its complement} = \text{cosec. } \angle HAC$.

155. Cosecant $\angle CAB' = AK = \text{sec. of its comp.} = \text{sec } \angle CAH$.

156. Versed sine $\angle CAB' = BB'$.

157. Covered sine $\angle CAB' = HI = \text{versed sine of its complement}$.

158. Chord $\angle CAB' = CB' = \text{twice the sine of } \frac{1}{2} \text{ the } \angle CAB'$.

158a. Complement of an angle is what it wants of being 90° .

158b. Supplement of an angle is what it wants of being 180° .

158c. Arithmetical complement is the log. sine of an angle taken from 10, or begin at left hand and subtract from 9 each figure but the last, which take from 10.

159. Let ACB (fig. 9) represent a right angled triangle, in which $AB = c$, $BC = a$, and $AC = b$, and A, B, C , the given angles.

160. Sine $\angle A = \frac{a}{b}$

161. Cos. $\angle A = \frac{c}{b}$

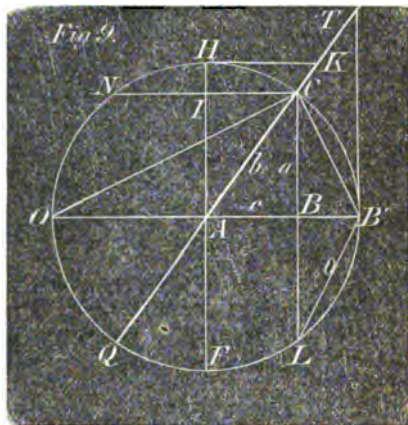
162. Tan. $\angle A = \frac{a}{c}$

163. Sine $C = \frac{c}{b}$

164. Cos. $C = \frac{a}{b}$

165. Tan. $C = \frac{c}{a}$

166. Sec. $A = \frac{b}{c}$



And the sides can be found as follows:

167. $a = c \tan. A$.

168. $a = b \sin. A$
 d

$$169. a = b \cos. C.$$

$$170. b = c \sec. A = a \sec. < C = \frac{c}{\cos. A} = \frac{a}{\cos. C} = \frac{a}{\sin A}$$

$$171. c = b \cos. A = b \sin C = a \tan. C = \frac{b}{\sec. A}$$

Examples. Let AC = the hypotenuse = 480, and the angle at A $53^\circ 8'$, to find the base AB and perpendicular BC .

By sec. 168, natural sine of $< A$, 8000 = departure of $53^\circ 8'$

$$AC = 480$$

$$BC = a = 384 = \text{product.}$$

Or by logarithms:

$$\text{Log. sine of } < A (53^\circ 8') = 8,9081084$$

$$\text{Log. of } b = \text{log. of } 480 \quad 2,6812412$$

$$BC = 384 = 2,5843496$$

And by having the $< A = 53^\circ 8'$ $\therefore$ the $< C = 36^\circ 52'$.

$$\text{Nat. sine of } 36^\circ 52' = ,6000$$

$$AC = 480$$

$$AB = 280 = \text{product.}$$

Otherwise,

$$36^\circ 52' \text{ Log. sine} = 9,7781186$$

$$\text{Log. of } 480 = 2,6812412$$

$$288 \text{ nearly} = 2,4593598$$

$$\text{or } 287,978 = AB.$$

171a. Let the side $BC = a = 384$, and the angle $C = 36^\circ 52'$ be given to find c , b , and the angle A .

$$90^\circ - 36^\circ 52' = < A = 53^\circ 8',$$

$$\text{and a } \tan. C = c, \text{ that is } 384 \times 0,7499 = AB = 288 \text{ nearly.}$$

171b. Let the sides be given to find the angles A and C .

$$\text{Sine } A = \frac{a}{b} (\text{per sec. } 160) = \frac{384}{480} = 0,8000 = 53^\circ 8' \text{ nearly.}$$

$$\text{Sec. } A = \frac{b}{c} (\text{per sec. } 166) = \frac{40}{288} = 1,6666 = 53^\circ 8' \text{ nearly.}$$

$$\text{Cos. } A = \frac{c}{b} (\text{per sec. } 161) = \frac{288}{480} = 0,6000 = 53^\circ 8' \text{ nearly.}$$

$$\text{Tan. } A = \frac{a}{c} (\text{per sec. } 162) = \frac{384}{288} = 1,3333 = 53^\circ 8' \text{ nearly.}$$

In like manner the angle C may be found.

These examples are sufficient to enable the surveyor to find the sides and angles.

The calculations may be performed by logarithms as follows:

$$\text{Log. } a = +, \text{ etc.}$$

$$\text{Log. } b = -, \text{ etc.}$$

$$\text{Sine of angle } A$$

$$\text{Log. sine of } < A.$$

OBLIQUE ANGLED TRIANGLES.

171c. The following are the algebraic values for the four quadrants:

	From 0 to 90.	From 90 to 180.	From 180 to 270	From 270 to 360
Sine,	+	+	—	—
Cosine,	+	—	—	+
Tangent,	+	—	+	—
Cotangent,	+	—	+	—
Secant,	+	—	—	+
Cosecant,	+	+	—	—
Versed sine,	+	+	+	+

	0°	90°	180°	270°
Sine,	0	1	0	—1
Cosine,	1	0	—1	0
Tangent,	0	inf	0	inf
Cotangent,	inf	0	inf	0
Secant,	1	inf	—1	inf
Cosecant,	inf	1	inf	—1
Versed sine,	0	1	2	1

Note. Here the symbol *inf* signifies a quantity which is infinitely great.

$$172. a^2 = b^2 + c^2 - 2bc \cdot \cos. A.$$

$$173. b^2 = a^2 + c^2 - 2ac \cdot \cos. B.$$

$$174. c^2 = a^2 + b^2 - 2ab \cdot \cos. C.$$

Now, from 172, 173, and 174, we find the cosines of the angles A, B, and C.

$$175. \cos. A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$176. \cos. B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$177. \cos. C = \frac{b^2 + a^2 - c^2}{2ba}$$

And by substituting $s = \frac{1}{2}$ the sum of the three sides $= \frac{1}{2}(a + b + c)$, we find—

$$178. \sin A = \frac{2}{bc} \sqrt{s \cdot (s - a) \cdot (s - b) \cdot (s - c)}$$

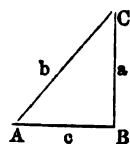
$$179. \sin B = \frac{2}{ac} \sqrt{s \cdot (s - a) \cdot (s - b) \cdot (s - c)}$$

$$180. \sin C = \frac{2}{ab} \sqrt{s \cdot (s - a) \cdot (s - b) \cdot (s - c)}$$

$$181. \cos. \frac{A}{2} = \sqrt{\frac{s \cdot (s - a)}{bc}}$$

$$182. \cos. \frac{B}{2} = \sqrt{\frac{s \cdot (s - b)}{ac}}$$

$$183. \cos. \frac{C}{2} = \sqrt{\frac{s \cdot (s - c)}{ab}} \quad \text{Also, we find in terms of the tangent—}$$



$$184. \quad \text{Tan.} \frac{A}{2} = \sqrt{\frac{(s-b) \cdot (s-c)}{s \cdot (s-a)}}$$

$$185. \quad \text{Tan.} \frac{B}{2} = \sqrt{\frac{(s-a) \cdot (s-c)}{s \cdot (s-b)}}$$

$$186. \quad \text{Tan.} \frac{C}{2} = \sqrt{\frac{(s-a) \cdot (s-b)}{s \cdot (s-c)}} \quad \text{We can find in terms of sine—}$$

$$187. \quad \text{Sine} \frac{A}{2} = \sqrt{\frac{(s-b) \cdot (s-c)}{bc}}$$

$$188. \quad \text{Sine} \frac{B}{2} = \sqrt{\frac{(s-a) \cdot (s-c)}{ac}}$$

$$189. \quad \text{Sine} \frac{C}{2} = \sqrt{\frac{(s-a) \cdot (s-b)}{ab}}$$

$$190. \quad \text{Radius of the inscribed circle in a triangle} = r = \frac{\sqrt{s \cdot (s-a) \cdot (s-b) \cdot (s-c)}}{s} \quad \text{which is the same as given in sec. 48.}$$

$$191. \quad \text{Radius of the circumscribing circle} = R = \frac{1}{4} \{s \cdot (s-a) \cdot (s-b) \cdot (s-c)\}^{\frac{1}{2}}.$$

192. By assuming D = the distance between the centres of the inscribed and circumscribed circles, we have $D^2 = R^2 - 2Rr$, and $D = (R^2 - 2Rr)^{\frac{1}{2}}$

193. Area of a quadrilateral figure inscribed in a circle is equal to $\{(s-a) \cdot (s-b) \cdot (s-c) \cdot (s-d)\}^{\frac{1}{2}}$, where s is equal to the sum of the sides.

Sides are to one another as the Sines of their Opposite Angles.

$$194. \quad a : c :: \text{sine } A : \text{sine } C.$$

$$195. \quad a : b :: \text{sine } A : \text{sine } B.$$

$$196. \quad b : c :: \text{sine } B : \text{sine } C. \quad \text{And by alternando—}$$

$$197. \quad a : \text{sine } A :: c : \text{sine } C.$$

$$198. \quad a : \text{sine } A :: b : \text{sine } B.$$

$$199. \quad b : \text{sine } B :: c : \text{sine } C. \quad \text{And by invertendo—}$$

$$200. \quad \text{Sine } A : a :: \text{sine } C : c.$$

$$201. \quad \text{Sine } A : a :: \text{sine } B : b.$$

$$202. \quad \text{Sine } B : b :: \text{sine } C : c.$$

Having two Sides and their contained Angle given to Find the other Side and Angles.

203. *Rule.* The sum of the two sides is to their difference, as the tangent of half the sum of the opposite angles is to the tangent of half their difference; i. e., $a + b : a - b :: \tan. \frac{1}{2} (A + B) : \tan. \frac{1}{2} (A - B).$

Here a is assumed greater than b . $\therefore$ the $\angle A$ is greater than B .—E. I., 19. (See fig. 12.)

Now, having half the difference and half the sum, we can find the greater and lesser angles of those required for half the sum, added to half the difference = greater $\angle$, and half the difference taken from the half sum = lesser $\angle$.

When the Three Sides of the Triangle are given to Find the Angles.

205. *Rule.* As twice the base or longest side $AC = b$ is to the other two sides, so is the difference of these two sides to the distance of a perpendicular from the middle of the base; that is, $2b : a + c :: a - c : DE$.

Here BD is the perpendicular, and BE the line bisecting the base; because $BC = a$ is greater than $AB = c$, CD is greater than AD ; because $\angle A$ is greater than $\angle C$, the $\angle ABD$ is less than $\angle CBD$; therefore, the area of the $\triangle CDB$ is greater than $\triangle ADB$; consequently, the base CD is greater than AD .

Let $DE = d$; now the $\triangle ABC$ is divided into two right angled triangles ABD and CBD , having two sides and an angle in each given to find the other angles.

$$\text{In the } \triangle ABD \text{ is given } AD = \frac{b}{2} - d = \frac{b - 2d}{2}$$

$$\text{And } AB = c, \text{ and } BC = a, \text{ and } CD = \frac{b}{2} + d = \frac{b + 2d}{2}$$

$$\text{By sec. 161, } \cos. A = \frac{b - 2d}{2c} \quad \text{And in like manner,}$$

$$\cos. C = \frac{b + 2d}{2a} \quad \text{And by Euclid I. 32, angle } B \text{ is found.}$$

Cosine A may be found by sec. 175, and cosine C by sec. 177.

206. *Example.* Let the $\angle A = 40^\circ$ (fig. 5), $\angle B = 50^\circ$, and the side BC equal to 64 chains, to find the side AC .

By sec. 194, $\sin 40^\circ : 64 \text{ chains} :: \sin 50^\circ : AC$.

Nat. sine 50°	= 0,76604
Nat. number	= 64
Product	= 49,02656
Nat. sine 40°	= 0,64279
Quotient, 76,272	= AC .

Or thus:

Log. sine 50°	= 9,884254
Log. 64	= 1,806180
Sum	11,690434
Log. sine 40°	= 9,882396
Dif.	1,882038

Nat. No. = 76,272 chains = AC .

In like manner, by the same section, AB may be found, because angles A and B together = $90^\circ \therefore \angle C = 90^\circ$.

207. In the $\triangle ABC$ (fig. 12), let the angle $A = 40^\circ$, angle $B = 60^\circ$, consequently, $\angle C = 80^\circ$. Let $BC = 64$, to find the side AC .

Nat. sine 60°	= 0,86602
Nat. number	= 64
Product,	= 55,42528
Nat. sine 40°	= 0,64279
Quotient 86,277	= side AC

Or thus:

Log. sine	= 9,937531
Log.	= 1,806180
Sum	= 11,743711
Log. sine	= 9,808068
Dif.	= 1,935643

Nat. No. = 86,227 = AC .

Or thus:

Log. sine	= 9,937531
Log.	= 1,806180
Ar. comp.	= 0,191982
Sum	= 1,935643
	= 86,227
	= AC .

AB may be found by sec. 200.

Note. Here ar. comp. signifies arithmetical complement. It is log. sine 40° taken from 10 (see sec. 158 c), or it is the cosecant of 40° .

Given Two Sides and the Contained Angle to Find the Other Parts.

208. *Example.* Let $AC = 120$, $BC = 80$, and $\angle ACB = 40^\circ$, to find the other side, AB , and angles A and B .

By sec. 203, $120 + 80 : 120 - 80 :: \tan. 70^\circ : \text{tangent of the half difference between the angles } B \text{ and } A$.

i. e., $200 : 40 :: \tan. 70^\circ : \tan. \frac{1}{2} \text{ dif. } B - A$.

i. e., $5 : 1 :: 2,747477 : 0,549495 = 28^\circ 47'$.

$\therefore 70^\circ + 28^\circ 47' = 98^\circ 47' = \angle B$.

And $70^\circ - 28^\circ 47' = 41^\circ 13' = \angle A$.

By sec. 194, $\text{sine } 41^\circ 13' : 80 :: \text{sine } 40^\circ : AB$.

Nat. sine 40°	0,64279	Or thus:		Or thus:	
Nat. number 80 =	80	Log. sine	9,808068	Log. sine 40°	9,808068
Product	51,42820	Log.	1,90,090	Log. 80	1,9,3090
Nat. sine $41^\circ 13'$	0,66891	Sum	11,71158	Ar. comp. $40^\circ 13' = 0,181175$	
Quotient, 78,043 = AB .		Log. sine	9,818825		1,892338
		Dif.	1,892338		= 78,043 = AB .
			= 78,043 = AB .		

Given the Three Sides to Find the Angles.

209. *Example.* $AB = b = 142,02$, $AC = c = 70$, and $BC = a = 104$, to find the angles at A , B and C . (See fig. 5.)

By sec. 205, $284,04 : 174 :: 84 : DE = 20,828$

But $AD = DB = 71,010$

Therefore, $AE = 91,838 = \cos. \angle A \times AC$.

And $BE = 50,182 = \cos. \angle B \times BC$.

Consequently $50,182 + 70 = 0,716885 = \cos. \angle A = 44^\circ 12'$

and $91,838 + 104 = 0,88805 = \cos. \angle C = 27^\circ 59'$

Having the angles A and C , the third angle at B is given.

Or thus by sec. 175:

$$b^2 = (142,02)^2 = 20169,6804$$

$$a^2 = (104)^2 = 10816,$$

$$\text{sum, } 30985,6804$$

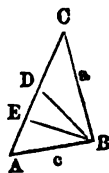
$$c^2 = (70)^2 = 4900,$$

$$2ba = 29540 \quad 26085,6804 \text{ quotient} = 0,88806$$

(Divisor.) (Dividend.)

Which is the cosine of the $\angle C = 27^\circ 59'$

210. Or thus by sec. 183:



$b = 142.02$, $b = 104$, and $a = 70$.

$a = 104$,

$c = 70$,

$2)316.02 = \text{sum.}$

$s = 158.01 = \text{half sum,} \quad \log. = 2.1986846$

$s - c = 88.01, \quad \log. = 1.9445320$

$a = 104, \log. = 2.0170833, \text{ ar. comp. } 7.9829667$

$b = 142.02, \log. 2.1523495, \text{ ar. comp. } 7.8476505$

$2)19.9738838$

$\cos. \frac{1}{2} < C = 18^\circ 59' 36'' = \log. \text{ sine } 9.986169$

$\therefore \text{ the angle } A = 27^\circ 59' 12''.$

In like manner, $\cos. \frac{1}{2} < B$ may be found by sec. 176.

The same results could be obtained by using the formulas in sections 184 and 188.

HEIGHTS AND DISTANCES.

211. In chaining, the surveyor is supposed to have his chain daily corrected, or compared with his standard. He uses ten pointed arrows or pins of iron or steel, one of which has a ring two inches in diameter, on which the other nine are carried; the other nine have rings one inch in diameter. The rings ought to be soldered, and have red cloth sewed on them. He carries a small axe, and plumb bob and line, the bob having a long steel point, to be either stationary in the bob or screwed into it, thus enabling the surveyor to carry the point without danger of cutting his pocket. A plumb bob and line is indispensable in erecting poles and pickets; and in chaining over irregular surfaces, etc., he is to have steel shod poles, painted white and red, marked in feet from the top; flags in the shape of a right angled triangle, the longest side under; some flags red, and some white. For long distances, one of each to be put on the pole. For ranging lines, fine pickets or white washed laths are to be used, set up so that the tops of them will be in a line. Where a pole has to be used as an observing station, and to which other lines are to be referred, it would be advisable to have it white-washed, and a white board nailed near the top of it.

His field books will be numbered and paged, and have a copious index in each. In his office he will keep a general index to his surveys, and also an index to the various maps recorded in the records of the county in which he from time to time may practice. In his field book he keeps a movable blotting sheet, made by doubling a thin sheet of drawing paper, on which he pastes a sheet of blotting paper, by having a piece of tape, a little more than twice the length of the field book. The sheet may be moved from folio to folio. One end of the tape is made fast at the top edge and back, brought round on the outside, to be thence placed over the blotting sheet to where it is brought twice over the tape on the outside, leaving about one inch projecting over the book. He has offset poles,—one of ten links, decimally divided, and another of ten or six feet, similarly divided, mounted with copper or brass on the ends. One handle of the

chain to have a large iron link, with a nut and screw, so as to adjust the chain when the correction is less than a ring. By this contrivance the chain can be kept of the exact length. Some surveyors keep their chains to the exact standard, but most of them allow the thickness of an arrow, to counteract any deflections—that is allowing one-tenth of an inch to every chain.

In surveying in towns and cities, where the greatest accuracy is required, the best plan is to have the chain of the exact length, and the fore chain bearer to draw a line at the end of the chain, and mark the place of the point at the middle of the handle. Turn the arrow so as to make a small hole, if in a plank or stone; if in the earth, hold the handle vertically, so as to make the mark on the handle come to the side of the arrow next the hind chainman. Where permanent buildings are to be located, surveyors use a fifteen feet pole, made of Norway pine, and decimally marked. This, with the plumb line, will insure the greatest accuracy.

In locating buildings, the surveyor gives lines five feet from the water table, so as to enable cellars or foundations to be dug. When the water table is laid, the surveyor *ought to go on the ground* and measure the distance from the water table and face of the walls from the true side or sides of the street or streets and sides of the lot.

In making out his plan and report of the survey, he ought to state the date, chainmen, the builder and owner of the lot and building, at what point he began to measure, and his data for making the survey. A copy of this he files in his office, in a folio volume of records, and another is given to him for whom the survey has been made, on the receipt of his fees. If any of his base lines used in measuring said land pass near any permanent object, he makes a note of it in his report.

In chaining in an open country, he leaves a mark, dug at every ten chains, made in the form of an isosceles triangle, the vertex indicating the end of the ten chains, or 1000 feet or links. Out of the base cut a small piece about two by four inches, to show that it is a ten chain mark, and to distinguish it from other marks made near crossings of ditches, drains, fences, or stone walls. Some of the best surveyors I have met in the counties of Norfolk, Suffolk and Essex, in England, amongst whom may rank Messrs. Parks, Molton and Eacles, had small pieces of wood about six inches long, split on the top, into which a folded piece of paper, containing the line and distance, was inserted. This was put at the pickets or triangular marks made in the ground, and served to show the surveyor where other lines closed.

In woodland, drive a numbered stake at every ten chains. In open country, note buildings, springs, water courses, and every remarkable object, and take minute measurements to such as may come within one hundred feet of any boundary lines, *for future reference.*

In laying out towns and villages, stones 4 feet long and 6 inches square, at least, ought to be put at every two blocks, either in the centre of the streets, or at convenient distances from the corners, such as five feet; the latter would be best, as paving, sewerage, gasworks or public travel would not interfere with the surveyor's future operations. All the angles from stone to stone ought to be given, and these angles referred, if possible, to some permanent object, such as the corner of a church tower, steeple, or brick building; or, as in Canada, refer them to the true meridian.

This latter, although troublesome, is the most infallible method of perpetuating these angles. When the hole is dug for the stone, the position of its centre is determined by means of a plumb line; a small hole is then made, into which broken delf or slags of iron or charcoal is put, and the same noted in the surveyor's report or *proces verbal*. These precautions will forever prevent 99-100th parts of the litigations that now take place in our courts of justice. The office of a surveyor being as responsible as it is honorable, he ought to spare no pains or expense in acquiring a theoretical and practical knowledge of his profession, and to be supplied with good instruments. Where a difference exists between them, it ought to be their duty to make a joint survey, and thus prevent a lawsuit. This appears indispensable when we consider the difficulty of finding a jury who is capable of forming a correct judgment in disputed surveys.

When in woodland, we mark trees near the line, blazing front, rear, and the side next the line, and cutting in the side next the line, a notch for every foot that the line is distant from the tree, which notches ought to be lower than where the trees will be cut, so as to leave the mark for a longer time, to be found in the stumps. State the kind of tree marked, its diameter, and distance on the line. Where a post is set in woodland, take three or four bearing trees, which mark with a large blaze, facing the post. Describe the kind of each tree, its diameter, bearing, and distance from the post. For further, see United States surveying.

In order to make an accurate survey, the surveyor ought to have a good transit instrument or theodolite, as *the compass cannot be relied on, owing to the constant changing of the position of the needle*. By a good theodolite, the surveyor is enabled to find the true time, latitude, longitude, and variation of any line from the true meridian. If packed in a box, covered with leather or oiled canvas, it can be carried with as little inconvenience as a soldier carries his knapsack,—only taking care to have the box so marked as to know which side to be uppermost. The box ought to have a space large enough to hold two small bull's eye lamps and a square tin oil can; this space is about 9 inches by 8. Also, a place for an oil cap covering for the instrument in time of rain or dust; two tin tubes, half an inch in diameter and five inches long; with some white lead to clean the tubes occasionally. These tubes are used when taking the bearing of a line at night, from the true meridian. One of the tubes is put on the top of a small picket, or part of a small tree: this we call the tell-tale. The other is made fast to the end of a pole or picket, and set in direction of the required line, or line in direction of the pole star when on the meridian, or at its greatest eastern or western elongation. Some spider's web on a thick wire, bent in the shape of a horse shoe, about six inches long and two and a half inches wide, having the tops bent about a third of an inch, and a lump of lead or coil of wire on the middle of the circular part. This put in a small box, with a slide a fourth of an inch over the wire, so as to keep the web clean. Have a small phial full of shellac varnish, to put in cross hairs when required. In order to have the instrument in good adjustment, have about two pounds of quicksilver, which put in a trough or on a plate, if you have no artificial horizon. In order to have the telescope move in a vertical position, place the instrument, leveled, so that you can see some remarkable point above the horizon, and reflected in

the mirror or quicksilver. Adjust the telescope so as to move vertically through these points. Mark on the lid of the box the index error, if any, with the sign +, if the error is to be added, and —, if it is to be subtracted.

On the last page of each field book pencil the following questions, which read before leaving home: Have I the true time,—necessary extracts from the Nautical Almanac,—latitude and longitude of where the survey is to be made,—expenses, axes, flags, poles, instrument, tripod, keys, necessary clothing, etc.,—field notes, sketches, and whatsoever I generally carry with me, according to the nature of the survey. It ought to be the duty of one of the chainmen every morning, on sitting to breakfast, to say, "*Wind your chronometer, sir.*" These precautions will prevent many mistakes. The surveyor ought to carry a pocket case filled with the necessary medicines for diarrhoea, dysentery, ague and bilious fever, and some salves and lint for cuts or wounds on the feet; some needles and strong thread, and all things necessary for the toilet; a copy of Simms or Heather on Mathematical Instruments, and McDermott's Manual, and the surveyor is prepared to set out on his expedition. If it so happens that he is to be a few days from home, he ought to have drawing instruments and cartridge paper, on which to make rough outlined maps every night, after which he inks his field notes. He makes no erasures in his report or field notes. When he commits an error, he draws the pen twice over it, and writes the initials of his name under it. This will cause his field book to be deserving of more credit than if it had erasures. The surveyor ought to leave no cause for suspecting him to have acted partially.

212. Let it be required at station A (fig. 12) to find the $\angle BAC$, where the points B and C are at long distances from A. Let the telescope be directed to C, and the limb read 0. Move the telescope to B; let the limb now be supposed to read $20^\circ +$. Direct the whole body with the index at $20^\circ +$ on C, clamp the under plate and loosen the upper. Bring the A telescope again on B, reading $40^\circ +$. Repeat the same operation, bringing the telescope a third time on B, and reading $60^\circ 23'$, which being three times the required angle, $\therefore$ the $\angle BAC = 20^\circ 7' 20''$.

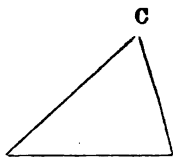


Fig. 12. B

By this means, with a five inch theodolite, angles can be taken to within twenty or thirty seconds, which is equal to six inches in a mile, if read to twenty seconds. In setting out a range of pickets, one of the cross hairs ought to be made vertical, by bringing it to bear on the corner of a building, on a plumb line suspended from a tree or window. The plumb-bob ought to be in water to prevent vibration. Two corresponding marks may be cut,—one on the Ys and the other on the telescope. These two marks, when together, indicate that the vertical hair is adjusted. Where the surveyor has an artificial horizon or quicksilver, he can, by the reflection of the point of a rod or stake, or any other well defined point, adjust the vertical hair, and then mark the Y and telescope for future operations.

213. All the interior angles of any polygon, together with four right angles, are equal to twice as many right angles as the figure has sides.

<i>Example.</i> Interior angles A, B, C, D, E, F = n°		
4 right angles,		360
Sum	= $n^{\circ} +$	360 $^{\circ}$
Number of sides = 6 $\therefore$ 6 $\times$ 2 right angles =		1080 $^{\circ}$
By subtraction	n° =	720 $^{\circ}$

Having the Interior Angles, to Reduce them to Circumferentor Bearings, and thence to Quarter Compass Bearings.

214. Assume any line whose circumferentor bearing is given. *Always keep the land on the right as you proceed to determine the bearings.*

Rule 1. If the angle of the field is greater than 180 degrees, take 180 from it, and add the remainder to the bearing at the foregoing station. The sum, if less than 360 degrees, will be the circumferentor bearing at the present station—that is, the bearing of the next line (forward). But if the sum be more than 360 $^{\circ}$, take 360 from it, and the remainder will be the present bearing.

Rule 2. If the angle of the field be less than 180, take it from 180, and from the bearing at the foregoing station take the remainder, and you will have the bearing at the present station. But if the bearing at the foregoing station be less than the first remainder to this foregoing bearing, add 360, and from the sum subtract the first remainder, and this last remainder will be the present bearing.

To Reduce Circumferentor Bearings to Quarter Compass Bearings.

Rule 3. If the circumferentor bearings are less than 90, they are that number in the N. W. Quadrant.

Rule 4. If the circumferentor bearings are between 90 and 180, take them from 180. The remainder is the degrees in the S. W. Quadrant.

Rule 5. If the degrees are between 180 and 270, take 180 therefrom, and the remainder is the degrees in the S. E. Quadrant.

Rule 6. If the circumferentor bearing is between 270 and 360, take them from 360, and the remainder is the degrees in the N. E. Quadrant.

Rule 7. 360, or 0, is N., 180 is S., 90 is W., and 270 is E.

These rules are from Gibson's Surveying, one of the earliest and best works on practical surveying. Why so many editions of his Surveying have been published omitting these rules, plainly shows, that too many of our works on Surveying have been published by persons having but little knowledge of what the practical surveyor actually requires.

We will give the same example as that given by Mr. Gibson in the unabridged Dublin edition, page 269:

The following example shows the angles of the field, and method of reduction. The bearing of the first line is given = 262 degrees.

Stat'n.	Angle Field.		Chr. B.	Q. C. B.
1 A	159			
2 B	200	$200 - 180 = 20, 262 + 20$	$= 282 =$	N. E. 78
3 C	270	$270 - 180 = 90, 282 + 90 = 372, 372 - 360 = 12 =$		N. W. 12
4 D	80	$180 - 80 = 100, 12 + 360 = 372, 372 - 100 = 272 =$		N. E. 88
5 E	98	$180 - 98 = 82, 272 - 82$	$= 190 =$	S. E. 10
6 F	100	$180 - 100 = 80, 190 - 80$	$= 110 =$	S. W. 70
7 G	230	$230 - 180 = 50, 110 + 50$	$= 160 =$	S. W. 20
8 H	90	$180 - 90 = 90, 160 - 90$	$= 70 =$	N. W. 70
9 I	82	$180 - 82 = 98, (70 + 360 - 98) = 430 - 98 = 332 =$		N. E. 28
10 K	191	$191 - 180 = 11, 332 + 11$	$= 343 =$	N. E. 17
11 L	120	$180 - 120 = 60, 343 - 60$	$= 283 =$	N. E. 77
Sum,	1620			
Add,	360	$180 - 159 - 21, 283 - 21$	$= 262 =$	S. E. 82

$90 \times 11 \times 2 = 1980$, which proves that the angles of the field have been correctly taken. Also finding 262 to be the same as the bearing first taken by the needle, is another proof of the correctness of the work.

215. Having selected one of the sides as meridian, for example, a line that is the most easterly. This may be called a north and south line; the north, or 360, or zero, being the back station, and 180 the forward station. Let the angles, as you proceed round the land, *keeping it on the right*, be A, B, C, D, E, and let the line A B be assumed N and S. A = north and B = south. Then the circumferentor bearing of the line A B from station A, is $= 180^\circ$. If the surveyor begins on the east side of the land, and sets his telescope at zero on the forward station, and then clamps the body, he then turns it on the back station. The reading on the limb will be the interior angle. But if the telescope be first directed to the back station, and then to the forward station, the difference of the readings will be the exterior angle of the field, which taken from 360 will be the interior angle.

The circumferentor is numbered like the theodolite, from north to east, thence south-west, etc., to the place of beginning. But the bearings found by the circumferentor are not the same as those found by the ordnance survey method, where any line is assumed as meridian, as A B.

ORDNANCE METHOD.

216. The following method is that which has been used on the ordnance survey of Ireland:

Assume any line as meridian or base, so as to keep the land to be surveyed *on the left* as you proceed around the tract to be surveyed. Let the above be the required tract, whose angles are at A, B, C, D, E, F, G, H, I, K and L. In taking the interior angles for to determine the circumferentor bearings, the land is *kept on the right*; but by this method the land is *kept on the left*. To determine by this method all the interior angles, we proceed from A to L, L to K, K to I, I to H, H to G, G to F, F to E, E to D, D to C, C to B, and B to A.

Let B to A be the first line, and B the first station. Let the magnetic or true bearing of A to B = S. 82° E.

Let the theodolite at A read on B	= 0	Angle. 0
on L read	= 159	A = 159°
Theodolite at L read on A	= 159	
on forward K, read	= 279	L = 120
Theodolite at K, read on L back	= 279	
read forward on I	= 110	K = 191
Theodolite at I, read back on K	= 110	
read forward on H	= 192	I = 82
Theodolite at H, read back on I	= 192	
read forward on G	= 282	H = 90
Theodolite at G, read back on H	= 282	
read forward on F	= 152	G = 280
Theodolite at F, read back on G	= 152	
read forward on E	= 252	F = 100
Theodolite at E, read back on F	= 252	
read forward on D	= 850	E = 98
Theodolite at D, read back on E	= 350	
read forward on C	= 70	D = 80
Theodolite at C, read back on D	= 70	
read forward on B	= 840	C = 270
Theodolite at B, read back on C	= 840	
read forward on A	= 180	B = 200

When at B, 860 was on station A, and 180 on station B. Now when at A, 180 is on B,—a proof that the traverse has been correctly taken.

217. In traversing by the ordnance method where the survey is extensive, it is necessary to run a check-line, or lines running through the survey, beginning at one station and closing on some opposite one. This will serve in measuring detail, such as fields, houses, etc., and will divide the field into two or more polygons, and enable the surveyor to detect in which part of the survey any error has been committed, and whether in chaining or taking the angles. I consider it unsafe for a surveyor to equate his northings and southings, eastings and westings, where the difference would be one acre in a thousand. When the error is but small, *equate or balance* in those latitudes and departures which increase the least in one degree.

DeBurgh's method—known in America as the Pennsylvania method—is as follows:

As the sum of the sides of the polygon is to one of its sides, so is the difference between the northing and southing to the correction to be made in that line.

Half the difference to be applied to each side; as, for example,

Let sum of the sides = 24000 feet, and one of them = 600 feet, whose bearing is N. 40° E.

And that the northings	= 56,20	equated	56,80
And sum of the southings	= 26,40	equated	56,80
		diff. 20 and half dif.	= 10

As 24000 : 600 :: 0,10 : cor. = 0,0025, correction to be added, because the northings is less than the southings.

218. TABLE. To Change Degrees taken by the Circumferentor to those of the Quartered Compass, and the Contrary.

Degrees.		Degrees.		Degrees.		Degrees.		Degrees.		Degrees.	
Cir.	Q. C.	Cir.	Q. C.	Cir.	Q. C.	Cir.	Q. C.	Cir.	Q. C.	Cir.	Q. C.
360	North.	60	N. W.	60	120	S. W.	60	180	South	240	S. E.
1	N. W. 1	61	61	121	59	181	S. E. 1	241	61	300	N. E. 60
2	2	62	62	122	58	182	2	242	62	002	58
3	3	63	63	123	57	183	3	243	63	303	57
4	4	64	64	124	56	184	4	244	64	304	56
5	5	65	65	125	55	185	5	245	65	305	55
6	6	66	66	126	54	186	6	246	66	306	54
7	7	67	67	127	53	187	7	247	67	307	53
8	8	68	68	128	52	188	8	248	68	308	52
9	9	69	69	129	51	189	9	249	69	309	51
10	10	70	70	130	50	190	10	250	70	310	50
11	11	71	71	131	49	191	11	251	71	311	49
12	12	72	72	132	48	192	12	252	72	312	48
13	13	73	73	133	47	193	13	253	73	313	47
14	14	74	74	134	46	194	14	254	74	314	46
15	15	75	75	135	45	195	15	255	75	315	45
16	16	76	76	136	44	196	16	256	76	316	44
17	17	77	77	137	43	197	17	257	77	317	43
18	18	78	78	138	42	198	18	258	78	318	42
19	19	79	79	139	41	199	19	259	79	319	41
20	20	80	80	140	40	200	20	260	80	320	40
21	21	81	81	141	39	201	21	261	81	321	39
22	22	82	82	142	38	202	22	262	82	322	38
23	23	83	83	143	37	203	23	263	83	323	37
24	24	84	84	144	36	204	24	264	84	324	36
25	25	85	85	145	35	205	25	265	85	325	35
26	26	86	86	146	34	206	26	266	86	326	34
27	27	87	87	147	33	207	27	267	87	327	33
28	28	88	88	148	32	208	28	268	88	328	32
29	29	89	89	149	31	209	29	269	89	329	31
30	N. W. 30	90	West. 150	S. W. 30	210	S. E. 30	270	East. 330	N. E. 30	30	
31	31	91	S. W. 89	151	29	211	31	271	N. E. 89	331	29
32	32	92	88	152	28	212	32	272	88	332	28
33	33	93	87	153	27	213	33	273	87	333	27
34	34	94	86	154	26	214	34	274	86	334	26
35	35	95	85	155	25	215	35	275	85	335	25
36	36	96	84	156	24	216	36	276	84	336	24
37	37	97	83	157	23	217	37	277	83	337	23
38	38	98	82	158	22	218	38	278	82	338	22
39	39	99	81	159	21	219	39	279	81	339	21
40	40	100	80	160	20	220	40	280	80	340	20
41	41	101	79	161	19	221	41	281	79	341	19
42	42	102	78	162	18	222	42	282	78	342	18
43	43	103	77	163	17	223	43	283	77	343	17
44	44	104	76	164	16	224	44	284	76	344	16
45	45	105	75	165	15	225	45	285	75	345	15
46	46	106	74	166	14	226	46	286	74	346	14
47	47	107	73	167	13	227	47	287	73	347	13
48	48	108	72	168	12	228	48	288	72	348	12
49	49	109	71	169	11	229	49	289	71	349	11
50	50	110	70	170	10	230	50	290	70	350	10
51	51	111	69	171	9	231	51	291	69	351	9
52	52	112	68	172	8	232	52	292	68	352	8
53	53	113	67	173	7	233	53	293	67	353	7
54	54	114	66	174	6	234	54	294	66	354	6
55	55	115	65	175	5	235	55	295	65	355	5
56	56	116	64	176	4	236	56	296	64	356	4
57	57	117	63	177	3	237	57	297	63	357	3
58	58	118	62	178	2	238	58	298	62	358	2
59	59	119	61	179	1	239	59	299	61	359	1
60	N. W. 60	120	S. W. 60	180	South	240	S. E. 60	300	N. E. 60	360	North.

218a. *Traverse surveying is to be preferred to triangulation.* In triangulation, the various lines necessary will have to pass over many obstacles, such as trees, buildings, gardens, ponds, and other obstructions; whereas in a traverse survey, we can make choice of good lines, free from obstructions, and which can be accurately measured, and the angles correctly taken, without doing much damage to any property on the land.

In every Survey which is truly taken, the sum of the Northings or North Latitudes is equal to the sum of the Southings or South Latitudes, and the sum of the Eastings or East Departure is equal to the sum of the Westings or West Departure.

219. Let A, B, C, D, E, F, G, H, I, K, be the respective stations of the survey, (see fig. 17b), and NS the meridian, N = north and S = south. Consequently, all lines passing through the stations parallel to this meridian will be meridians; and all lines at right angles to these meridians, and passing through the stations, will be east and west lines, or departures.

Let fig. 17b represent a survey, where the first meridian is assumed on the west side of the polygon.

Here we have the northings = $AB + Bc + Cd + d o + RA = RQ$, and the southings = $nF + FG + mI + iL = PL$.

But $RQ = PL$ ∴ the sum of the northings = sum of the southings, and the eastings $Cc + oE + En + Gm$.

But $Cc = Dd + Dh$. Therefore the

eastings = $Dd + Dh + Qn + Gm = QP + Dh$,

and westings = $Dh + LR$; but $LR = QP$, and $Dh = Dh$. Consequently the sum of the eastings is equal to the sum of the westings.

Example 2. Let fig. 17c, being that given by Gibson at page 228, and on plate IX, fig. 1, represent the polygon $abcdefg$. Let a be the first station, b the second, c the third, etc. Let NS be a meridian line; then will all lines parallel thereto which pass through the several stations be also meridians, as ao, bs, cd , etc., and the lines bo, cs, dc , etc., perpendicular to those, will be east or west lines or departures.

The northings are $ei + go + hq = ao + bs + cd + fr$, the southings.

Let the figure be completed,—then it is plain that $go + hq + rk = ao + bs + cd$, and $ei - rk = fr$. If we add $ei - rk$ to the first, and fr to the latter, we have $go + hq + rk + ei - rk = ao + bs + cd + fr$.

i. e., $go + hq + ei = ao + bs + cd + fr$. Hence the sum of the northings = sum of the southings.

The eastings $cs + qa = ob + de + if + rg + oh$, the westings.

For $aq + yo = aq + az = de + if + rg + oh$, and $bo = cs - yo$; therefore $aq + yo + cs - yo = de + if + rg + oh + bo$.

i. e., $aq + cs = bo + de + if + rg + oh$; that is, the sum of the eastings = the sum of the westings.

220. *Method of Finding the Northings and Southings, and Eastings and Westings. (Fig. 17b.)*

	Bearing.	Distance.	Northing.	Southing.	Eastng.	Westng.
A B	North	29,18	29,1800			
B C	N. 40° E.	8,00	6,1283		5,1428	
C D	N. 10° W.	9,00	8,8633			1,5629
D E	N. 50° E.	12,00	7,7185	8,6608	9,1925	
E F	S. 30° E.	10,00		17,0000	5,0000	
F G	South	17,00				
G H	East	11,00			11,0000	
H I	S. 20° E.	20,00		18,7938	6,8404	
I K	S. 60° W.	21,00		10,5000		18,1866
K A	N. 80° W.	17,69	8,0726			17,4257
			54,9577	54,9541	87,1752	87,1552

If the above balance or trial sheet showed a difference in closing, we proceed to a resurvey, if the error would cause a difference of area equal to one acre in a thousand. But if the error is less than that, we equate the lines, as shown in sec. 217.

By Assuming any Station as the Point of Beginning, and Keeping the Polygon on the Right, to Find the most Easterly or Westerly Station.

221. Let us take the example in section 220, and assume the station F as the place of beginning (see fig. 17b).

		Eastng.	Total Eastng.	Westng.	Total Westng.	
F G	South	11,00				
G H			11,00			
H I		6,84	17,84			I = most easterly station.
I K				18,19	18,19	
K A				17,43	35,62	
A B	North					A and B the most westerly stations.
B C		5,14				
C D				1,56		
D E		9,19				
E F		8,66				

Here we see that the point I has a departure east = 17,84
 after which follow west departure to A = 35,62
 Therefore the point A is west of F = 17,78

Then follows E. dep. 5,14, and W. dep. = 1,56, which leaves points A and B west of C, D, E and F. Consequently point I is the most easterly, and points A and B, or line A B, the most westerly.

In calculating by the traverse method, the first meridian ought to pass through the most easterly or westerly station. This will leave no chance of error, and will be less difficult than in allowing it to pass through the polygon or survey. However, each method will be given; but we ought to adopt the simplest method, although it may involve a few more figures, in calculating the content. For the first method, see next page.

INACCESSIBLE DISTANCES.

Let AB (Fig. 17a) be a Chain Line, CD , a part of which passes through a house, to find CD .

221a. Find where the line meets the house at C ; cause a pole to be held perpendicularly at D , on the line AB ; make $De = Cf$; then Euclid I, 34, $fe = CD$.

221b. When the pole cannot be seen over the house, measure any line, AR , and mark the sides of the building; if produced, meet the line AK , in the points i and K . Then by E. VI, 4, $Ai : Ci :: AK : KD$. KD is now determined. Let Ci be produced until $Cm = DK$. Measure mK , which will be the length required. Distance CD .

221c. Or, at any points, A and G on the line AB , erect the perpendiculars AO and GH equal to one another, and produce the line OH far enough to allow perpendiculars to be erected at the points L and M , making $LB = MN = AO = HG$. $\therefore$ the line BN will be in the continuation of the line AB ; and by measuring DN and AC , and taking their sum from OW , the difference will be equal to CD .

222. When the obstruction is a river. In fig. 18, take the interior angles at C and D ; measure CD ; then $\sin \angle E : CD :: \sin \angle D : CE$. When the line is clear of obstructions to the view, make the $\angle D$ equal to half the complement of the $\angle C$. Then the line $CE = CD$.

As, for example, when the $\angle$ at C is 40° , the half of the complement is $70^\circ = \text{angle at } D = \angle CED$; consequently (E. I, 5), $CE = CD$. In this case the flagman is supposed to move slowly along the line AB , until the surveyor gives him the signal to halt in direction of the line DE , the surveyor having the telescope making $\angle CDE = 70^\circ$.

223. Or, take (fig. 19) CD perpendicular to AB . If possible, let CD be greater than CE . Take the $\angle$ at D ; then, by sec. 167, $CD \times \tan. \angle D = CE$. Or by the chain only (fig. 20), erect CD and KL perpendicularly to AB ; make $CF = FD$ and $KL = CD$; produce EF to meet DL in G ; then $GD = CE$, the required distance. See Euclid I, prop. 15 and 26.

224. Let AC (fig. 20a) be the required distance. Measure AB any convenient distance, and produce AB , making $BE = AB$; make EG parallel to AC ; produce CB to intersect the line EG in F . Then it is evident, by Euclid VI, 4, that $EF = AC$ and $BF = BC$.

225. Let fig. 21 represent the obstruction (being a river). Measure any line $AB = c$, and take the angles HAC , CAB , and ABC , C being a station on the opposite shore. Again, at C take the $\angle ACG$ and ACB , E being the object. Now, by having the length to be measured from C towards $G = CE$, E will be a point on the line AF .

By sec. 194 we find AC , and having the angles EAC and ACE , we find (E. I, 32) the $\angle AEC = \angle$ at E . Then $\sin \angle E : AC :: \sin \angle ACE : AE$, and $\sin \angle E : AC :: \sin \angle CAE : CE$; but in the $\triangle CDE$ we have the $\angle$ at D , a right angle, and the $\angle E$ given, $\therefore$ the $\angle ECD$ may be found. Now, CD being given = to the cosine of the $\angle ECD = \sin$ of $\angle E = CD$, we have found AE , CE , and the perpendicular CD ; consequently, the line ADE may be found, and continued towards H , and the distances aH , Hb , and bD , may be found. $DE = \cos. E \cdot CE$.

f

226. Let the line $A F$ (fig. 22) be obstructed from a to b . Assume any point D , visible from A and C ; measure the lines $A D$ and $D C$; take the angles $A C D$, $C A D$, $A D C$, and $C D Y$, Y being a station beyond the required line, if possible. In the triangle $B C D$ we have one side $C D$, and two angles, $C B D$ and $C D B$, to find the sides $C B$ and $D B$, which may be found by sec. 194.

227. Or, measure any line $A D$ (fig. 22); take the angle $C A D$, and make the angle $A D G = 180^\circ - \angle C A D$; i.e., make the line $D H$ parallel to $A C$; take two points in the line $A H$, such as E and G , so that the lines $E B$ and $G F$ shall be parallel and equal to $A D$, and such that the line $E B$ will not cut the obstruction $a b$, and that the lines $G F$ parallel to $E B$ will be far enough asunder from it to allow the line $B F$ to be accurately produced.

As a check on the line thus produced, take the angle $F B E$, which should be equal to the angle $B E D = \angle C A D$.

228. Let the obstruction on the line $A W$ (fig. 23) be from a to b , and the line running on a pier or any strip of land. At the point C measure the line $C D = 800$, or any convenient distance, as long as possible; make the $\angle A C D =$ any $\angle$, as 140° , and the interior $\angle C D E =$ any angle, as 180° ; measure $D E = 400$; make the $\angle D E Y = 70^\circ$, Y being some object in view beyond the line, if possible.

To find the line $E B$, and the perpendicular $E H$. In the figure $C B E D$, we have the interior angles $B C D = 40^\circ$

$$\begin{array}{r} C D E = 180 \\ D E Y = D E B = 70 \\ \hline 240^\circ \end{array}$$

$$\begin{array}{r} \text{Let the interior angle } C B E = x^\circ \\ \text{Sum, } 240^\circ + x^\circ \end{array}$$

$$\begin{array}{r} \text{To which add four right angles, } 360 \\ \hline 600^\circ + x^\circ \end{array}$$

$$\begin{array}{r} \text{Should be, by E. I, 32, } 720 \\ \hline \end{array}$$

That is, $600^\circ + x^\circ = 720^\circ \therefore x^\circ = 120^\circ = \angle A B E$; therefore, the angle $H B E = 60^\circ$.

By E. I, 16, the $A B E = \angle H B E + \angle H E B$, but the angle $H B E = 60^\circ \therefore \angle H E B = 30^\circ$; consequently, the interior $\angle D E H = 100^\circ = 70^\circ + 30^\circ$.

Now, we have the interior angles $H C D = 40^\circ$, bearing $N. 40^\circ E$.

$$\begin{array}{r} C D E = 180 \\ D E B = 70 \\ A B E = 120 \\ D E H = 100 \\ C H E = 90 \end{array}$$

The bearings of these lines are found by sec. 218. We assume the meridian $A H$, making A the south, or 180° , and H the north, or 0° , and keeping the land invariably on the right hand, as we proceed, to find the bearings.

$$\begin{array}{r} 180 \quad 360 \\ 120 \quad 60 \\ \hline 60 \quad 300 = N. 60^\circ E. = \text{bearing of } B E, \text{ per quarter compass table.} \\ \text{(See this table, sec. 218.)} \end{array}$$

180	360
70	110
110	190 = S. 10° E. = bearing of E D.
180	190
180	50
50	140 = S. 40° W. = bearing of D C.
180	140
40	140
140	000 = north = bearing of C B or C H.

Now we have, by reversing these bearings, and finding the northings and southings by traverse table—

Sine.	Chains.	Bearing.	Northing.	Southing.	Eastng.	Westing.
C D	8,00	N. 40° E.	6,1288 = C d		5,1423	
D E	4,00	N. 10° W.	3,9392 = d H			0,6946
E B		S. 60° W.		x = B H		y = B H
B C		South.		10,0675 — x		
			10,0675	10,0675 — x	5,1423	0,6946 + y

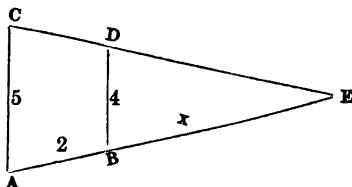
But as the eastings, per sec. 218a, is equal to the westings, $y = 5,1423 - 0,6946 = 4,4477 = E H$. Also, from the above, the $\angle H E B = 30$, and the $\angle B H E = 90^\circ \therefore$ we have, in the triangle B H E, given the angles, and side E H, to find E B and B H. For the angle B E H, its latitude or cosine = 0,866, and its sine or departure = 0,500; therefore E H = 4,4477, divided by 0,866, gives 5,136 = E B, and $5,136 \times 0,500 = 2,5680 = B H$; and by taking B H from C H, i.e., $10,0675 - 2,5680 = C B = 7,4995$; and by calling the distances links, we have C B = 749,95 links, and E B = 513,6.

Note. If, instead of having to traverse but three lines, we had to traverse any number of lines, the line E H, perpendicular to the base A W, will always be the difference of departure, or of the eastings and westings, and B H = difference of latitudes, or of the northings and southings.

229. Chain A C (fig. 25), and at the distance A B, chain B D parallel to A C, meeting the line C E in D; then, by E. VI, 4, and V, prop. D, convertendo, $A E : B E :: A C - B D : B D \therefore$ (E. VI, 16)

$$B E = \frac{A B \times B D}{A C - B D}, \text{ which is a convenient method.}$$

Example. Let B E be required. Let A C = 5, B D = 4, and A B = 2, to find B E. By the last formula, $B E = \frac{2 \times 4}{5 - 4} = 8$ chains.



230. In fig. 26, the line O L is supposed to pass over islands surrounded by rapids, indicated by an arrow. The lines O A, O B, and E F, are measured. From the point B erect the perpendicular B G, and take a point H, from which flag-poles can be seen at O, A, B, C, D, E, and F. Take the angles O H A, A H B, B H C, D H B, E H B, F H B.

The tangents of these angles multiplied by B H, will give the lines B A, O B B C B D, B E, B F, and B L.

H B is made perpendicular to O L, and the $\angle O H B$ is given $\therefore$ the angle B O H is given, whose tangent, multiplied by O B, will give the distance B H; consequently, B H multiplied by the tangents of the angles B H C, B H D, B H E, etc., will give the sides B C, B D, B E, etc.

281. If one of the stations, as L, be invisible at H, from L run any straight line, intersecting the line B G in K; take the angle B K L and measure H K; then we have the side B K, and the angle B K L, to find B L in the right angled triangle B K L.

$$\therefore B L = B K \times \tan. \angle B K L.$$

282. But if the line B G cannot be made perpendicular, make the $\angle O B G$ any angle; then having the $\angle O B G$, we have the $\angle L B K$, and having observed the $\angle B K L$, and measured the base B K, we find the distance B L by sec. 181.

In this case we have assumed that B K could be measured; but if it cannot be measured, take the $\angle B O H$ and $O H B$; measure O B; then we have all the angles, and the side O B given in the $\triangle O H B$ to find B H, which can be found by sec. 181. Having B H, measure the remaining part H K.

283. Let the inaccessible distance A B (fig. 27) be on the opposite side of a river. Measure the base C D, and take angles to A and B from the stations C and D, also to D from C, and to C from D. Let $s = C D$, $a = \angle A C B$, $b = \angle B C D$, $c = \angle A D C$, $d = \angle A D B$, $e = \angle C A D$, and $f = \angle C B D$.

$$\text{Sine } e : s :: \text{sine } c : A C.$$

$$\text{Sine } f : s :: \text{sine } b : B D.$$

$$\text{Sine } f : s :: \text{sine } (c + d) : B C.$$

Now having A C and B C, and the included angle, we find (sec. 140) the required line A B.

284. If it be impracticable to measure a line from B (fig. 26), making any angle with the base O L, in order to find the inaccessible distance B C, assume any point H, from which the stations A, B and C are visible.

$$\text{Let } A B = g, B C = x.$$

$$\angle C A H = a = \angle B A H. \quad \angle A H B = c.$$

$$\angle A C H = b. \quad \angle C H B = d.$$

$$\text{Therefore, } \angle A B H = 180 - a - c.$$

$$\text{By sec. 181, sine } c : g :: \text{sine } a : H B = \frac{g \cdot \text{sine } a}{\text{sine } c}$$

$$\text{sine } b : H B :: \text{sine } d : x = \frac{H B \cdot \text{sine } d}{\text{sine } b}$$

Substituting the value of H B in the last equation, we have

$$x = \frac{g \cdot \text{sine } a \cdot \text{sine } d}{\text{sine } c \cdot \text{sine } b} = B C.$$

This formula can be used, by either using the natural or logarithmic sines.

Example. Let $A B = 400$ links $= g$,

$$\text{the angle } A H B = c = 60^\circ$$

$$B A H = a = 80^\circ$$

$$\therefore \text{E. I, 82, } A B H = 40^\circ$$

$$C H B = d = 10^\circ \therefore \angle A H C = 70^\circ.$$

$$180 - (B A H + C H B + B H A) = 180 - (80 + 10 + 60) = 30^\circ = A C H = b.$$

$$\text{Log. } g = \text{log. } 400 = 2,6020600$$

$$\text{Log. sine } a = \text{log. sine } 80^\circ = 9,9983515$$

$$\text{Log. sine } d = \text{log. sine } 10^\circ = 9,2396702$$

$$\text{Sum, } 21,8350817$$

$$\text{Log. sine } c = \text{log. sine } 60^\circ = 9,9375306$$

$$\text{Log. sine } b = \text{log. sine } 30^\circ = 9,6989700$$

$$19,6365000$$

$$2,1985811 = 157,98 = B = x.$$

And, as in sec. 163, we have $A B = 400$, and $B C = x = 157,98$, and the included angle $A H C$, the lines $A H$ and $B H$ may be found.

235. Let the land between $C D$ and the river be wood land (see fig. 28). Assume any two random lines, traced from the stations A and B through the wood; let these lines meet at the point C ; trace the lines $C E$ and $E D$ in any convenient direction, so that the point A be visible from E , and the point B visible from the point D ; take the angles $A E C$, $A C E$, $A C B$, $B C D$, and $C D B$, $\therefore$ by E. I, 32, the angles $E A C$ and $C B D$ can be found; and by sec. 131, the sides $A C$ and $C B$ are found; and having the contained angle $A C B$, we find, by sec. 140, the side $A B$.

Note. This case is applicable to hilly countries.

236. The line $A B$ may be found as follows: In direction of the point B (fig. 29) run the random line $P B$, and from A run the lines $A D$ and $A C$ to meet the line $P B$; measure the distance $D C$, and take the angles $A D C = a$, $A C B = c$, $A C D = b$; let the $\angle C A D = d$, and $\angle C A B = e$, and the $\angle A B D = f$. Now, as the angles d , e and f have not been taken, we find them as follows: The angles a and c are given $\therefore$ by E. I, 16, $\angle c = \angle a + \angle d \therefore \angle d = \angle c - \angle a$, and by E. I, 16, we have $\angle b = \angle e + \angle f$, and 180° — the sum of the angles a , d , $e = \angle f$. Now, by sec. 131, $\text{sine } \angle d : D C = s :: \text{sine } \angle a : A C$.

$$\text{i. e., } \text{sine } \angle d : s :: \text{sine } \angle a : \frac{s \cdot \text{sine } \angle a}{\text{sine } \angle d} = A C.$$

$$\text{Also } \text{sine } \angle f : \frac{s \cdot \text{sine } \angle a}{\text{sine } \angle d} :: \text{sine } \angle c : \frac{s \cdot \text{sine } \angle a \cdot \text{sine } \angle c}{\text{sine } \angle d \cdot \text{sine } \angle f} = A B.$$

237. *By the Chain only.* Let it be required to measure the distance $A B$, on the line $O R$ (fig. 30). Measure $A G = G E$ any convenient distances, 50 or 100 links; describe the equilateral triangles $G E D$ and $A G C$ equal to one another; produce $G D$ and $B C$ to meet one another at F ; measure $D F$. Now, because $G F$ and $A C$ are parallel to one another, the $\triangle F D C$ is similar or equiangular to the $\triangle B A C$ (E. VI, 4). $F D : D C :: A C : A B$, but $A C = C D$, because $D C = A C$.

$\therefore F D : D C :: D C : A B$, and by E. VI, 16.

$$F D \times A B = D C^2.$$

$$\therefore A B = \frac{D C^2}{F D} = \frac{A G^2}{F D} \text{ which is a convenient formula.}$$

Example. Let $AC = 100$, and $DF = 120$;
 then $AB = \frac{1000}{120} = 83\frac{1}{3}$ links.

This is a practical method, and is the same as that given by Baker in his *Surveying*, London, 1850.

238. The following problem, given by Galbraith in his *Mathematical and Astronomical Tables*, pp. 47 and 48, will be often found of great use in trigonometrical surveying (see fig. 81):

From a convenient station P there could be seen three objects, A , B and C , whose distances from each other were $AB = 8$ miles, $AC = 6$ miles, $BC = 4$ miles. I took the horizontal angles $APC = 33^\circ 45'$, $BPC = 22^\circ 30'$. It is hence required to determine the respective distances of my station P from each object.

Because equal angles stand upon equal or on the same circumferences, the $\angle BPC = \angle DAB$, and $\angle APC = \angle ABD$. In this case the point D is supposed to fall in the original $\triangle ABC$. From this the construction is manifest.

Make the $\angle BAD = \angle ABD$ as above; join C and D , and produce it indefinitely, say to Q ; about the $\triangle ADB$ describe a circle, cutting the line CQ in P ; join A and P , and B and P ; then, by E. III, 21, the $\angle CPB = \angle DAB$, and $\angle APD = \angle ABD$. In this case, the $\angle CPB$ is assumed less than the $\angle CAB$, and the $\angle APB$ less than ABC . Now having the three sides of the $\triangle ABC$ by sec. 142, we find the angles A , C and B of the $\triangle ABC$; consequently the $\angle CAD$ is found; also the $\angle CBD$, because, by observation, the $\angle BPC = \angle BAD$, and $\angle ATC = \angle ABC$. In the $\triangle ADB$ are given the side AB and the angles DAB and DBA , to find the sides AD and BD and $\angle ADB$, all of which can be found by sec. 131. Now having the sides AD and AC , and the contained angle BAD , we find (sec. 140) the $\angle ACB$ and the side DC ; and having the angles ACP and ATC given, we find the $\angle CAP$; but above we have found the $\angle CAB$. $\therefore$ the $\angle CAP = \angle CAB = \angle BAP$. In like manner we find the $\angle ABP$; and by sec. 130, and E. I, 82, we find the distances AP and BP . In like manner we proceed to find CP .

COMPUTATION.

$AC = 6$ miles $= b$, and $APC = 33^\circ 45'$.

$CB = 4 = a$, and $CPB = 22^\circ 30'$.

$BA = 8$ miles $= c$.

By sec. 125, $\sin \frac{1}{2} \angle A = \left(\frac{(s-b) \cdot (s-c)}{bc} \right)^{\frac{1}{2}}$

Here $s = 9$ miles.

$b = 6$.

$s - b = 3$.

$s - c = 9 - 8 = 1$.

$(s-b) \cdot (s-c) = 3 \times 1 = 3$.

And $bc = 6 \times 8 = 48$; consequently the value of half the angle $A =$

$\left(\frac{3}{48} \right)^{\frac{1}{2}} = \sqrt{\frac{1}{16}} = \frac{1}{4}$, but $\frac{1}{4} = .25 = \sin 14^\circ 28' 39''$; therefore

$\angle BAC = 28^\circ 57' 18''$.

By sec. 126, we find $\angle A B C = 46^{\circ} 34' 03''$
 and by sec. 127, $\angle A C B = 104^{\circ} 28' 39''$
 Now we have the $\angle C A B = 28^{\circ} 57' 18''$
 and by observation, the $\angle D A B = 22^{\circ} 30' 00'' = \angle C P B$.
 $\therefore$ the $\angle C A D = 6^{\circ} 27' 18''$
 By observation, we have the $\angle D A B = 22^{\circ} 30' 00''$
 The $\angle D B A = 83^{\circ} 45' 00''$
 Their sum $= 56^{\circ} 15' 00''$
 $\therefore 180^{\circ} - 56^{\circ} 15' = \angle A D B = 123^{\circ} 45' 00''$
 And as the $\angle C A D = 6^{\circ} 27' 18''$, this taken from 180, leaves the $\angle$
 $A D C + \angle A C D = 273^{\circ} 32' 42''$
 and half the sum of these $= 86^{\circ} 46' 21''$
 By sec. 131. As sine $A D B 123^{\circ} 45'$ (arith. complement) $= 0,0801536$
 is to the side $A B 8$ miles, $\log. 0,9030900$
 so is the sine of the $\angle A B D = 38^{\circ} 45'$ $\log. \text{ sine } 9,7447390$
 to $A D = 5,84543$. $\text{Sum } 0,7279826$

$A C = 6$, by hypothesis.

As the sum $= 11,34543$ $\log. 1,0548110$
 is to the difference $0,65457$, $1,8159561$
 so is $\tan. \frac{1}{2} (\angle A D C + \angle A C D) =$
 $86^{\circ} 46' 21''$ $\tan. 11,2487967$

to the $\tan.$ of half the difference of the
 angles $A D C$ and $A C D$. $16,0099818 = 45^{\circ} 39' 18''$

$\therefore$ by sec. 140, the $\angle A C P = 41^{\circ} 07' 03''$
 and the $\angle A D C = 132^{\circ} 25' 39''$
 As sine $\angle A P C 33^{\circ} 45'$ $\text{arith. comp. } 0,2552610$
 is to $A C = 6$ miles, $\log. 0,9781513$
 so is $\angle A C P = 41^{\circ} 7'$ $\text{sine } 9,8179654$

to the distance $A P 7,10195$. $\log. 0,8518777$
 Now we have the $\angle A C B = 41^{\circ} 07' 03''$
 The $\angle A P C = 33^{\circ} 45' 00''$
 Their sum $= 74^{\circ} 52' 03''$
 $180^{\circ} - 74^{\circ} 52' 3'' = \angle P A C = 105^{\circ} 07' 57''$
 By sec. 131, sine $\angle A C P = 41^{\circ} 7' 3''$ $\text{arith. comp. } 0,1820846$
 is to $P A = 7,10195$, $\log. 0,8518777$
 so is sine $\angle P A C = 105^{\circ} 7' 56''$ $\text{sine } 9,9846734$
 to the side $P C = 10,42523$ $\log. 1,0180857$

We have found the $\angle A B C = 46^{\circ} 34' 03''$
 $\angle B A C = 28^{\circ} 57' 18''$
 Their sum $= 75^{\circ} 31' 21''$, which taken from 180, gives
 the $\angle A C B = 104^{\circ} 28' 39''$.

But the $\angle A C B$ has been found $= 41^{\circ} 07' 03''$
 $\therefore$ the $\angle B C P = 63^{\circ} 21' 86''$
 and by hypothesis $\angle C P B = 22^{\circ} 30' 00''$
 the sum of the two last angles $= 94^{\circ} 09' 24''$
 $\therefore$ the sine of $\angle C P B = (22^{\circ} 30') \text{ arith. comp. } = 0,4171603$
 is to $B C, 4$ miles, $\log. = 0,6020600$
 so is sine $\angle B C P (63^{\circ} 21' 36'')$ $\text{sine } 9,9512605$

to $P B, 9,342879$ miles. $\log. 0,9704808$

Galbraith finds $9,342850$ miles by a different method of calculation.

239. *Second Case.* Let us assume the three stations, A, B, W, to be on the same straight, and the angles A P W and W P B to be given (see fig. 81), as in the last example. We find the sides A D and D B. And having the sides A D and A W, and the contained angle, we find the $\angle A D P = \angle A D W$, and the $\angle A P D$ is given by hypothesis $\therefore$ by E. I, 32, we find the $\angle D A P$, and all the angles, and the side A D being given, in the $\triangle A D P$ we can find, by sec. 181, the sides A P and P W. In like manner we find the side P B.

240. *Third Case.* Let us assume the station P to be within the $\triangle A B C$, fig. 82. The $\angle A B D$ is made equal to the supplement of the $\angle A P C$, and the $\angle B A D =$ the supplement of the $\angle B P C$ $\therefore$ as above, we find the sides A D and B D, and having the sides A B, B C, and A C, we find the angles B A C and A B C; consequently, we have the $\angle D A C$. And by sec. 140, we find the angles A D C and A C D, and the $\angle A P C$ being given by hypothesis, $\therefore$ the $\angle C A P$ is found; and by sec. 180, we find the sides P A and P C. In like manner we find the side P B.

Note. When the sum of the two angles at P is 180° , the point P is on the same straight line connecting the stations A, B and C. And when the sum is less than 180° , the point P is without the $\triangle A B C$. When the sum is greater than 180° , the point P is within the $\triangle A B C$.

241. In fig. 83, the sum of the angle B P C is supposed $=$ to the sum of the angles C A B + C B A, making the $\angle C A B = C P B$, and the $\angle C B A = A P C$; consequently, the point P is in the circumference of the circumscribing circle about $\triangle A B C$ $\therefore$ the point P can be assumed at any point of the circumference of the segment A P B, and consequently, the problem is indeterminate.

242. The following equation, given by Lacroix in his Trigonometry, and generally quoted by subsequent writers on trigonometry, enables us to find the angles P A C and P B C, and, consequently, the sides A P, C P, and B P. Let $P = \angle A P C$.

Let $a = A C$. $P' = \angle B P C$.

$$b = B C. \quad R = 360^\circ - P - P' - c.$$

$$x = \angle P A C.$$

$$y = \angle P B C.$$

$$c = \angle A C B.$$

$$x = \cot. R \left(\frac{a \cdot \sin P'}{b \cdot \sin P \cdot \cos. R} + 1 \right)$$

$$243. \quad x = \frac{a}{b} (\sin P' \cdot \operatorname{cosec}. P \cdot \sec. R \cdot \cot. R + \cot. R)$$

In the problem now discussed, we have

$$a = 6, \text{ and } P = 38^\circ 45' 00''$$

$$b = 4, \text{ and } P' = 22^\circ 30' 00''$$

$$\text{by sec. 288, } 104^\circ 28' 39'' = \angle A C B.$$

$$\text{Sum, } \frac{160^\circ 43' 39''}{360^\circ}$$

$$R = 199^\circ 16' 21''$$

$$\text{By sec. 242, } \frac{a}{b} = \frac{6}{4} = \frac{3}{2}$$

From the equation $\cot. x = \cot. R \left(\frac{a \cdot \sin P'}{b \cdot \sin P \cdot \cos. R} + 1 \right)$ (see sec. 242), we have—

3 log.	= 0,4771212
2 ar. comp.	= 9,6989700
$P' = 22^\circ 30'$ sine	= 9,5828397
$P = 33^\circ 45'$ ar. comp. sine	= 0,2552610
$R = 199^\circ 16' 21''$ neg. ar. comp. cos.	= 0,0250452
— 1,09458 log.	= 0,0392371
+ 1,	
0,09458 log.	= 2,9757993
$\cot. R = + 199^\circ 16' 21''$	= 10,4563594
$\cot. x, (- 105^\circ 8' 10'')$	= 9,4321587
By sec. 181, as $\sin 33^\circ 45'$ ar. comp.	= 0,2552610
is to $\sin < P A C, (105^\circ 8' 10'')$ log. sine	= 9,9846660
so is 6	log. = 0,7781513
to $P C = 10,4251$	log. = 1,0180783

By sec. 241, $R - x = y = 199^\circ 16' 21'' - 105^\circ 8' 10'' = 94^\circ 8' 11''$.

By sec. 181, we can find the lines $A P$ and $P C$.

Note. — $0,09458 \times$ by $+ 199^\circ 16' 21'$, gives a negative product; $\therefore$ the cot. is negative, and the arc is to be taken from 180 , by sec. 108a.

REDUCTION TO THE CENTRE.

244. It frequently happens in extensive surveys that we take angles to spires of churches, corners of permanent buildings, etc. From such points, angles cannot be taken to those stations from which angles were observed. Let C (fig. 34) be the spire of a church. Take any station D , as near as possible to observed station C , from which take the $\angle C D B = D$. Let $\log. \sin 1'' = 4,6855749$; let $\angle C D A = a$, $\angle A D B = b$, and the distance $C D = g$, and $\angle A C B = x$;
then $x = b + \frac{g \sin (b + a)}{B C \cdot \sin 1''} - \frac{g \cdot \sin a}{A C \cdot \sin 1''}$

Great care is required in taking out the sine of the sine of the angles $(a + b)$, and sine of a . The first term, $\frac{g \cdot \sin (b + a)}{B C \cdot \sin 1''}$, will be positive when $(a + b)$ is less than 180° , and the sine of a will be negative.

245. Let A be a station in a ravine, from which it is required to determine the horizontal; distance $A H$ the height of the points D and C above the horizontal line $A H$ (fig. 35).

Trace a line up the hill in the plane of $A D H$, making $A B = g$ feet = 600; take the angles $\angle C A H = 3^\circ 10'$, $\angle D A H = 5^\circ 20'$.

Therefore $\angle C A D = 2^\circ 10'$
 $\angle G A B = \angle E B A = 2^\circ 7'$
and $\angle C B E = 1^\circ 7'$
 $\angle A H C = 90^\circ 0'$
 $\angle A C H = 86^\circ 50'$
 $\angle A D C = 84^\circ 40'$

In the triangle $A B C$ are given $A B = 600$.

The $\angle A B C = \angle E B A + \angle C B E = 3^\circ 14'$

The $\angle B A C = 180^\circ - \angle C A H - \angle B A G = 174^\circ 43'$

Consequently, $\angle A C B = 2^\circ 3'$

By sec. 131, the sides $A C$ and $B C$ may be found.

And $A C \cdot \cos. C A H = A H$.

And $A C \cdot \sin. C A H = H C$.

And $H A \cdot \tan. C A H = H D$. And by taking the $\angle C B D$, and multiplying its tangent by the line $B C$, we find the line $D C$, which added to $H C$, will give the line $H D$.

Otherwise,

We have the angles $D A C$, $C A H$, and angle at H a right angle.

$180 - 90 - \angle C A H = \angle A C H = 86^\circ 50' = \angle A D C + \angle C A D$.
But $\angle C A D$ being $2^\circ 10'$, $\therefore \angle A D C = 84^\circ 40'$, and $\angle C A D = 2^\circ 10'$, and the side $A C$ may be found; and by sec. 131, $C D$ can be found.

As $\sin 2^\circ 3' (< B C A)$	arith. comp. = 1,4464614
is to $A B$ (600),	log. = 2,7781513
so is $\sin 3^\circ 14' (< A B C)$	log. sine = 8,7512973
to $A C = 946,04$,	log. = 2,9759100
$\sin 3^\circ 10' (< C A H)$	= 8,7422686
$C H$ 52,26	log. = 1,7181686
Also log. $A C$	= 2,9759100
$\cosine (< C A H = 3^\circ 10')$	= 9,9998364
$A H = 944,597$	log. = 2,9952464
$\tan (< H A D = 5^\circ 20')$	= 8,9701350
$H D = 88,182$	log. = 1,9453814
$C H = 52,26$.	
$\therefore C D = 35,922$.	

Or, $C D$ may be found as follows:

As $\sin (A D C = 84^\circ 40')$	arith. comp. 0,0018842
is to the log. $A C$ from above,	2,9759100
so is $\sin (< D A C = 2^\circ 10')$	sine 8,5775660
to $C D = 35,922$	log. 1,5553602

INACCESSIBLE HEIGHTS.

246. When the line $A B$ is in the same horizontal plane (fig. 37), required the height $B C$.

$$A B \cdot \tan. < C A B = B C.$$

247. Let the point B be inaccessible (see fig. 37a). Measure $A D = m$ in the direction of B ; take the $\angle C A B = f$, and $C D B = g$; then, by E. I, 16, $A C D = g - f = h$; and, by E. I, 32, $\angle B C D = 90^\circ - g = k$.

$$\text{By sec. 131, } C D = \frac{m \cdot \sin. f}{\sin. h}$$

$$B C = \frac{m \cdot \sin. f \cdot \sin. g}{\sin. h}$$

$$D B = \frac{m \cdot \sin. f \cdot \cos. g}{\sin. h}$$

248. Let the inaccessible object CE be on the top of a hill, whose height above the horizontal plane is required (fig. 38).

$$\begin{aligned} \text{As in sec. 246, let } \angle CAB &= f &= 44^\circ 00' \\ &\angle CDB &= g &= 67^\circ 50' \\ \text{and E. I. 16, } \angle ACD &= g - f = h &= 23^\circ 50' \\ &\angle EDB &= k &= 51^\circ 00' \\ &\angle BCD &= p &= 22^\circ 10' \end{aligned}$$

And the horizontal distance $AD = m = 184$ yards.

$$\text{By sec. 246, } CD = \frac{m \cdot \sin f}{\sin h}$$

$$BC = \frac{m \cdot \sin f \cdot \sin g}{\sin h}$$

$$BD = \frac{m \cdot \sin f \cdot \cos g}{\sin h} = BC \cdot \tan. \angle BCD.$$

And by substituting the value of BC , we have—

$$BD = \frac{m \cdot \sin f \cdot \sin g \cdot \tan. p}{\sin h}$$

$$BE = \frac{m \cdot \sin f \cdot \cos g \cdot \tan. k}{\sin h}$$

$$* \text{ or, } BE = \frac{m \cdot \sin f \cdot \sin g \cdot \tan. p \cdot \tan. k}{\sin h}.$$

Now having BC and BE given, their difference, CE , may be found.

$m = 184$ yards,	log.	2,1271048
$f = 44^\circ 00'$	log. sine	9,8417718
$g = 67^\circ 50'$	log. sine	9,9666533
$h = 23^\circ 50'$	cosec. (ar. comp.	0,8935353
$BC = 213,86$ yards	log.	2,8290649
$\angle BCD = p = 22^\circ 10'$	tan.	9,6100359
$\angle BDE = k = 51^\circ 00'$	tan.	10,0916308
$BE = 107,38$ yards	log.	2,0807814
$BC = 213,86$		

$\therefore CE = 106,08 =$ height required over the top of the hill.

Note. I have used the formula or value of BE , marked *, which is very convenient. The data of this problem is from Keith's Trigonometry, chap. iii, example 37.

249. Let BC be the height required, situated on sloping ground AB (see fig. 39). At A and D take the vertical angles $CAB = a$, equal the angle above the horizontal line AF .

$$\angle CAB = f.$$

$$\angle CDB = k.$$

$$\angle ACD = h = \angle BDC - CAB.$$

$$\angle ACB = i = 90^\circ - \angle CAF.$$

$$\angle FAB = b.$$

$$\angle AD = m, \text{ and } DB = n, \therefore AB = m + n.$$

$$BF = (m + n) \cdot \sin b.$$

$$AF = (m + n) \cdot \cos b.$$

$$CF = (m + n) \cdot \cos b \cdot \tan a.$$

Second Method.

250. Measure on the slope A B the distance A D = m; take the $\angle C A B = f$, and the vertical angles $E D B = p$ and $\angle C D E = q$.

$$C D = \frac{m \cdot \sin f}{\sin h}$$

$$C E = \frac{m \cdot \sin f \cdot \cos. q}{\sin h}$$

$$D E = \frac{m \cdot \sin f \cdot \cos. q}{\sin h}$$

$$B E = \frac{m \cdot \sin f \cdot \cos. q \cdot \tan. p}{\sin h}$$

Consequently $C E - B E = C B$.

In this case the distance B D is assumed inaccessible.

Third Method.

251. Having found $C D = \frac{m \cdot \sin f}{\sin h}$, we measure on the continuation of the slope D B = n, making the $\angle E D B =$ as above = p, and the $\angle E D C = q$. We find $B E = n \cdot \sin b$.

$$C E = \frac{m \cdot \sin f \cdot \sin q}{\sin h}$$

$$\therefore B C = \frac{m \cdot \sin f \cdot \sin q}{\sin h} - n \cdot \sin b.$$

252. Let the land, from A towards B, be too uneven and impracticable to produce the line B A (see fig. 89).

Measure any line, as A G = m; take the horizontal $\angle C G A = a$.

$$\angle C A G = b.$$

Then $180^\circ - a - b = x$

$$= \angle A C G = c.$$

Let the vertical angle $C A F = o$.

$$\angle C A B = f.$$

$$\angle B A F = l.$$

$$\text{By sec. 131, } A C = \frac{m \cdot \sin a}{\sin c}$$

$$C F = \frac{m \cdot \sin a \cdot \sin o}{\sin c}$$

$$B F = \frac{m \cdot \sin a \cdot \cos. o \cdot \tan. b}{\sin c}$$

Consequently, $C F - B F = B C =$ the required height.

Example. Let $a = 64^\circ 30'$

$$\angle o = 58^\circ$$

$$\angle b = 72^\circ 10'$$

$$\angle l = 33^\circ$$

$$\angle c = 43^\circ 20'$$

$m = 52$ yards, to find C B.

To find C F. We have from this article $C F = \frac{m \cdot \sin a \cdot \sin o}{\sin c}$

$$m = 52 \text{ yards,}$$

$$\log. \quad 1,71600$$

$$a = 64^\circ 30'$$

$$\log. \sin \quad 9,95549$$

$$o = 58^\circ 00'$$

$$\sin \quad 9,92842$$

$$c = 43^\circ 20'$$

$$\text{ar. comp.} \quad 0,16352$$

$$C F = 58,1$$

$$\log. \quad 1,76348$$

To find the height B F. We find the value of B F by the last equation of this article.

m =		1,71600
< a	sine	9,95549
< o	cosine	9,72421
< c	ar. comp.	0,16352
< l	tan. =	9,81252
B F = 23,536,	log. =	1,36174
∴ 58 - 23,536 = 34,464 yards = B C.		

253. At sea, at the distance of 20 miles from a lighthouse, the top of which appeared above the horizon; height of the observer's eye above the sea, 16 feet. Required, the height of the lighthouse above the level of the sea. Here 16 feet = 0,003 miles.

Assuming the circumference of the earth 25020 miles, and its semi-diameter 2982 miles.

As 417 : 120 :: 20 miles : 0° 17' 16'' nearly = < B C D.

And because the angle at D is right angled,

90 - 0° 17' 16'' = 89° 42' 44'' = < C B D.

∴ by sec. 181, as sine < B : C D :: rad. : B C.

= 3982,003 = C D, . log. = 3,6001018
rad. = 10

89° 42' 44'' log. sine = $\frac{13,6001018}{9,9999945}$
3,6001068

B C = 3982,05

A C = 3982

A B = .05 miles.
5280

A B = 264 feet, 26400

By sec. 107, < C D · sec. < B C D = B C. But as the secant in small angles change with little differences, it would be unsafe to use it. In this example, < B C D = 0° 17' 16'', the secants 17' and 18' show no difference for 1'.

254. When the altitude is 45°, the error will be the least possible; in which case 1' would make an error of $\frac{1}{1716}$ part of the altitude; and generally the error in altitude is to the error committed in taking the altitude, as double the height is to double the observed angle.—*Keith's Trigonometry, chap. iii., example xxix.*

TRAVERSE SURVEYING.

255. Let the figure A, B, C, D, E, F and G (see fig. 17d) be the polygon. This is the same figure given by Gibson on plate 9, fig. 3. Let S N be a meridian assumed west of the polygon; let A W = meridian distance of the point A from the assumed meridian; then M B = mer. dist. of the point B, N C = mer. dist. of point C, D Z = mer. dist. of point D, T E = mer. dist. of E, Q F = mer. dist. of the point F, and G S¹ = mer. dist. of G. Let Y I = mer. dist. to middle of A B, O K = mer. dist. to the middle of B C, L L¹ = mer. dist. to middle of C D, X M = mer. dist. to middle of D E, R R² = mer. dist. to middle of E F, P a = mer. dist. to middle of F G.

It also appears that $W M =$ northing of $A B$, $M N =$ the northing of $B C$, $N Z =$ southing of $C D$, $Z T =$ southing of $D E$, $Q F =$ southing of $E F$, and $Q S^1 =$ the northing of $F G$.

By the method of finding the areas of the trapeziums (sec. 24), we have as follows:

	North Area.	South Area.
$W M \cdot Y I =$ area of $A B M W =$	$W M \cdot Y I$	
$M N \cdot O K =$ area of $B C N M =$	$M N \cdot O K$	
$N Z \cdot L L^1 =$ area of $C D Z N =$		$N Z \cdot L L^1$
$Z T \cdot M X =$ area of $D E T Z =$		$Z T \cdot M X$
$T Q \cdot R R^1 =$ area of $E F Q T =$		$T Q \cdot R R^1$
$Q S^1 \cdot P a =$ area of $F G S Q =$	$Q S^1 \cdot P a$	

Hence appears the following rule, which is substantially the same as Gibson's Theorem III, section v:

256. *Rule.* Multiply the meridian distance taken in the middle of every stationary or chain line by the particular northing or southing of that line.

Put the product of southings in the column of south areas, and the product of northings in the column of north areas. The difference of the area columns will be the required area of the polygon; to which add the offsets, and from the sum take the inlets. The remainder will be the area of the tract which has been surveyed.

To Find the Numbers for Column B, entitled Meridian Distance.

257. Let $A W$ (fig. 17*d*) represent the first number—viz., 61,54 chains, and $N Q$ the first meridian line; and since the map is on the east side of this meridian, all those lines that have east departure will lie farther from the first meridian than those that have west departure; therefore, knowing the length of the line $A W$, the length of the other lines, $I Y$, $B M$, etc., may be found by adding the eastings and subtracting the westings.

The first meridian is supposed to be the length of the whole departure, or the entire easting or westing from the first station; for should the first station be at the eastermost point of the land, the first meridian will then pass through the most westerly point, and the map will entirely be on the east of the first meridian.

But if the meridian distance be assumed less than the whole easting or westing from the most easterly point of the land, then it is plain that the first meridian will pass through the polygon or map, and that part of the land will be east and part west of that meridian. In this case, in that part which would be east of the meridian, we would add the eastings and subtract the westings; but in that part west of the meridian, we would add the westings and subtract the eastings.

In method 1, the sum of all the east departures is assumed as the first meridian distance.

In method 2, the first meridian is made to pass through the most westerly station.

In method 3, the first meridian is made to pass through the most northerly station of the polygon, as station E (see fig. 17*b*).

258. METHOD I.—Commencing Column B with the Sum of all the East Departures (see fig. 17b).

Bearing.	Dist.	N. lat.	S. lat.	E. dep.	W. dep.	A or lat. and $\frac{1}{2}$ dep.	B or mer. dis.	N. Area.	S. Area.
	Ch'na.						37,1752		
North.	29.18	29.178		0.0000		N. 29.178	37,1752 E.	1084,6979	
N. 40° E.	8.00	6.128		5.1428		0.000	37,1752 E.		
N. 10° W.	9.00	8.968			1.5629	N. 6.128	39,74635 E.	243,5659	
N. 50° E.	12.00	7.714		9.1925		E. 2,57115	42,8175 E.		
S. 30° E.	10.00		8.661	5.0000		N. 8.863	41,53805 E.	368,1386	
South.	17.00		17.001			W. 0.78145	40,7546 E.		
East.	11.00		East.	11.0000		N. 7.714	45,35085 E.	349,8376	
S. 20° E.	20.00		18.794	6.8404		E. 4,59625	49,9471 E.		
S. 60° W.	21.00		10,500		18,1866	S. 8.661	52,4471 E.		454,2445
N. 80° W.	17.694	8.078			17,4257	E. 2,5000	54,9471 E.		
						S. 17,001	54,9471 E.		934,1556
						0.0000	54,9471 E.		
						0.0000	60,4471 E.		
						E. 5,5000	66,9471 E.		
						S. 18,794	69,3673 E.		1303,6890
						E. 3,4202	72,7875 E.		
						S. 10,500	68,6942 E.		668,7891
						W. 9,0933	54,6009 E.		
						N. 3,073	45,888 E.	141,0133	
						W. 8,71235	37,1752 E.		
								2187,2488	3360.8780
									2187.2488
									1173,6292

In column A, the top line of each pair is the north or south latitude, and the under number is half the corresponding departure.

In column B, the sum of all the east departures is assumed as the first meridian distance, thus making the first meridian to be west of the most westerly station.

The meridian distance is found by adding half the eastings twice, and subtracting half the westings twice. These give the meridian distances at half the lines.

Example. The first line is N. lat. 29.178, and departure = 0, $\therefore$ 0

added to 37,152 gives the meridian distance = 37,152, and $37,152 + 0 = 37,152 =$ lower number of the first pair in column B. The next half departure is = 5,57115 east, $\therefore 2,57115 + 37,152 =$ meridian distance = 39,7463; add 2,57118 to 39,7463; it will give the under line of second pair = 42,3175. From 42,3175 take half the next departure, 0,78145, and it gives meridian distance = 41,53805, etc., always adding the eastings and subtracting the westings.

The product of the upper numbers in columns A and B will give the areas. If the upper number in column A is north latitude, the product is put under the heading, north area; but if the upper number in column A be south latitude, then the product is put under the heading, south area.

Having found the last number in column B to agree with the first meridian distance at top, is a proof that the calculation is correct.

The difference between the north area and south area columns determine the area of the given polygon in square chains.

The area could be found in like manner by assuming the principal meridian east of the polygon, and adding the westings, or west departures, and subtracting the eastings, or east departures.

$$\text{Area} = 117 \frac{2420}{1000} \text{ acres.}$$

259. METHOD II.—*The First Meridian passes through the Most Westerly Station (see fig. 17b).*

Bearing.	Dist.	N. lat.	S. lat.	E. dep.	W. dep.	A or lat, and $\frac{1}{2}$ dep.	B or mer. dist.	N. Area.	S. Area.
							0,0000		
North.	29,178	29,178		0,0000		N. 29,178	0,0000		
N. 40° E.	8,00	6,128		5,1428		0,000	0,0000		
N. 10° W.	9,00	8,863			1,5629	N. 6,128	2,57115 E.	15,7568	
N. 50° E.	12,00	7,714		9,1926		E. 2,57115	5,14230 E.		
S. 30° E.	10,00		8,661	5,0000		N. 8,863	4,86085 E.	38,6507	
South.	17,00		17,001	0,0000		W. 0,78145	3,57940 E.		
East.	11,00		East.	11,0000		N. 7,714	8,17565 E.	68,0678	
S. 20° E.	20,00		18,794	6,8404		E. 4,59625	12,77190 E.		
S. 60° W.	21,00		10,500		18,1866	S. 8,661	15,27190 E.		132,2699
N. 80° W.	17,694	3,0780			17,4257	E. 2,5000	17,77190 E.		
						S. 17,001	17,77190 E.		302,1401
						0,0000	17,77190 E.		
						0,0000	22,27190 E.		
						E. 5,5000	23,77190 E.		
						S. 18,794	32,19210 E.		606,0183
						E. 3,4202	35,61230 E.		
						S. 10,500	28,51900 E.		278,4455
						W. 9,0683	17,42570 E.		
						N. 8,0730	18,71285 E.	26,7747	
						W. 8,71285	10,0000		
								144,2490	1317,8766

In this example we take the corrected distances and correct balance sheet; that is, the numbers are such as to give the northings equal to the southings, and the eastings equal to the westings (see sec. 220).

By sec. 221, the point or station A is found to be the most westerly station on the survey.

By making the first meridian pass through the most easterly station, we find the area by adding the westings and subtracting the eastings.

North area = 144,2490

Area of the polygon = 117,86288 acres.

*

By first method = 117,86292 acres.

By second method = 117,86288 acres.

This is satisfactory proof.

Note. The surveyor ought to adopt some uniform system, as by this means he will be in less danger of committing errors. I have invariably made the principal meridian pass through the most westerly station of the polygon according to this method, and checked it by the third method, thereby making one method check the other. Making the first meridian pass through the polygon requires less figures, but more care in passing from east to west, and *vice versa*; also in entering the areas in their proper columns, as sometimes the north area is to be put in the south area columns, and the contrary. But in the first and second methods, the north area is always put in north area column, and the south area in south area column.

260. METHOD III.—*The First Meridian passes through the Most Northern Station of the Polygon, as through Station E (see fig. 17b).*

Bearing.	Dist.	N. lat.	S. lat.	E. dep.	W. dep.	A or lat., and $\frac{1}{2}$ dep.	B or mer. dist.	N. Area.	S. Area.
							0,0000		
S. 30° E.	10,00		8,661	5,0000		S. 8,661	2,5000 E.		21,525
South.	17,00		17,001	0,0000		E. 2,5000	5,0000 E.		
East.	11,00		0,000	11,0000		S. 17,001	5,0000 E.		85,0050
S. 20° E.	20,00		18,794	6,8404		0,0000	5,0000 E.		
S. 60° W.	21,00		10,500		18,1866	0,0000	10,5000 E.		
N. 80° W.	17,694	3,0730			17,4257	E. 5,5000	16,0000 E.		
North.	29,178	29,178				S. 18,794	19,4202 E.		384,9832
N. 40° E.	8,00	6,128		5,1423		E. 8,4202	22,8404 E.		
N. 10° W.	9,00	8,863			1,5629	S. 10,500	18,7471 E.		144,3446
N. 50° E.	12,00	7,714		9,1925		W. 9,0933	4,6533 E.		
						N. 3,0730	4,0591 W.		12,4736
						W. 8,7129	12,7720 W.		
						N. 29,178	12,7720 W.		372,6614
						0,000	12,7720 W.		
						N. 6,128	10,2009 W.		62,5111
						E. 2,5711	7,6298 W.		
						N. 8,863	8,4112 W.		74,5486
						W. 0,7814	9,1927 W.		
						N. 7,714	4,5985 W.		35,4574
						E. 4,5962	0,0003		

In this method, everything is the same as in methods 1 and 2, except finding the areas.

Rule. The north or south multiplied by their respective east meridian distances, are put in their respective columns of areas, as in methods 1 and 2; but north and south latitudes multiplied by their respective west meridian distances, are put in contrary area columns.

That is, S. lat. $\times$ E. mer. dist. is put in south area column; N. lat. $\times$ E. mer. dist. is put in north area column; S. lat. $\times$ W. mer. dist. is put in north area column; N. lat. $\times$ W. mer. dist. is put in south area column.

Area in acres = 117,3637

Second method = 117,3629

First method = 117,3629

•

The proof of the above rule will appear from the following (see fig. 17b). Draw the meridian E W through the point or station E; let p F, g H, r D, s K, R s, C w, and D x, be the departures respectively.

	North Area Column.	South Area Column.
$n F \times \frac{1}{2} F p$ = south $\times$ by east = a		a
$P G \times \frac{1}{2} (F p + G q)$ = south $\times$ by east = a'		a'
$m I \times \frac{1}{2} (H q + I r)$ = south $\times$ by east = a''		a''
$l L \times \frac{1}{2} (I r + K L)$ south $\times$ by east = a'''		a'''
This includes figure $I r v K + \triangle V K S$, S K being the east meridian distance of K; then $S K - \frac{1}{2} (K A)$ = mer. dist. of the middle of the line A K, which is — or east, if S K is more than $\frac{1}{2} A K$; but if S K is more than $\frac{1}{2} A K$, then the meridian distance will be + or east, and if the mer. dist. S K is equal to $\frac{1}{2} A K$, then the mer. dist. of line K A = o.		b
		b'
		b''
		b'''
h		

We now suppose that SK is less than KA ; therefore mer. distance to middle of $KA = SK - \frac{1}{2} Ag =$ west or negative, and $(SK - \frac{1}{2} Ag) \cdot gK =$ figure $gKsy - \triangle AgK =$ figure $gKvy + Kvs - \triangle AgK$; but the meridian distance being negative, $\therefore$ the product must be negative; that is, the above product $\triangle AgK - gKvy + Kvs$, which is equal to the $\triangle Ayv$, because we have to deduct $gKvy + Kvs$, which have been including the figure $Kirs$; consequently north by west is to be added or put in south area column. Let this area be equal to b , and entered in the south area column. The mer. dist. of A is the same as that of B , and is found by adding $\frac{1}{2} Ag$ to the last mer. dist. to the middle of AK . That mer. dist. $\times$ by AB , gives an area to be added $=$ figure $gABb = b'$, which is put in south area column. Also the mer. dist. in middle of BC is west, which multiplied by BC , will give the area $BCwb = b''$, which put in south area column. In like manner we find the area $C Dxw = b'''$, which put in south area column; and the area of DEx is west of the meridian b'''' , and is to be put in south area column.

Hence it appears that those areas derived from east meridian distances are put under their respective heads, S. and N.; but those having west meridian distances, are put in their contrary columns.

261. Calculating the Offsets and Inlets. (See fig. 17a.)

Line 1.	Base.	Sum of offsets	Double area. add.	Double area. Sub't
On a to b	40	14	1960	
	107	78	8346	
	103	84	8652	
	116	14	1604	
On b to F	98	16		1568
	190	46		8740
	102	50		5100
	94	80		2820
Sum of addition,			20562	18228
Sum of subtraction,			18228	
Difference,			2384,	to be added to the area of the polygon.

The letters a, b, etc., show between what points on the line the areas are calculated.

When the area, and not the double area, of the polygon is given, then we take half the double area of the difference of the offset and inlet columns, and add or subtract to or from the area of the polygon, as may be the case.

In making out the bases, we subtract 150 from 190; put the difference, 40, in base column, and opposite which, in offset column, and opposite which, in offset column, put 14; then 40×14 will give double the area of the $\triangle$ between 150 and 190.

Again, take 190 from 297; the difference, 107, is put in base column, opposite to which, in offset column, is put $78 = 14 + 64$; then $107 \times 78 =$ double the area of the trapezium between 190 and 297.

This method of keeping field notes facilitates the computation of offsets and plotting detail.

We begin at the bottom of the page or line, and enter the field notes as we proceed toward the top or end of the line. The chain line may be a space between two parallel lines, or a single line, as in fig. 17a. If the field book is narrow, only one line ought to be on the width of every page, and that up the middle (see sec. 211).

262. *Field Book, No. 16, Page 64.*

On the first day of May, 1838, I commenced the survey of part of Flaskagh, in the parish of Dunmore, and county of Galway, Ireland, surveyed for John Connolly, Esq.

MICHAEL McDERMOTT, C. L. S.

THOMAS LYNSEY, } *Chain bearers.*
THOMAS KING, }

The angles have been taken by a theodolite, the bearing of one line determined, from which the following bearings have been deduced (see fig. 17e). *Land kept on the right.*

We begin at the most northerly station, as by this means we will always add the south latitudes and subtract the north latitudes.

Explanation. On line 1, at distance 210, took an offset to the left, to where a boundary fence or ditch, etc., jutted. The dotted line along said fence shows that the face next the dots is the boundary.

At 297, offset of 64 links to Mr. JAMES ROGER's schoolhouse.

At 340, offset of 70 links to south corner of do.

The width = 80, set down on the end of do.

At 400, offset to the left of 14 links to a jutting fence.

From 150 to 400, the boundary is on the inside or right, as shown by the characters made by dots and small circles joined. See characters in plates. From this point, 400, the boundary continues to the end of the line, to be on the left side of fence.

At 804, met creek 30 links wide, 5 deep, clear water, running in a southern direction.

At 820, met further bank of do.

At 830, dug a triangular sod out of the ground, making the vertex the point of reference. Here I left a stick 6 inches long, split on top, into which split a folded paper having line 1—830 in pencil marks. This will enable us to know where to begin or close a line for taking the detail.

At 960, offset to the right 20 links.

At 1000, met station F, where I dug 3 triangular sods, whose vertexes meet in the point of reference. This we call *leveling mark*.

The distance, 1000 links, is written lengthwise along the line near the station mark.

The station mark is made in the form of a triangle, with a heavy dot in the centre.

Distances from which lines started or on which lines closed, are marked with a crow's foot or broad arrow, made by 3 short lines meeting in a point.

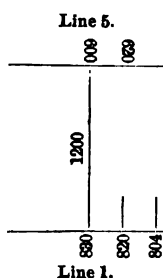
Along the line write the number of the line and its bearing.

Line 2 may be drawn in the field book as in this figure, or it may be continued in the same line with line 1, observing to make an angle mark on that side of the line to which line 2 turns. This may be seen in lines 4 and 5, where the angle mark is on the right, showing that line 5 turns to the right of line 4.

Line 2, total distance to station G = 1700 links. The distance from the station to the fence, on the continuation of line 2, is 10 links, which is set correctly on the line.

Key offset. See where line 2 starts from end of line 1. At the end of line 1, offset to corner of fence = 10. At 10 links on line 2, offset to corner = 2. This is termed the key offset, and is always required at each station for the computation of offsets and inlets.

Running from one line to another. We mention the distance of the points of beginning and closing as follows:



This shows that the line started from 880, on line 1, and closed on 600, line 5. It also shows, from the manner in which distances 804, 820 and 830 are written, that the line turns to the right of line 1. When we use a distance, as 830, etc., we make 2 broad arrows opposite the distance. This will enable us to mark them off on the plotting lines for future reference.

We take detail on this line—it will serve as a check when the scale is 2, 3, or 4 chains to 1 inch scale. We number it and enter it on the diagram, which must always be on the first page of the survey. The diagram will show the number of the line; the distances on which it begins and ends; the reference distances. This will enable the surveyor to lay down his plotting or chain lines, and test the accuracy of the survey. Having completed the plotting plan, we then fill in the detail, and take a copy or tracing of it to the field, and then compare it with the locality of the detail.

This comparison is made by seeing where a line from a corner of a building, and through another corner of a fence or building, intersects a fence; then from the intersection we measure to the nearest permanent object. We draw the line in pencil on the tracing, and compare the distance found by scale with the measured distance. Some surveyors can pace distances near enough to detect an error. On the British Ordnance Survey, the *sketchers* or examiners seldom used a chain, unless in filling in omitted detail.

On Supplying Lost Lines or Bearings.

263. It would be unsafe to depend on this method, unless where the line or lines would be so obstructed as to prevent the bearings and distances to be taken. The surveyor seeing these difficulties, will take all the available bearings and measure the distances with the greatest accuracy, leaving no possible doubt of their being correctly taken. Then, *and not till then*, can he proceed to supply the omissions.

Case 1. In fig. 17*b*, we will suppose that all the lines and bearings have been correctly taken, but the distance I K has been obliterated, and that its bearing is given to *find the distance I K*.

Let the bearing of I K be S. 60 W. From sec. 259, method 2, we have
calculated the departure of K from the line A B = 17,4257
departure of I from do. = 35,6123
consequently the departure of line I K is = K L = 18,1866

We have the angle K I L = 60°, therefore the $\angle$ I K L = 30°, and its departure = ,5000

The product of the last two numbers will give (by sec. 167) I L = 9,0933

By E. I, 47, from having I K and K L we find 10,50 = I K

or I L = 9,1933, divided by the lat. or cos. of 60° or ,86603 = 10,50 = I K

Case 2. The bearing and distance of the line I K is lost.

Here we have to find the lines I L and L K. From the above sec., method 2, we have—

Lat. K A = 3,0726 N.	Lat. E F = 8,6610 S.
Lat. A B = 29,1780 N.	Lat. F G = 17,0010 S.
Lat. B C = 6,1280 N.	Lat. H I = 18,7940 S.
Lat. C D = 8,8630 N.	44,456 S.
Lat. D E = 7,7140 N.	
54,9556 N.	
44,4560 S.	

Lat. I L = 10,4996, and from above K L = 18,1866.

Therefore, by E. 1, 47, $K L^2 + L I^2 = K I^2$; consequently K I is found. But $I K \cdot \cos. < K I L = I L$.

Therefore $\frac{I L}{I K} = \cosine < K I L$, which take from table of lat. and dep., and it gives $< K I L = 60^\circ$. Consequently the bearing is S. 60° W., or $\frac{K L}{I K} = \frac{9,0933}{10,50} = ,8662 = \cos. < I K L$; $\therefore$ the $< I K L = 30^\circ$, and the bearing of the line K I = N. 60° E. from station K.

Case 3. Let there be two lines wanted whose bearings are known to be S. 60° W. and N. 80° W.

Here the station K may be obstructed by being in a pond, in a building, or that buildings are erected on part of the lines I K and A K (see fig. 17b).

We find from case 2 that A is south of F = 51,8830
I is south of F = 44,4560
A is south of I = t g = A a = 7,4270
We have above, a I = dep. of I = d = 85,6123

Now we have A a and a I, $\therefore$ we find the line A I.
And A a divided by a I gives the tangent of $< A I a = ,2085$.
And the $< A I a = 11^\circ 47'$.

$\therefore$ I a divided by the cosine $10^\circ = A I = 35,6123 \div ,9789 = 36,38$.

Now we have the $< A I a = 11^\circ 47'$
and the $< A a I = 90^\circ$; $\therefore$ the $< a A I = 78^\circ 13'$
consequently the $< g A I = 11^\circ 47'$
but the $< g A K = 10^\circ 00'$, $\therefore < K A I = 21^\circ 47'$.
Again the $< K I a = 30^\circ 00'$
and the $< A I a = 11^\circ 47'$, $\therefore = A I K = 18^\circ 13'$.

And by Euclid I, 32, we have the $< A K I = 140^\circ 30'$.

By sec. 194, we have $\sin < A K I : A I :: \sin < A I K : A K$.
 $\sin < A K I : A I :: \sin < K A I : K I$.

Case 4. Let all the sides be given, and all the bearings, except the bearings of I K and A K, to find these bearings.

By the above methods we can find the departure a i of the point I, east of the meridian A B.

We also have the difference of lat. of the points A and I = t g = A a.
 $\therefore (A a)^2 + (I a)^2 =$ the square of A I; $\therefore$ A I may be found.
Or, $A a \div I a =$ tangent of the $< A I a$; $\therefore < A I a$ may be found.
And $I a \div \cos. < A I a$, will give the side A I.

Now having the sides A I, A K and K I, by sec. 205, we can find the angles K A I and K I A. And the $< A I a$ and $< A I K$ are given; $\therefore$ their sum $< A I K$ is given; $\therefore$ the bearing of the line I K is given.

264. *Calculation of an Extensive Survey (fig. 17c), where the First has been made. Calculated*

Line.	Bearing.	Dist. in chains	N. lat.	S. lat.	E. dep.	W. dep.	Equated N. lat.	Equated S. lat.
B C	N. 40° E.	8,00	6,1283		5,1423		6,128	
C D	N. 10° W.	9,00	8,8633			1,5629	8,863	
D E	N. 50° E.	12,00	7,7135		9,1925		7,714	
E F	S. 30° E.	10,00		8,6603	5,0000			8,660
F G	South.	17,00		17,0000				17,000
G H	East.	11,00			11,0000			
H I	S. 20° E.	20,00		18,7938	6,8404			18,794
I K	S. 60° W.	21,00		10,5000		18,1866		10,500
K A	N. 80° W.	17,69	3,0727			17,4260	3,073	
A L	North.	7,00	7,0000				7,000	
L M	West.	8,00				8,0000		
M N	N. 55° W.	9,00	5,1622			7,3724	5,162	
N O	N. 75° W.	7,00	1,8117			6,7615	1,812	
O P	N. 27° W.	6,00	5,3461			2,7239	5,346	
P Q	N. 33° E.	10,00	8,8867		5,4464		8,887	
Q R	N. 77° W.	9,00	8,9330			1,0968	8,933	
R S	N. 37° W.	9,00	7,1878			5,4163	7,188	
S T	N. 43° E.	11,00	8,0449		7,5020		8,045	
T U	S. 52° E.	13,00		8,0036	10,2441			8,003
U B	S. 29° E.	16,80		14,6936	8,1448			14,694
			77,6502	77,6512	68,5125	68,5466	77,651	77,651

Here we find that line K A, which theoretically should close on A, wants but 1,8 links.

To find the Most Westerly Station.

By looking to fig. 17d, it will appear that either the point S or P is the most westerly.

L M = 8,000 west.

M N = 7,370 W.

N O = 6,766 W.

O P = 2,722 W.

Point P = 24,858 west of the assumed point L.

P Q = 5,448 E.

19,410.

Q R = 1,095 W.

R S = 5,414 W.

Point S = 25,919 west of the assumed point L.

Therefore the point S is the most westerly station, through which, if the first meridian be made to pass the area, can be found by the second method.

To Find the Meridian Distances.

When the first mer. passes through the most westerly station, we add the eastings and subtract the westings.

When the first mer. is through the most easterly station, we add the westings and subtract the eastings.

When the first mer. passes through the polygon, we add the eastings in that part east of the first mer., and subtract them in that part west of that mer. We also subtract the westings in that part east of that mer., and add them west of it.

Meridian is made the Base Line A B, at each of which a Station by the Third Method.

Equated K. dep.	Equated W. dep.	A or latitude, and half departure.	B. or Meridian dist.	North area.	South area.
5,146	1,561	N. 6,128	2,572½ E.	15,7648	
		E. 2,572½	5,146		
9,195		N. 8,863	4,864½ E.	38,6826	
5,002		W. 0,780½	8,584		
11,002	18,184	N. 7,714	8,181½ E.	68,1121	
8,842		E. 4,697½	12,779		
		S. 8,660	15,280 E.		132,3248
		E. 2,501	17,781		
	17,428	S. 17,000	17,781 E.		802,2770
		0,000	17,781		
	8,000	0,000	23,282 E.		
	7,370	E. 5,501	28,783		
	6,760	S. 18,794	32,204 E.		605,2420
	2,722	E. 3,421	35,625		
5,448	5,414	S. 10,500	26,538 E.		278,5965
		W. 9,092	17,441		
		N. 8,078	8,729½ E.	26,8258	
		W. 8,711½	0,018		
7,503	5,414	N. 7,000	0,018 E.	0,1260	
10,246		0,000	0,018		
8,146		0,000	8,982 W.		
68,529	68,529	W. 4,000	7,982		
		N. 5,162	11,667 W.		60,2251
		W. 3,685	15,352		
		N. 1,812	18,732 W.		33,9424
		W. 3,880	22,112		
		N. 5,346	23,473 W.		125,4867
		W. 1,861	24,834		
		N. 8,387	22,110 W.		185,4866
		E. 2,724	19,386		
		N. 8,983	19,938 W.		178,0660
		W. 0,547½	20,481		
		N. 7,188	23,188 W.		166,6758
		W. 2,707	25,895		
		N. 8,045	22,143 W.		178,1445
		E. 8,751½	18,892		
		S. 8,003	13,269 W.	106,1915	
		E. 5,123	8,146		
		S. 14,694	4,073 W.	59,8487	
		E. 4,073	0,000		
				310,5513	2246,4179
					810,5518

Required area = 1985, ⁸⁹⁹⁵/₁₀₀₀ chains, or 198,5867 acres.

VARIATION OF THE COMPASS.

264a. In surveying an estate such as that shown in fig. 17c, we run a base line through it, such as A M. We find the magnetic bearing, and its variation from the true meridian. We measure it over carefully, then take a fly-sheet and remeasure the same, then compare, and survey a third time if the two surveys differ. With good care in chaining, it is possible to make two surveys of a mile in length to agree within one foot. With a fifteen feet pole they agree very closely.

We refer the base line A M to permanent objects as follows :

Theodolite at station A, read on station M,	0° 00'
On the S.W. corner of St. Paul's tower,	15° 11'
On the S.E. corner of the Court House (main building),	27° 10'
On the S.W. corner of John Cancannon's Mill,	44° 16'
On the N.E. corner of John Doe's stone house,	276° 15'
On the N.W. corner of Charles Roe's house,	811° 02'

Any two or three of these, if remaining at a future date, would enable us to determine the base A M, to which all the other lines may be referred.

The variation of the compass is to be taken on the line at a station where there is no *local attraction*, the station ought to be at same distance from buildings.

We find the magnetic bearing of A M = N. 64° 10' E., as observed at the hour of 8 A. M., 8th December, 1860, at a point 671 links north of station A, on the base line A M. Thermometer = 40°, and Barometer 29 inches.

Let the latitude of station = 53° 45' 00''

Polar distance of Pole Star (*Polaris*) = 1° 25' 30''

(Declination of *Polaris* being = 88° 34' 30'', ∴ its polar distance is found by taking the declination from 90.)

To Find at what time Polaris will be at its Greatest Azimuth or Elongation.

264b. *Rule.* To the tan. of the polar dist. add the tan. of the lat.; from the sum take 10. The remainder will be the cosine of the hour angle in space, which change into time. The time here means sidereal.

To Find the Greatest Azimuth or Bearing of Polaris.

264c. *Rule.* To radius 10 add sine of the polar distance; from the sum take the cosine of the latitude. The remainder will be the sine of the greatest azimuth.

To Find the Altitude of Polaris when at its Greatest Azimuth.

264d. *Rule.* To the sine of the latitude add 10; from the sum take the cosine of the polar distance. The difference will be the log. sine of the altitude.

In the above example we have lat. = 53° 45', and its tan. = 10,1857596
Polar distance = 1° 25' 30'', and its tangent = 8,8957818
88° 3' 05'' = hour angle in space, whose cosine = 8,5315414
This changed into time gives 5 h., 52 m., 12,3 s. This gives the time from the upper meridian passage to the greatest elongation.

To Find when *Polaris* will Culminate or Pass the Meridian of the Station on Line A M, being on the Meridian of Greenwich on the 8th Dec., 1860.

264e. From Naut. Almanac, star's right ascension	=	1h. 08m. 48.5s.
Sun's right ascension of mean sun (sidereal time)	=	17 09 59.9
Sidereal time, from noon to upper transit	=	7 58 52.6 ●
Sidereal time, from upper transit to greatest azimuth	=	5 00 01
Sidereal time from noon to greatest eastern azimuth	=	2 58 52

Now, as this is in day time, we cannot take the star at its greatest eastern elongation, but by adding 5h. 52m. 12.3s. to 7h. 58m. 52.6s., we find the time of its greatest western azimuth = 13h. 51m. 4.9s. from the noon of the 8th December, and by reducing this into mean time, by table xii, we have the time by watch or chronometer.

To Find the Altitude and Azimuth in the above.

264f. Lat. 53° 45' N., sine + 10 + 19,906575	cos. =	9,771815
N. polar dist. 1° 25' 80'', cos. = 9,999866	sine + 10 + 18,395648	
sine = 9,906709	sine =	8,623883
True altitude = 53° 46' 27''.	Greatest azimuth =	2° 24' 37''.

Alpha and Beta are termed the pointers, or guards, because they point out the Pole star, which is of the same (second) magnitude, and nearly on the same line.

Benetnach *	Alcor *	Alioth * or Epsilon	Delta *	* Gamma	Beta * Alpha * or Dubhe
-------------	---------	------------------------	---------	---------	----------------------------

The distance from Alpha Ursamajor to the Pole star is about five times the distance between the two pointers.

URSAMAJOR, or DIPPER, or THE PLOUGH,
at its under transit.

When Alioth and *Polaris* are on the same vertical line, the Pole star is supposed to be on the meridian. Although this is not correct, it would not differ were we to run all the lines by assuming it on the meridian; but as we sometimes take *Polaris* at its greatest azimuth, both methods would give contradictory results.

264g. *Alioth* and *Polaris* are always on opposite sides of the true pole. This simple fact enables us to know which way to make the correction for the greatest azimuth. (For more on this subject, see *Sequel Canada Surveying*, where the construction and use of our polar tables will be fully explained.)

Variation of the Compass.

264A. Variation of the compass is the deviation shown by the north end of the needle when pointing on the north end of the mariner's compass and the true north point of the heavens; or, it is the angle which is made by the true and magnetic meridians.

When the magnetic meridian is west of the true meridian, the variation is westerly.

Let S N = true meridian, S = south, and N = north.

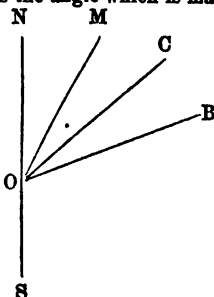
Let M O = magnetic meridian through station O.

Let the true bearing of B = N. 60° 40' E.

Let the magnetic do. = N. 50° 50' E.

Variation east = 9° 50'

In this case, the true bearing is to the right of the magnetic.



Let M = magnetic and N = true North Pole. M

Let the true bearing of B = N. $60^{\circ} 50'$ E.

Let the magnetic do. = N. $70^{\circ} 40'$ E.

Variation west = $9^{\circ} 50'$

● Here the true bearing is to the left of the magnetic.

In the first example we protract the $\angle NOC$ = $\angle MOB$, which show that B is to the right of C.

In the second example we make the $\angle NOD$ = $\angle MOB$, which shows that B is to the left of D.

Hence appears the following rule:

Rule 1. Count the compass and true bearings from the same point north or south towards the right.

Take the difference of the given bearings when measured towards the east or towards the west; but their sum when one bearing is east and the other west.

When the true bearing is to the right of the magnetic, the variation is east. When the true bearing is to the left of the magnetic, the variation is west.

Example 3. Let the true bearing = N. 60° W. = 300° ,
and the magnetic bearing = N. 70° W. = 290° .

Variation east = 10° .

Here we have the true bearing at 300° , counting from N. to right, and the magnetic bearing at 290° , counting from N. to right.

10° variation east, because the true bearing is to the east of the magnetic.

Example 4. Let true bearing = N. 60° W. = 300° , from N. to right
and magnetic bearing = N. 70° W. = 290° , from N. to right.

Variation 10° west, because the true bearing is to the right of the magnetic.

Example 5. Let true bearing = N. 5° E. = 5 from N. to right,
and the magnetic bearing = N. 5° W. = 365 from N. to right.

Variation 10° east, because the true bearing is to the right of the magnetic.

Rule 2. From the true bearing subtract the magnetic bearing. If the remainder is +, the variation is east; but if the remainder or difference is -, the variation is west.

Example 6. True bearing = N. $60^{\circ} 40'$ E.

Magnetic bearing = N. $50^{\circ} 50'$ E.

+ $9^{\circ} 50'$ = variation east.

Example 7. True bearing = N. 5° E. = +,

Magnetic bearing = N. 5° W. = -.

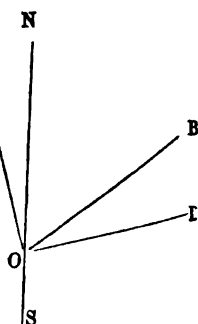
+ 10° east.

Here we call the east +, and the west negative -; and by the method of subtracting algebraic quantities, we change the sign of the lower line, and add them.

Example 8. Let true bearing = N. 16° W. = -,

and magnetic bearing = N. 6° W. = -.

- 10° = variation 10° west.



Let us now find the true bearing of the line A M in fig. 17c.

By sec. 264a, we have the magnetic bearing of A M = N. $64^{\circ} 10'$ E., < from Polaris, at its greatest western elongation, to the base line A M, as determined = $80^{\circ} 40'$. The work will appear as follows:

On the evening of the 8th December, 1860, we proceeded to the station mentioned in sec. 264a. Set up the theodolite on the line A M. At a distance of 10 chains, I set a picket fast in the ground, whose top was pointed to receive a polished tin tube, half an inch in diameter. Not wishing to calculate the necessary correction of Polaris from the meridian, I preferred to await until it came to its greatest western azimuth, being that time when the star makes the least change in azimuth in 6 minutes, and the greatest change in altitude, this being the time best adapted for finding the *greatest azimuth and true time* of any celestial object. The station is assumed on the meridian of Greenwich. If on a different meridian, we correct the sun's right ascension. (See our Sequel Spherical Astronomy, and Canada Surveying.)

On the morning of 9th December, 1860, at 1h. 51m. 5s., found the base line A M to bear from Polaris =	N. $80^{\circ} 40' 00''$ E.
Magnetic bearing of line A M =	N. $64^{\circ} 10' 00''$ E.
Polaris at its greatest azimuth =	N. $80^{\circ} 40' 00''$ E.
Greatest azimuth from sec. 264f =	$2^{\circ} 24' 37''$
Bearing of the line A M from true meridian =	N. $78^{\circ} 15' 23''$ E.
Magnetic bearing of line A M =	N. $64^{\circ} 10' 00''$ E.

By rule 2, the variation = N. $14^{\circ} 05' 23''$ E.

From sec. 264f, we have the star's altitude when at its greatest azimuth.

True altitude =	$53^{\circ} 46' 27''$
Correction from table 14 for refraction =	$42''$
Apparent altitude =	$53^{\circ} 47' 09''$

We had the telescope elevated to the given apparent altitude until the star appeared on the centre, then clamped the lower limb, and caused a man to hold a lamp behind the tin tube on the line A M. Found the < $80^{\circ} 40'$, as above. Here the vernier read on Polaris at its greatest western azimuth =

Read on the tin tube and picket on the line A M =	$00^{\circ} 00' 00''$
On the true meridian =	$281^{\circ} 44' 37''$

The last bearing taken from 360° will give the true bearing of A M = N. $78^{\circ} 15' 23''$ E.

After having taken the greatest azimuth, we bring the telescope to bear on A M; if the vernier read zero, or whatever reading we at first assume, the work is correct. If it does not read the same, note the reading on the lower limb, and, without delay, take the bearing of the Pole star, which is yet sufficiently near to be taken as correct, and thus find the angle between it and the base line. The surveyor, having two telescopes, will be in no danger of committing errors by the shifting of the under plate, can have one of the telescopes used as a *tell-tale*, fixed on some permanent object, on which he will throw the light shortly before taking the azimuth of Polaris, to ascertain if the lower limb remained as first adjusted.

264i. A second telescope can be attached to any transit or theodolite, so as to be taken off when not required for tell-tale purposes, as follows: To the under plate is riveted a piece of brass one inch long, three-fourths

inch wide, and two-tenths thick. On this there is laid a collar or washer, about one-eighth inch thick. To these is screwed a right angled piece in the form of L, turned downwards, and projecting one inch outside of the edge of the parallel plates. Into the outer edge of the L piece is fixed a piece having a circular piece three-fourths inch deep, having a screw corresponding to a thread on the telescope of the same depth. This screw piece is fastened on the inside of the L piece by a screw, and has a vertical motion. When we use this as a tell-tale, we bring it to bear on some well defined object, and then clamp the lower plate. We then bring the theodolite telescope to bear on the above named object or tin tube, and note the reading of the limb. After every reading we look through the tell-tale telescope to see if the lower plate or limb is still stationary. If so, our reading is correct; if not, *vice versa*.

The expense of a second telescope so attached will be about twelve dollars, or three pounds sterling. The instrument will be lighter than those now made with two telescopes, such as six or eight inch instruments. This adjustment attached to one of Troughton and Simm's five inch theodolite has answered our purposes very well during the last twenty-two years. We prefer it to a six inch, as we invariably, for long distances, repeat the angles. (See sec. 212.)

265. *To Light the Cross Hairs.* Sir Wm. Logan, Provincial Geologist of Canada, has invented the following appendage: On the end of the telescope next the object is a brass ring, half an inch wide, to which a second piece is adjusted, at an angle of 45° . This second piece is elliptical, two inches by two and three-eighths, in the centre of which is an elliptical hole, one inch by three-eighths. This is put on the telescope. The surface of the second piece may be silvered or polished. Our assistant holds the lamp so as to illuminate the elliptical surface, which then illuminates the cross hairs. He can vary the light as required.

This simple appendage will cost one and a half dollars, and will answer better than if a small lamp had been attached to the axis of the telescope, as in large instruments. Those surveyors who have used a hole in a board, and other contrivances, will find this far more preferable.

We have a reflector on each of our telescopes. The tell-tale being smaller is put into the other, and both kept clean in a small chamois leather bag, in a part of the instrument box. (See sec. 211.)

UNITED STATES SURVEYING.

The following sections are from the Manual of Instructions published by the United States Government in 1858, which are called *New Instructions*, to distinguish them from those issued between 1796 and 1855, which are called the Old Instructions. The notes are by M. McDermott.

SYSTEM OF RECTANGULAR SURVEYING.

266. The public lands of the United States are laid off into rectangular tracts, bounded by lines conforming to the cardinal points.

These tracts are laid off into townships, containing 23040 acres. These townships are supposed to be square. They contain 36 tracts, called sections, each of which is intended to be 640 acres, or as near that as possible. The sections are one mile square. A continuous number of townships between two base lines constitutes a range.

267. The law requires that the lines of the public surveys shall be *governed by the true meridian, and that the township shall be six miles square*—two things involving a mathematical impossibility, by reason of the convergence of the meridians. The township assumes a trapezoidal form, which unequally develops itself more and more as the latitude is higher.

* In view of these circumstances, the act of 18th May, 1796, sec. 2, enacts that the sections of a mile square shall contain 640 acres, *as nearly as may be*.

* The act 10th May, 1800, sec. 8, enacts "That in all cases where the exterior lines of the townships thus to be subdivided into sections, or half sections, shall *exceed, or shall not extend* six miles, the excess or deficiency shall be specially noted, and added to or deducted from the *western and northern* ranges of sections or half sections in such township, according as the error may be, in *running the lines from east to west or from south to north*.

268. The sections and half sections bounded on the northern and western lines of such townships, shall be sold as containing only the quantity expressed in the returns and plats respectively, and all others as containing the complete legal quantity."

The accompanying diagram, marked A (see sec. 271), will illustrate the method of running out the exterior lines of townships, as well on the *north* as on the south side of the base line.

OF MEASUREMENTS, CHAINING AND MARKING.

269. "Where uniformity in the variation of the needle is not found, the public surveys *must be made* with an instrument operating independently of the magnetic needle. *Burt's Solar Compass*, or other instrument of equal utility, *must* be used of necessity in such cases; and it is deemed best that such instruments should be used under all circumstances. Where the needle can be relied on, however, the ordinary compass may be used in subdividing and meandering."—Note Traversing.

BASE LINES, PRINCIPAL MERIDIANS, AND CORRECTION OR STANDARD LINES.

• 270. *Base Lines* are lines run due east and west, from some point assumed by the Surveyor General. North and south of this base line, townships are laid off, by lines running east and west.

Standard or Correction Lines are lines run east and west, generally at 24 miles north of the base line, and 80 miles south of it. These lines, like the townships, are numbered from the base line north or south, as the case may be.

Principal Meridians are lines due north and south from certain given points, and are numbered first, second, third, etc. Between these principal meridians the tiers of townships are called ranges, and are numbered 1, 2, 3, 4, etc., east or west of a given principal meridian.

All these lines are supposed to be run astronomically; that is, they are run in reference to the true north pole, without reference to the magnetic pole. In proof of this, it is well to state that the Old Instructions has shown, in the specimen field notes, that the true variation has been found. See pages 13 and 18, and in the New Instructions, pages 28 to 35, both inclusive. Here the method of finding the greatest azimuth is not given, although there is a table of greatest azimuths for the first day of July for the years 1851 to 1861, and for lat. 32° to 44° . At page 30 is given the mean time of greatest elongation for every 6th day of each month, and shows whether it is east or west of the true meridian.

At page 27 are given places near which there is no variation. At page 29 are given places with their latitudes, longitudes, and variation of the compass, with their annual motion.

The method of finding these for other places and dates is not given in either manual. For these, see sequel Canadian method of surveying side lines. For formulas and example, see sections 264a and 264b of this manual.

Principal Meridians. The 1st principal meridian is in the State of Ohio.

The 2nd principal meridian is a line running due north from the mouth of the Little Blue River, in the State of Indiana.

The 3d principal meridian runs due north from the mouth of the Ohio River to the State line between Illinois and Wisconsin.

The 4th principal meridian commences in the middle of the channel, and at the mouth of the Illinois River; passes through the town of Galena; continues through Illinois and Wisconsin, until it meets Lake Superior, about 10 chains west of the mouth of the Montreal River. For further information, see Old Instructions, page 49.

Ranges are tiers of townships numbered east or west from the established principal meridian, and these lines run north or south from the base line. They serve for the east and west boundary lines of townships. On these lines, section and quarter section corners are established. These corners are for the sections on the west side of the line, but not for those on the east side. (See Old Instructions, page 50, sec. 9.)

Note. This is not always the case. There are many surveys where the same post or corners on the west line of the township have been made common to both sides. This is admitted in the Old Instructions, page 54, sec. 21.

Townships are intended to be six miles square, and to contain 36 sections, each 640 acres. They are numbered north and south, with reference to the base line. Thus, Chicago is in township 39 north of the base line, and in range 14 east of the third principal meridian.

Township lines converge on account of the range lines being run toward the north pole, or due north. This convergency is not allowed to be corrected, but at the end of 4 townships north, and 5 south of the base line, this causes the north line of every township to be 76,15 links less than the south line, or 304,6 links in $\frac{1}{4}$ townships.

The deficiency is thrown into the west half of the west tier of sections in each township, and is corrected at each standard line, where there is a jog or offset made, so as to make the township line on the standard line six miles long. In surveying in the east 5 tiers of sections, each section

is made 80 chains on the township lines. In the east tier of quarter sections of the west tier, each quarter section is 40 chains on the east and west township and section lines.

Example. Let 1, 2, 3 and 4 represent 4 townships north of the base line. Township number 1 will be 6 miles on the base line, and the

North boundary of section 6, in township 1	= 7923,8 links.
North boundary of section 6, in township 2	= 7847,7 links.
North boundary of section 6, in township 3	= 7771,5 links.
North boundary of section 6, in township 4	= 7695,4 links.

Here we make the south line of sec. 36, in township 5 = 8000 links.

271. Townships are subdivided into 36 sections, numbered from east to west and west to east, according to the annexed diagram. Lot 1 invariably begins at the N.E. corner, and lot 6 at the N.W.; lot 36 at S.W., and lot 36 at the S.E. corner.

Surplus or deficiency is to be thrown into the north tier of quarter sections on the north boundary, and in the west tier of quarter sections on the west boundary of the township.

78,477	5	4	3	2	1	T. 2 N.
7	8	9	10	11	12	
18	17	16	15	14	13	
19					24	
30					25	
81					36	
79,238						T. 1 N.
80	R. I E.				R. II E.	

Base

Line.

Base

Line.

North and South Section Lines How to be Surveyed.

272. Each north and south section line must be made 1 mile, except those which close to the north boundary line of the township, so that the excess or deficiency will be thrown in the north range of quarter sections; viz., in running north between sections 1 and 2, at 40,00 chains, establish the quarter section corner, and note the distance at which you intersect the north boundary of the township, and also the distance you

fall east or west of the corresponding section corner for the township to the north; and at said intersection establish a corner for the sections between which you are surveying.—Old Instructions, p. 9, sec. 28.

East and West Section Lines. Random or Trial Lines.

* 278. All east and west lines, except those closing on the west boundary of the township, or those crossing navigable water courses, will be run from the proper section corners *east* on *random* lines (without blazing), for the corresponding section corners. At 40 chains set *temporary* post, and note the distance at which you intersect the range or section line, and your falling north or south of the corner run for. From which corner you will correct the line west by means of offsets from stakes, or some other marks set up, or made on the random line at convenient distances, and remove the *temporary* post, and place it at average distance on the *true* line, where establish the quarter section corner. The random line is not marked but as little as possible. The brushwood on it may be cut. The true line will be blazed as directed hereafter. The east and west lines in the west tier are by some run from corner to corner, and by others at right angles to the north and south adjacent lines.

East and West Lines Intersecting Navigable Streams.

274. Whenever an east and west section line other than those in the west range of sections crosses a navigable river, or other water course, you will not run a random line and correct it, as in ordinary cases, where there is no obstruction of the kind, but you will run east and west on a *true* line (at right angles to the adjacent north and south line) from the proper section corners to the said river or navigable water, and make an *accurate* connection between the corners established on the opposite banks thereof; and if the error, neither in the length of the line nor in the falling north or south of each other of the fractional corners on the opposite banks, exceeds the limits below specified in these instructions for the closing of a whole section, you will proceed with your operations. If, however, the error exceeds those limits, you will state the amount thereof in your field notes, and proceed forthwith to ascertain which line or lines may have occasioned the excess of error, and reduce it within proper bounds by re-surveying or correcting the line or lines so ascertained to be erroneous, and note in your field book the whole of your operations in determining what line was erroneous, and the correction thereof. (See Old Instructions, p. 10, sec. 32.) Limits in closing = 160 links.

Note. From sec. 272 we find that the north and south lines are intended to be on the true meridian from the south line of the township to its north boundary. This is the intention of the act Feb., 1805. From sec. 278 we find that in the east 5 tiers of sections of every township, a *true* line is that which is run *from post to post*, or from "a corner to the corresponding corner opposite."

But in the west tier of sections, a *true* line is that which is run at right angles to the adjacent north and south line; that is, the north and south line must be run before the east and west line can be established. This agrees with the above act, which requires that certain lines are to be run due east or west, as the case may be.—Old Instructions, p. 10.

Note 2. In many of the old surveys, the field notes show that the surveyor ran from corner to corner on the west and north tiers of sections. This appears to be the case in town. 89 N., range 14 E., of the third principal meridian, surveyed by Mr. John Wall in 1821. This will also appear from the Old Instructions, p. 54, sec. 21, where it is stated as follows: "Previous to 1828, some of the deputies considered, in making the calculations of the area of the north and west tiers of quarter sections in a township, that the quarter section corners on the township and range lines were common to the sections on both sides of the line, whilst others adopted the method now in use, and particularly explained in diagram numbers 4 and 5." At one time, some of the deputy surveyors, in subdividing a township through which a navigable stream passed, ran a random line east between the proper sections, and corrected it west, making the corner to the fractional sections on both banks of the river, and on the true line. Others pursued the method as now required.

Note 3. From this appears the necessity of having a copy of the original Field Notes, so as to know which method to pursue in order to re-establish any line or corner, and to know if the surveyor deposited charcoal at the corner or corners.

Insurmountable Obstacles. Witness Points.

275. Whenever your course may be obstructed by insurmountable obstacles, such as ponds, swamps, marshes, lakes, rivers, creeks, etc., you will prolong the line across such obstacles, by taking the necessary right angle offsets, or, if this is inconvenient, by a traverse or trigonometrical operation, until you regain the line on the opposite side. And in case a north and south, or a true east and west line is regained in advance of any obstacle, you will prolong and mark the line back to the obstacle so passed, and state all the particulars in relation thereto in your field notes; and at the intersection of lines with both margins of impassable obstacles, you will establish a witness point (for the purpose of perpetuating the intersections therewith), by setting a post, and giving in your field notes the course and distance therefrom to two trees on opposite sides of the line, each of which trees you will mark with a blaze, and notch facing the post, except on the margins of navigable water courses or navigable lakes. In these cases you will mark the trees with the proper number of fractional section, township and range.—O. I., p. 7, sec. 22. This sec. is the same as that in the N. I., p. 3.

Note. In fractional townships it appears that the north and south lines are first established, and afterward the east and west lines are run due east and west, or at right angles to the north and south lines. (See the U. S. act 11 Feb., 1805, and sections 272 and 278.)

Note 2. It appears from the manuals that, where a north and south line meets a navigable stream, it runs directly across it, and that some surveyors run directly across navigable waters on east and west lines, and others close on it, showing the jog or difference in closing, which must not be more than 150 links from a straight line. Hence it appears that where the official field notes do not show a jog, none exists.

Limits in Closing.

276. Any excess or deficiency in the length of any township boundary
j

line, or excess of error in the falling off from the corner to which any closing township line shall be run, that may exceed five chains, or any excess or deficiency exceeding one chain in the length of any section line, or excess of error in falling off from the corner to which any section line shall be run, that shall exceed one chain in closing the lines of a whole section; and at the same rate for the section lines, and at the same rate of one chain and fifty links per mile of the meanders (*traverse survey*), in closing the meanders of a navigable river, or other water course, with the line or lines of a fractional section, must be corrected by you, and reduced within those limits, before leaving the ground, by resurveying the line or lines which may have occasioned the excess or deficiency in the length of such township or section line, or excess of error in closing the lines of a township, or of a whole or fractional section.—O. L., p. 8, sec. 25.

Meandering of Navigable Streams.

277. Standing with the face looking down the stream, the bank on the left is termed the "left bank," and that on the right, the "right bank." Both these banks are to be meandered (traversed) by taking the bearings and distances of their sinuosities, and the same entered in the field book.

Where township or section lines cross navigable waters, the line is marked by posts, and bearing trees, and mounds, as the case may require at the time of running those lines.

In meandering, you are to commence at one of those corners and meander to the next meander corner, carefully noting your intersections with all meander corners. The opposite bank is to be meandered in like manner.

The crossing distance *between the meander corners, on the same line*, is to be ascertained by triangulation, in order that the river may be protracted with entire accuracy. The particulars to be given in the field book.

All lakes and deep ponds of the area of 25 acres and upwards, also all bayous, shallow ponds readily to be drained, are not to be meandered.

Notice all streams of water falling into the river, lake or bayou you are surveying, stating the width at their mouth; also all springs, noting their size, depth, and kind of water; also the head and mouth of all bayous, and all islands, rapids and bars are to be noticed, with intersections to their upper and lower points to establish their exact situation. Note the elevation of the banks of rivers and streams, the heights and falls of cascades, and the length of rapids.

Trees are Marked for Line and Bearing Trees.

278. All trees left standing on a line are marked with 8 marks or blazes on front and rear; the distance on the line, the kind of timber, and diameter, is marked in the field book.

Trees are also marked on both sides of the line, making one blaze on the front, another on the rear, and one on the side next the line. By these means the situation of the line may be determined approximately at a future time. (See sec. 211.)

Note. It is hard to expect a true line from corner to corner where trees are allowed to stand on the line. The Canadian plan of cutting everything off the line is preferable.

279. *Township corners* are marked by a post, whose 4 sides are marked

by 6 notches, directed to the 4 cardinal points. Posts to be 3 feet above the ground, not pointed but set in the ground. There are to be, when possible, 4 bearing trees marked, 1 in each township, each having a large blaze facing the post, having marked thereon with a marking iron the range, township and section.

280. *Section corners* have 4 bearing trees, one in each section. *Quarter section corners* have but two bearing trees. The bearings, distances, wood, and diameter of these are entered in the field book.

The section corner post will be notched on two sides.

Example. The post corner sections 9, 10, 15, 16, will have 2 marks or notches on the north side and 4 on the south side, to signify that this is 2 miles from the north boundary and 4 miles from the south boundary. Bearing trees have the sections cut on them.

Note. It would be well in this example to have 3 marks on the east side, to show that it is three miles from the east boundary of the township. These notches would at any future time enable us to know which corner we are at in the absence of bearing trees.

281. *Township corners in a prairie*, or other situation where bearing or witness trees are not at hand, will be perpetuated by depositing in the ground, and at least 3 inches beneath the natural surface thereof, a portion of charcoal (the quantity to be specified in your field notes), not less than two quarts at the place of such corner, over which you will erect a mound of earth 3 feet high, 5 feet square at the base, and 2 feet square at the top, the sides whereof must be revetted or faced with sods laid horizontally and in successive layers on each other, each of said layers having an offset inwards, corresponding to the general slope of the face of the mound; in this there is to be a post which must rise 1 foot above the top of the mound. Or you will deposit at the place of the corner 3 stones, not less than 5 inches square by 3 inches thick, all of which you will describe in the field notes. The top of the uppermost stone to be 8 inches below the natural surface of the ground, and the other 2 successively and beneath the first; and over said stone erect a mound, as if it were made over charcoal or stone. You may perpetuate the corner by inserting endways into the ground, a stone of the dimensions marked, and set in the manner mentioned, over which no mound need be erected.

282. *If a township corner, where bearing or witness trees are not to be found within a reasonable distance therefrom*, shall fall within a ravine, or any other situation unfavorable to the erection of a mound, you will select in the vicinity thereof a suitable plot of ground as a site for a bearing or witness mound, and erect thereon a mound of earth, as if it were at the township corner, with charcoal or stone deposited beneath, as before directed for township corner, and measure, and state in your field notes the distance and course from the position of the true corner, of the bearing or witness mound so placed and erected.

Witness Mounds to Section Corners.

283. *Section corners, where bearing or witness trees cannot be had*, will be made as if for a township corner, except that where mounds are made they need be only 2 feet 6 inches high, by 4 feet square at the base, and 2 feet at the top.

Witness mounds to quarter section corners are 3 feet 6 inches at base, 1 foot 6 inches square at top. Under the post no charcoal or stone is put. Old Instructions, sec. 18, 19, 20 and 21.

In the above we do not find the manner of digging the pits for the mounds, but the following from the New Instructions shows the manner now adopted:

"At the point or corner dig out a spadeful or two, and in the cavity put a stone or charcoal, which note in the field book. A charred stake is driven 12 inches in the ground, and in the centre of the cavity. Either of these will be a witness for the future. The spot from which the earth for the mound is taken is called a pit. The pits are made at uniform distances from the centre on each side of every mound, so as to indicate hereafter the place of the mound, when it may become obliterated by time or accident. Mounds are to present a conical shape. There is dug around the base a quadrangular trench, 1 spadeful deep of earth. At about 18 inches from this trench, dig pits at the 4 cardinal points; the length of the north and south pits to be east and west, and those of the east and west pits to be north and south.

284. *For corner mounds common to 2 townships, or 2 sections, the quadrangular trenches at north and south are at right angles to the north and south line; but where the mound is common to 4 townships, or 4 sections, the north and south line cuts the quadrangle diagonally.*" (See New Instructions, diagram C.)

Courses and Distances to Witness Trees.

285. You will ascertain and state in your field notes the course and distance from the several section and township corner posts, trees or stones, to a tree in each section for which they stand as a corner. Each of said trees you will mark with a notch and blaze, facing the post, tree or stone; the notch to be at the lower end of the blaze, and on it, which must be neatly made. Mark with a marking iron, in a plain, permanent manner, the letter S, with the number of the section, and over it the letter T, with the number of the township, and above this the letter R, with the number of the range. Where there is no tree within reasonable distance of the corner, note that in the field notes.—O. I., p. 5, sec. 17.

286. From this we infer that at township corners 4 trees are marked, and also at section corners; but at quarter section corners only 2 are marked.

Note. This does not point out the necessity of describing the kind of tree marked, and its diameter, although we find that all the Government field notes give the kind of tree, and its diameter. In the specimen field book, the bearing trees are described with reference to their diameter and wood.

Quarter section corners have simply a blaze, and $\frac{1}{4}$ S marked on it, and no bearing trees marked.

Note 2. Surveyors, in getting out of wood into open land or prairie, are always accustomed to mark line trees, and to describe the kind of wood and their diameters, and, if possible, they mark another tree or trees on the opposite side of the mound, on the same line. By these means the place of the mound can be determined, if these trees are to be found.

287. *Corner Stones.* These stones not to be less than a stone 14 inches long by 12 inches wide, and 3 inches thick; are set with the edge being north and south, for north and south lines, and east and west, for east and west lines; they are to be 7 or 8 inches in the ground. Where stones are placed as range and township lines, as many notches will be distinctly cut with a chisel or pick on the two sides, in direction of the line, as the corner is sections from the nearest section corners. At township corners, 6 notches will be cut on each edge or side toward the cardinal points. At section corners in the interior of a township, as many notches will be cut on the south edge and east side as the corner is sections distant from the south and east boundaries of the township. At the corners subdivisional intersections with the north boundaries of the townships, notches on the south edge, and at the intersections with the west boundary, 6 notches on the east edge, and as many notches on the north or south edges, as the case may require, as the corner is sections distant from the township corner. Quarter section corner stones will have $\frac{1}{4}$ S on the west side, on north and south lines; and on the north side, on east and west lines.—Old Instructions, p. 5, sec. 16 and 17.

88. *Note.* These two sections from the Old Instructions are the same in substance as those given in the New Instructions. 7 or 8 inches without ground will not be deep enough; better have it at least 1 foot in the ground, and then build a heap of stones around it, and afterwards a mound. Under every stone there ought to be some charcoal, broken delf, glass or a piece of iron, or a stone, into which a hole 1 inch deep is cut, to perpetuate the centre or true corner.

The manuals do not mention which part of the stone is to be taken, the usual custom is to take the middle, unless something to the contrary is mentioned.

The New Manual of Instructions gives a specimen field book of township No. 2, range 2 W., Willamette meridian.

From these it appears that the corners on the north and west boundaries of the township have been made common to the adjacent townships.

That in every case the north and south lines crossed rivers and lakes on the same line, excepting on the line between 4 and 5, where the river crossed within 4,00 links of the north boundary. Here he ran south as follows:

From the corner to sections 4, 5, 32 and 33, on the north boundary of township, I run—

North on a true line, between sections 4 and 5.

Variation 18° east.

A white oak, 15 inches diameter.

Set a post on the right bank of Chickeeles River, for corner to original sections 4 and 5, from which a bur oak, 16 inches diameter, N. 25° E., $3\frac{1}{2}$ links distance; a black oak, 20 inches diameter, bears S. W., 21 links.

From these field notes we find that the north and south lines have crossed a navigable stream and lakes on 8 lines, and on straight lines, the above exception.

That on the east and west lines, 4 lines crossed a navigable river, 2 of which are on a straight line, one makes a jog of 12 links, and the other of 16 links.

Manual of Old Instructions gives specimen field notes, showing that the variation has been taken, and that compared with the east boundary of the township. That the north and south section lines are run straight across navigable water.

That jogs are made on the north and west boundary lines. That the amount of each jog or offset is invariably given.

That on east and west lines crossing navigable rivers, jogs are made, and the amount shown, how determined, and the distance between the corresponding corners on the river.

Both Instructions show that in meandering, the surveyor begins at some fractional corner, and traverses along the stream to the next corner, and so on from section line to section line.

Note 2. Hence it appears that a traverse survey is not to be used in determining where a section line crossed or met a river. (See Re-establishing Lost Corner.)

SUMMARY OF OBJECTS AND DATA REQUIRED TO BE NOTED.

289. Lengths of all lines, with the offsets thereon.

The kind and diameter of all bearing trees, with the courses and distances of the same from their respective corners.

The material of which mounds are constructed; the bearing of the pits from which the earth was dug from the centre; the material put under the post—if charcoal, etc.; how much.

Trees on the line; their names, diameters and distance on the line.

Intersections by the line of *land objects*, and the distance to each settler's claim, and also at leaving it.

Distances to and at leaving woods, prairies, rivers, creeks, lakes, ravines, top and bottom of hills, bottom or swamp lands, underbrush or brushwood.

Describe the *surface* of the land—whether level, rolling, or hilly.

The *soil*—whether first, second, or third rate quality.

The several sorts of *timber* and undergrowth, in the order in which they predominate.

Bottom lands—whether wet, dry, or subject to inundation. If subject to be inundated, show to what depth.

Note. It would be well, as in Europe, to show the high and low water mark.

Springs of water—if fresh, saline, or mineral; with the course of the stream flowing from them.

Note. This supposes that the surveyor has a knowledge of chemistry. To those who have not such knowledge, we refer them to the *Sequel* for our sections on analysis of soils, water, and ashes of plants.

Lakes and ponds—describe their banks; give their height, depth of water, and whether the water is pure or stagnant.

Note. State if there is any inlet or outlet, and the course of these coming to or leaving the lakes or ponds.

Improvements, towns and villages, Indian towns and wigwams, houses, cabins, fields, sugar tree groves, sugar camps, mill seats, forges and factories.

Coal banks or beds, *peat* or *turf* grounds, *minerals* and *ores*; with a description of their quality and extent, and all diggings therefor; also salt springs and lakes.

Roads and trails (or pathways), with their directions—whence they come and whither they go.

Rapids, cataracts, cascades, or falls of water, with the height of their falls in feet.

Precipices, caves, sink holes, ravines, stone quarries, ledges of rocks, with the kind of stone they afford.

Natural curiosities, interesting fossils, petrifications, organic remains, etc.; also all ancient work of art, such as mounds, fortifications, embankments, ditches, or objects of like nature.

The variation of the needle must be noted at all points or places on the lines where there is found any material change of variation, and the positions of such points must be perfectly identified in the field notes.

Besides the ordinary notes taken on the line (and which must always be written down on the spot, leaving *nothing* to be supplied by memory), the deputy will subjoin, at the conclusion of his book, such further description or information touching any matter or thing connected with the township (or other survey), which he may be able to afford, and may deem useful or necessary to be known, with a general description of the township in the aggregate, as respects the face of the country, its soil and geological features, timber, minerals, water, etc.—New Instructions, pp. 17, 18.

Note. Surveyors keep a loose or random field book, where they keep the notes in running lines from corner to corner, and crossing rivers, lakes, etc.; and when they have performed the work, copy the result into the proper field book. That this has been the custom with surveyors will appear by referring to their field books, where the crossing distances on rivers, etc., are given without the necessary notes.

290. *Field Notes of the Subdivision Lines and Meanders of Chickeeles River, in Township 25 N., Range 2 W., Willamette Meridian.*

Chains.

To determine the *proper adjustment* of my compass for subdividing this township, I commence at the corner to townships 24 and 25 N., range 1 and 2 W., and run *north* on a blank line along the east boundary of section 36,

Variation $17^{\circ} 51' E.$

40,05
80,09

To a point 5 links west of the quarter section corner,

To a point 12 links west of the corner to sections 25 and 36.

To retrace this line, or run parallel thereto, my compass must be adjusted to a variation of $17^{\circ} 46' E.$

Note. 12 divided by 8009 gives ,00149, which is the natural sine of 5 minutes; and because the blank line was run west of the true line, the correction is to be taken from $17^{\circ} 51'$, which will give the true magnetic variation of the east side of section 36. The variation here is not that from the true meridian, but is that from the section line assumed as base or meridian.

Subdivision Commenced February 1, 1851.

291. From the corner to sections 1, 2, 3, 5 and 36, on the south boundary of the township, I ran—

Chains.	
9,19	A beech, 30 inches diameter (marked 3 blazes on front and rear).
29,97	A beech, 30 inches diameter (on line marked as above).
40,00	Set a post for quarter section corner, from which— A beech, 8 inches diameter, bears N. 23° W., 45 links dist. A beech, 15 inches diameter, bears S. 48° E., 12 links dist.
51,00	A beech, 18 in. diam. (this is a line tree, and marked as above).
71,00	A sugar tree (maple), 30 inches diameter.
80,00	Set a post for corner to sec. 25, 26, 35 and 36, from which— A beech, 28 inches diameter, bears N. 60° E., 45 links dist. A beech, 24 inches diameter, bears N. 62° W., 17 links dist. A poplar, 20 inches diameter, bears S. 70° W., 50 links dist. A poplar, 36 inches diameter, bears S. 66° E., 34 links dist. Land level; second-rate quality. <i>Timber</i> —poplar, beech, sugar tree, and some oak, undergrowth and hazel.
Chains.	
	East on a random line, between sections 25 and 36, Variation 17° 46' E.
9,00	A brook, 20 links wide, runs north,
15,00	To foot of hill; bears north and south.
40,00	Set a post for temporary quarter section corner,
55,00	To opposite foot of hill; bears north and south.
72,00	A brook, 15 links wide, runs north.
80,00	Intersected the east boundary at the corner post to sections 25 and 36, from which corner I ran— West, on a true line, between sections 25 and 36. Variation 17° 46' E.
40,00	Set a post on top of hill; bears north and south; from which— A hickory, 14 inches diameter, bears N. 60° E., 27 links dist. A beech, 15 inches diameter, bears S. 74° W., 9 links dist.
80,00	The corner to sections 25, 26, 35 and 36. Land—east and west parts level; first-rate. Middle part broken, and third-rate. <i>Timber</i> —beech, oak and ash; undergrowth; same and spice in the branch bottoms.
Chains.	
	We proceed as above, N. bet. 25 and 26, then E. bet. 25 and 36. Again, N. between 23 and 24, then E. between 24 and 25. Again, N. between 14 and 13, then E. between 13 and 24. Again, N. between 11 and 12, then E. between 12 and 13. Again, N. between 1 and 2, as follows: North, on a random line. Variation 17° 46' E.
40,00	Set a temporary post for quarter section corner.
80,11	Intersected the north boundary of the township 82 links east of corner to sections 1 and 2, from which corner I ran— South, on a true line, between sections 1 and 2. Variation 18° E.
40,11	Set a post for quarter section corner, from which— A white oak, 20 inches diameter, bears N. 31° W., 65 links dist.

Chains. A sugar tree, 14 inches diameter, bears S. 49° E., 32 links dist.
 80,11 The corner to sections 1, 2, 11 and 12.
 Land level; good; rich soil.
Timber—walnut, sugar tree, beech, and various kinds of oak;
 open woods. February 2, 1851.

Note. Here we find that the line between sections 1 and 2 is run from post to post, making no jog or offset on the north boundary of the township; and that the south quarter sections in the north tier of sections are 40 chains, from south to north, leaving the surplus of 11 links in the north tier of quarter sections.

Field Notes of a Line Crossing a Navigable Stream on an East and West Line.

292. West, on a *true line*, between sections 30 and 31, knowing that it will strike the Chickeeles River in less than 80,00 chains. Variation $17^{\circ} 40'$ E.

8,41 A white oak, 15 inches diameter.
 5,00 Leave upland, and enter creek bottom, bearing N.E. and S.W.
 8,00 Elk creek, 200 links wide; gentle current; muddy bottom and banks; runs S.W.

Ascertained the distance across the creek on the line as follows:

Cause the flag to be set on the right bank of the creek, and in the line between sections 30 and 31. From the station on the left bank of creek, at 8,00 chains, I run *south* 245 links, to a point from which the flag on the right bank bears N. 45° W., which gives for the distance across the creek, on the line between sections 30 and 31, 245 links.

25,17 A bur oak, 24 inches diameter.

40,00 Set a post for quarter section corner, from which—
 A buck-eye, 24 inches diameter, bears N. 15° W., 8 links dist.
 A white oak, 30 inches diameter, bears S. 65° E., 12 links dist.
 41,90 Set a post on the left bank of Chickeeles River, a navigable stream, for corner to fractional sections 30 and 31, from which—
 A buck-eye, 16 inches diameter, bears N. 50° E., 16 links dist.
 A hackberry, 15 inches diameter, bears S. 79° E., 14 links dist.
 Land and timber described as above.

Note. We find this part of the line between sections 30 and 31 in the *Manual of New Instructions*, page 35, and the other part in page 42, as follows:

From the corner to sections 30 and 31, on the west boundary of the township, I ran—
 East on a *true line*, between sections 30 and 31.

Variation 18° E.

16,10 A white oak, 16 inches diameter.
 23,50 Intersected the right bank of Chickeeles River, where I set a post for corner to fractional sections 30 and 31, from which—
 A black oak, 16 inches diameter, bears N. 60° W., 25 links dist.
 A white oak, 20 inches diameter, bears S. 85° W., 32 links dist.
 k

Chains. From this corner I run south 12 links, to a point west of the corner to fractional sections 30 and 31, on the left bank of the river. Thence continue south 314 links, to a point from which the corner to fractional sections 30 and 31, on the left bank of the river, bears N. 72° E., which gives for the distance across the river 9,65 chains. The length of the line between sections 30 and 31, is as follows:

Part east of the river,	41,90 chains.
Part across the river,	9,65 "
Part west of the river,	23,50 "
Total,	75,05 chains.

Note. Here the method of finding the distance across the river, and of showing the amount of the jog or deviation from a straight line, is shown.

MEANDERING NOTES. (*New Manual*, p. 42.)

293. Begin at the corner to fractional sections 25 and 30, on the range line. I chain south of the quarter section corner on said line, and run thence down stream, with the meanders of the left bank of Chickeeles River in fractional section 30, as follows:

	Chains.	
S. 41° E.	20,00	At 10 chains discovered a fine mineral spring.
S. 49° E.	15,00	Here appeared the remains of an Indian village.
S. 42° E.	12,00	
S. 123° E.	5,80	To the fractional sections 30 and 31.
		Thence in section 31,
S. 12° W.	13,50	To mouth of Elk River, 200 links wide; comes from the east.
S. 41° W.	9,00	At 200 links (on this line) across the creek.
S. 58° W.	11,00	
S. 35° W.	11,00	
S. 20° W.	20,00	At 15 chains, mouth of stream, 25 links wide, comes from S.E.
S. 283° W.	8,80	To the corner, to fractional sections 31 and 36, on the range line, and 8,56 chains north of the corner to sections 1, 6, 31 and 36, or S.W. corner to this township.
		Land level, and rich soil; subject to inundation.
		Timber—oak, hickory, beech, elm, etc.

RE-ESTABLISHING LOST CORNERS. (*New Instructions*, p. 27.)

294. Let the annexed diagram represent an east and west line between

Sec. 31.	Sec. 32.	Sec. 33.	Sec. 34.	Sec. 35.	Sec. 36.
	<i>d</i>	<i>a</i>			
	<i>c</i>	<i>b</i>			
Sec. 6.	Sec. 5.	Sec. 4.	Sec. 3.	Sec. 2.	Sec. 1.

two townships, and that all traces of the corner to sections 4, 5, 32 and 33 are lost or have disappeared. I restored and re-established said corner in the following manner:

Begin at the quarter section corner marked *a* on diagram, on the line between sections 4 and 33. One of the witness trees to this corner has fallen, and the post is gone.

The black oak (witness tree), 18 inches diameter, bearing N. 25° E., 32 links distance, is standing, and sound. I find also the black oak station or line tree (marked *b* on diagram), 24 inches diameter, called for at 37.51 chains, and 2.49 chains west of the quarter section corner. Set a new post at the point *a* for quarter section corner, and mark for witness tree. A white oak, 20 inches diameter, bears N. 34° W., 37 links dist. West with the *old marked line*.

Variation 18° 25' E.

At 40.00 chains, set a post for temporary corner to sections 4, 5, 32 and 33.

At 80.06 chains, to a point 7 links south of the quarter section corner (marked *c* on diagram), on line between sections 5 and 32. This corner agrees with its description in the field notes, and from which I run east, on a *true line*, between sections 5 and 32.

Variation 18° 22'.

At 40.03 chains, set a lime stone, 18 inches long, 12 inches wide, and 3 inches thick, for the *re-established corner* to sections 4, 5, 32 and 33, from which—

A white oak, 12 inches diameter, bears N. 21° E., 41 links dist.

A white oak, 16 inches diameter, bears N. 21° W., 21 links dist.

A black oak, 18 inches diameter, bears S. 17° W., 32 links dist.

A bur oak, 20 inches diameter, bears S. 21° E., 37 links dist.

Note 1. The diagram, and letters *a*, *b*, *c*, and that part in parentheses, are not in the Instructions.

Note 2. Hence it appears that the surveyor has run between the nearest undisputed corners, and divided the distance *pro rata*, or in proportion to the original subdivision. Although in this case the line has been found blazed, and one line or station tree found standing, the required section corner is not found by producing the line from *a*, through *b*, to *d*. Although I have met a few surveyors who have endeavored to re-establish corners in this manner, I do not know by what law, theory or practice they could have acted. It is in direct violation of the fundamental act of Congress, 11 Feb., 1806, which says that lines are to be run "*from one corner to the corresponding corner opposite*." (See sequel *Geodetical Jurisprudence*.)

Re-establishing Lost Corners. (From *Old Instructions*, p. 63.)

295. Where old section or township corners have been completely destroyed, the places where they are to be re-established may be found, in *timber*, where the old blazes are tolerably plain, by the *intersections* of the east and west lines with the north and south lines.

If in *prairie*, in the following manner:

Let the annexed diagram represent part of the township. This example is often given: Suppose that the corner to sections 25, 26, 35 and 36 to be missing, and that the quarter section corner on the line between sections 25 and 36 to be found. Begin at the said quarter section corner, and run north on a *random line* to the *first* corner which can be identified, which we will suppose to be that of sections 23, 24, 25 and 26.

15	1 4	1 3
22	2 3	2 4
27	2 6	2 5
34	3 5	3 6

At the end of the first 40 chains, set a *temporary* post corner to sections 25, 26, 35 and 36. At 80 chains, set a temporary quarter section corner post, and suppose also that 121,20 chains would be at a point due east or west of said corner 23, 24, 25 and 26. Note the falling or distance from the corner run for, and the distance run. Thence from said corner run south on a *true line*, dividing the surplus, 1,20 chains, equally between the three half miles, viz.: At 40,40 chains, establish a quarter section corner. At 80,80 chains, establish the corner to sections 25, 26, 35 and 36. Thence to the quarter section corner, on the line between sections 35 and 36, would be 40,40 chains.

The last mentioned section corner being established, east or west random or true lines can now be ran therefrom, as the case may require.

This method will in most cases enable the surveyor to renew missing corners, by re-establishing them in the right place.

But it may happen that after having established the north and south line, as in the above case, the corner to sections 26, 27, 34 and 35 can be found; also the quarter section corner on the line between 26 and 35. In this case it might be better to extend the line from the corner 26, 27, 34 and 35, to said quarter section corner, *straight* to its intersection with the north and south line already established, and there establish the corner to sections 25, 26, 35 and 36. If this point should differ much from the point where you would place the corner by the first method laid down, it might be well to examine the line between sections 25 and 36.

Note 1. Hence it appears that the north and south lines are first established, in order that the east and west lines may be run therefrom; and that when the east and west lines can be correctly traced to the north and south line, that the point of intersection would be the required corner. It is also to be inferred that where the lines on both sides can be traced to the north and south line, a point equidistant between the points of intersection would be the required corner.

Note 2. It will not do to run from a section or quarter section corner on the west side of a north and south line, to a section corner, or quarter section, on the east side of the line, and make its intersection with the north and south line, the required corner, unless that these two lines were originally run on the same variation, which is seldom the case.

Note 3. Having found approximately the missing corner, we ought to

search diligently for the remains of the old post, mound, bearing trees, or the hole where it stood.

Bearing trees are sometimes so healed as to be difficult to know them. By standing about 2 feet from them, we can see part of the bark cut with an even face. We cut obliquely into the supposed blaze on the tree to the old wound. We count the layers of growth, each of which answers to one year. By these means we find the years since the survey has been made, which, on comparing with the field notes, we will always find not to differ more than one year.

Remains of a post, or where it once stood, may be determined as follows: Take the earth off the suspected place in layers with a sharp spade. By going down to 10 or 12 inches, we will find part of the post, or a *circular surface*, having the *soil black and loose*, being principally composed of vegetable matter. By putting an iron pin or arrow into it, we find it partially hollow. We dig 6 feet or more around the suspected place. Where such remains are found, we make a note of it, and of those present. Put charcoal, glass, delf, or slags of iron, in the hole, and re-establish the corner, noting the circumstances in the field book.

Ditches or lockspitting are sometimes made on the line to perpetuate it. This will be an infallible guide, and we only require to know if the edge or centre of the ditch was the line or boundary, or was it the face or top of the embankment. These answers can be had from the record, or from the persons who have made the ditch, or for whom it has been made. Should this ditch be afterwards ploughed and cultivated, we can see in June a difference in the appearance of the plants that grow thereon, being of a richer green than those adjoining the ditch. Or, we dig a trench across the suspected place. The section will plainly show where the old ditch was, for we will find the black or vegetable mould in the bottom of the old ditch. We may have the line pointed out by the oldest settlers, who are acquainted with the locality. Surveyors ought to spare no pains to have all things so correctly done as to prevent litigation, and to bear in mind that "*where the original line was, there it is, and shall be.*"

ESTABLISHING CORNERS. (*Old Instructions*, p. 62.)

296. In surveying the public lands, the United States Deputy Surveyors are required to *mark only the true lines*, and establish on the ground the corners to townships, and sections, and quarter sections, on the range, township and sectional lines.

There are, no doubt, many cases where the corners are not in the right place, more particularly on *east* and *west* sectional lines, which, doubtless, is owing to the fact that some deputy surveyors did not always run the random lines the whole distance and close to the section corner, correct the line back, and establish the quarter section corner on the true line, and at average distance between the proper section corner; but only ran east or west (from the proper section corner) 40,00 chains, and there established the quarter section corner.

In all cases where the land has been sold, and the corners can be found and *properly identified*, according to the original approved field notes of the survey, *this office has no authority to remove them.*

RE-ESTABLISHING CORNERS IN FRACTIONAL SECTIONS, AND ALSO THE INTERIOR CORNER SECTIONS. (*Old Instructions*, p. 55.)

Present Subdivision of Sections.

297. None of the acts of Congress, in relation to the public lands, make any special provision in respect to the manner in which the subdivisions of sections should be made by deputy surveyors.

The following plan may, however, be safely adopted in respect to all sections, excepting those adjoining the north and west boundaries of a township, where the same is to be surveyed:

Let the annexed diagram represent an interior section, as sec. 10. B, D, H and F are quarter section corners. Run a true line from F to D; establish the corner E, making $DE = EF$; then make straight lines from E to B and from E D to H, and you have the section divided into quarters.

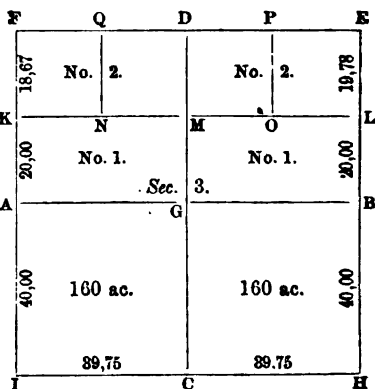
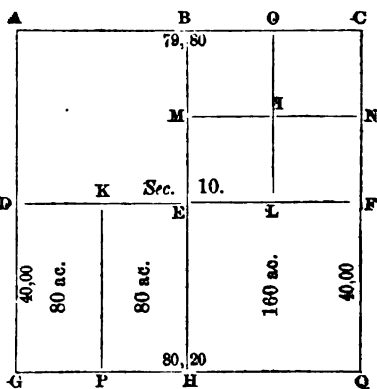
If it is required to subdivide the N. E. quarter into 40 acre tracts, make $EL = LF$, and $BO = OC$, and $GP = PH$, and $DK = KE$; also $EM = MB$, and $FN = NC$. Run from M to N on a true line, and make $MI = IN$. Here the N. E. quarter section is divided into 4 parts, and the S. W. quarter section into two halves.

Note. As the east and west sides of every regular section is 80 chains, and that the quarter section corners on the north and south sides are at average distances, it is evident that the line BH will bisect DF , or any line parallel to GQ . Consequently the method in the section is the same in effect as that in the next.

But if, by a re-survey, we find that AB is not equal to BC , or that GH is not equal to HQ , then we measure the line from D to F , and establish the point E at average distance.

298. Let the annexed diagram represent a subdivision of section 3, adjoining the north boundary of a township, being a fractional section.

In this case, we have on the original map $AF = 38,67$, $BE = 39,78$, $DE = 39,75$, $FD = 39,95$, $IC = 39,75$, and $CH = 39,75$. The S. E. and S. W. quarter sections each equal to 160 acres. Lot No. 1 each equal to 80 acres. In the N. W. quarter section the west half of lot 2 = 87,41 acres, and the east half



of lot 2 = 37,96 acres. These areas are taken from the original survey. In the N.E. quarter section, the west half of lot 2 = 38,23 acres, and the east half of lot 2 = 38,78.

In this example, there can be but *one rule* for the subdivision, to make it agree with the manner in which the several areas are calculated. You will observe that the line I H is 79,50 chains, and that the one half of it, = 39,75, is assumed as the distance from E to D, which last distance, 39,75, is deducted from 79,50, the length of the line E F leaving 39,95 chains between the points F and D. Consequently the line C D must be *exactly* parallel to the line H E, without paying any respect to the quarter section corner near D, which belongs entirely to section 84 of the township on the north. Run the line A B in the same manner as that of D F on diagram sec. 297, except that the corner G is to be established at the point where the line A B intersects the line C D. After surveying thus far, if the S.E. and S.W. quarters are to be subdivided, it can be done as in diagram sec. 297. In this case, to subdivide the N.E. and N.W. quarters, the line K L *must be parallel to* A B. The two lines ought to be 20 chains apart. The corner, M, is made where K L is intersected by C D. But as *two surveyors seldom agree exactly* as to distances, there might be found an excess or deficiency in the contents of the N.E. and N.W. quarters. If so, the line K L should be so far from A B as to *apportion the excess or deficiency* between lots 1 and 2, not equally, but in proportion to the quantities sold in each. If the lots numbered 2 are divided on the township plat by north and south lines, then that of the N.W. quarter must have its south end equidistant between K and M, and its north end equidistant between F and D. The N.E. quarter will be subdivided by a line parallel to M D and L E, *exactly* half way between them.

Note. Here we have the quarter section corners A, B, C and D given, and where the line A B intersects C D, gives the interior quarter section corner.

We find also that $A K = B L = 20$ chains generally, and that $K N = N M$, and $F Q = Q D$. Also $M O = O L$, and $D P = P E$.

Let us suppose that the original map or plat in this example gave the N.E. quarter 157 acres—that is, lot 1 = 80 and lot 2 = 77 acres, and that in surveying this quarter section we find the area = 159 acres, then we say, as 157 : 159 :: 80 to the surplus for lot 1, or, as 157 : 159 :: 77 to surplus in lot 2; and having the corrected area of lot 1, and the lengths of B G and L M, we can easily find the width B L.

Note 2. The above method of establishing the interior corner, M, is according to the statutes of the State of Wisconsin, and appears to be the best, as the original survey contemplates that the lines I F, H E, F E, I H, A B and C D are straight lines.

Government Plats or Maps.

299. The plats are drawn on a scale of 40 chains to one inch. The section lines are drawn with faint lines; the quarter section lines are in dotted lines; the township lines are in heavy lines. The number of the section is above the centre of each section, and its area in acres under it. On the north side of each section is the length thereof, excepting the south section lines of sections 82, 83, 84, 85 and 86. The section corners on the township lines are marked by the letters A, B, C, D, etc., A being at

the N.E. corner, G at the N.W., N at the S.W., and T at the S.E. The quarter section corners are marked by a, b, c, d, etc., a being between A and B, f between G and F, n between N and O, and s between S and T. (See New Instructions, diagram B.)

Note. On the maps or plats which we have seen, A begins at N.W. corner and continues to the right, making F at the S.W. corner of the township. The quarter section corner on the north side of every section is numbered 1, 2, 3, 4, 5 and 6, beginning on the east side, and running to the west line. Number 1 is at the quarter section corner on the north side of each section, 12, 13, 24, 25 and 36. Number 6 is at the quarter section corners on the north side of each, of sections 7, 18, 19, 30 and 31.

There is a large book of field notes, showing only where mounds and trees are made landmarks. The kind of trees marked as witness trees; their diameter, bearing and distances, are given for A, a, B, b, C, c, to X, x, Y, y.

For interior section corners, begin at S.E. corner, showing the notes to sections 25, 26, 35, 36; 23, 24, 25, 26; and two after two to sections 5, 6, 7, 8, at N.W. corner of the township.

For interior quarter section corners, begin at M, the N.E. corner of section 36, and run to U, N.W. corner of section 31, thus:

M to U, at 1, post in mound.

2, bur oak, 18 inches diameter, bears N. 3° E. 80 links.

bur oak, 12 inches diameter, bears S. 89° W. 250 links.

6, post in mound.

Next run L to V, K to W, I to X, and H to Y, giving the witness trees, if any, at quarter section corners numbered 1, 2, etc., as above. Then begin to note from south to north, by beginning at O and noting to F, then P to E, Q to D, R to C, and S to B.

The plats show by whom the outlines and subdivisions have been surveyed; date of contract; total area in acres; total of claims or land exempt from sale; the variation of the township and subdivision lines; and the detail required by section.

SURVEYS OF VILLAGES, TOWNS AND CITIES.

800. A. lays out a village, which may be called after him, as Cleaver-ville, Kilbourntown, Evanston; or it may be named after some river, Indian chief, etc., as Hudson, Chicago. This village is laid out into blocks, streets and alleys. The blocks are numbered 1, 2, 3, etc., generally beginning at the N.E. corner of the village. The lots are laid off fronting on streets, and generally running back to an alley. The lots are numbered 1, 2, 3, etc., and generally lot 1 begins at the N.E. corner of each block. The streets are 80, 66, 50 and 40 feet—generally 66 feet. In places where there is a prospect of the street to be of importance as a place for business, the streets are 80 feet. Although many streets are found 40 feet wide, they are objectionable, as in large cities they are subsequently widened to 60 or 66 feet. This necessarily incurs expenses, and causes litigations.

Sidewalks. The streets are from the side of one building to that of another on the opposite side of the street; that is, the street includes the carriage way and two sidewalks. Where the street is 80 feet wide, each

sidewalk is usually 16 feet. When the street is 66 feet, the width of the sidewalk is usually 14 feet. Where the street is 40 feet, the width of the sidewalk is usually 9 feet.

Corner stones. The statutes of each State generally require corner stones to be put down so as to perpetuate the lines of each village, town, or addition to any town or city.

Maps or plats of such village, town or addition, is certified as correct by the county or city surveyor, as the State law may require. The map or plat is next acknowledged by the owner, before a Justice of the Peace or Notary Public, to be his act and deed.

Plat recorded. The plat is then recorded in a book of maps kept in the Recorder's or Registrar's office, in the county town or seat.

Dimensions on the map. Show the width of streets, alleys and lots; the depths of lots; the angles made by one street with another; the distances from corner or centre stones to some permanent objects, if any. These distances are supposed to be mathematically correct, and according to which the lots are sold.

Lots are sold by their number and block, as, for example: "All that parcel or piece of land known as lot number 6, in block 42, in Matthew Collins' subdivision of the N.E. quarter section 25, in township 6 north, and range 2 east, of the third principal meridian, being in the county of —, and State of —."

All plats are not certified by county or city surveyors. In some States, surveyors are appointed by the courts, whose acts or valid surveys are to be taken as *prima facie* evidence. In other States, any competent surveyor can make the subdivision, and swear to its being correct before a Justice of the Peace.

Lots are also sold and described by metes and bounds, thus giving to the first purchasers the exact quantity of land called for in their deeds, leaving the surplus or deficiency in the lot last conveyed.

Metes and bounds signify that the land begins at an established point, or at a given distance from an established point, and thence describes the several boundaries, with their lengths and courses.

Establishing lost corners. When some posts are lost, the surveyor finds the two nearest undisputed corners, one on each side of the required corners. He measures between these two corners, and divides the distance *pro rata*; that is, he gives each lot a quantity in proportion to the original or recorded distance. Where there is a surplus found, the owners are generally satisfied; but where there is a deficiency, they are frequently dissatisfied, and cause an inquiry to be made whether this deficiency is to be found on either side of the required lots, or in one side of them. As mankind is not entirely composed of honest men, it has frequently happened that *posts, and even boundary stones, have been moved out of their true places* by interested parties or unskilful surveyors.

In subdividing a tract into rectangular blocks, we measure the outlines twice, establish the corners of the blocks on the four sides of the tract, and, by means of intersections, establish the corners of the interior blocks.

Let us suppose a tract to be divided into 36 blocks, and that block 1 begins at the N.E. corner, and continues to be numbered similar to township surveys. We erect poles at the N.W. corners of blocks 1, 2, 3, 4 and 5, and at the N.E. corners of blocks 12, 13, 24, 25 and 36. We set the in-

strument on the south line at S.W. corner of block 36: direct the telescope to the pole at the N.W. corner of block 1. Let the assistant stand at the instrument. We stand at the N.W. angle of 31, and make John move in direction of the pole at the N.W. angle of 36, until the assistant gives the signal that he is on his line. This will give the N.W. angle of 36, where John drives a post, on the top of which he holds his pole again on line, and drives a nail in the true point. We then move to the N.W. angle of 30, and cause John to move until he is on our assistant's line, thereby establishing the N.W. corner of 25, and so on for the N.W. corners of 24, 13 and 12. We move the instrument to the S.W. corner 35, and set the telescope on the pole at N.W. corner of 2, and proceed as before. This method is strictly correct, and will serve to detect any future fraud, and enable us to re-establish any required corner. Where the blocks are large, the lots may be surveyed as above.

Where the ground is uneven, or woodland, this method is not practicable. However, proving lines ought to be run at every three blocks.

CANADA SURVEYING.

301. No person is allowed to practice land surveying until he has obtained license, under a penalty of £10, one-half of which goes to the prosecutor.

Each Province has a Board of Examiners, who meet at the Crown Land Office, on the first Monday of January, April, July and October.

The candidate gives one week's notice to the Secretary of the Board. He must have served as an apprentice during three years. He must have first-rate instruments, (a theodolite, or transit with vertical arch, for finding latitude and the true meridian.) He must know Geometry. (six books of Euclid,) Trigonometry, and the method of measuring superficies, with Astronomy sufficient to enable him to find latitude, longitude, true time, run all necessary boundary lines by infallible methods, and be versed in Geology and Mineralogy, to enable him to state in his reports the rocks and minerals he may have met in his surveys. He must have standard measures, one five links long, and another three feet. He gives bonds to the amount of 1000 dollars. His fees, when attending court, is four dollars per day. He keeps an exact record of all his surveys, which, after his death, is to be filed with the clerk of the court of the county in which he lived. Said clerk is to give copies of these surveys to any person demanding them on paying certain fees, one-half of which is to be paid to the heirs of the surveyor.

The Government have surveyed their townships rectangularly, as in the United States, except where they could make lots front on Government roads, rivers and lakes. This has been a very wise plan, as several persons can settle on a stream; whereas, in the United States, one man's lot may occupy four times as much river front as a man having a similar lot in Canada.

302. Lines are run by the compass in the original survey, but all subsequent side lines are run astronomically. In the United States, lines are run from post to post, which requires to have two undisputed points,

and that a line should be invariably first run and then corrected back for the departure from the rear post. In the Canada system, we find the post in front of the lot, and then run a line truly parallel to the governing line, and drive a post where the line meets the concession in rear.

The annexed Fig. represents a part of the town of Cox; *b c*, *a d*, etc., are concession lines. Heavy lines are concession roads, 66 feet wide, always between every two concessions. There is an allowance of road generally at every fifth lot.



The front of each concession is that from which the concessions are numbered; that is, the front of concession II is on the line *a d*.

Where posts were planted, or set on the river, the front of concession B is the river, and that of concession A is on the concession line *n f*, etc.

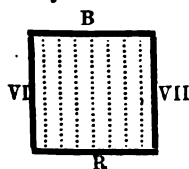
303. Side lines are to be run parallel to the township line from which the lots are numbered.

The line between lots 7 and 8, in concession II; is to be run on the same true bearing as the township line *a b*; but if the line *m, n, o, p, s*, etc., be run in the original survey as a proving line, then the line between 7 and 8 is to be run parallel to the line *p s*, and all lines from the line *p s* to the end are to be run parallel to *p s*, and lines from *a* to *p* are to be run parallel to *a b*. When there is no proving or township line where the lots are numbered from, as in con. A, we must run parallel to the line *v w*; but if there is a proving line as *m n*, all lines in that concession shall be run parallel to it.

When there is no town line at either end of the concession, as in con. B, the side lines are ran parallel to the proving line, if any.

When there is neither proving line or township line at either end, as in concession B, we open the concession line *k w*, and with this as base, lay off the original angle.

Example. The original bearing of *k w* is N. 16° W., and that of the side lines N. 66° E. To run the line between lots 14 and 15, in con. B, we lay off from the base *k w* an angle of 82°, and run to the river. The



original posts are marked on the four sides thus. This shows that the allowance for road is in rear of con. C; that is, the concession line between concessions B and C is on the west line of allowance of road. The original field notes are kept as in the United States, showing the quality of timber, soil, etc.

If the concessions were numbered from a river or lake, and that no posts were set on the water's edge, then the lines shall be run from the rear to the water.

When concession lines are marked with two rows of posts, and that the land is described in half lots, then the lines shall be drawn from both ends parallel to the governing line, and to the centre of the concession if the lots were intended to be equal, or proportional to the original depths.

When the line in front of the concession was not run in the original survey, then run from the rear to a proportionate depth between said rear line and the adjacent concession. (See Act, 1849, Sec. XXXVI.)

Example. The line *a d* has not been run, but the lines *b c* and *t v* have been ran.

Let the depth of each concession = 8000 links. Road, on the line *a d*, 100 links. Run the line between 7 and 8, by beginning at the point *h*, and running the line *h g* parallel to *a b*, and equal to half the width of concession I and II. Measure *h g*, and find it 8200 links. Suppose that the allowance for road is in the rear of each concession; that is, the west side of each concession road allowance is the concession line; then 8200 links include 100 links for one road, leaving the mean depth of concession II = to be 8100 links = *h g*. In like manner we find the depth of the line between 8 and 9, and the straight line joining these points is the true concession line. (See Act, May, 1849, Sec. XXXVI.)

304. Maps of towns or villages are to be certified as correct by a land surveyor and the owner or his agent, and shall contain the courses and distances of each line, and must be put on record, as in the United States, within one year, and before any lot is sold. These maps, or certified copies of them, can be produced as evidence in court, provided such copy be certified as a true copy by the County Registrar.

When A got P. L. surveyor S, to run the line between 6 and 7 in concession II, and finds that the line has taken part of his lot 6, on which he has improved; that is, he finds part of B's lot 7 included inside his old boundary fence. The value of his improvements is 400 dollars, belonging to A, and the value of the land to be recovered by B is 100 dollars. Then, if B becomes plaintiff to recover part of his lot 7, worth 100 dollars, he has to pay A the amount of his damages for improvement, viz. 400 dollars, or sell the disputed piece to A for the assessed value. (See Act of 1849, Sec. L.)

305. In the *Seigniories*, fronting on the St. Lawrence, the true bearing of each side line is N. 45° W., with a few exceptions about the vicinity of St. Ignace, below Quebec.

In the Ottawa *Seigniories*, the true or astronomical bearing is N. 11° 15' E. This makes it easier than in the townships, as there is no occasion to go to the township line for each concession.

306. *Where the original posts or monuments are lost.*

"In all cases when any land surveyor shall be employed in Upper Canada to run any side line or limits between lots, and the original post or monument from which such line should commence cannot be found, he shall in every such case, obtain the best evidence that the nature of the case will admit of, respecting such side line, post or limit; but if the same cannot be satisfactorily ascertained, then the surveyor shall measure the true distance between the nearest undisputed posts, limits or monuments, and divide such distance into such number of lots as the same contained in the original survey, assigning to each a breadth proportionate to that intended in such original survey, as shown on the plan and field-notes thereof, of record in the office of the Commissioner of Crown Lands of this Province; and if any portion of the line in front of the concession in which such lots are situate, or boundary of the township in which such

concession is situate, shall be obliterated or lost, then the surveyor shall run a line between the two nearest points or places where such line can be clearly and satisfactorily ascertained, in the manner provided in this Act, and in the Act first cited in the preamble to this Act, and shall plant all such intermediate posts or monuments as he may be required to plant, in the line so ascertained, having due respect to any allowance for a road or roads, common or commons, set out in such original survey; and the limits of each lot so found shall be taken to be, and are hereby declared to be the true limits thereof; any law or usage to the contrary thereof in any wise notwithstanding."

[This is the same as Sec. XX of the Act of May, 1849, respecting Lower Canada, and of the Act of 1855, Sec. X.]

GEODEDICAL JURISPRUDENCE.

The general method of establishing lines in the United States, may be taken from the United States' Statutes at Large, vol. II, p. 313, passed Feb. 11, 1805.

CHAP. XIV., Feb. 11, 1805.—An Act concerning the mode of Surveying the Public Lands of the United States.

[See the Act of May 18, 1796, chap. XXIX, vol. I, p. 465.]

Be it enacted by the Senate and House of Representatives of the United States of America, in Congress assembled, That the Surveyor General shall cause all those lands north of the river Ohio which, by virtue of the act intitled "An Act providing for the sale of the lands of the United States in the territory N.W. of the river Ohio, and above the mouth of the Kentucky River," were subdivided by running through the townships parallel lines each way, at the end of every two miles, and by marking a corner on each of the said lines at the end of every mile, to be subdivided to sections, by running straight lines from those marked to the opposite corresponding corners, and by marking on each of the said lines intermediate corners, as nearly as possible equidistant from the corners of the sections on the same. And the said Surveyor General shall also cause the boundaries of all the half sections which had been purchased previous the 1st July last, and on which the surveying fees had been paid, according to law, by the purchaser, to be surveyed and marked, by running straight lines, from the half mile corners heretofore marked, to the opposite corresponding corners; and intermediate corners shall, at the same time, be marked on each of the said dividing lines, as nearly as possible equidistant from the corners of the half section on the same line.

Provided, That the whole expense of surveying and marking the lines shall not exceed three dollars for every mile which has not yet been surveyed, and which will be actually run, surveyed and marked by virtue of this section, shall be defrayed out of the moneys appropriated, or which may be hereafter appropriated for completing the surveys of the public lands of the United States.

SEC. 2. And be it further enacted, That the boundaries and contents of several sections, half sections and quarter sections of the public lands of the United States shall be ascertained in conformity with the following principles, any Act or Acts to the contrary notwithstanding:

1st. All the corners marked in the surveys returned, by the Surveyor General, or by the surveyor of the land south of the State of Tennessee respectively, shall be established as the proper corners of sections or divisions of sections which they were intended to designate; and the corners of half and quarter sections, not marked on the said surveys, shall be placed as nearly as possible equidistant from those two corners which stand on the same line.

2nd. The boundary lines, actually run and marked in the surveys re-

turned by the Surveyor General, or by the surveyor of the land south of the State of Tennessee, respectively, shall be established as the proper boundary lines of the sections or subdivisions for which they were intended, and the length of such lines as returned by either of the surveyors aforesaid shall be held and considered as the true length thereof.

And the boundary lines which shall not have been actually run and marked as aforesaid, shall be ascertained by running straight lines from the established corners to the opposite corresponding corners; but in those portions of the fractional townships where no such corresponding corners have been or can be fixed, the said boundary lines shall be ascertained by running from the established corners due north and south, or east and west lines, as the case may be, to the water course, Indian boundary line, or other external boundary of such fractional township.

An Act passed 24th May, 1824, authorizes the President, if he chooses to cause the survey of lands fronting on rivers, lakes, bayous, or water courses, to be laid out 2 acres front and 40 acres deep. (See United States' Statutes at Large, vol. IV, p. 84.)

An Act passed 29th May, 1830, makes it a misdemeanor to prevent or obstruct a surveyor in the discharge of his duties. Penalties for so doing, from \$50 to \$3000, and imprisonment from 1 to 3 years.

SEC. 2 of this Act authorizes the surveyor to call on the proper authorities for a sufficient force to protect him. (*Ibid*, vol. IV, p. 417.)

The Act for adjusting claims in Louisiana passed 15th Feb., 1811, gave the Surveyor General some discretionary power to lay out lots, fronting on the river, 58 poles front and 65 poles deep. (*Ibid*, vol. II, p. 618.)

FROM THE ALABAMA REPORTS.

307. Decision of the Supreme Court of Alabama in the case of *Lewin v. Smith*.

1. The land system of the United States was designed to provide advance with mathematical precision the ascertainment of boundaries, and the second section of the Act of Congress of 1805 furnished the rule of construction, by which all the disputes that may arise about boundaries or the contents of any section or subdivision of a section of land, shall be ascertained.

2. When a survey has been made and returned by the Surveyor, shall be held to be *mathematically true*, as to the lines run and marked, and the corners established, and the contents returned.

3. Each section, or separate subdivision of a section, is *independent* of any other section in the township, and must be governed by its marked and established boundaries.

4. And should they be obliterated or lost, recourse must be had to the best evidence that can be obtained, showing their former situation and place.

5. The purchaser of land from the United States takes by metes and bounds, whether the actual quantity exceeds or falls short of the amount estimated by the surveyor.

6. Where a navigable stream intervenes in running the lines of a section the surveyor stops at that point, and does not continue across the river; the fraction thus made is complete, and its contents can be ascertained. Therefore, where there is a discrepancy between the corners of a section as established by the United States' Surveyor, and the lines as run and marked—the latter does not yield to the former.

7. Whether this would be the case where a navigable stream does cross the lines.—Query.

This is the case of *Lewin v. Smith*:

Error to the Circuit Court of Tuscaloosa. Plaintiff—an action of trespass on portion of fractional sec. 26, town. 21, range 11 W., lying north and west of the Black Warrior River.

Line *a b* claimed by Lewin.

Line *b c* claimed by Smith.

Field Notes. Beginning at N. W. corner, south $73^{\circ} 50'$, to a post on N. bank of the river, from which north $80^{\circ} W.$ 0.17, box elder—S. $66^{\circ} E.$, 0.18, do. Thence with the meander of the river

S. $74^{\circ} E.$, 7.50.

N. $32^{\circ} E.$, 10.

N. $9^{\circ} W.$, 20.

N. $10^{\circ} E.$, 22.

N. $4^{\circ} W.$, 24.50.



to a poplar on the south boundary of sec. 23; thence west 13.70, to S. W. corner, containing $100\frac{55}{100}$ acres.

NOTE.—Here the line claimed by Smith was established, by finding the original corners, *b* and *c*. Lewin claimed that, although there was no monument to be found at *a*, that such would be legally established by the intersection of a line from *b* to *d*, *d* being a fractional corner at the stockade fence supposed to be correct. The Court decided that the line *b* to *c* was the true line, as the line and bearing trees corresponded with the field notes, and therefore decided in favor of Smith. The disputed gore or triangle, *a b c*, contained 9 acres, and the jog, *a c* = 207 links.—McD.

FROM THE KENTUCKY REPORTS.

308. From the Kentucky Reports, by Thomas B. Monroe, vol. VII, p. 333. *Baxter v. Evett*. Government survey made in 1803. Patent deed issued in 1812. Ejectment instituted in 1825. Decision in 1830.

The rule is, that visible or actual boundaries, natural or artificial, called for in a certificate of survey, are to be taken as the abutments, so long as they can be found or proved. The legal presumption is, that the surveyor performed the duty of marking and bounding the survey by artificial or natural abutments, either made or adopted at the execution of the survey. And if this presumption could be destroyed by undoubted testimony, yet, as this was the fault of the officer of the Government, and not of the owner of the survey, his right ought not to be injured, when the omission can be supplied by any rational means, and descriptions furnished by the certificate of survey.

In locating a patent, the inquiry first is for the demarkation of boundary, natural or artificial, alluded to by the surveyor. If these can be found extant, or if not now existing, can be proved to have existed, and their locality can be ascertained, these are to govern. The courses and distances specified in a plat and certificate of survey, are designed to describe the boundaries as actually run and made by the surveyor, and to assist in preserving the evidence of their local position, to aid in tracing them whilst visible, and in establishing their former position in case of destruction, by time, accident or fraud. As guides for these purposes, the courses and distances named in a plat and certificate of survey are useful; but a line or corner established by a surveyor in making a survey, upon which a grant has issued, cannot be altered because the line is longer or shorter than the distance specified, or because the relative bearings between the abutments vary from the course named in the plat and certificate of survey: so, if the line run by the surveyor be not a right line, as supposed from his description, but be found, by tracing it, to be a curved line, yet the actual line must

govern, the visible actual boundary the thing described, and not the ideal boundary and imperfect description, is to be the guide and rule of property.

These principles are recognized in *Beckley v. Bryan*, prim. dec. 107, and Litt. sel. Cas. 91; *Morrison v. Coghill*, prin. dec. 382; *Lyon v. Ross*, 1 Bibb. p. 467; *Cowan v. Fauntelroy*, 2 Bibb. p. 261; *Shaw v. Clement*, 1 Call, p. 438, 3d point; *Herbert v. Wise*, 3 Call, p. 239; *Baker v. Glasscocke*, 1 Hen. & Munf., p. 177; *Helm v. Smallhard*, p. 369.

From the same State Reports.

5 Dana, p. 543-4. *Johnson v. Gresham*. Here Gresham found the section to contain 696 acres; had it surveyed into four equal parts, thus embracing 1 to 3 acres of Johnson's land, which extended over the line run, with other improvements. Gresham had purchased that which Johnson had pre-empted.

Opinion of the Court by Judge Ewing, Oct. 19, 1837.

1. Though the Act of 1820, providing for surveying the public lands west of the Tennessee River, directs that it shall be laid off into townships of 36 miles square, and divided into sections of 360 acres each, yet it is well known, through the unevenness of the ground, the inaccuracy of the instruments, and carelessness of surveyors, that many sections embrace less, and many more, than the quantity directed by the Act. The question therefore occurs, how the excess or deficiency shall be disposed of among the quarters. The statute further directs that in running the lines of townships, and the lines parallel thereto, or the lines of sections, "that trees, posts, or stones, half a mile from the corners of sections, shall be marked as corners of quarter sections." So far, therefore, as the corners or lines of the quarters can be ascertained, *they should be the guides and constituted boundaries* and abutments of each quarter. In the absence of such guides, and of all other indices directing to the place where they were made, the sections should be divided, as near as may be, between the four quarters, observing, as near as practicable, the courses and distances directed by the Act. When laid down according to these rules, the quarter in contest embraces 174 acres, and covers a part of the field of the complainant, as well as his washhouse.

FROM THE ILLINOIS REPORTS.

809. From the Illinois Reports, vol. XI. *Rogers v. McClintock*.

The corners of sections on township lines were made when the township was laid out. They became fixed points, and if their position can now be shown by testimony, these must be retained, although not on a straight line—from A to B. The township line was not run on a straight line from A and B. It was run mile by mile, and these mile points are as sacred as the points A to B. (Land Laws, vol. I, pages 50, 71, 119 and 120.)

Therefore, if the actual survey, as ascertained by the monuments, show a deflected line, it is to be regarded as the true one.—*Baker v. Talbott*, 6 Monroe, 182; *Baxter v. Evett*, 7 Monroe, 333.

Township corners are of no greater authority in fixing the boundary of the survey than the section corners.—*Wishart v. Crosby*, 1 A. R. Marsh, 383.

Where sections are bounded on one side by a township line, and the line cannot be ascertained by the calls of the plat, it seems quite clear that if the corners of the adjacent section corners be found, this is better evidence to locate the township line than a resort to course merely.—1 Greenleaf Evidence, p. 369, sec. 301, note 2; 1 Richardson, p. 497.

Chief Justice Caton's Opinion.

All agree that courses, distances and quantities must always yield to the monuments and marks erected or adopted by the original surveyor, as indicating the lines run by him. Those monuments are facts. The field notes and plats, indicating courses, distances and quantities, are but descriptions which serve to assist in ascertaining those facts. Established

monuments and marked trees not only serve to show the lines of their own tracts, but they are also to be resorted to in connection with the field notes and other evidence, to fix the original location of a monument or line, which has been lost or obliterated by time, accident or design.

The original monuments at each extreme of this line, that is, the one five miles east, and the other one mile west of the corner, sought to be established, are identified, but unfortunately, none of the original monuments and marks, showing the actual line which was run between townships 5 and 6, can be found; and hence we must recur to these two, as well as other original monuments which are established, in connection with the field notes and plats, to ascertain where those monuments were; *for where they were, there the lines are.*

Much of the following is from Putnam's U. S. Digest:

809A. A survey which starts from certain points and lines not recognised as boundaries by the parties themselves, and not shown by the evidence to be *true points* of departure, *cannot be made* the basis of a judgment establishing a boundary. 12 La. An. 689 (18.) See also U. S. Digest, vol. 18, sec. 23, *Martin vs. Breaux*.

a. A party is entitled to the lands actually apportioned, and where the line marked out upon actual survey differs from that laid in the plat, the former controls the latter. 1 Head (Tenn.) 60, *Mayse vs. Lafferty*.

b. When a deed refers to a plat on record, the dimensions on the plat *must govern*; and if the dimension on the plat do not come together, then the *surplus* is to be *divided* in proportion to the dimensions on the plat. *Marsh vs. Stephenson*, 7 Ohio, N. S. 264.

c. Courses and distances on a plat referred to, are to be considered as if they were recited in the deed. *Blaney vs. Rice*, 20 Pick. 62.

d. Where, on the line of the same survey between remote corners, the length varies from the length recorded or called for, in re-establishing intermediate monuments, marking divisional tracts, it is to be presumed that the error was distributed over the whole, and not in any particular division, and the variance *must be* distributed proportionally among the various subdivisions of the whole line according to their respective lengths. 2 Iowa (Clarke) p. 189, *Moreland vs. Page*. *Bailey vs. Chamblin*, 20 Ind. 83.

e. Where the same grantor conveys to two persons, to each one a lot of land, limiting each to a certain number of rods from opposite known bounds, running in direction to meet if extended far enough, and by admeasurement the lots do not adjoin, when it appears from the same deeds that it was the intention they should, *a rule should be* which will divide the surplus over the admeasurement named in the deeds ascertained to exist by actual measurement on the earth, between the grantees in proportion to the length of their respective lines as stated in their deeds. 28 Maine 279, *Lincoln vs. Edgecomb*. *Brown vs. Gay*, 8 Greenl. 118. *Wolf vs. Scarborough*, 2 Ohio St. Rep. 868.

Deficiency to be divided *pro rata*. *Wyatt vs. Savage*, 11 Maine 481.

f. Angel on Water Courses, sec. 57, says of dividing the surplus: "By this process justice will be done, and all interference of lines and titles prevented."

No person can, under different temperatures, measure the same line into divisions a, b, c and d, and make them exactly agree; but if the difference is divided, the points of division will be the same.

When we compare the distance on a map, and find that the paper expanded or contracted, we have to allow a proportionate distance for such variance. (See Table II, p. 165.)

809b. The system of dividing *pro rata* is embodied in the Canada Surveyors' Act, and quoted at sec. 306 of this work. It is also the French system.

By the French Civil Code, Article 646, all proprietors are obliged to have their lines established. In case it may be subsequently found that the survey was incorrect, and that one had too much, if the excess of one would equal the deficit of the other, then no difficulty would occur in dividing the difference.

If the excess in one man's part is greater than the deficit in the other, it ought to be divided *pro rata* to their respective quantities, each participating in the gain as well as the loss, in proportion to their areas. This is the opinion of the most celebrated lawyers.

The following is the French text:

"Le terrain excédant au celui qui manque devra être partagé entre les parties, au *pro rata* de leur quantité respective, en participant au gain comme à la perte, chacun proportionnellement à leur contenance; c'est l'avis de plus célèbres jurisconsultes."

Adverse possession or prescriptive right, does not interfere when the encroachment was made clandestinely or by gradual anticipation made in cultivating or in mowing it.

For prescriptive right, see the French Civil Code, Article 2262:

"Cependant la prescription ne sera jamais invoquée dans le cas où la possession sera clandestine. C'est-à-dire lorsqu'elle est le résultat d'une anticipation faite graduellement en labourant ou en fauchant." Cours Complet. D'Arpentage. Paris, 1854. Par. D. Puille, p. 250.

a. No one has a right to establish a boundary without his contiguous owner being present, or satisfied with the surveyor employed.

The expense of survey is paid by the adjacent owners.

The loser in a contested survey has to pay all expenses. In a disputed survey, each appoints a surveyor, and these two appoint a third. If they cannot agree on the third man, the case is taken before a Justice of the Peace, who is to appoint a third surveyor.

The surveyors then read their appointments to one another, and to the parties for whom the survey is made. They examine the respective titles, original or old boundaries, if any exist, all land marks, and then proceed to make the necessary survey, and plant new boundaries. On their plan and report, or *process verbal*, they show all the detail above recited, mark the old boundary stones in black, and the new ones in red.

A stone is put at every angle of the field, and on every line at points which are visible one from another. The stones are in some places set so as to appear four to six inches over ground; but where they would be liable to be damaged, they are set under the ground.

b. Boundary Witnesses. Under each stone is made a hole, filled with delf, slugs of iron, lime or broken stones, and on or near this, is a piece of slate on which the surveyor writes with a piece of brass some words called a *mule witness*.

Witness. He then sets the stone and places four other stones around it corresponding to the cardinal points. The *mule witness* or expression can be found after an elapse of one hundred years, provided it has been kept from the atmosphere. *Ibid.* p. 252 and 253.

The United States take pains in establishing a corner where no witness tree can be made. Under the stake or post is placed charcoal. The mound and pits about it are made in a particular manner. (See sec. 281.)

In Canada, if in wood land, the side lines from each corner is marked or blazed on both sides of the line to a distance of four or five chains, to serve as future witnesses.

309c. When the number of a lot on a plan referred to in the deed, is the only description of the land conveyed, the courses, distances, and other particulars in that plan, are to have the same effect as if recited in the deed. *Thomas vs. Patten*, 1 Shep. 329.

In ascertaining a lost survey or corner, help is to be had by considering the system of survey, and the position of those already ascertained. See *Moreland vs. Page*, 2 Clarke (Iowa) 189.

a. Fixed monuments, control courses and distances. 3 Clarke (Iowa) 143, *Sargent vs. Herod*.

b. Metes and bounds control acres; that is, where a deed is given by metes and bounds, which would give an area different from that in the deed, the metes and bounds will control. *Dalton vs. Rust*, 22 Texas 188.

c. Metes and bounds must govern. 1 J. J. Marsh, *Wallace vs. Maxwell*.

d. Marked lines and corners control the courses and distances laid down in a plat. 4 McLean 279.

e. If there are no monuments, courses and distances must govern. U.S. Dig., vol. 1, sec. 47.

f. So frail a witness as a stake is scarcely worthy to be called a monument, or to control the construction of a deed. *Cox vs. Freedley*, 38 Penn. State B. 124.

g. Stakes are not considered monuments in N. Carolina, but regarded as imaginary ones. 3 Dev. 65, *Reed vs. Schenck*.

h. Lines actually marked must be adhered to, though they vary from the course. 2 Overt. 304, and 7 Wheat. 7, *McNairy vs. Hightour*.

i. It is a well settled rule, that where an actual survey is made, and monuments marked or erected, and a plan afterwards made, intended to delineate such survey, and there is a variance between the plan and survey, the survey must govern. 1 Shep. 329, *Thomas vs. Patten*.

j. The actual survey designated by lines marked on the ground, is

the true survey, and will not be affected by subsequent surveys. 7 Watts 91, *Norris vs. Hamilton*.

309d. In locating land, the following rules are resorted to, and generally in the order stated:

1. Natural boundaries, as rivers.
2. Artificial marks, as trees, buildings.
3. Adjacent boundaries.
4. Courses and distances.

Neither rule however occupies an inflexible position, for when it is plain that there is a mistake, an inferior means of location may control a higher. 1 Richardson 491, *Fulwood vs. Graham*.

a. Description in a boundary is to be taken strongly against the grantor. 8 Connecticut 369, *Marshall vs. Niles*.

b. *Between*, excludes the termini. 1 Mass. 91, *Reese vs. Leonard*.

b. Where the boundaries mentioned in a deed are inconsistent with one another, those are to be retained which best subserve the prevailing intention manifested on the face of the deed. Ver. 511, *Gates vs. Lewis*.

309e. The most material and most certain calls shall control those that are less certain and less material. 7 Wheat. 7, *Newsom vs. Pryor*. *Thomas vs. Godfrey*, 8 Gill & Johnson 142.

a. What is most material and certain controls what is less material. 36 N. H. 569, *Hale vs. Davis*.

b. The least certainty in the description of lands in deeds, must yield to the greater certainty, unless the apparently conflicting description can be reconciled. 11 Conn. 335, *Benedict vs. Gaylord*.

309f. Where the boundaries of land are fixed, known and unquestionable monuments, although neither course nor distance, nor the computed contents correspond, the monuments must govern. 6 Mass. 181. 2 Mass. 380. *Pernan vs. Wead*. *Howe vs. Bass*.

a. A mistake in one course does not raise a presumption of a mistake in another course. 6 Litt. 93, *Bryan vs. Beekley*.

b. When there are no monuments and the courses and distances cannot be reconciled, there is no universal rule that requires one of them to yield to the other; but either may be preferred as best comports with the manifest intent of parties, and with the circumstances of the case. U. S. Dig., vol. 1, sec. 18.

c. The lines of an elder survey prevail over that of a junior. Ib. 77.

d. Boundaries may be proved on hearsay evidence. Ibid. 167.

e. The great principle which runs through all the rules of location is, that where you cannot give effect to every part of the description, that which is more fixed and certain, shall prevail over that which is less. 1 Shobhart 143, *Johnson vs. McMillan*.

309g. A line is to be extended to reach a boundary in the direction called for, disregarding the distance. U. S. Dig. vol. 7, 16.

a. Distances may be increased and sometimes courses departed from, in order to preserve the boundary, but the rule authorizes no other departure from the former. *Ibid.* 13.

b. If no principle of location be violated by closing from either of two points, that may be closed from which will be more against the grantor, and enclose the greater quantity of land. *Ibid.* sec. 14.

809H. What are boundaries described in a deed, is a question of law, the place of boundaries is a matter of fact. 4 Hawks 64, *Doe vs. Paine*.

a. What are the boundaries of a tract of land, is a mere question of construction, and for the court; but where a line is, and what are facts, must be found by a jury. 18 Ind. 379, *Burnett vs. Thompson*.

b. It is not necessary to prove a boundary by a plat of survey or field notes, but they may be proved by a witness who is acquainted with the corners and old lines, run and established by the surveyor, though he never saw the land surveyed. 17 Miss. 459, *Weaver vs. Robinett*.

c. A fence fronting on a highway for more than twenty years, is not to be the true boundary thereof under Rev. St. C. 2, if the original boundary can be made certain by ancient monuments, although the same are not now in existence. 11 Cush (Mass.) 487, *Wood vs. Quincy*.

d. The marked trees, according to which neighbors hold their distinct land when proved, ought not to be departed from though not exactly agreeing with the description. 3 Call. 239. 7 Monroe, 333. *Herbert vs. Wise*. *Baxter vs. Evett*. *Rockwell vs. Adams*.

e. Where a division line between two adjoining tracts exists at its two extremities, and for the principal part of the distance between the two tracts, and as such is recognized by the parties, it will be considered a continuous line, although on a portion of the distance there is no improvement or division fence. 6 Wendell 467.

f. If the lines were never marked, or were effaced, and their actual position cannot be found, the patent courses so far must govern. 2 Dana 2. 1 Bibb. 466. *Dimmet vs. Lashbrook*. *Lyon vs. Ross*.

g. Or, if the corners are given, a straight line from corner to corner must be pursued. Dig. vol. 1, sec. 33.

h. Abutments are not to be disregarded. *Ibid.* vol. 12, sec. 4.

809L. Where there is no testimony on variation, the court ought not to instruct on that subject. *Wilson vs. Inloes*, 6 Gill 121.

a. The beginning corner has no more, or the certificate of survey has no greater, dignity than any other corner. 4 Dan. 332, *Pearson vs. Baker*.

b. Sec. 34. Where no corner was ever made, and no lines appear running from the other corners towards the one desired, the place where the courses and distances will intersect, is the corner. 1 Marsh 382. 4 Monroe 382. *Wishart vs. Crosby*. *Thornberry vs. Churchill*.

c. The land must be bounded by courses and distances in the deed where there are no monuments, or where they are not distinguishable from other monuments. Dig., vol. 1, sec. 47, 48, 49.

d. Seventy acres in the S. W. corner of a section, means that it must be a square. 2 Ham. 827, Walsh vs. Ruger.

809j. The plat is proper evidence. Dig., vol. 1, sec. 61, and Sup. 4, sec. 51.

a. Mistake in the patent may be corrected by the plat on record. The survey is equal dignity with the patent. Dig., vol. 1, sec. 60.

b. A survey returned more than twenty years, is presumed to be correct. 7 Watts 91, Norris vs. Hamilton.

809k. Declaration by a surveyor, chain carrier, or other persons present at a survey, of the acts done by or under the authority of the surveyor, in making the survey, if not made after the case has been entered, and the person is dead, is admissible. U. S. Dig., vol. 12, Boundary, sec. 10. See also English Law Reports, vol. 88, p. 140.

a. An old map, thirty years amongst the records, but no date, and the clerk, owing to his old age, could give no account of it, map admissible. Gibson vs. Poor, 1 Foster (N. H.) 240.

809l. The order of the lines in a deed may be reversed. 4 Dana 822, Pearson vs. Baxter.

a. Trace the boundary in a direct line from one monument to another, whether the distance be greater or less. 41 Maine 601, Melche vs. Merryman.

Norw. This is the same as the U. S. Act of 11th February, 1805.

b. *Northward* means due north. Haines 298. Dig., vol. 1, sec. 4.

Northerly means north when there is nothing to indicate the inclination to the east or west. 1 John 156, Brandt vs. Ogden.

c. It is a well settled fact, where a line is described as running towards one of the cardinal points, it must run directly in that course, unless it is controlled by some object. 8 Porter 9, Hogan vs. Campbell.

e. A survey made by an owner for his own convenience, is not admissible evidence for him or those claiming under him. 1 Dev. 228, Jones vs. Huggins.

809m. Parties, to establish a conventional boundary, must themselves have good title, or the subsequent owners are not bound by it. 1 Sneeds (Tenn.) 68, Rogers vs. White.

a. Parties are not bound by a consent to boundaries which have been fixed under an evident error, unless, perhaps, by the prescription of thirty years. 12 La. An. 780, Gray vs. Cawvillon.

b. The admission by a party of a mistaken boundary line for a true one, has no effect upon his title, unless occupied by one or both for fifteen years. 10 Vermont 88, Crowell vs. Bebes.

c. A hasty recognition of a line, does not estop the owner. *Overton vs. Cannon*, 2 Humph. 264.

d. In a division of land between two parties, if either was deceived by the innocent or fraudulent misrepresentation of the other, or there was any mistake in regard to their right, the division is not binding on either. 14 Georgia 884, *Bailey vs. Jones*.

e. A division line mistakenly located and agreed on by adjoining proprietors, will not be held binding and conclusive on them, if no injustice would be done by disregarding it. U. S. Digest, vol. 18, sec. 82. See, also, 29 N. Y. 892, *Coon vs. Smith*. English Reports 42, p. 807.

f. A mistaken location of the line between the owners of contiguous lots is not conclusive between the immediate parties to such location, but may be corrected. App. 412, *Colby vs. Norton*.

g. If 8 surveys for A, A is not estopped from claiming to the true line. 9 Yerg. 455, *Gilchrist vs. McGee*.

h. When owners establish a line and make valuable improvements, they cannot alter it. *Lavery vs. Moore*, 38 N. Y. 650.

309w. A fence between tenants, in common, if taken down by one of them, the others have no cause of action in trespass. 2 Bailey 880, *Gibson vs. Vaughn*.

309o. A line recognized by contiguous owners for thirty years, controls the courses and distances in a deed. 32 Penn. State R. 802, *Dawson vs. Mills*.

a. A line agreed on for thirty years, cannot be altered. 10 Watts 321, *Chew vs. Morton*.

b. Adjacent owners fixed stakes to indicate the boundary of water lots. One filled the part he supposed to belong to him; the other, being cognizant of the progress of the work, held that the other and his grantees were estopped to dispute the boundary. 32 Barb. (N. Y.) 847, *Lavery vs. Moore*.

c. To establish a consentable line between owners of adjoining tracts, knowledge of, and assent to the line as marked, must be shown in both parties. 4 Barr. 234, *Adamson vs. Potts*.

d. When two parties own equal parts of a lot of land, in severalty, but not divided by any visible monuments, if both are in possession of their respective parts for fifteen years, acquiescence in an imaginary line of division during that time, that line is thereby established as a divisional line. 9 Vernon 852, *Beecher vs. Parmalee*.

e. Sec. 29. Where parties have, without agreement, and ignorant of their right, occupied up to a division line, they may change it on discovering their mistake. *Wright* 576, *Avery vs. Baum*.

f. Where A and B and their hired man built a fence without a compass, and acquiesced in the fence for fifteen years, it was held to be the true line in Vermont. 18 Verm. 395, *Ackley vs. Nuck*.

- g. Quantity generally cannot control a location. Dig. vol. 10, sec. 49.
- A. Long and notorious possession infer legal possession. *Newcom vs. Leary*, 8 Iredell 49.
- i. A hasty, ill-advised recognition is not binding. *Norton vs. Cannon*, Dig., vol. 4, sec. 78.
- j. The line of division must be marked on the ground, to bring it within the bounds of a closed survey. Ibid. sec. 106.
- k. *Bounded by a water course*, according to English and American decisions, means to the centre of the stream. (See *Angel on Water Courses*, ch. 1, sec. 12.)
- l. East and north of a certain stream includes to the thread thereof. *Palmer vs. Mulligan*, 8 Caines (N. Y.) 819.
- m. *Bank and water* are correlative, therefore, to a monument standing on the bank of a river, and running by or along it, or along the shore, includes to the centre. 20 Wend. (N.Y.) 149. 12 John. (N.Y.) 252.
- n. Where a map shows the lots bounded by a water course, the lots go to the centre of the river. *Newsom vs. Pryor*, 7 Wheat. (U. S.) 7.
- o. To the bank of a stream, includes the stream itself. *Hatch vs. Dwight*, 17 Mass. 299.
- p. Up a creek, means to the middle thereof. 12 John. 252.
- q. Where there are no controlling words in a deed, the bounds go to the centre of the stream. *Herring vs. Fisher*, 1 Sand. Sup. Co. (N.Y.) 844.
- r. Land bounded by a river, not navigable, goes to the centre, unless otherwise reserved. *Nicholas vs. Siencocks*, 34 N. H. 345. 9 Cushing 492. 3 Kernan (N.Y.) 296. 18 Barb. (N. Y.) 14. *McCullough vs. Wall*, 4 Rich. 68. *Norris vs. Hill*, 1 Mann. (Mich.) 202. *Canal Trustees vs. Havern*, 5 Gilman 548. *Hammond vs. McLaughlin*, 1 Sandford Sup. Ct. R. 323. *Orindorf vs. Steel*, 2 Barb. Sup. Ct. R. 126 3 Scam. Ill. 510. *State vs. Gilmanton*, 9 N. Hamp. 461. *Luce vs. Cartey*, 24 Wend. 541. *Thomas vs. Hatch*, 8 Sumner 170.
- s. On, to, by a bank or margin, cannot include the stream. 6 Cow. (N. Y.) 549.
- t. A water course may sometimes become dry. *Gavett's Administrators vs. Chamber*, 8 Ohio 495. This contains important reasons for going to the centre of the stream.
- u. Along the bank, excludes the stream. *Child vs. Starr*, 4 Hill 869.
- v. A corner standing on the bank of a creek; thence down the creek, etc. Boundary is the water's edge. *McCulloch vs. Allen*, 2 Hamp. 309, also *Weakley vs. Legrand*, 1 Overt. 205.
- w. To a creek, and down the creek, with the meanders, does not convey the channel. *Sanders vs. Kenney*, J. J. Marsh 187. (See next page, which has been printed sometime in advance of this.)

monuments and marked trees not only serve to show with certainty the lines of their own tracts, but they are also to be resorted to in connection with the field notes and other evidence to fix the original location of a monument or line which has been lost, or obliterated by time, accident, or design.

The original monuments at each extreme of this line—that is, the one five miles east, and the other one mile west of the corner—sought to be established, are identified; but, unfortunately, none of the original monuments and marks, showing the actual line which was run between townships 5 and 6, can be found, and hence we must recur to these two, as well as other original monuments, which are established in connection with the field notes and plats, to ascertain where those monuments were, *for where they were, there the lines are.*

WATER COURSES.

809a. *Eminent domain* is the right retained by the government over the estates of owners, and the power to take any part of them for the public use. First paying the value of the property so taken, or the damages sustained to their respective owners. 8 Paige, N. Y. Chancery Rep. 45.

The British Crown has the right of eminent domain over tidal rivers and navigable waters, in her American colonies. Each of the United States have the same. See Pollard v. Hogan, 8 Howe, Rep. 228; Goodtitle v. Kibbe, 9 Howe Rep. 117; Stradar v. Graham, 10 Howe Rep. 96; Doe v. Beebe, 13 Howe Rep. 25. From these appear that the State has jurisdiction over navigable waters, provided it does not conflict with any provision of the general government. The Constitution of the U. States reserves the power to regulate commerce—which jurists admit to include the right to regulate navigation, and foreign and domestic intercourse, on navigable waters. On those waters the general government exercises the power to license vessels, and establish ports of entry, consequently it can prevent the construction of any material obstruction to navigation, and declare what rules and regulations are required of vessels navigating them.

Prescriptive right must set forth that the occupier or person claiming any *easement*, has been in an open, peaceable and uninterrupted possession of that which is claimed, during the time prescribed by the statute of limitation of the country, or state in which the *easement* is situated.

In England, the prescribed time is 20 years. Balston v. Bensted, 1 Campbell Rep., 468; Bealey v. Shaw, 6 East. Rep. 215.

In the United States the time is different—in New Hampshire, 20; Vermont and Connecticut, 15; and South Carolina, 5 years.

Water Course, is a body of water flowing towards the sea or lake, and is either private or public. It consists of bed, bank and water.

Public water course, is a navigable stream formed by nature, or made and dedicated to the public as such by artificial means. Navigable streams may become sometimes dry.

A stream which can be used to transport goods in a boat, or float rafts of timber or saw logs, is deemed a navigable stream, and becomes a public highway. But a stream made navigable by the owners, and not dedicated to the public, is a private water course. See Wadsworth v. Smith, 2 Fairfield, Maine Rep. 278.

The owners of the adjoining lands have a title to the bed of the river; each proprietor going to the centre, or thread thereof, when the river is made the boundary.

Should the river become permanently dry on account of being turned off in some other direction; or other cause, then the adjoining riparian owners claim to the centre of the bed of the stream, the same as if it were a public highway.

Bounded by a water course—signifies that the boundary goes to the centre of the river. *Morrison v. Keen*, 3 Greenleaf, Maine Rep. 474; 1 Randolph, Va., Rep. 420; *Waterman v. Johnson*, 8 Pickering, Mass. R., 261; *Star v. Child*, 20 Wendell, N. Y. Rep., 149.

To a swamp, means to the middle of the stream or creek, unless described to the edge of the swamp. *Tilder v. Bonnet*, 2 McMull South Carolina Report, 44.

Any unreasonable or material impediment to navigation placed in a navigable stream, is a public nuisance. 12 Peters, U. S. Rep. 91. The legislature cannot grant leave to build an obstruction to navigation. 5 Ohio Rep., 410.

A winter way on the ice, dedicated to the public for 20 years, becomes a highway, and cannot be obstructed. 6 Shepley, Maine Rep., 488.

The legislature cannot declare a river navigable which is not really so, unless they pay the riparian owners for all damages sustained by them. 16 Ohio Rep. 540.

Rivers in which the tide ebbs and flows are public, both their water and bed as far as the water is found to be affected by local influences, but above this, the riparian owners own to the centre of the river, and have the exclusive right of fishing, etc., the public having the right of highway. See 26 Wendell, N. Y. Rep. 404.

Banks of a navigable river are not public highways, unless so dedicated, as the banks of the Mississippi, in Illinois and Tennessee, and the rivers of Missouri for a reasonable time. See 4 Missouri Rep. 343; 3 Scammon 510.

This last decision had reference to a place in an unbroken forest, where it was admitted that the navigators had a right to land and fasten to the shore. It would be unfair to give a captain and crew of any vessel the right to land on a man's wharf, or in his enclosure without his permission; therefore, it would appear "that the public have the privilege to come upon the river bank so long as it is vacant, although the owner may at any time occupy it, and exclude all mankind." *Austin v. Carter*, 1 Mass. Rep. 281.

Obstructing navigation by building bridges without an act of the legislature, sinking impediments or throwing out filth, which would endanger the health of those navigating the river, is a nuisance. See *Russel on Crimes* 485. Although an obstruction may be built under an act of the legislature in navigable waters, he who maintains it there, is liable for any damage sustained by any vessel or navigator navigating therein. 4 Watts, Pennsylvania Rep. 487.

Bridges can be built over navigable rivers by first obtaining an act of the legislature, *Commonwealth v. Breed*, 4 Pick, Massachusetts R. 460; *Strong v. Dunlap*, 10 Humphrey, Tenn. R. 423. See *Angel on Highways*, sec. 4.

The State of Virginia, authorized a company to build a bridge at Wheeling, across the eastern channel of the Ohio river, it was suspended so low as to obstruct materially the navigation thereof. The Superior Court ordered its removal, but gave them a limited time to remove it to the other channel, where the company proposed to have sufficient depth of water and a drawbridge of 200 feet wide. The Court did not consider the additional length of channel nor the necessary time in opening the draw a material impediment. Subsequently an act of Congress declared the first bridge built on the eastern channel not to be a *material or unreasonable obstruction*, and ordered that captains and crews of vessels navigating on the river should govern themselves accordingly by lowering their chimneys, etc. 18 Howe Rep. 518; 18 Howe Rep. 421.

If a bridge is built across a river in a reasonable situation, leaving sufficient space for vessels to pass through, and causing no unreasonable delay or obstruction, and is built for the public good, it is not deemed a nuisance. *Rex v. Russel*, 6 Barn. and Cresw. 566; 15 Wendell, 138.

For further, see Judge Caton's decision in the Rock Island Bridge case, delivered in 1862.

Canals. If after being built, a new road is made over it, the canal company is not obliged to erect a bridge. *Morris Canal v. State*, 4 Zab-riskie, N. Y. Rep. 62.

In America, when two boats meet, each turns to the right. They carry lights at the bow. Freight boats must give away to packet or passenger boats. *Farnsworth v. Groot*, 6 Cowen, N. Y. Rep. 698.

In Pennsylvania, the descending boat has preference to the ascending. Act of Pennsylvania, April 10, 1826.

Ferries. The owner of a public ferry ought to own the land on both sides of the river. Savill 11 pl. 29. A ferry cannot land at the terminus of a public highway, without the consent of the riparian owners. *Chambers v. Ferry*, 1 Yeates. A use of twenty years, does not confer the right to land on the opposite side without the consent of the adjacent owners.

If A erects a dam or ditch on his own land, provided it does not overflow the land of his neighbor B, or diverts the water from him, he is justified in so doing. *Colborne v. Richards*, 18 Mass. Rep. 420. But if A injures B, by diverting the water or overflowing his land, B is empowered to enter on A's land and remove the obstructions when finished, but not during the progress of the work, doing no unnecessary damage, or causing no riot. In this case, B cannot recover damages for expense of removal, etc. If B enters suit against A, he recovers damages, and the nuisance is abated. *Gleason v. Gary*, 4 Connecticut Rep. 418; 3 Blackstone Comm. 9 Mass. Rep., 216; 2 Dana, Kentucky Rep. 158.

If B, C and D, as separate owners, cause a nuisance on A's property, A can sue either of the offending party, and the non-joinder of the others cannot be pleaded in abatement. 1 Chitty's Pleadings, 75.

The tenant may sue for a nuisance, even though it be of a temporary nature. Angel on Water Courses, chap. 10, sec. 398.

The reversioners may also have an action where the nuisance is of a permanent one. *Ibid.*

If A and B own land on the same river, one above the other, one of them cannot erect a dam which would prevent the passage of fish to the other. *Weld v. Hornby*, 7 East. R. 195; 5 Pickering, Mass. Rep. 199.

One riparian owner cannot divert any part of the water dividing their estate, without the consent of the other; as each has a right to the use of the whole of the stream. 18 Johnson, N. Y. Rep. 212.

It is not lawful for one riparian owner to erect a dam so as to divert the water in another direction, to the injury of any other owner. 3 Scammon, Illinois Rep. 492.

Where mills are situate on both banks of a river, each having an equal right; one of them, in dry weather, is not allowed to use more than his share of the water. See Angel on Water Courses, chap. 4. p. 105.

One mill cannot detain the water from another lower down the stream, nor lessen the supply in a given time. 13 Connecticut Rep. 808.

One riparian owner cannot overflow land above or below him by means of a dam or sluices, etc., or by retaining water for a time, and then letting it escape suddenly. See 7 Pickering, Massachusetts Rep. 76, and 17 Johnson, N. Y. Rep. 806.

Hence appears the legality of constructing works to protect an owner's land from being overflowed. Such work may be dams or drains leading to the nearest natural outfall; for it is evident, that if by making a drain, ditch or canal, to carry off any overflow to the nearest outlet, such proceedings would be legal, and the party causing the overflow would have no cause of complaint. Merrill v. Parker Coxe, New Jersey Rep. 460.

For the purpose of Irrigation. A man cannot materially diminish the water that would naturally flow in a water course. Hall v. Swift, 6 Scott R. 167. He may use it for motive power, the use of his family, and watering his cattle; also for the purpose of irrigating his land, provided it does not injure his neighbors or deprive a mill of the use of the water. That which is made to pass over his land for irrigation if not absorbed by the soil, is to be returned to its natural bed. Arnold v. Foot, 12 Wendell, N. Y. Rep. 830; Anthony v. Lapham, 5 Pickering, Mass. Rep. 175.

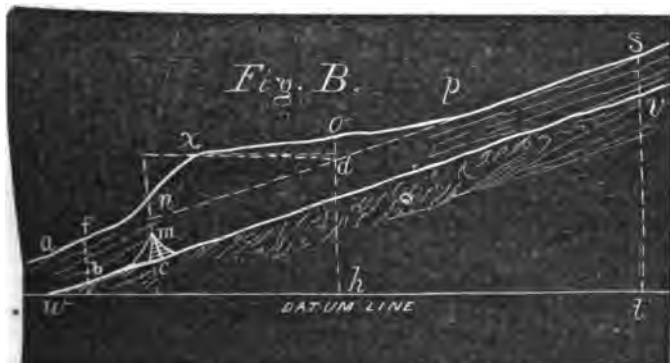
A riparian owner has no right to build any work which would in ordinary flood cause his neighbor's land to be overflowed, even if such was to protect his own property from being destroyed. Angel on Water Courses, chap. 9, p. 884.

In several countries, the law authorizes A to construct a drain or ditch from the nearest outlet of the overflow on his land, along the lowest level through his neighbor's land, to the nearest outfall. This is the law in Canada. Callis on Sewers, 186.

If A raises an obstruction by which B's mill grinds slower than before, A is liable to action. 7 Con. N. Y. Rep. 266, and 1 Rawle, Penn. Rep. 218.

Back water. No person without a grant or license is allowed to raise the water higher than where it is in its natural state, or, unless the so doing has been uninterruptedly done for twenty years. Regina v. North Midland Railway Company, Railway Cases, vol. 2, part 1. p. 1.

No one can raise the level of the water where it enters his land, nor lower it where it leaves it. Hill v. Ward, 2 Gill. Ill. Rep. 285.



Let s represent the surface of a uniform channel, and w its bottom. Let $w t$ = datum line, parallel to the horizon; $f b$, $g m$, $h d$ and $t s$ the respective heights above datum. Let from a to b belong to A, b to d belong to B, and d to s belong to C. B found that on his land he had 10 feet of a fall from d to n , and the same from n to f . He built a dam = $c m$, making the surface of the water at x the same height as the point d , and claimed that he did no injury to the owner C. If C had a peg or reference mark at d , before B raised his dam, he could prove that B caused back water on him. When this is not the case, recourse must be had to the laws of hydraulics. Mr. Neville, County Surveyor of Louth, Ireland, in his Hydraulics, p. 110, shows that (practically) in a uniform channel, when the surface of the water on the top or crest of the dam is on the same level with d , the water will back up to p , making $x p$ = 1.5 to 1.9 times $x d$.

The latter is that given by Du Buat, and generally used. See Encyclopedia Britannica, vol. 19. The former, 1.5, by Funk. See D'Aubuisson's Hydraulics by Bennett, sec. 167.

When the channel is uniform, the surface $x o p$ is nearly that of a hyperbola, whose asymptote is the natural surface; consequently, the dam would take effect on the whole length of the channel. All agree that the effect will be insensible, when the distance, $x p$, from the dam is more than 1.9 times the distance $x d$. Let x be the point behind the dam where the water is apparently still, then $m n$ is half the height of x above m , as the water, in falling from x , assumes the hydraulic curve, which is practically that of a parabola. As we know the quantity of water passing over in a given time, and the length of the dam, we can find the height $m n$, twice of which added to $c m$ gives the height of x above c . Let this height of x above $c = H$. Find where the same level through x , will meet the natural surface as at d , then measure $d p =$ nine-tenths of $d x$, the point p will be the practical limit of back water, or removal. *Within this limit we are to confine our inquiries, as to whether B has trespassed on C , and if the dam will cause greater damage in time of high water than when at its ordinary stage.* For further, see sections on Hydraulics.

Owners of Islands, own to the thread of the river on each side. *Hendrick v. Johnson*, 6 Porter, Alabama Rep. 472. The main branch or channel is the boundary, if nothing to the contrary is expressed. *Doddridge v. Thompson*, 9 Wheel, U. S. Report, 470. Above the margin goes to the centre. N. Y. Rep. 6 Cow. 518.

Natural and permanent objects are preferred to courses and distances. Hurley v. Morgan, 1 Devereaux and Bat. N. Carolina Report, 425.

Boundary may begin at a post or stake on the land, by the river, then run on a given course, a certain distance to a stake standing on the bank of the river, and so along the river. The law holds that the centre of the river or water course, is the boundary. 5 New Hampshire Rep. 520; see also Lowell vs. Robinson, 4 Maine Rep. 357.

A grant of land extending a given distance from a river, must be laid off by lines equidistant from the nearest points on the river. Therefore a survey of the bank of the river is made, and the rear line run parallel to this at the given distance. Williams v. Jackson, N. Y. Rep. 489.

PONDS AND LAKES.

309b. Land conveyed on a lake, if it is a natural one, extends only to the margin of the lake. But if the lake or pond is formed by a dam, backing up the water of a stream in a natural valley, then the grant goes to the centre of the stream in its natural state. State v. Gilmanton, 9 N. Hampshire R. 461.

The beds of lakes, or inland seas with the islands, belong to the public. The riparian owners may claim to low water mark. Land Commissioners v. People, 5 Wend. N. Y. R. 428. Where a pond has been made by a dam across a stream, evidence must be had by parol, or from maps showing where the centre of the river was; for if the land was higher on one side than on the other, the thread of the original stream would be found nearer to the high ground.

Island in the middle of a stream not navigable, is divided between the riparian owners, in proportion to the fronts on the river. 2 Blackstone, 1. But if the island is not in the middle, then the dividing line through it, is by lines drawn in proportion to the respective distances from the adjacent shores. 18 Wendell, N. Y. Rep. 255. If no part of the island is on one side of the middle of the river, then the whole of the island belongs to the riparian owners nearest to the island. See Cooper, Justice, lib. 2, t. 3, and Civil Code of Louisiana, art. 505 to 507.

An island between an island and the shore, is divided as if the island was main land, for if it be nearer the main land than the island, it is divided in proportion as above. Fleta, lib. 3, c. 11. § 8.

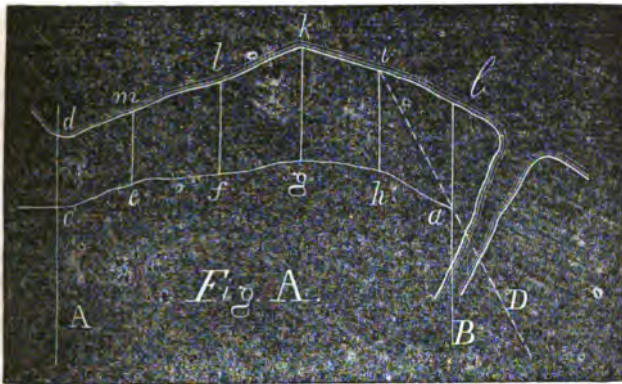
Where there are channels surrounding one or more islands, one has no right to place dams or other obstructions, by which the water of one channel may be diverted into another. 10 Wendell, N. Y. Rep. 260.

If a river or water course divides itself into channels, and cuts through a man's land, forming an island, the owner of the land thus encircled by water can claim his land. 5 Cowen, 216.

ACCRETION OR ALLUVION.

309c. Accretion or alluvion is where land is formed by the accumulation of sand or other deposits on the shore of the sea, lake or river. Such accretions being gradual or imperceptibly formed, so that no one exactly can show how much has been added to the adjacent land in a given time, the adjacent owner is entitled to the accretion. 2 Blackstone Com. 262. See also Cooper Justice, lib. 2, tit. 1.

In subdividing an accretion, find the original front of each of the adjacent lots, between the respective side lines of the estates; then measure the new line of river between the extreme side lines, and divide *pro rata*, then draw lines from point to point, as on the annexed diagram.



The meandered lines are taken from corner to corner of each lot, without regard to the sinuosities of the shore as *b i*.

It is sometimes difficult to determine the position of the lines *cd* and *eb*. As some may contend that *A c* produced in a straight line to the water, would determine the point *d*, also *B a* produced, would determine *b*, from the above diagram appears that by producing *B a* to the water, it would intersect near *i*, thus cutting off one owner from a part of the accretion, and entirely from the water.

The plan adopted in the States of Maine and Massachusetts, in determining *b* and *d*, is as follows: From *a* draw a perpendicular to *B a*, and find its intersection on the water's edge, and call it *Q*. From *a* with *a h* as base, draw a perpendicular, and find its intersection on the water's edge, and call it *P*. Bisect the distance *P Q* in the point *r*, then the line *a r*, determines the point *b*. In like manner we determine the point *d*. Having *b* and *d*, we find *i*, *k*, etc., as above.

In Maine and Massachusetts the point *i*, *k*, *l* and *m* are found as we have found *b* and *d*, erecting two perpendiculars from each abuttal, on the main land, one from each adjacent line and bisecting their distance apart for a new abuttal. 6 Pickering, Mass. Rep. 168; 9 Greenleaf, Maine Rep. 44.

When *A c* and *B a* are township lines, as in the Western States, they are run due East and West, or North and South. In this case, *d* and *b* would be found by producing *A c* and *B a* due East and West, or North and South, as the case may be. Now, let *B a c* be the original shore, and *d*, *b*, *n*, *a* and *B* the present shore, making *c*, *x*, *n*, *d* the accretion or alluvion. It is evident that it would be incorrect to divide the space *a*, *n*, *b*, *d*, between the riparian owners, that only *b d* should be so divided. When *A c* and *B a* are township lines run East and West, or North and South, as in the Western States, they are run on their true courses to the water's edge, intersecting at the points *d* and *b*. Here it would be plain that the space *b d* should be divided in proportion to the fronts *c e*, *e f*, etc., by the above method.

We do not know a case in Wisconsin or Illinois, where a surveyor has adopted this method. They run their lines at right angles to the adjacent section lines, which many of them take for a due East and West, or North and South line, as required by the act of Congress, passed 1805.

The accretion *b, a, n*, in our opinion, would belong to him who owns front *a h*. There is a similar case to this pending for some time in Chicago, where some claim that the water front, *a, n, b, d* should be divided; others claim that only *b* to *d*, as the part *a, b, n* may be washed away, by the same agent which has made it.

"Where land is bounded by water, and allusions are gradually formed, the owner shall still hold to the same boundary, including the accumulated soil. Every proprietor whose land is thus bounded, is subject to a loss by the same means that may add to his territory, and as he is without remedy for his loss in this way, he cannot be held accountable for his gain." *New Orleans v. United States*, laid down as a fundamental law by Judge Drummond, Oct. 1858, in his charge to the jury in the Chicago sand bar case.

When the river or stream changes its course. If it changes suddenly from being between *A* and *B*, to be entirely on *B*, then the whole river belongs to *B*. But if the recession of a stream or lake be gradual or imperceptible, then the boundary between *A* and *B* will be on the water, as if no recession had taken place. 2 Blackstone, Com. 262; 1 Hawkes, North Carolina R. 56.

When a stream suddenly causes A's soil to be joined to B's, A has a right to recover it, by directing the river in its original channel, or by taking back the earth in scows, etc., before the soil so added becomes firmly incorporated with *B's* land. 2 Blackstone Com. 262.

HIGHWAYS.

809d. *Highway* is a public road, which every citizen has a right to use. 8 Kent Comm. 32. It has been discussed in several States, whether streets in towns and cities are highways; but the general opinion is that they are. *Hobbs v. Lowell*, 19 Pick. Mass. Rep. 405; *City of Cincinnati v. White*, 8 Peters, U. S. Rep. 481. A street or highway ending on a river or sea, cannot be "blocked up" as to prevent public access to the water. *Woodyer v. Hadden*, 5 Taunton R. 125.

When a road leads between the land of *A* and *B*, and that the road becomes temporarily or unexpectedly impassable, the public has a right to go on the adjoining land. *Absor v. French*, 2 Show, 28; *Campbell v. Race*, 7 Cushing, Mass. Rep. 411.

Width of public highways is four rods, if nothing to the contrary is specified, or unless by user for twenty years, the width has been less. *Horlan v. Harriston*, 6 Cow. 189.

Twenty years uninterrupted user of a highway is *prima facie* evidence of a prescriptive right. 1 Saund., 323 a, 10 East 476.

Unenclosed lands adjoining a highway, may be travelled on by the public. *Cleveland v. Cleveland*, 12 Wend. 376.

Owners of the land adjoining a public highway, own the fee in the road, unless the contrary is expressed. The public having only an easement in it. When the road is vacated, it reverts to the original owners. Comyn digest Dig. tit. Chemin A 2; *Chatham v. Brand*, 11 Conn. R. 60; *Kennedy v. Jones*, 11 Alabama R. 63; *Jackson v. Hathaway*, 15 Johnson's Rep. 947.

A road is dedicated to the public, when the owners put a map on record showing the lots, streets, roads or alleys. Manly et al v. Gibson, 18 Illinois, 308.

In Illinois the courts have decided, that in the county the owners of land adjoining a road have the fee to the centre of it, and that they have only granted an easement, or right to pass over it, to the public. Country roads are styled highways. In incorporated towns and cities, roads are denominated streets, the fees of which are in the corporations or city authorities. The original owner has no further control over that part of his land. *Huntley v. Middleton, 13 Illinois, 54.*

In Chicago, however, the adjacent owners build cellars under the streets, and the corporation rents the ends of unbridged streets on the river, for dock purposes. Where streets are vacated, they revert to the original owners, as in other States. The adjacent owners must grade the streets and build the sidewalks, yet by the above decision they have no claim to the fee therein. It appears strange that Archer road outside the city limits is a highway, and inside the limits, is a street. The road outside and inside is the same. Part of that now inside, was in January, 1863, outside; consequently, what is now a street, was 10 months ago a highway. Then, the fee in the road was in the adjacent owners, now by the above decision, it is in the corporation. It seems difficult to determine the point where a highway becomes a street, and *vice versa*.

Footpaths. Cul-de-sac are thoroughfares leading from one road to another, or from one road to a church or buildings. The latter is termed a *cul-de-sac*. These, if used as a highway for 20 years, become a highway. *Wellbeloved on Highways, page 10. See Angel on Highways, sec. 29.*

A cannot claim a way over B's land.

A cannot claim a way from his land through B's; but may claim a way from one part of his land to another part thereof, through B's, that is when A's land is on both sides of B's. *Cruises' English Digest, vol. 3, p. 122.*

If A sells part of his land to B, which is surrounded on all sides by A's, or partly by A's and others, a right of way necessarily passes to B. 2 *Boll's Abridgment, Co. P. L. 17, 18.*

If A owned 4 fields, the 3 outer ones enclosing the fourth, if he sells the outer three, he has still a right of way into the fourth. *Cruise, vol. 3, p. 124*; but he cannot go beyond this enclosure. *Ibid, 126.* When a right of way has been extinguished by unity of possessions, it may be revived by severance. *Ibid, p. 129.*

Boundaries on highways, when expressed as bounded by a highway, it means that the fee to the centre of the road is conveyed. 3 *Kent Comm. 432.*

Exceptions to this rule are found in *Canal Trustees v. Haven, 11 Illinois R. 554*, where it is affirmed that the owner cannot claim but the extent of his lot.

By, on, or along, includes the middle of the road. 2 *Metcalf, Mass. R. 151.*

By the line of, by the margin of, by the side of, does not include the fee to any part of the road. 15 *Johnson, N. Y. R. 447.*

The town that suffers its highways to be out of repair, or the party who obstructs the same, is answerable to the public by indictment, but not to an individual, unless he suffers damage by reason thereof in his *person or property*. *Smith v. Smith*, 2 Pick. Mass. Rep. 621; *Forman v. Concord*, 2 New Hampshire Rep. 292. Individuals and private corporations are likewise liable to pay damages. 6 Johnson, N. Y. Rep. 90.

Lord Ellenborough says two things must concur to support this action; an obstruction in the road by the fault of the defendant, and no want of ordinary care to avoid it on the part of the plaintiff. *Butterfield v. Forrester*, 11 East. Rep. 60.

Towns, or corporations, are primarily liable for injuries, caused by an individual placing an obstruction in the highway. The town may be indemnified for the same amount. In Massachusetts the town or corporation is liable to double damages after reasonable notice of the defects had been given, but they can recover of the individual causing it but the single amount. *Merrill v. Hampden*, 26 Maine Rep. 224; *Howard v. Bridgewater*, 16 Pick. Mass. Rep. 189; *Lowell v. Boston and Lowell Railroad corporation*, 28 Pick. Mass. R. 24.

By the extension of a straight line, is to be understood, that it is produced or continued in a straight line. *Woodyer v. Hadden*, 5 Faunl. Rep. 125.

Plankroads, if made on a highway, continue to be highways, the public have the right to pass over them, by paying toll. *Angel on Highways*, sec. 14.

The Court has the jurisdiction to restrain any unauthorized appropriation of the public property to private uses; which may amount to a public nuisance, or may endanger, or injuriously affect the public interest. Where officers, acting under oath, are intrusted with the protection of such property, private persons are not allowed to interfere. 6 Paige, Chancery Rep. 188.

Railroads may be a public nuisance, when their rails are allowed to be 2 to 3 inches above the level of the streets, as now in Chicago,—thereby requiring an additional force to overcome the resistance. See *Manual*, 819c, where it has been shown, that the rail was 8 inches above the level of the street, and required a force of 969 pounds to overcome the resistance. This state of things would evidently be a public injury, and be sufficient reasons to prevent a recurrence of it in any place where it had previously existed. It may be a private injury, when the track is so near a man's sidewalk, as to prevent a team standing there for a reasonable time to load or unload.

When a road is dedicated to the public at the time of making a town plat or map, it is held that the street must have the recorded width though the adjoining lots should fall short, because the street has been first conveyed.

When a new street is made, the expense is borne by the adjacent owners or parties benefitted. Subsequent improvements are usually made by a general city or town tax; sometimes by the adjacent owners—the city paying for intersections of streets and sidewalks. In February, 1864, Judges Wilson and Van Higgins, of the Cook County (Illinois) Superior Court, decided that a lot cannot be taxed for more than the actual increase in its value, caused by the improvement in front thereof.

SIR RICHARD GRIFFITH'S SYSTEM OF VALUATION.

NOTE.—All new matter introduced is in italics or enclosed in parenthesis.

309c. *The intention of the General Valuation Act* was, that a valuation of the lands of Ireland, made at distant times and places, should have a relative value, ascertained on the basis of the prices of agricultural produce, and that though made at distant periods, should be the same. The 11th section of the Act, quoted below, gives the standard prices of agricultural produce, according to which the uniform value of any tenement is to be ascertained, and all valuations made as if these prices were the same, at the time of making the valuation.

309f. *Act 15 and 16 Victoria, Cap. 68, Sec. XI.*—Each tenement or rateable hereditament shall be separately valued, taking for basis the net annual value thereof with reference to prices of agricultural produce hereinafter specified; all peculiar local circumstances in each case to be taken into consideration, and all rates, taxes and public charges, if any, (except tithes) being paid by the tenant.

NOTE.—(The articles in italics are not in the above section, but inserted so as to extend the system as much as possible to America and other places.)

General average prices of 100 lbs. of

Wheat,	6s. 9d. or \$1.62	Mutton,	36s. 11d. or \$8.86
Oats,	4s. 4d. " 1.04	Pork,	28s. 10d. " 6.91
Barley,	4s. 11d. " 1.19	Flax,	44s. 1d. " 10.58
<i>Maize,</i>		<i>Hemp,</i>	
<i>Rice,</i>		<i>Tobacco,</i>	
Butter,	58s. 10d. or 14.11	<i>Cotton,</i>	
Beef,	35s. 3d. or 7.65	<i>Sugar,</i>	
&c.	&c.	&c.	

To find the price of live weights.—Deduct one-third for beef and mutton, and one-fifth for pork.

Houses and Buildings shall be valued upon the annual estimated rent which may be reasonably expected from year to year, the tenant paying all incidental charges, except tithes.

Sections 12 to 16, inclusive, of the act, treat of the kind of properties to be valued.

309g. *Lands and Buildings* used for scientific, charitable or other public purposes, are valued at half their annual value, all improvements and mines opened during seven years; all commons, rights of fishing, canals, navigations and rights of navigation, railways and tramways; all right of way and easement over land; all mills and buildings built for manufacturing purposes, together with all water power thereof. But the valuation does not extend to the valuation of machinery in such buildings.

A tenement is any rateable hereditament held for a term of not less than one year.

Every rateable tenement shall be separately valued.

The valuator shall have a map showing the correct boundary of each tenement, which shall be marked or numbered for references. The map shall show if half streets, roads or rivers are included.

The Field Book is to contain a full description of every tenement in the townland (or township), the names of the owners and occupiers, together with references to the corresponding numbers on the plan or map. The book to be headed with the name of the county, parish (or township), each townland (or section).

Gentlemen of property, learning, or the law, should have "Esquire" attached to their names.

Land, is ground used for agricultural purposes.

Houses and Offices, are buildings used for residences.

Other tenements, such as brickfield, brewery, &c.

To determine the value of land, particular attention must be paid to its geological and geographical position, so far as may be necessary to develop the natural and relative power of the soil.

NATURE OF SOILS.

309A. *Examine the soil and subsoil* by digging it up, in order to ascertain its natural capabilities; for if guided by the appearance of the crops, the valuator may put too high a price on bad land highly manured. This would be unjust, as it is the intrinsic and not the temporary value which is to be determined.

To obtain an average value, where the soil differs considerably in short distances; examine and price each tract separately, and take the mean price.

The value of soil depends on its composition and subsoil.

Subsoil may be considered the *regulator* or *governor* of the powers of the soil, for the nature of its composition considerably retards or promotes vegetation.

In porous or sandy soil, the necessary nutriment for plants is washed away, or absorbed below the roots of the plants.

In clayey soils, the subsoil is impervious, the active or surface soil is cold and late, and produces aquatic plants. Hence appears the necessity of strict attention to the subsoil.

Soils are compounded of organic and inorganic matter: the former derived from the disintegration and decomposition of rocks. The proportion in which they are combined is of the utmost importance.

A good soil may contain six to ten per cent. of organic matter; the remainder should have its greater portion silica; the lesser alumina, lime, potash, soda, &c.—(See tables of analysis at the end of these instructions.)

Soils vary considerably in relation to the physical aspect; thus in mountain or hilly districts, where the rocks are exposed to atmospheric influence, the soils of the valleys consist of the disintegrated substance of the rocks, whilst that of the plains is composed of drifted materials, foreign to the subjacent rock. In the former case the soil is characterised by the locality; in the latter it is not.

By reference to the *Geological Map of Ireland*, it will be seen that the mountain soil is referable to the granite, schistose rocks and sandstone.

The fertility of the soil is to some extent dependent on the proportion or combinations which exist between the component minerals of the rocks from which it may have been formed; thus granite in which feldspar is in excess when disintegrated, usually forms a deep and easily improved soil, whilst that in which it is deficient will be comparatively unproductive.

The detritus of mica slate and the schistose rocks form moderately friable soils fit for tillage and pasture.

Sandstone soils derived from sandstone, are generally poor.

The most productive lands in Ireland are situate in the carboniferous limestone plain, which, as shown on the Geological Map, occupies nearly two-thirds of that country. When to the naturally fertile calcareous soils of this great district, foreign matters are added, derived from the disintegration of granite and trappean igneous rocks, as well as from mica slate, clay slate, and other sedimentary rocks, soils of an unusually fertile character are produced. Thus the proverbially rich soil of the *Golden-val*, situate in the limestone district extending between Limerick and Tipperary, is the result of the intermixture of disintegrated trap derived from the numerous igneous protusions which are dispersed through that district, with the calcareous soil of the valley.

Lands of superior fertility occur near the contacts of the upper series of the carboniferous limestone and the shales of the millstone grit, or lower coal series; important examples of this kind will be found in the valley of the Barrow and Nore, etc., etc.

For geological arrangement the carboniferous limestone of Ireland has been divided into four series.

1st Series beginning from below the yellow sandstone and carboniferous slate.

2d Series, the lower limestone.

3d Series, the calp series.

4th Series, the upper limestone.

Soil derived from 1st Series is usually cold and unproductive, except where beds of moderately pure limestone are interstratified with the ordinary strata, consisting of sandstone and slate-shale.

The 2d Series, when not converted by drift, consisting chiefly of limestone-gravel intermixed with clay, usually presents a friable loam fit for producing all kinds of cereal and green crops, likewise dairy and feeding pastures for heavy cattle, and superior sheep-walks.

The 3d Series consists of alternations of dark grey shale, and dark grey impure argillo-siliceous limestone, producing soil usually cold, sour, and unfit for cereal crops; but in many districts naturally dry, or which has been drained and laid down for pasture. This soil produces superior feeding grasses, particularly the cock's foot grass. These pastures improve annually, and are seldom cultivated, because they are considered the best for fattening heavy cattle.

The 4th Series produces admirable sheep pasture, and, in some localities, superior feeding grounds for heavy cattle, and produces every variety of cereal and green crops.

809i. It is of the utmost importance that the valuator should carefully attend to the mineral composition of the soil in each case, and a reference to the Geological Map will frequently assist his judgment in this respect, the relative position of the subjacent rocks having been determined upon sectional and fossiliferous evidence. He should carefully observe the changes in the quality and fertility of the soil near to the boundaries of different rock formations, and should expect and look for sudden transitions from cold, sterile, clayey soils, as in the millstone grit districts, into the rich unctuous loams of the adjoining limestone districts, which

usually commence close to the line of boundary; and similar rapid changes will be observed from barrenness to fertility, along the boundaries of our granite, trap, and schistose districts, and likewise on the border of schistose and limestone districts, the principle being that every change in the composition of the subjacent rocks tends to an alteration in the quality both of the active and subsoils.

As it appears from the foregoing that the detritus of rocks enters largely into the composition of soils and other formations, the most trustworthy analysis is supplied, which, compared with the crops usually cultivated, will show their relative value and deficiencies.

NOTE.—(The table of analysis given by Sir Richard Griffith is less than one page. Those given by us in the following pages of these instructions are compiled from the most authentic sources, and will enable the valuator or surveyor to make a correct valuation. The surveyor will be able, in any part of the world, to give valuable instructions to those agriculturists with whom he may come in contact. We also give the method of making an approximate analysis of the rocks, minerals and soils which he may be required to value. Where a more minute analysis is required, he may give a specimen of that required to be analysed to some practical chemist—such as Jackson, of Boston; Hunt, of Montreal; Blaney, Mariner, or Mahla, of Chicago; Kane, or Cameron, of Dublin; Muspratt, or Way, of England, etc. etc.)

Table in section 310 contains the analysis of rocks and grasses.

Section 310a, analysis of trees and grasses.

Section 310b, analysis of grains, hemp and flax.

Section 310c, analysis of vegetables and fruit.

Section 310d, analysis of manures.

Section 310e, comparative value of manures; the whole series making several pages of valuable information.

In Canada, the law requires that Provincial Land Surveyors should know a sufficient share of mineralogy, so as to enable them to assist in developing the resources of that country. In Europe, all valuations of lands are generally made by surveyors, or those thoroughly versed in that science; but in the United States a political tinsmith may be an assessor or valuator, although not knowing the difference between a solid and a square. This state of things ought not to be so, and points out the necessity of forming a Civil Engineers' and Surveyors' Institute, similar to those in other countries.)

From these tables it will appear what materials are in the formation of the soil, and the requirements of the plants cultivated; thus, in corn and grasses, silica predominates. Seeds and grain require phosphoric acid. Beans and leguminous plants require lime and alkalis. Turnips, beets and potatoes require potash and soda.

The soils of loamy, low lands, particularly those on the margins of rivers and lakes, usually consist of finely comminuted detrital matter, derived from various rocks; such frequently, in Ireland, contain much calcareous matter, and are very fertile when well drained and tilled. The rich, low-lying lands which border the lower Shannon, etc., are alluvial, and highly productive.

It is necessary that the valuator should enter into his book a short, accurate description of the nature of the soil and subsoil of every

tenement which may come under his consideration, and that all valuers may attach the same meaning or descriptive words to them. The following classification will render this description as uniform as possible :

Classification of soils, with reference to their composition, may be comprehended under the following heads, viz :

Argillaceous or clayey—clayey, clayey loam, argillaceous, alluvial.

Silicious or sandy—sandy, gravelly, slaty or rocky.

Calcareous—limey, limestone gravel, marl.

Peat soil—moor, peat.

The color of soils is derived from different admixtures of oxide or rust of iron.

Argillaceous earths, or those in which alumina is abundant, as brick and pipe clays.

The soil in which alumina predominates is termed clay.

When a soil consists chiefly of *blue* or *yellow* tenacious clay upon a retentive subsoil, it is nearly unfit for tillage ; but on an open subsoil it may be easily improved. Clayey soils containing a due admixture of sand, lime and vegetable matter, are well adapted to the growth of wheat, and are classed amongst the most productive soils, *where the climate* is favorable. Soils of this description will, therefore, graduate from cold, stiff clay soils to open clay soils, in proportion as the admixture of sand and vegetable matter is more or less abundant, and the subsoil more or less retentive of moisture.

Loams are friable soils of fine earth, which, if plowed in wet weather, will not form clods.

A *strong clayey loam* contains about one-third part of clay, the remainder consisting of sand or gravel, lime, vegetable and animal matters, the sand being the predominating ingredient.

A *friable clayey loam* differs from the latter by containing less clay and more sand. In this case the clay is more perfectly intermixed with the sand, so as to produce a finer tilth, the soil being less retentive of moisture, and easier cultivated in wet weather.

Sandy or gravelly loams is that where sand or gravel predominates, and the soil is open and free, and not sufficiently retentive of moisture.

A *stiff clay soil* may become a rich loam by a judicious admixture of sand, peat, lime and stable manure, but after numerous plowings and exposure to winter frosts in order to pulverize the clay, and to mix with it the lime, peat, sand, etc.

Alluvial soils are generally situated in flats, on the banks of rivers, lakes, or the sea shore, and are depositions from water, the depositions being fine argillaceous loam, with layers of clay, shells, sand, etc. The subsoil may be different.

On the sea shore and margin of lakes, the the clay subsoils usually contain much calcareous matter in the form of broken shells, and sometimes thick beds of white marl.

The value of the soil and subsoil depend on the proportion of lime it may contain. This may be found by an analysis. (See *sequel* for analysis.)

Rich alluvial soils are the most productive when out of the influence of floods. These soils are classed as clayey, loamy, sandy, etc., according to their nature.

Flat lands or holms, on banks of rivers, are occasionally open and sandy, but frequently they are composed of most productive loams.

SILICEOUS SOILS.

309j. *Sandy soils* vary very much in their grade, color and value, according to the quality of the sand. White shelly sands, which are usually situated near the sea shore, are sometimes very productive, though they contain but a very small portion of earthy matter.

Gravelly soils are those in which coarse sand or gravel predominates; these, if sufficiently mixed with loam, produce excellent crops.

Slatey soils occur in mountains composed of slate rock, either coarse or fine grained. In plowing or digging the shallow soils on the declivities of such places, a portion of the substratum of slate intermixes with the soil, which thus becomes slatey.

Rocky soils. Soil may be denominated rocky where it is composed of a number of fragments of rock intermixed with mould. Such soils are usually shallow, and the substratum consists of loose broken rock, presenting angular fragments.

CALCAREOUS SOILS.

309k. *Calcareous or limestone soils*, are those which contain an unusual quantity of lime, and are on a substratum of limestone. These lands form the best sheepwalks.

Limestone gravel soil, is where we find calcareous or limestone gravel forming a predominant ingredient in soils.

Marly soils are of two kinds, clayey marl, or calcareous matter combined with clay and white marl, which is a deposition from water, and is only found on the margins of lakes, sluggish rivers and small bogs.

On the banks of the River Shannon, beds of white marl are found 20 feet deep. When either clayey or white marl enters into the composition of soils, so as to form an important ingredient, such soils may be denominated *marly*.

PEATY SOILS.

309l. *Flat, moory soils* are such as contain more or less peaty matter, assuming the appearance of a black or dark friable earth. When the peat amounts to one-fourth, and the remainder a clayey loam, the soil is productive, especially when the substratum is clay or clayey gravel.

When the peat amounts to one-half, the soil is less valuable.

When the peat amounts to three-fourths of the whole, the soil becomes very light, and decreases in value in proportion to the increase of the peat in the soil.

Peaty or boggy soils are composed of peat or bog, which, when first brought into cultivation, present a fibrous texture and contain no earthy matter beyond that which is produced by burning the peat.

The quantity of ashes left by burning is red or yellow ashes, about one-eighth of the peat, generally one-tenth or one-twelfth in shallow bogs.

In deep bogs the ashes are generally white, and weigh about one-eightieth of the peat. Such land is of little value unless covered with a heavy coat of loamy earth or clay. Hence it appears that the value of peaty soil depends on the amount of red ashes it contains. For this reason peaty soils are valued at a low price.

NOTE.—(Bousingsault, in his "Rural Economy," says: "*The quality of an arable land depends essentially on the association of its clay and sand or gravel.*"

Sand, whether it be siliceous, calcareous or felspathic, always renders a soil friable, permeable and loose; it facilitates the access of the air and the drainage of the water, and its influence depends more or less on the minute division of its particles.)

The following table, given by Sir Richard Griffith, is from Von Thae'r's Chemistry, as found by him and Einhoff:

300m.	CLASS.	Clay, per cent	Sand, or Gravel, per cent	Carbonate of Lime, per cent	Humus per cent	Compa- rative Value.
1	First class strong wheat land..	74	10	4.5	11.5	100
2	Do.....	81	6	4	8.7	98
3	Do.....	79	10	4	6.5	96
4	Do.....	40	22	86	4	90
5	Rich light land in natural grass	14	49	10	27	—
6	Rich barley land.....	20	67	8	10	78
7	Good wheat land.....	58	86	2	4	77
8	Wheat land.....	56	80	12	2	75
9	Do.....	60	88	} In small proportions.	2	70
10	Do.....	48	50		2	65
11	Do.....	68	80		2	60
12	Good barley land.....	88	60		2	60
13	Do. second quality.....	88	65		2	
14	Do. Do.....	28	70		2	
15	Oat lands.....	23½	75		1.5	
16	Do.	18½	80		1.5	
17	Rye land.....	14	85		1	
18	Do do	9	90		1	
19	Do do	4	95		0.75	
20	Do do	2	97.5		0.5	

Under the head clay, has been included alkalis, chlorides, and supposed to be in fair proportions. The soil in each case supposed to be uniform to the depth of six inches.

In the *Field Book* the following explanatory terms may be used as occasion may require:

Stiff.—Where a soil contains a large proportion (say one-half or even more) of tenacious clay; this cracks in dry weather, forming into lumps.

Friable.—Where it is loose and open, as in sandy, gravelly or moory lands.

Strong.—Where it has a tendency to form into clods.

Deep.—Where the depth is less than 8 inches.

Dry.—No springs. Friable soil, and porous subsoil.

Wet.—Numerous springs; soil and subsoil tenacious.

Sharp.—A moderate share of gravel or small stones.

Fine or soft.—No gravel: chiefly composed of very fine sand, or soft, light earth, without gravel.

Cold.—Grows on a tenacious clay subsoil, and has a tendency, when in pasture, to produce rushes and other aquatic plants.

Sandy or gravelly.—A large proportion of sand or gravel.

Slatey.—Where the slatey substratum is much mixed with the soil.

Worn.—Where it has been a long time cultivated without rest or manure.

Poor.—When of a bad quality.

Hungry.—When consisting of a great proportion of gravel or coarse sand resting on a gravelly subsoil. On such land manure does not produce the usual effect.

The color of the soil and the features of the land ought to be mentioned, such as steep, level, rocky, shrubby, etc., etc.

Indigenous plants should be observed, as they sometimes assist to indicate particular circumstances of soil and subsoil.

<i>Name of Plant.</i>	<i>Indicates</i>
Thistle.....	Strong, good soil.
Dockweed and nettle.....	Rich, dairy land.
Sheep sorrell.....	Gravelly soil.
Trefoil and vetch.....	Good dry vegetable soil.
Wild thyme.....	Thinness of soil.
Ragweed.....	Deep soil.
Mouse-ear hawk-weed.....	Dryness of soil.
Iris, rush and lady's smock.....	Moisture of soil.
Purple red nettle and naked horsetail.....	Retentive subsoil.
Great Ox-eye.....	Poverty of soil.

CLASSIFICATION OF SOILS WITH REFERENCE TO THEIR VALUE.

809*n*. All lands to be valued may be classed under arable and pasture.

Arable land may be divided into three classes, viz :

Prime soils, rich, loamy earth.

Medium soils, rather shallow, or mixed.

Poor soils, including cultivated moors.

Pasture, as fattening, dairy and stone land pastures.

The prices set forth in the Act (see sec. 809*f*) is the basis on which the relative and uniform valuation of all lands used for agricultural purposes must be founded. It is incumbent on the valuator to ascertain the depth of soil and nature of subsoil, to calculate the annual outlay per acre. He should calculate the value per acre of the produce, according to the scale of the Act, and from these data deduce the net annual value of the tenement.

309*o*. *Tables of produce, etc.*, formulæ for calculation, and an acreable scale of prices, supplied in the following sections, are given as auxiliaries with a view to produce uniformity among the valutors employed. Thus, if the valuator finds it necessary to test his scale of prices for a certain quality of land, he may select one or more farms characteristic of the average of the neighborhood. Their value should be correctly calculated and an average price per acre obtained, from which he deduces the standard field price of such description of land. The farms (or fields) thus examined will serve as points of comparison for the remainder of the district.

SCALE FOR ARABLE.

<i>Class and Description.</i>		<i>Average price per acre.</i>			
		<i>From. To.</i>			
PRIME SOILS.	1. Very superior, friable clayey loam, deep and rich, lying well, neatly fielded, on good, sound clayey subsoil, having all the properties that constitute a superior subsoil, average produce 9 barrels (or stones = lbs. = bushels) per acre.....	s.	d.	s.	d.
		30	0	26	0
	2. Superior, strong, deep and rich, with inferior spots deducted, lying well on good clay subsoil.....	27	0	24	0
MEDIUM SOILS.	3. Superior, not so deep as the foregoing, or good alluvial soils—surface a little uneven.....	25	0	22	0
	4. Good medium loams, or inferior alluvial land of an even quality.....	21	0	18	0
	5. Good loams, with inferior spots deducted.....	17	6	15	0
	6. Medium land, even in quality, rather shallow, deep and rocky.....	14	0	10	0

POOR SOILS. CULTIVATED MOORS OR BOGS.	7. Cold soil, rather shallow and mixed, lying steep on cold clayey, or cold, wet, sandy subsoil.....	9 0 7 0
	8. Poor, dry, worn, clayey or sandy soil, on gravelly or sandy subsoil.....	6 6 5 0
	9. Very poor, cold, worn, clayey, or poor, dry, shallow, sandy soil, or high, steep, rocky, bad land.....	4 0 1 0
	10. Good, heavy moor, well drained, on good, clayey subsoil	12 0 10 0
	11. Medium moory soil, drained, and in good condition.....	9 0 6 0
	12. Poor moory, or boggy arable, wet, and unmixed with earth.....	5 6 1 0

The above prices opposite each class is what the valuator's field price should be in an ordinary situation, subject to be increased or decreased for local circumstances, together with deductions for rates and taxes.

303p. *Of Arable land.*—The amount of crop raised depends on the system of tillage, and the crops raised. The system of cultivation should be such as would maintain an adequate number of stock to manure the farm, and the crops should be suited to the soil; thus, lands on which oats or rye could be profitably grown, may not repay the cost of cultivating it for wheat.

The following tables show the average maximum cost, produce and value of crops in ordinary cultivation for one statute acre.

TABLE OF PRODUCE.

	Potatoes	Mangel Wurzel.		Turnips	Vetches (Green.)	Cabbage (Kale.)	Beans.
		Longred or Orange.	Leaves.				
Total produce in tons.....	7	22	1	20	4	20	cwt. 20
Price per ton.....	s. d. 40 0	s. d. 10 0	s. d. 5 0	s. d. 8 0	s. d. 30 0	s. d. 5 0	s. d. 8 0
Total val. of produce pr acr.	£ s. 14 0	£ s. 11 5		£ s. 8 0	£ s. 6 0	£ s. 5 0	£ s. 8 0
Total cost of culture pr acr.	8 10	6 15		7 0	3 3	1 18	5 10

	Wheat.		Barley.		Oats.		Rye.		Flax.	Meadow.		Clover.
	Grain.	Straw.	Grain.	Straw.	Grain.	Straw.	Grain.	Straw.		Hay.	After grass.	
Total produce pr. acre.....	Brls. 8	Tns 2 10	Brls. 14	Tns 1 14	Brls. 11	Tns 1 14	Brls. 10	Tns 2	Cwt. 5	Tns 2 14	Tns 2 14	Tns. 3
Price per.....	s. d. 18 9	s. d. 15	s. d. 11	s. d. 13	s. d. 8 5 14	s. d. 17	s. d. 6	s. d. 14	s. d. 45	s. d. 30	s. d. 5	s. d. 30
Total val of pr'duce	£ s. d. 9 0 0	£ s. d. 6 10 9	£ s. d. 6 3 0	£ s. d. 4 8 0	£ s. d. 11 5 4	£ s. d. 7 6 4	£ s. d. 11 5 4	£ s. d. 7 6 4	£ s. d. 11 5 4	£ s. d. 7 6 4	£ s. d. 7 6 4	£ s. d. 11 5 4
Total cost of culture	3 9 0	3 2 0	3 11 0	3 0 0	7 8 1	9 6 2	0	0				0

NOTE.—The barrel is pounds, and the ton = 2,240 pounds.

From this table it appears that the cost of cultivating turnips, and other broad-leaved plants, is greater than that for grain crops.

309g.

SCALE OF PRICES FOR PASTURE.

Classes and Description.	Acres, Statute.	Average.			Price per Acre.	Observations.
		Stock in				
		Cattle.	Sheep.	Swine.		
FEEDING LAND.					s. s.	
1. Very superior fattening land, soil composed of finely comminuted loam, producing the most succulent qualities of grass, exclusively used for finishing heavy cattle and sheep,	10	Two sets of cattle of six each set, from 1st April to 1st Sept. finished.	One set of sheep 5 to each acre for 8 months.	—	35 to 31	This soil being used for "finishing" cattle and sheep, the latter replace the former when finished for market.
DAIRY LAND.						
2. Superior dairy pasture or fattening land, with verges of prime heavy moors, all having a grassy tendency,	15	Six and 3 calves.	—	3	30 to 24	This land is calculated at 3½ firkins of butter to each cow.
3. Good dairy pasture on clay or sandy soils, or good rocky pasture, each adapted to dairy purposes or fattening sheep,	20	Six and 3 calves.	—	3	23 to 17	This soil is calculated at 2½ firkins of butter to each cow.
4. Tolerable mixed clayey or moory pastures, or good rocky pasture, adapted to dairy purposes or the rearing of young cattle or sheep,	25	Six and 3 calves.	—	3	16 to 11	This description of soil is calculated at 2½ firkins of butter to each cow.
STORE PASTURE.						
5. Coarse sour rushy pasture on shallow clayey or moory soil, or dry rocky shrubby pasture, adapted to the rearing of young cattle or store sheep,	30	These two divisions are calculated at 18½-year old two-year old heifers, equal to 6 collops for each subdivision.	Or 36 one-year old sheep in each case for one year.	—	10 to 5	This description of soil is calculated for the purpose of rearing young cattle or sheep.
6. Inferior coarse sour pasture on cold shallow clayey or shallow moory soil, or dry rocky shrubby pasture, adapted chiefly to winterage for young cattle or store sheep,	35			—	6 to 4	The description of land that this brace includes ranges from coarse sour verges, inferior dry rocky pastures, and mixed green and heathy pastures, chiefly adapted and generally used for the rearing of young cattle of an inferior description.
7. Good mixed green and heathy pasture in the homestead of mountains or inferior dry rocky shrubby pasture, adapted to the rearing of light dry cattle or sheep,	40			—	3	
8. Mixed green and heathy mountain pasture, or inferior close rocky or shrubby pasture, adapted to the rearing of young cattle or sheep,	45			—	2	
9. Mixed brown heathy pastures with spots of green intermixed, or very inferior bare rocky pastures, or steep shrubby banks near homestead,	50			—	1s. to 9d	
10. Heathy pastures high and remote, or cut away bog, partly pasturable.	—	—	—	—	8d to 4d	
11. Red bog or coarse high remote mountain tops,	—	—	—	—	3d to 1d	
12. Precipitous cliffs,	—	—	—	—	—	

NOTE.—The price inserted opposite each class of lands, according to its respective produce, is what the valuator's field price should be in an ordinary situation, subject to be increased or reduced for particular local circumstances, together with deductions for rates and taxes.

In the calculations for testing his scale price, the valuator should tabulate, as above, at the prices per ton or barrel, the average produce per acre of the district under consideration. These values he will again tabulate according to the system of farming adopted.

The following may serve as a formula :

**ONE HUNDRED STATUTE ACRES UNDER FIVE YEARS' ROTATION
AS FOLLOWS:**

		Acres. Stat.	Cost of Tillage.			Value of Tillage.		
			£	s.	d.	£	s.	d.
1st Year, $\frac{1}{5}$ or 20 acres,	Potatoes, . . .	3	25	10	0	42	0	0
	Vetches, . . .	2	6	6	0	12	0	0
	Mangel Wurtzel, . . .	3	20	5	0	38	15	0
	Turnips, . . .	12	84	0	0	96	0	0
2d Year, $\frac{1}{5}$ or 20 acres,	Winter Wheat, . . .	12	41	0	0	108	0	0
	Spring Wheat, . . .							
	Barley, . . .	8	24	17	0	52	0	0
3d Year, $\frac{1}{5}$ or 20 acres,	Hay, . . .	6	8	17	0	26	5	0
	Clover, . . .	1	2	0	0	4	10	0
	Pasture, . . .	18	41	0	0	95	0	0
4th Year, $\frac{1}{5}$ or 20 acres,	Pasture, . . .	20						
5th Year, $\frac{1}{5}$ or 20 acres,	Potato Oats, . . .	20	70	13	0	128	0	0
	Common Oats, . . .							
		100	324	16	0	592	10	0
Allow for wear and tear of implements, . . .			10	0	0			
" Five per cent. on £500 capital, . . .			25	0	0			
Deduct Expenses, . . .						359	16	0
Nett Annual Value of Produce, . . .						282	14	0

FATTENING LANDS.

309r. It has been ascertained that the fat in an ox is one-eighth of the lean, and is in proportion of the fatty matter to the saccharine and protein compounds in the herbage. The method of grazing, too, has some influence. The best lands will produce about ten tons of grass per acre, in one year. One beast will eat from seven to nine stones in one day. Six sheep will eat as much as one ox. One Irish statute acre of prime pasture will finish for the market two sets of oxen from April to September. From September until December it is fed by sheep. The general formulæ may be as follows:

SUPERIOR FINISHING LAND.

Mode of Farming and Description of Stock.	Nett Increase.	Act. Price.	Am't.
	cwt.qrs.lbs	s. d.	£ s. d.
Two sets of cattle to be finished in the season, the lands preserved during the months of January, February and March.			
A four-year old heifer, weighing about 5 cwt., well wintered, and coming on in good condition, in the first two months of April and May, will increase,	1 2 0	35 6	2 18 3
A heifer in the same condition, in the months of June, July and August, will increase,	1 2 0	"	2 13 3
On the same land, 5 sheep to the Irish acre will increase at the rate of 2 lb. per week, for October, November and December,	1 1 0	41 0	2 11 8
Gross produce on one Irish acre, or 1A. 2R. 19P. statute measure,			7 17 9

Expenses.	£	s.	d.
Interest on capital for one beast to the Irish acre, at 5 per cent. for £10,	0	10	0
Herd, per Irish acre, (a herd will care 150 Irish acres,) at 2s. per acre,	0	2	0
Contingencies,	1	10	0
Commission on the sale of 2 beasts and 7 sheep, at 2½ per cent.,	1	9	0
Extra expenses,	0	8	0
Deduct expenses,		3	19 0
Nett produce per Irish acre, or 1A. 2R. 10P., statute measure, .			3 18 9

Cattle in good condition will fatten quicker on this description of land during the early months than under the system of stall-feeding.

DAIRY PASTURE.

309a. *Dairy pastures* are more succulent than fattening lands. The average quantity of butter which a good cow will give in the year may be taken at 3½ firkins = 218 lbs.; or, allowing nine quarts to the pound of butter, the milk will be 1,960 quarts. If the stock be good, under similar circumstances its produce may be considered to vary with the quantity and quality of the herbage. This and the quality and suitability of the stock must be carefully discriminated and considered.

The general formula is as follows:

In column A, set the cows and produce; the hogs, and increase in weight; the calves, when reared; the milk used by the family. In column B, set the weight of the produce. In column C, set the Act price. And in column D, the amount. The sum of column D will be the gross receipts, from which deduct the sum of all the expenses, rent of land under tillage, and the difference will be the nett annual produce for that part used as a dairy pasture.

STORE PASTURE.

309b. The value of store pasture depends on the amount of stock it can feed. The valuator will estimate the number of acres which would feed a three years beast for the season, from which the number of stock for the whole tenement may be ascertained, which, calculated at an average rate for their increase or improvement, will give the gross value. This valuation must be checked for all incidental expenses and local circumstances—in general, *two-thirds of the gross produce* may be considered as a fair value.

In mountain districts, it is divided into inside and remote grazing. The inside is allotted for milch cattle and winter grass. The remote or outside pasture is for summer grazing for dry cattle and sheep.

The annual value of these pastures is to be obtained from the herds or persons living on or adjacent to them, taking for *basis* the number of *sums* grazed and the rate per sum.

The following will enable the valuator to estimate the number of *sums* on any tenement:

One three years old heifer is called a "*sum*" or *collop*; one sum is = to three yearlings = one two years old and one, one year old = four

ewes and four lambs = five two years old sheep = six *hoggets* (one year old sheep) = to two-thirds of a horse.

LAND IN MEDIUM SITUATION.

309u. The above classifications, scales of prices, etc., for different kinds of land, have been calculated with reference to the quality of the soil and its productive capabilities, arising from the composition, depth and nature of the subsoil, without taking into consideration the extremes of position in which each particular kind may occasionally be found. The value thus considered may be defined as *the value of land in medium or ordinary situation*.

Land in an ordinary or medium situation. Should not be distant more than five or six miles from a principal market town, having a fair road to it, not particularly sheltered or exposed, not very conveniently or very inconveniently circumstanced as to fuel, lime and manures; not remarkably hilly or level, the greatest elevation of which shall not exceed 300 feet above the level of the sea.

When the valuation of the property is made, he will enter in the first column the valuations obtained, and in the second column the valuations corrected for local circumstances.

LOCAL CIRCUMSTANCES.

309v. The local circumstances may be divided into two classes, viz: natural and artificial.

Natural, is that which aids or retards the natural powers of the soil in bringing the crop to maturity.

Artificial, is that which afford or deny facilities to maintain or increase the fertility of the soil, and such as involve the consideration of remunerations for labor of cultivation. Local circumstances may, therefore, be classed under—*climate, manure, and market*.

309w. *Climate* includes all the phenomena which affect vegetation, such as temperature, quantity of atmospheric moisture, elevation, prevailing winds, and aspect. Various combinations of these, and other external causes, are what cause diversity of climate.

The germination of plants, and the amount of atmospheric moisture, are considerably dependent on temperature; hence the advantage of a locality in which its mean is greatest. Its average in Ireland varies from 48° (Fahrenheit) in the north to 51° in the south, the corresponding atmospheric moisture being from 4.27 to 4.88 grains to the cubic foot. These are considerably modified by elevation, which produces nearly the same effect as latitude, every 350 feet in height being equivalent to one degree of temperature.

309x. The average depth of rain which falls in one year in Ireland, varies from 40 inches on the west coast to 33 on the east. The proportion of the rain fall is greater for the mountain districts than for the low lands. The general effect of elevation on arable lands in this case are, that the soluble and fine parts of the soil are washed out, and ultimately carried down by the streams. Such elevated districts are also frequently exposed to high winds, etc. The prevailing winds, and how modified, are to be taken into consideration.

309y. *In Ireland, on land exposed to westerly winds, the crops are fre-*

quently injured in the months of August and September. A suitable deduction should therefore be made for such lands, although the intrinsic value may be similar to land in a more sheltered situation.

To determine the influence of climate requires considerable care and extensive comparison. Thus, the soil which in an elevated district is worth 10s. per acre, will be worth 15s. if placed in an ordinary situation, about 300 feet above the level of the sea, and not particularly sheltered or exposed. The same description of lands, however, in a more favorable situation, say from 50 to 100 feet above the sea, distant from mountains, and having a south-east aspect, may be worth 20s. per acre.

In making deductions from cultivated lands, in mountainous districts, the following table will be found useful, and may be applied in connection with heights given in Ordnance Survey maps:

Altitude in feet.	Deduct per £.
800 to 900 feet.....	5 shillings.
700 " 800 "	4 " "
600 " 700 "	3 " "
500 " 600 "	2 " "
400 " 500 "	1 " "

Arable land in the interior of mountains, may be considered 100 feet of altitude, worse than on the exterior declivities on the same height; so also those on the north may be taken 100 worse than those having a southern aspect, both having the same height.

In mountain districts, take the homestead pasture at 3, the outer at 2, and the remote at 1.

Deduct for steepness in proportion to the inconvenience sustained by the farmer in plowing and manuring.

Deduct for bad roads, fences, and for difference in the soils of a field where it is of unequal quality.

MANURE.

809z. *Manures* are that which improve the nature of the soil, or restore the elements which have been annually consumed by the crops. The most important of these, in addition to stable manure and that produced from towns, consist of limestone, coal turbarry, sea weed, sea sand, etc.

In a limestone country, where the soil usually contains a sufficient quantity of calcareous matter, the value of lime as a manure is trifling when compared to its striking effects in a drained clayey or loamy argillaceous soil. It promotes the decomposition of vegetable or animal matter existing in the soil, and renders stiff clay friable when drained, and more susceptible of benefit from the atmosphere, by facilitating the absorption of ammonia, carbonic acid gas, etc.; decomposes salts injurious to vegetation, such as sulphate of iron, (which it converts into sulphate of lime and oxide of iron, and known here as gypsum or plaster of Paris,) and further it improves the filtering power of soils, and enables them to retain what fertilizing matter may be contained in a fluid state.

Lime may therefore be used in due proportion, either on moory arenaceous or argillaceous soils; hence the vicinity of limestone quarries is to be considered relatively to the value of lime as a manure to the lands

under consideration: say from sixpence to two shillings STERLING per pound to be added according to circumstances.

The vicinity of coal mines and turf bogs are likewise an important consideration affecting the value of land, for the expense of hauling fuel for burning lime and domestic purposes, must be considered. The percentage should vary from sixpence to two shillings and sixpence per pound.

Sea manure includes sea weed and sea sand, containing shells, both of which are highly valuable, especially the former.

Where sea weed of good quality is plentiful and easy of access, the land within one mile of the strand is increased in value 4s. in the pound at least. Where the soil is a strong clay or clayey loam, shelly sea sand, when abundant, will increase the value of the land 2s. 6d. in the pound, for the distance of one mile.

The valuator will consider whether sea weed is cast on the shore or brought in boats, and the nature of the road. If hilly, reduce them to level by table at p. 72515. The following will enable the valuator to ascertain the value at any distance from the strand:

Supply rather scarce at one mile, 2s.	For every one-half mile
“ middling “ 3s.	deduct 6d.
“ plentiful “ 4s.	

The proximity to towns, as a source of manure and market farm, garden and dairy produce, is to be considered.

MARKET.

310. To this head may be referred the influence of cities, towns and fairs; these possess a topical influence in proportion to their wealth and population. The following is a classification of towns:

Villages, from 250 to 500 inhabitants.

Small market towns, from 500 to 2000.

Large market towns, from 2000 to 19,000.

Cities, from 19,000 to 75,000, and upwards.

Small villages, of from 250 to 500 inhabitants, do not influence the value of land in the neighborhood beyond the gardens or fields immediately behind the houses. The increase in such cases above the ordinary value of the lands will rarely exceed 2s. in the pound.

Large villages and small towns, having from 500 to 1000 inhabitants, usually increase the value of land around the town to a distance of three miles. For the first half mile, the increase is 3s. in the pound; for the next half mile, 2s.; next, 16d. etc., deducting one-third for each half mile, making, for three miles distant, 6d. in the pound, or one-fortieth.

Market towns, having from 8000 to 75,000 inhabitants, town parks, or land within one mile, is 10s. in the pound higher than in ordinary situations. Beyond this the value decreases proportionately to 6s. at the distance of three miles from the town. Thence, in like manner, to a distance of seven miles, where the influence of such town terminates.

Cities and large towns, having a population of from 19,000 to 75,000 inhabitants. The annual value of town parks will exceed by about 14s. in the pound the price of similar land in ordinary situations; and this increased value will extend about two miles in every direction from the houses of the town, beyond which the adventitious value will gradually decrease for the next mile to 12s. in the pound; at the termination of four miles, to 6s.; at seven miles, to 4s.; and at nine and a half miles, its influence may be considered to end.

Its increase to be made for the vicinity of towns, is tabulated as follows :

Class.	Population.	Distance in Miles.																																								
		Town lots.	1.				2.				3.				4.				5.				6.				7.				8.				9.				10.			
			s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.								
9	From 250 to 500,	—	2	0	1	0	0	0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—								
8	" 500 " 1,000,	—	3	0	2	0	1	0	0	6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—								
7	" 1,000 " 2,000,	—	4	0	3	0	2	0	0	0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—								
6	" 2,000 " 4,000,	—	6	0	5	0	3	0	0	0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—								
5	" 4,000 " 8,000,	—	—	8	0	6	0	0	0	0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—								
4	" 8,000 " 15,000,	—	—	10	0	8	0	0	0	0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—								
3	" 15,000 " 19,000,	—	—	12	0	10	0	0	0	0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—								
2	" 19,000 " 75,000,	—	—	—	14	0	12	0	10	0	0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—								
1	" 75,000 and upwards,	—	—	—	—	—	22	0	20	0	18	0	15	0	10	0	6	0	3	0	2	0	0	1	0	0	6	0	3	0	2	0	1	0								

In applying the above table, *the population must be used only for a general index*, as it is the wealth and commercial influence which principally fixes the class; the valuator must use his judgment, combining the comparative wealth with the population, and raise it one class in the tables, or even more. If there be a large poor class, he should take a class lower.

The general influence of markets and towns includes the effects of railways, canals, navigable rivers, and highways; thus, of two districts equally distant from a market, and equal in other respects, that which is intersected by or lies nearer to the best and cheapest mode of communication for sale of produce, is the most valuable.

Bleach greens, fair greens, orchards, osieries, etc., should be valued according to the agricultural value of the land which they occupy.

Plantations and woods, are valued according to their agricultural value.

(NOTE.—We have made up the following section from Sir Richard Griffith's instructions, and Brown on American Forest Trees. The latter is a very valuable work.)

310a. The condition of trees is worthy of attention, as indicating the nature of the soil, thus :

Acer. Maple. Requires a deep, rich, moist soil, free from stagnant water; some species will thrive in a drier soil.

Alnus. Alder. A moist damp soil.

Betula. Birch. In every description—from the wettest to the driest, generally rocky, dry, sandy, and at great elevation.

Carpinus. Ironwood and Hornbeam. Poor clayey loams, incumbent on sand and chalky gravels.

Castanea. Chestnut. Deep loam, not in exposed situations. A rich, sandy loam and clayey soils, free from stagnant water.

Cupressus. Cypress. A sandy loam, also clayey soil.

Chamerops. Cabbage Tree. A warm, rich, garden mould.

Gleditschia. Locust. A sandy loam.

Juglans. Hickory. Grows to perfection in rich, loamy soils. Also succeeds in light siliceous, sandy soils, as also in clayey ones.

Larix. Larch. A moist, cool loam, in shaded localities.

Laurus. Sassafras. A soil composed of sand, peat and loam.

Lyriodendron. Poplar, or Tulip Tree. A sandy loam.

Pinus. Pine. Siliceous, sandy soils; rocky, and barren ones.

Platanus. Buttonwood, or Sycamore. Moist loam, free from stagnant moisture.

Quercus. Oak. A rich loam, with a dry, clayey subsoil. It also thrives on almost every soil excepting boggy or pent.

Robinia. Locust. Will grow in almost any soil; but attains to most perfection in light and sandy ones.

Tilia. Lime Tree. Will thrive in almost any soil provided it is moderately damp.

(For further, see Brown on Forest Trees, Boston: 1832.)

It would be well, in every instance, to make sublots of plantations.

In some instances, plantations may be a direct inconvenience or injury to the occupying tenant. In such cases, the circumstances should be noted, and a corresponding deduction be made for the valuation of the farm so affected.

Bogs and turbarry should be valued as pasture. The vicinity of turf, as well as coal, is one of the local circumstances to be considered as increasing the value of the neighboring arable land.

Where the turf is sold, the bog is valued as arable, and the expense of cutting, saving, etc. of turf deducted from the gross proceeds, will give the net value.

Bogs, swamps, and morasses, included within the limits of a farm, should be made into sublots, if of sufficient extent.

Mines, quarries, potteries, etc. The expense of working, proceeds of sales, etc., should be ascertained from three or four yearly returns.

Mines, not worked during seven years previous, are not to be rated.

Tolls. The rent paid for tolls of roads, fairs, etc., should be ascertained, and also the several circumstances of the tolls. If no rent be paid, the value must be ascertained from the best local information.

Fisheries and ferries. From the gross annual receipts deduct the annual expenses for net proceeds. It will be necessary to state if the whole or part of a fishery or ferry is in one township, or in two, etc., and to apportion the proceeds of each.

Railways and canals. "The rateable hereditament," in the case of railways, is the land which is to be valued in its existing state, as part of a railway, and at the rent it would bring under the conditions stated in the Act. The profits are not strictly rateable themselves, but they enter materially into the question of the amount of the rate upon the lands by affecting the rent which it would bring, or which a tenant would give for the railway, etc., not simply as land, but as a railway, etc., with its peculiar adaptation to the production of profit; and that rent must be ascertained by reference to the uses of it (with engines, carriages, etc., the trading stock), in the same way as the rent of a farm would be calculated, by reference to the use of it, with cattle, crops, etc. (likewise trading stock). In neither cases would the rent be calculated on the dry possession of the land, without the power of using it; and in both cases, the profits are derived not only from the stock, but from the land so used and occupied.

It will be necessary, therefore, to ascertain the gross receipts for a

year or two, taken at each station along the line; also the amount of receipts arising from the intermediate traffic between the several stations. From the total amount of such receipts, the following deductions are to be made, viz.: interest on capital; tenants' profits; working expenses; value of stations: depreciation of stock.

It is to be observed, that the valuation of railway station houses, etc., should be returned separately.

WASTE.

The value of the ground under houses, yards, streets, and small gardens, is included in their respective tenements. So also in the country, roads, stackyards, etc., are included in the tenements. The area of ground occupied by these roads should be entered as a deduction at the foot of the lot in which they occur.

When a farm is intersected by more roads than is necessary to its wants, the surplus may be considered waste. Also deduct small ponds, barren cliffs, beaches along lakes, and seashores.

OF THE VALUATION OF BUILDINGS.

310b. By a system analogous to that pursued in ascertaining the value of land, the value of buildings may be worked out; the one being based on the scale of agricultural prices, and modified by local circumstances; the other, on an estimate of the intrinsic or absolute value, modified by the circumstances which govern house letting.

The absolute value of a building is equivalent to a fair percentage on the amount of money expended in its construction, and it varies directly in proportion to the solidity of structure, combined with age, state of repair, and capacity, as shown in the following classification:

Buildings are divided into two classes: those used as houses, and those used as offices. In addition to the distinction of tenements already noticed in sec. 309g, it may here be observed that houses and offices, together with land, frequently constituted but one tenement. All out-buildings, barns, stables, warehouses, yards, etc., belonging or contiguous to any house, and occupied therewith by one and the same person or persons, or by his or their servants, as one entire concern, are to be considered parts of the same tenement, and should be accounted for separately in the house book, such as herd's house, steward's house, farm house, porter's house, gate house, etc.

A part of a house given up to a father, mother, or other person, without rent, does not form a separate tenement.

Country flour mills, with miller's house and kiln, form one tenement.

310c. CLASSIFICATION OF BUILDINGS WITH REFERENCE TO THEIR SOLIDITY.

Buildings,	Slated,	House or office (1st class),	Built with stone or brick, and lime mortar.
		Basements to do. (4th),	
		House or office (2nd),	
	Thatched,	House or office (3rd),	Stone walls with mud mortar.
		Offices (5th),	Dry stone walls, pointed.
			Good mud walls.
			Dry stone walls.

The above table comprises four classes of houses and five of offices, of each of which there may be three conditions, viz., new, medium, and old, which may also be classified and subdivided, as follows:

CLASSIFICATION OF BUILDINGS WITH REFERENCE TO AGE AND REPAIR.

Quality.	Description.	
NEW,	A. +	<i>Built or ornamented with cut stone, or of superior solidity and finish.</i>
	A.	<i>Very substantial building, and finished without cut stone ornament.</i>
	A. —	<i>Ordinary building and finish, or either of the above, when built twenty years.</i>
MEDIUM,	B. +	<i>Not new, but in sound order and good repair.</i>
	B.	<i>Slightly decayed, but in good repair.</i>
	B. —	<i>Deteriorated in age, and not in perfect repair.</i>
OLD,	C. +	<i>Old, but in repair.</i>
	C.	<i>Old, out of repair.</i>
	C. —	<i>Old, dilapidated, scarcely habitable.</i>

The remaining circumstance to be considered is capacity or cubical content, from which, in connexion with the foregoing classifications, tables have been made for computing the value of all buildings used either as houses or offices. (See sequel for tables.)

Houses of one story are more valuable, in proportion to their cubical contents, than those of two stories. Those more than two stories diminish in value, as ascertained by their cubical contents, in proportion to their height.

Tables are calculated and so arranged on a portion of a house 10 feet square and 10 feet high, = 100 cubic feet, so that a proportionate price given for a measure of 100 cubic feet, as above, is greater than for a similar content 20 feet high, or for 10 square feet and 80 or 40 feet high. For example, in an ordinary new dwelling house, the price given by the table for a measure containing 10 square feet and 10 feet high, is 7½ pence; for the same area and 20 feet high, the price is 1s. 0¾d.; for the same area and 80 feet high, 1s. 4¼d.; and for the same area and 40 feet high, the price is 1s. 6¼d.

OF THE MEASUREMENT OF BUILDINGS.

810d. Ascertain the number of measures (each 100 square feet) contained in each part of the building. Measure the height of each part, and examine the building with care. Enter in the field book the quality letter, which, according to the tables, determines the price at which each measure containing 10 square feet is to be calculated.

The houses are to be carefully lettered as to their age and quality. Addition or deduction is to be made on account of unusual finish or want of finish, etc. Such addition or deduction is to be made by adding or deducting one or more shillings in the pound to meet the peculiarity, taking care to enter in the field book the cause of such addition or deduction.

Enter also the rent it would bring in one year in an ordinary situation.

If any doubts remain as to the quality letter, examine the interior of the building.

In measuring buildings, the external dimensions are taken—length, breadth and height—from the level of the lower floor to the eaves. In

attic stories formed in the roof, half the height between the eaves and ceiling is to be taken as the height.

Basement stories or cellars, both as dwellings and offices, are to be measured separately from the rest of the building.

Main house is measured first, then its several parts in due form.

Extensive or complicated buildings should have a sketch of the ground plan on the margin of the field book, with reference numbers from the plan to the field book.

If a town land boundary passes through a building, measure the part in each.

MODIFYING CIRCUMSTANCES.

310e. The chief circumstances which modify the tabular value are deficiencies, unsuitableness, locality, or unusual solidity.

Deficiencies.—In large public buildings, such as for internal improvements, an allowance of 10 to 30 per cent. is made; also in stables and fuel houses. When the walls of farm houses exceed 8 or 12 feet in height, but have no upper flooring, they should not be computed at more than 8 feet, except in the cases of grain houses, factories, barns, foundries, etc. The full height is, however, to be registered in each case.

Unsuitableness.—Houses found too large, or superior to the farm and locality—where there are too many offices or too few.

All buildings are to be valued at the sum or rent they would reasonably rent for by the year.

Buildings erected near bleach greens, or manufactories which are now discontinued, or if they were built in injudicious situations, should be considered an incumbrance rather than a benefit to the land; consequently, only a nominal value should be placed on them.

The tabular amount for large country houses, occupied by gentlemen, usually exceeds the sum they could be let for, and this difference increases with the age of the building. The following is to correct this defect:

Houses amounting from	to	Reduction per Pound.	Reduction per cent.
£10	£35	None.	None.
35	40	0s. 6d.	0.025
40	50	1 0	0.05
50	60	1 6	0.075
60	70	2 0	0.10
70	80	2 6	0.125
80	90	3 0	0.150
90	100	3 6	0.175
100	110	4 0	0.200
110	120	4 6	0.225
120	140	5 0	0.250
140	160	5 6	0.275
160	200	6 0	0.300
200	300	7 0	0.350
300 and upwards,		8 0	0.400

Where any improvements have been made to gentlemen's houses, care should be taken to ascertain whether any part of the original house was made useless, or of less value. If so, deduct from the price given by the table as the case may require.

Locality includes aspect, elevation, exposure to winds, means of access, abundance or scarcity of water, town influence, etc., each of which is to be carefully considered on the ground.

In determining the value of buildings immediately adjoining large towns, ascertain the percentage which the town valuator has added to the tabular value of these on the limits of the town lot. Those in the town lot are referred to another heading, as will appear from sec. 310f.

Solidity.—In large mills, storehouses, factories, etc., well built with stone or brick, and well bonded with timber, a proportional percentage should be added to the tabular value for unusual solidity and finish, which will range from 30 to 50 per cent. The value thus found may be checked by calculating the tabular value of the ground floor, and multiplying this amount by the number of floors, not including the attic.

VALUATION OF HOUSES IN CITIES AND TOWNS.

210f. In valuing houses in cities and towns, there are circumstances for consideration in addition to those already enumerated, viz., arrangement of streets, measurement, comparative value, gateways, yards, gardens, etc. To effect this object, each town should be measured according to a regular system; and the following appears to be a convenient arrangement for the purpose:

Arrangement of streets.—The valuator should commence at the main street or market square, and work from the centre of the town towards the suburbs, keeping the work next to be done on his right hand side, measuring the first house in the street, and marking it No. 1 on his field map and in his field book. Afterwards proceed to the next house on the same side, marking it No. 2, and so on till he completes the measurement of the whole of the houses on that side of the street. He is then to turn back, proceeding on the other side, keeping the work to be done still at his right hand. The main street being finished, he proceeds to measure the cross streets, lanes or courts that may branch from it, commencing with that which he first met on his right hand in his progress through the main street. This street is measured in the same manner as the main street; and all lanes, courts, etc., branching from it are measured in like manner, observing the same rule of measurement throughout.

Having finished the first main street, with all its branches, he is to take the next principal street to his right hand, from the first side of the first main street, and proceed as in the first, measuring all its branches as above.

(NOTE.—Let Clark and Lake streets, in the city of Chicago, be the two principal streets, and their intersection one block north of the Court House, the principal or central point of business. Clark street runs north and south; Lake street, east and west. Nearly all the other principal streets run parallel to these. We begin at the west side of Clark and north side of Lake, and run west to the city limits, and return on the south side of the street, keeping the buildings on the right, to Clark street. We continue along the south side of Lake, east to the city limits, and then return on the north side of Lake, keeping the buildings on the right, to the place of beginning. Having finished all the branches leading into this, we take the next street north of Lake, and measure on the north side of it west to the city limits, and so proceed as in the first main street. Having finished all the east and west streets north of the first or Lake street, we proceed to measure those east and west streets south of the first or Lake street, as above. We now proceed to measure the

north and south streets, taking first the one next west of Clark, and run north to city limits; then return on the west side of the street to Lake, and continue south to the city limits; return on the east side of the street to the place of beginning. Thus continue through the whole city.)

In measuring buildings, the front dimensions, and that of returns, is set in the first column of his book, the line from front to rear is placed in the second column, and the height in its own place.

In offices, the front is that on which the door into the yard is situated.

In houses with garrets, measure the height to the eave, and set in the field book, under which set the addition made on account of the attic, and add both together for the whole height.

Every house having but one outside door of entrance, is to be numbered as one tenement. Where there are two doors, one leading to a shop or *store*, to which there is internal access from the house, the whole is to be considered as one tenement; but if the shop and other part of the house be held by different persons, the value of each part should be returned.

Where a number of houses belonging to one person are let from year to year to a number of families, each house is to be returned as one tenement.

Buildings in the rear of others in towns are to be valued separately from those in front.

COMPARATIVE VALUE.

310g. In towns, a shop for the sale of goods is the most-valuable part of a house; and any house having much front, and affords room for two or three shops, is much more valuable than the same bulk of house with only one shop.

When a large house and a small one have each a shop equally good, the smaller one is more valuable in proportion to its cubical contents, as ascertained by measurement, and a proportionate percentage should be added to the lesser building to suit the circumstances of the case.

Where large houses and small mean ones are situated close to each other, the value of the small ones are advanced, and that of the large ones lessened. In such cases, a proportionate allowance should be made.

Stores (warehouses) in large towns do not admit of so great a difference for situation as shops—a store of nearly equal value, in proportion to its bulk, in any part of a town, unless where it is adjoining to a quay, railway depot or market; then a proportionate additional value should be added.

Gateways.—In stores or warehouses in a commercial street, where there is a gateway underneath, no deduction is made.

In shops or private dwellings, a gateway under the front of the house is a disadvantage, compared to a stable entrance from the rear. In such cases, a proportionate deduction should be made on account of the gateway.

In measuring gateways, take the height the same as that of the story of which it is a part.

Passages in common are treated similar to gateways.

Where any addition or deduction is made on account of gateways, it should be written in full at the end of the other dimensions, so as to be added or subtracted as the case may be.

Where deductions are made on account of want of finish in any house, state the nature of the wants, and where required.

Stores do not want the reductions for large amount, which has been directed in the case of gentlemen's country seats.

OF TOWN GARDENS AND YARDS.

310A. In large towns, the open yard is equal to half the area covered by the buildings; if more, an additional value is added, but subtracted if less. Allowance is made if the yard is detached or difficult of access.

The quantity of land occupied by the streets, houses, offices, warehouses, or other back buildings belonging to the tenements, together with the yards, is to be entered separately at the end of the town lots in which they occur, the value of such land being one of the elements considered in determining the value of the houses, etc.

A timber yard, or commercial yard, is to be valued. If large, state the area, and if paved, etc., the kind of wall or enclosure, and if any offices are in it, their value is to be added to that of the yard.

Gardens in towns.—In towns, the yards attached to the houses are to be considered as one tenement; but the garden, in each case, is to be surveyed separately, and not included in the value of the tenement. The gardens in towns are to be valued as farming lands under the most favorable circumstances.

OF THE SCALE FOR INCREASING THE TABULAR VALUE OF HOUSES FOR TOWN INFLUENCE.

310i. Ascertain the rents paid for some of the houses in different parts of the city. This will enable one to determine the tabular increase or decrease.

As it is better to have a house rented by a lease than by the year or half year, therefore a difference is made between a yearly rent and a lease rent: for a new house, two shillings in the pound in favor of the lease rent; for a medium house, about three shillings in the pound; and for an old house, about four shillings in the pound.

In all houses whose annual value is under ten pounds, the rent from year to year is higher in proportion to the cubical contents than in larger houses let in the same manner, but the risk of losing by bad tenants is greater for small houses, therefore in reducing such small houses, when let by the year or half year, to lease rents, five shillings in the pound at least should be deducted.

In villages and small market towns, an addition of twenty-five per cent. to the prices of the tables will generally be found sufficient.

In moderate sized market towns, the prices given in the tables may be trebled for the best situations in the main street, near the market or principal business part of the town; and in the second and third classes, the prices will vary from one hundred to fifty per cent. above the tables; and in large market towns, the prices for houses of the first class, in the best situations, will be about three and one-half times those of the tables.

In dividing the streets or houses of any town into classes, the valuator is, in the first instance, to fix on a medium situation or street, and having ascertained the rents of a number of houses in it, he is, by measurement, to determine what percentage, in addition to the country tables, should

be made, so as to produce results similar to the average of the ascertained rents.

Having determined the percentage to be added to the price given in the tables for houses in medium situations, the standard for the town about to be valued may be considered as formed; and from this standard, percentages in addition are to be made for better and best situations, or for any number of superior classes of houses, or of situations which the size of the town may render necessary.

In towns, the front is the most invaluable, therefore value the front and rear of the building separately, so as to make one gross amount. It is impossible to determine accurately the proportion between the value of the front and rear buildings; but it has been found that in revising the valuations of several towns, that the proportion of five to three was applicable to the greater number of houses in good situations; that is, the country price given by the tables should be multiplied by five for the front, and three for the back buildings, stores and offices.

WATER-POWER.

310y. Ascertain the value of the water power, to which add that of the buildings.

A horse-power is that which is capable of raising 33,000 pounds one foot high in one minute.

The horse-power of a stream is determined by having the mean velocity of the stream, the sectional area, and the fall per mile.

The fall, is the height from the centre of the column of water to the level of the wheel's lower periphery. The weight of a cubic foot of water is 62.25 pounds.

Total weight discharged per minute = $V \cdot A \cdot 62.25$. Here A = sectional area, and V = mean velocity in feet per minute.

A body falling through a given space acquires a momentum capable of raising another body of equal weight to a similar height; therefore, the total weight discharged per minute, multiplied by the *modulus* of the wheel, and this product divided by 33,000 pounds, will give the required horse-power.

<i>Modulus for overshot wheel</i>	0.75
“ “ breast wheel, No. 1, with buckets.....	.66
“ “ “ “ No. 2, with float boards....	.55
“ “ turbine.....	.65 to 78
“ “ undershot wheel.....	.38

NOTE.—James Francis, Esq., C.E., has found at Lowell, Massachusetts, as high as 90 to 94, from Boyden's turbines.

Fourneyron and D'Aubuisson give the modulus for turbine of ordinary construction and well run = 0.70.

To measure the velocity of a stream. Assume two points, as A and B, 528 feet apart; take a sphere of wax, or tin, partly filled and then sealed, so as to sink about one-third in the water; drop the sphere in the centre of the water, and note when it comes on the line A-A, and on the line B-B. A and A may be on opposite sides of the river, or on the river, or on the same side at right angles to the thread of the stream. Let the time in passing from the line AA to the line BB be six minutes. Then as six min. : 528 ft. : 60 min. to 5280 ft.; that is, the measured surface velocity is one mile per hour.

M. Prony gives V = surface, W = bottom, and U = mean velocity, and
 $U = 0.80 V$ = mean velocity,
 $W = 0.60 V$ = bottom velocity;
 therefore, as 6 minutes gives a surface velocity of 88 ft.; this multiplied
 by 0.80, gives 70.4 ft. per minute as the mean velocity.

310k. The following may serve as an example for entry of data and calculation:

Date.	Ft.	In.	
A Breast Wheel, No. 1.			36 8
Mean velocity of stream per min- ute,	144		288 — 2 feet — Sectional area — A. 144
Breadth of stream in trough,	—	36	288 — Cubic content per minute. 62.25 — Weight of one foot.
Depth of do.	—	8	18000 lbs. — Weight discharged. 12 — Fall of water.
Fall of water,	12	—	216000 — Total available power. 66 — Modulus.
			1425600 This divided by 33000, gives 4.32 — effective horse-power.

Otherwise:

Date.	Ft.	In.	
Breast wheel No. 1. Revolutions per minute, 6.6.			36 x 8.6 = 312 feet — sectional area of bucket. 14 x 12 = 168, and 168 — 8.6 = 159.6 — 13.29 — reduced diameter at centre of buckets.
Diameter of wheel,	14	—	13.29 x 3.1416 = circumference at centre of buckets — 41.761, and 41.761 x 6.6 = 275.522 — 292 cub. ft. in buckets half full.
Breadth of do.	—	36	292 x 62.25 = 18350 12 — fall of water.
Depth of shroud- ing,	—	8.6	219000 66 — modulus.
Fall of water,	12	—	33000) 144540.00 (= 4.38 effective horse-power.

For undershot wheels, the data are as follow:

Date.	Ft.	In.	
Revolutions per minute, 6.2.			Ft. In. 4 6 — Breadth of float boards. 10 — Depth of do. acted on.
Diameter of wheel,	16	—	3.75 — Area of float boards. 798 — Velocity of stream.
Breadth of float board,	4	6	2992 62.25 — Weight of one cubic foot.
Depth of do.,	2	—	187081.25 2.75 — Height of fall due to velocity.
Velocity of stream per minute,	798	—	514335.9 33 — Modulus.
Height of fall due to vel- ocity,	2	9	169730 33000 — 5.14 horse-power.
Depth of do. under wheel,	—	—	

310l. It is to be observed that the horse-power deduced from measure-
 ment of a bucket-wheel may be found in some instances rather greater
 than that from the velocity and fall of water, as it is necessary that space
 should be left in the buckets for the escape of air, and also to economize
 the water.

When a bucket-wheel is well constructed, multiply the cubic content
 of water discharged per minute by .001825, and by the fall; the product
 will be the effective horse-power approximately.

For turbines, the effective cubical content of water discharged per min-
 ute multiplied by the height of the fall, and divided by 700, will be equal
 to the effective horse-power.

In practice, twelve cubic feet of water falling one foot per second, is considered equal to a horse-power.

When the water is supplied from a reservoir, and discharged through a sluice, measure from the centre of the orifice to the surface of the water, and note the dimensions of the orifice.

Head of water.—The velocity due to a head of water is equal to that which a heavy body would acquire in falling through a space equal to the depth of the orifice below the free surface of the fluid; that is, if V = velocity, and $M = 16\frac{1}{2}$ feet, or the space fallen through in one second, and H = the height, the velocity may be represented thus:

$V = 2 \sqrt{MH}$; thus the natural velocity for .09 feet head of water will be $V = 2 \sqrt{16\frac{1}{2} \times .09} = 2.4$ feet per second. In practice, $V = 8 \sqrt{H}$. The effective velocity = five times the square root of the height. (See sec. 312.)

VALUE OF WATER-POWER.

310m. The water-power is to be valued in proportion as it is used, and the time the mill works.

One horse running twenty-two hours per day during the year, is valued at £1 15s. This amount multiplied by the number of horses' power, will give the value of the water-power.

The annexed table is calculated with reference to class of machinery and time of working.

Quality of Machinery.	Number of Working Hours.							
	8	10	12	14	16	18	20	22
	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.
New,	18 8	18 6	23 3	26 9	28 9	30 9	33 0	35 0
Medium, ...	12 0	16 9	21 0	24 3	26 0	27 9	29 6	31 6
Old,	10 6	15 0	18 9	21 6	23 8	24 9	26 6	28 0

In this, two hours are allowed for contingencies and change of men.

The highest proportionate value is set on 14 hours' work, as during that time sufficient water can be had, and one set of men can be sufficient.

Where the supply of water throughout the year is not the same, the valuator is to determine for each period by the annexed table.

Description of Mill,..... Class of Machinery,..... A.				
Horses' Power.	Working Time.		Value of Water-power.	Observations.
	Number of Months per Year.	Number of Hours per Day.		
9	8	22	£ s. d. 10 10 0	For 8 months the full power of the wheel is used, but for the remaining 4, not more than two-thirds of the water-power can be calculated on.
6	4	12	2 6 6	
			12 16 6	

Where a mill is worked part of the year by water and another part by steam, care must be taken to determine that part worked by water, and also to value the machinery, as it sometimes happens that the mill may be one quality letter and the machinery another—higher or lower.

MODIFYING CIRCUMSTANCES.

310a. The wheel may be unsuitable and ill-contrived; the power may be injudiciously applied; the supply may be scarce, may overflow, or have backwater.

In gravity wheels, the water should act by its own weight—the principle upon which its maximum action depends being that the water should enter the wheel without impulse, and should leave it without velocity. The water should, therefore, be allowed to fall through such a space as will give it a velocity equal to that of the periphery of the wheel when in full work, thus: if the wheel move at the rate of five feet per second, the water must fall on it through not less than two-fifths of a foot; for the space through which a falling body must move to acquire a given velocity is expressed thus: $\frac{V^2}{4M} = \frac{V^2}{64.888}$

For mills situate in inland towns of considerable importance, such as Armagh, Carlow, Navan, Kilkenny, etc., in a good wheat country, where wheat can be bought at the mill, and the flour sold there also, five shillings in the pound may be added on the water-power for the advantage of situation.

The vicinity of such towns, say within three to four miles, may be called an ordinary situation. Beyond this distance, where the wheat has to be carried from, and flour to, the market, the water-power gradually decreases in value; and from such a town to ten miles distance from it, the water-power may be rated according to the following table.

		s.	d.	
Add to water-power,	{	10	0	per pound within the town lot.
		8	0	when distant from 0 to 1 mile.
		6	0	" " " 1 to 8 "
		4	0	" " " 8 to 5 "
		2	0	" " " 5 to 8 "
		1	0	" " " 8 to 10 "
		0	0	" " " 10 and upwards.

Beyond ten miles from a good local market, a flour mill can rarely require percentage for market.

But this rule of increase does not apply to small mills, such as flour mills, where only one pair of millstones is used; in this case, only half the above percentage is to be added within three miles of a large town; beyond that distance, no addition is to be made.

In the case of bleach mills, they should be as near to their purchasing or export market as flour or corn mills, and the valuator should make deductions for a remote situation, especially where the chief markets for buying linen are distant, or add a percentage to the water-power where the situation has unusual advantages in these respects.

310c. HORSE-POWER DETERMINED FROM THE MACHINERY DRIVEN.

In a flax mill, each stock is equivalent to one horse-power. The bruising machine of three rollers = $1\frac{1}{2}$ stocks.

The numbering of horse-power in the mill may thus be counted, and the value ascertained from the table for horse-power from sec. 310d.

In spinning mills, the horse-power may be determined from the number of spindles driven, and the degree of fineness spun, for in every spinning mill the machinery is constructed to spin within certain range of fineness. Therefore ascertain the range of fineness and number of spindles.

Yarn is distinguished by the degree of fineness to which it is spun, and known by the number of leas or cuts which it yields to the pound.

One lea or cut = 800 lineal yards.

12 leas = 1 hank; 200 leas = 16 hanks; and 8 leas = 1 bundle = 60000 yards.

Leas to the pound.	No. of Spindles.
From 2 to 8,	40 throstles require one horse-power.
From 12 to 30,	60 " " " "
From 70 to 120,	120 " " " "

In cotton mills, the throstle spindle is used for the coarser yarns, and for the finer kinds the mule spindle.

Leas to the pound.	No. of Spindles.
From 10 to 80,	180 throstles equal one horse-power.
From 10 to 50,	500 mules " " "

In bleaching mills, ascertain the number of beetling engines; measure the length of the wiper beam in each, together with the length of beetles, and their depth, taken across the direction of the beam; also the height the beetles are raised in each stroke.

From these data, the horse-power of such engine can be found by inspection of the table calculated for this purpose. Ascertain the number of pairs of washing feet, and if of the ordinary kind; the pairs of rub-boards, starching mangle, squeezing machine, calender, or any other machine worked by water, and state the horse-power necessary to work each.

The standard for a horse-power in a beetling mill is taken as follows: Beam, furnished with cogs for lifting the beetles, 10 feet long. The wiper beam makes 30 revolutions in a minute; and being furnished with two sets of cogs on its circumference, raises the beetle 60 times per minute, working beetles 4 feet 4 inches in length, and 3 inches in depth, from front to rear, making 30 revolutions per minute, or lifting the beetles 60 times in a minute one foot high, is equal to *one horse-power*. This includes the power necessary to work the traverse beam and guide slips, which retain the beetle in a perpendicular position.

Taking the wiper beam at 10 feet long, and height lifted as 1 foot, making 30 revolutions per minute, the following table will show, by inspection, the proportionate horse-power required to raise beetles of other dimensions 60 feet in one minute, assuming the weight of a cubic foot of dry beach wood = 712 ounces.

When the engine goes faster or slower, a proportionate allowance must be made.

Inches from front to rear.	LENGTH OF BEETLES.											
	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.
	4 4	4 6	4 8	4 10	5 0	5 2	5 4	5 6	5 8	5 10	6 0	
	Number of Horse Power.											
8	1.00	1.08	1.06	1.10	1.18	1.16	1.20	1.24	1.28	1.32	1.36	
3½	1.07	1.10	1.14	1.18	1.22	1.26	1.30	1.34	1.38	1.42	1.46	
3¼	1.16	1.19	1.23	1.27	1.32	1.36	1.40	1.45	1.49	1.53	1.58	
3½	1.28	1.27	1.32	1.37	1.41	1.45	1.49	1.54	1.58	1.63	1.69	
4	1.81	1.86	1.41	1.45	1.50	1.55	1.60	1.65	1.70	1.75	1.80	
4½	1.40	1.44	1.49	1.54	1.59	1.64	1.70	1.75	1.80	1.85	1.91	
4½	1.48	1.53	1.58	1.64	1.69	1.75	1.80	1.85	1.91	1.97	2.03	

From this table it appears that a ten feet wiper beam, having its beetles four inches in depth, five feet long, and to lift those beetles one foot high sixty times in a minute, would require the power of one and one-half horses.

If the wiper beam be more or less than ten feet in length, or if the lift of the beetles be more or less than one foot, a proportionate addition or deduction should be made.

The following is given to assist the valuator in determining the value of the other machinery in a bleaching mill:

One pair of rub-boards,	= 0.5 to 0.7 horse-power.
.. starching mill,	1 ..
.. drying and squeezing machine,	1 ..
.. pair of wash-feet,	1.5 to 2 ..
.. calender (various),	3 to 8 ..

In *beetling mills*, the long engine, with a ten feet wiper beam, is considered the most eligible standard for computing the water-power. Such a beam, having beetles four inches long and three inches deep, is equal to one horse-power. On these principles, the value of water-power may be ascertained from the table, sec. 310l.

310p. In *flour mills*, the power necessary to drive the machinery night and day for the year round, has been determined as follows:

The grinding portion, or flour millstones, have been considered to require, for each pair, four horses-power. The flour dressing machine of ordinary kind, together with the screens, sifters, etc., or cleansing machinery, require, on an average, four horses-power. Some machines, however, from their size and feed with which they are supplied, will require more or less than four horses-power, and should be noted by the valuator.

Every dressing, screening and cleansing machine is equal to one pair of stones.

(NOTE.—In Chicago, ten horses power is estimated for one pair of stones, together with all the elevating and cleansing machinery.—M. M'D.)

The following table has been made for one pair of millstones, four feet four inches diameter, for one year:

Quality of Machine.	Number of Working Hours per Day.															
	8.				10.				12.				14.			
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.	
New, A	2	18	0	3	14	0	4	13	0	5	7	0	5	15	0	6
Medium, B	2	8	0	3	7	0	4	4	0	4	17	0	5	4	0	5
Old, C	2	2	0	3	0	0	3	15	0	4	6	0	4	18	0	4

If more than one pair of millstones be used in the mill, multiply the above by the number of pairs usually worked, and if they are more or less than four feet four inches in diameter, make a proportional increase or decrease.

In flour mills, the valuator will state the kind of stones, how many French burrs, their diameter, the number worked at one time, the number of months they are worked, the number of months that there is a good supply, a moderate one, and a scarcity of supply.

FORM FOR FLOUR MILLS.—No. 1.

Description of Mill,.....Flour Mill. Class of Machinery,.....A.					
Dressing Machine, Screens, etc.	Millstones, No. of Pairs Worked.	Working Time.		Value of Water-power.	Observations.
		No. of Months per Year.	No. of Hours per Day.		
-	4	6	22	£ 14 0 0	In this mill there are five pairs of stones, one pair always up, being dressed; machine and screens and sifters only used when one or two pairs of stones are stopped, and not worked in summer, except one or two days in the week. Two sets of elevators used along with the millstones.
-	2	3	16	2 18 0	
-	1	3	10	0 18 0	
1	Only used when one or two pairs of stones are thrown out.				
-				17 16 0	

No. 2.

Description of Mill,.....Flour Mill. Class of Machinery,.....B.					
Dressing Machine, Screens, etc.	Millstones, No. of Pairs Worked.	Working Time.		Value of Water-power.	Observations.
		No. of Months per Year.	No. of Hours per Day.		
-	2	4	22	£ 4 4 0	In this mill there are three pairs of stones—one pair generally up, two driven for four months along with machines, screens and sifters, and one for one month with them also; during three months the machines and one pair of millstones must be worked alternate days, and during the other four months there is no work done. One set of elevators used along with the millstones.
-	1	1	22	0 11 0	
-	1	3	9	0 14 0	
1	-	5	22	2 13 0	
				8 2 0	

310g. In oatmeal mills, one pair of grinding stones require three horse-power; one pair of shelling stones, fans and sifters, require two horse-power. Elevator is taken at one-eighth of the power of the stones.

The following table, for one pair of millstones for one year, is to be used as the table for flour mills:

Quality of Machinery.	Number of Working Hours per Day.																							
	8			10			12			14			16			18			20			22		
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
New, A.....	2	0	0	2	16	0	3	10	0	4	0	0	4	6	0	4	12	0	4	19	0	5	5	0
Medium, B.	1	16	0	2	10	0	3	8	0	3	18	0	3	18	0	4	8	0	4	9	0	4	15	0
Old, C.....	1	12	0	2	5	0	2	16	0	3	4	0	3	10	0	3	14	0	3	19	0	4	4	0

310r. In corn mills, ascertain the number of pairs of grinding and shelling millstones and other machinery, and note the time each is worked. Where there are two pairs—one of which is used for grinding and the other for shelling; if there be fans and sifters, the shelling and sifters is = to two horses' power = two-thirds of a pair of grinding stones. Where one pair is used to shell and grind alternately, it is reckoned at three-fourths pair of grinding stones, unless the fans and sifters be used at the same time. In this case they will be counted as seven-eighths pair of stones. Where there are two pairs of grinding, with one pair of shelling with fans and sifters, the water power is equal to two and two-thirds pairs of millstones; but if one pair is idle, then the power = one and two-thirds pairs of grinding millstones, etc.

FORM No. 1.

Description of Mill,.....Corn Mill.									
Class of Machinery,.....A.									
Millstones, No. of Pairs Worked.				Working Time.		Value of Water-power.			Observations.
Grind'g	Shelling	Grind'g and Shelling	Equivalent in Grinding Stones.	No. of Months per Year.	No. of Hours per Day.	£	s.	d.	
2	1	—	2½	8	22	9	6	0	In this mill there are three pairs of stones, with elevators, fans, and sifters. Horse-power for 8 months equal to 8 or 2½ grinding stones; and for 4 months 5 horse power, or 1½ grinding stones.
1	1	—	1½	4	12	1	19	0	
					Add ⅛ for Elevators, .	11	5	0	
						1	8	0	
						12	13	0	

FORM No. 2.

Description of Mill,.....Corn Mill.									
Class of Machinery,.....B.									
Millstones, No. of Pairs Worked.				Working Time.		Value of Water-power.			Observations.
Grind'g	Shelling	Grind'g and Shelling	Equivalent in Grinding Stones.	No. of Months per Year.	No. of Hours per Day.	£	s.	d.	
1	1	—	1½	6	16	2	18	6	In this mill there are two pairs of stones, but no fans, sifters, or elevators.
1	1	—	1½	8	7	0	12	0	

FORM No. 8.

Description of Mill,.....Corn Mill. Class of Machinery.....C.								Observations.
Millstones, No. of Pairs Worked.			Equivalent Intiriding Stones.	Working Time.		Value of Water- power.		
Grind'g	Shelling	Grind'g and Shelling		No. of Months per Year	No. of Hours per Day.	£	s.	
-	-	1	$\frac{1}{4}$	4	16	1	0	In this mill there are two pairs of stones, only one pair can be worked at a time: there are fans and sifters in use, but no elevators. This mill works merely for the supply of the neighborhood, and is distant four miles from a market town.
-	-	1	$\frac{1}{4}$	4	8	0	9	

When there are two or more mills in a district, compare the value of one with the other.

Three stocks in a flax mill is equal to the power necessary to work a pair of millstones in a corn mill. Note the quantity ground annually as a further check, for it has been ascertained that a bushel of corn requires a force of 31,500 lbs. to grind, the stones being about 5 feet in diameter, and making 95 revolutions per minute.

310s. *In fine*, it should be borne in mind, that for each separate tenement a similar conclusion is ultimately to be arrived at, viz., that the value of land, buildings, etc., as the case may be, when set forth in the column for totals, is the rent which a liberal landlord would obtain from a solvent tenant for a term of years, (*rates, taxes, etc., being paid by the tenant;*) and that this rent has been so adjusted with reference to those of surrounding tenements that the assessment of rates may be borne equably and relatively by all.

The valuator, therefore, should endeavor to carry out fairly the spirit of the foregoing instructions, which have been arranged with a view to promote similarity of system in cases which require similarity of judgment.

As it may appear difficult to apply Griffith's System of Valuation to American cities, on account of the number of frame or wooden buildings, we give a table at p. 72b58, showing the comparative value of frame and brick houses. All the surveyors and land agents, to whom we have shown and explained this system of valuation, have approved of it, and expressed a hope of seeing such a system take the place of the present hit or miss valuations, too often made by men who are unskilled in the first rudiments of surveying and architecture.

FORM OF FIELD BOOK.

County of Clare, Barony of Bunratty Lower, Parish of St. Patrick, Townland of Shanakyle.—Ordnance Survey, 68.

Original Map.	Names.		Description.		Area.	Value per Acre.				Nett Annual Value.			Observations.
	Occupiers.	Immediate Lessors.	Of Tenement.	Of Soil.		In ordi- nary sit- uations.	s. d.	s. d.	With al- lowance for local circum- stances.	Land.	Buildings.	Totl.	
Part 7	Michael Mason, .	Hamilton Jackson, Esq.,	House,	-	A. R. P.	s. d.	s. d.	s. d.		£ s. d.	£ s. d.		} Improved. Old deduction.
"	Patrick Connolly, .	"	Garden,	-	0 0 32	-	-	13 0		-	0 13 4	-	
"	William Connors, .	"	Gardens,	-	0 1 10	-	-	14 0		-	-	-	
"	James Dalton, .	William Connors, .	House,	-	-	-	-	-		-	0 12 6	-	
"	John Costelloe, .	Hamilton Jackson, Esq.,	House and Land,	-	-	-	-	27 0		-	1 5 0	-	} Improved. Old deduction.
Part 2	{ Mattw. Hanrahan, Patk. Kelly, (Pat) Pat. Kelly, (John,) }	Hamilton Jackson, Esq.,	{ House, Office & Land, House, Office & Land, House, Offices & Land, }	{ - - - }	-	-	-	-		-	1 5 9 1 5 0 2 17 6	-	
	"	"	Land,	-	9 1 38	-	-	0 6		-	-	-	
	"	"	Land,	-	4 0 24	-	-	6 0		-	-	-	
	"	"	Land,	-	-	-	-	9 0		0 9 9	-	-	
"	"	"	-	-	3 0 0	-	-	-		-	-	-	Too high. Worn.
Part 8	"	"	Land,	-	-	-	-	16 0		-	-	-	Rather low. Bad; part of deduction.
"	"	"	-	-	1 2 0	-	-	11 0		-	-	-	

(310a.)

FORM OF TOWN BOOK.

Town of Killarney.										Parish of Killarney.						Townland of Killarney.													
High Street.																													
No.	Name and Description.	Quality.	Height.	Length.	Breadth.	Measure of.	Rate per Measure.	Amount of Items.	Area of garden.	Value of garden or Yard.	Gross tabular Amount with Multiplier.	Value for Public'n.	Yearly Rent.	Valuer's Estimate.	Immediate Lessor.	Observations.													
			Ft. In.	Ft. In.	Ft. In.	s. d.	f. s. d.	f. s. d.	A. R. P.	£. s. d.	£. s. d.	£. s. d.	£. s. d.	£. s. d.															
Lot 1	Daniel Mahony,																												
	House, offices, and yard	1 B	27	634	0 23	0 78	1 1	4 4 6			Good front,		40 0 0	Two	Christ. Galwey,	Has back gate-													
	House and shop, .	1 B	27	630	0 22	0 68	0 11	8 4 7			Multiplier,		in 1861.	shops,		way to yard.													
	House Return, .	1 B	12	200	0 8	0 16	0 61	0 8 8			Multiplier, 3 8			goodsitu-		Built some													
	Do., do., .	1 B	10	200	0 2	0 4	0 6	0 2 0			£14 1 8			ation,		offices lately,													
	Kitchen, .	1 B	14	614	0 15	0 21	0 71	0 13 1						25 0 0		and otherwise													
	Office (Bake-house),	1 B	14	615	0 15	0 22	0 81	0 6 10			Rare and offices,			to		improved the													
	Do., .	1 B	7	0 9	0 5	0 4	0 2	0 0 8			Multiplier, 2			80 0 0		concern.													
	Addition to Bake-house,	1 B	7	0 29	0 15	0 43	0 2	0 7 2								New portions.													
	Sheds, .	2 B	10	0 18	0 17	0 30	0 21	0 6 3			11 2 6																		
3	Stable, .							9 15 9		0 10 0	£25 4 2	28 0 0																	
	Richard Murphy,																												
	House, yard, and small																												
	garden.																												
	House, .	1 C	17	613	6 28	0 31	0 71	1 0 0		Small	Front house, { 1 0 0				W. W. Murphy,	Tenant has the													
	Return, .	1 B	6	6 9	6 16	0 13	0 31	0 4 8		garden,	Multiplier, { 0 6 0					portion of No. 4													
										add 3s.	Multiplier, 5 1/2					which is over													
	Office, .	1 C	6	0 14	0 17	6 24	0 1	0 2 0		6d. to	£7 5 9					the shop.													
	Do., .	1 C	6	6 10	0 6	0 5	0 11	0 0 6		house.	Rare and offices, 0 9 2																		
Deduct shop,	Old office, .	—	—	—	—	—	—	0 2 0			Multiplier, 2																		
	Part of adjoining house,	1 C	17	6 9	6 23	0 21	0 71	0 13 6			0 18 4																		
Deduct shop,		1 C	8	6 9	6 23	0 21	0 4	0 7 0			Small garden,																		
											8 4 1																		
											0 3 6																		
											£8 7 7	8 0 0																	

NOTE.—The Length multiplied by the Breadth and divided by 10 = Measures.

TABLES

FOR ASCERTAINING THE

ANNUAL VALUE OF HOUSES IN THE COUNTRY.

(310c.)

I.—SLATED HOUSES,

WALLS BUILT WITH STONE, OR BRICK, AND LIME MORTAR.

Height.	A+	A	A—	B+	B	B—	C+	C	C—
Ft. Inch	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	d.	d.
6 0	0 5 $\frac{1}{4}$	0 5	0 4 $\frac{3}{4}$	0 4 $\frac{1}{4}$	0 3 $\frac{3}{4}$	0 3 $\frac{1}{4}$	0 3	2 $\frac{1}{4}$	1
3	0 5 $\frac{1}{4}$	0 5 $\frac{1}{4}$	0 4 $\frac{3}{4}$	0 4 $\frac{1}{4}$	0 4	0 3 $\frac{1}{4}$	0 3 $\frac{1}{4}$	2 $\frac{1}{4}$	1
6	0 5 $\frac{1}{4}$	0 5 $\frac{1}{4}$	0 5	0 4 $\frac{1}{4}$	0 4	0 3 $\frac{1}{4}$	0 3 $\frac{1}{4}$	2 $\frac{1}{4}$	1 $\frac{1}{4}$
9	0 6	0 5 $\frac{1}{2}$	0 5	0 4 $\frac{1}{4}$	0 4 $\frac{1}{4}$	0 3 $\frac{1}{4}$	0 3 $\frac{1}{4}$	2 $\frac{1}{4}$	1 $\frac{1}{4}$
7 0	0 6 $\frac{1}{4}$	0 5 $\frac{3}{4}$	0 5 $\frac{1}{2}$	0 4 $\frac{3}{4}$	0 4 $\frac{1}{4}$	0 4	0 3 $\frac{1}{4}$	2 $\frac{1}{4}$	1 $\frac{1}{4}$
3	0 6 $\frac{1}{4}$	0 6	0 5 $\frac{1}{2}$	0 5	0 4 $\frac{1}{4}$	0 4	0 3 $\frac{1}{4}$	2 $\frac{1}{4}$	1 $\frac{1}{4}$
6	0 6 $\frac{1}{4}$	0 6 $\frac{1}{4}$	0 5 $\frac{3}{4}$	0 5	0 4 $\frac{1}{4}$	0 4 $\frac{1}{4}$	0 3 $\frac{3}{4}$	3	1 $\frac{1}{4}$
9	0 6 $\frac{1}{4}$	0 6 $\frac{1}{4}$	0 6	0 5 $\frac{1}{2}$	0 4 $\frac{3}{4}$	0 4 $\frac{1}{4}$	0 3 $\frac{3}{4}$	3	1 $\frac{1}{4}$
8 0	0 6 $\frac{3}{4}$	0 6 $\frac{1}{2}$	0 6	0 5 $\frac{1}{2}$	0 5	0 4 $\frac{1}{2}$	0 4	3	1 $\frac{1}{4}$
3	0 7	0 6 $\frac{3}{4}$	0 6 $\frac{1}{4}$	0 5 $\frac{3}{4}$	0 5	0 4 $\frac{1}{2}$	0 4	3 $\frac{1}{4}$	1 $\frac{1}{4}$
6	0 7 $\frac{1}{4}$	0 6 $\frac{3}{4}$	0 6 $\frac{1}{2}$	0 5 $\frac{3}{4}$	0 5 $\frac{1}{4}$	0 4 $\frac{3}{4}$	0 4	3 $\frac{1}{4}$	1 $\frac{1}{4}$
9	0 7 $\frac{1}{4}$	0 7	0 6 $\frac{1}{2}$	0 6	0 5 $\frac{1}{4}$	0 4 $\frac{3}{4}$	0 4 $\frac{1}{4}$	3 $\frac{1}{4}$	1 $\frac{1}{4}$
9 0	0 7 $\frac{3}{4}$	0 7 $\frac{1}{4}$	0 6 $\frac{3}{4}$	0 6	0 5 $\frac{1}{2}$	0 5	0 4 $\frac{1}{2}$	3 $\frac{1}{4}$	1 $\frac{3}{4}$
3	0 7 $\frac{3}{4}$	0 7 $\frac{1}{2}$	0 6 $\frac{3}{4}$	0 6 $\frac{1}{4}$	0 5 $\frac{3}{4}$	0 5	0 4 $\frac{1}{2}$	3 $\frac{1}{4}$	1 $\frac{3}{4}$
6	0 8	0 7 $\frac{1}{4}$	0 7	0 6 $\frac{1}{4}$	0 5 $\frac{3}{4}$	0 5 $\frac{1}{4}$	0 4 $\frac{3}{4}$	3 $\frac{1}{4}$	1 $\frac{3}{4}$
9	0 8 $\frac{1}{4}$	0 7 $\frac{3}{4}$	0 7 $\frac{1}{4}$	0 6 $\frac{1}{2}$	0 6	0 5 $\frac{1}{2}$	0 4 $\frac{3}{4}$	3 $\frac{1}{4}$	1 $\frac{3}{4}$
10 0	0 8 $\frac{1}{2}$	0 8	0 7 $\frac{1}{4}$	0 6 $\frac{1}{2}$	0 6	0 5 $\frac{3}{4}$	0 5	3 $\frac{1}{4}$	1 $\frac{3}{4}$
3	0 8 $\frac{1}{2}$	0 8	0 7 $\frac{1}{2}$	0 6 $\frac{1}{2}$	0 6 $\frac{1}{4}$	0 5 $\frac{3}{4}$	0 5	3 $\frac{1}{4}$	1 $\frac{3}{4}$
6	0 8 $\frac{3}{4}$	0 8 $\frac{1}{4}$	0 7 $\frac{1}{2}$	0 6 $\frac{3}{4}$	0 6 $\frac{1}{4}$	0 5 $\frac{3}{4}$	0 5	3 $\frac{1}{4}$	2
9	0 9	0 8 $\frac{1}{2}$	0 7 $\frac{3}{4}$	0 7 $\frac{1}{4}$	0 6 $\frac{1}{2}$	0 5 $\frac{3}{4}$	0 5 $\frac{1}{4}$	3 $\frac{1}{4}$	2
11 0	0 9 $\frac{1}{4}$	0 8 $\frac{3}{4}$	0 8	0 7 $\frac{1}{4}$	0 6 $\frac{3}{4}$	0 6	0 5 $\frac{1}{4}$	4	2
3	0 9 $\frac{1}{4}$	0 8 $\frac{3}{4}$	0 8	0 7 $\frac{1}{2}$	0 6 $\frac{3}{4}$	0 6	0 5 $\frac{1}{2}$	4	2
6	0 9 $\frac{1}{2}$	0 9	0 8 $\frac{1}{4}$	0 7 $\frac{3}{4}$	0 6 $\frac{3}{4}$	0 6 $\frac{1}{4}$	0 5 $\frac{3}{4}$	4	2
9	0 9 $\frac{1}{2}$	0 9 $\frac{1}{4}$	0 8 $\frac{1}{2}$	0 7 $\frac{3}{4}$	0 7	0 6 $\frac{1}{4}$	0 5 $\frac{3}{4}$	4	2
12 0	0 10	0 9 $\frac{1}{4}$	0 8 $\frac{1}{2}$	0 7 $\frac{3}{4}$	0 7 $\frac{1}{4}$	0 6 $\frac{3}{4}$	0 5 $\frac{3}{4}$	4 $\frac{1}{4}$	2
6	0 10 $\frac{1}{4}$	0 9 $\frac{1}{2}$	0 8 $\frac{3}{4}$	0 8	0 7 $\frac{1}{4}$	0 6 $\frac{3}{4}$	0 6	4 $\frac{1}{4}$	2 $\frac{1}{4}$
13 0	0 10 $\frac{1}{2}$	0 10	0 9	0 8 $\frac{1}{4}$	0 7 $\frac{3}{4}$	0 6 $\frac{3}{4}$	0 6 $\frac{1}{4}$	4 $\frac{1}{4}$	2 $\frac{1}{4}$
6	0 11	0 10 $\frac{1}{4}$	0 9 $\frac{1}{4}$	0 8 $\frac{1}{2}$	0 7 $\frac{3}{4}$	0 7	0 6 $\frac{1}{4}$	4 $\frac{1}{4}$	2 $\frac{1}{4}$
14 0	0 11 $\frac{1}{4}$	0 10 $\frac{1}{4}$	0 9 $\frac{3}{4}$	0 9	0 8	0 7 $\frac{1}{4}$	0 6 $\frac{3}{4}$	4 $\frac{1}{4}$	2 $\frac{1}{4}$
6	0 11 $\frac{1}{4}$	0 10 $\frac{3}{4}$	0 10	0 9 $\frac{1}{4}$	0 8 $\frac{1}{4}$	0 7 $\frac{1}{4}$	0 6 $\frac{3}{4}$	5	2 $\frac{1}{4}$
15 0	1 0	0 11	0 10 $\frac{1}{4}$	0 9 $\frac{3}{4}$	0 8 $\frac{3}{4}$	0 7 $\frac{3}{4}$	0 6 $\frac{3}{4}$	5	2 $\frac{1}{4}$
6	1 0 $\frac{1}{4}$	0 11 $\frac{1}{4}$	0 10 $\frac{1}{2}$	0 9 $\frac{3}{4}$	0 8 $\frac{3}{4}$	0 8	0 7	5 $\frac{1}{4}$	2 $\frac{1}{4}$
16 0	1 0 $\frac{3}{4}$	0 11 $\frac{3}{4}$	0 10 $\frac{3}{4}$	0 10	0 9	0 8	0 7 $\frac{1}{4}$	5 $\frac{1}{4}$	2 $\frac{1}{4}$
6	1 1	0 11 $\frac{3}{4}$	0 11	0 10 $\frac{1}{4}$	0 9 $\frac{1}{4}$	0 8 $\frac{1}{4}$	0 7 $\frac{1}{4}$	5 $\frac{1}{4}$	2 $\frac{1}{4}$
17 0	1 1 $\frac{1}{4}$	1 0	0 11 $\frac{3}{4}$	0 10 $\frac{1}{2}$	0 9 $\frac{1}{2}$	0 8 $\frac{1}{2}$	0 7 $\frac{1}{2}$	5 $\frac{1}{4}$	2 $\frac{1}{4}$
6	1 1 $\frac{1}{2}$	1 0 $\frac{1}{4}$	0 11 $\frac{3}{4}$	0 10 $\frac{3}{4}$	0 9 $\frac{1}{2}$	0 8 $\frac{1}{2}$	0 7 $\frac{1}{2}$	5 $\frac{1}{4}$	3

I.—SLATED HOUSES.

WALLS BUILT WITH STONE, OR BRICK, AND LIME MORTAR—continued.

Height.	A+		A		A—		B+		B		B—		C+		C	C—
Ft Inch.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	d.	d.
18 0	1	2	1	0 $\frac{1}{2}$	0	11 $\frac{1}{2}$	0	10 $\frac{3}{4}$	0	10	0	9	0	7 $\frac{1}{2}$	6	3 $\frac{1}{2}$
6	1	2	1	1	1	0	0	11	0	10	0	9	0	8	6	3
19 0	1	2 $\frac{1}{2}$	1	1 $\frac{1}{2}$	1	0 $\frac{1}{2}$	0	11 $\frac{1}{2}$	0	10 $\frac{1}{2}$	0	9 $\frac{1}{2}$	0	8 $\frac{1}{2}$	6 $\frac{1}{2}$	3
6	1	2 $\frac{1}{2}$	1	1 $\frac{1}{2}$	1	$\frac{1}{2}$	0	11 $\frac{1}{2}$	0	10 $\frac{1}{2}$	0	9 $\frac{1}{2}$	0	8 $\frac{1}{2}$	6 $\frac{1}{2}$	3
20 0	1	2 $\frac{3}{4}$	1	1 $\frac{3}{4}$	1	0 $\frac{3}{4}$	0	11 $\frac{3}{4}$	9	10 $\frac{1}{2}$	0	9 $\frac{1}{2}$	0	8 $\frac{1}{2}$	6 $\frac{1}{2}$	3 $\frac{1}{2}$
6	1	3	1	2	1	1	0	11 $\frac{3}{4}$	0	10 $\frac{3}{4}$	0	9 $\frac{3}{4}$	0	8 $\frac{3}{4}$	6 $\frac{1}{2}$	3 $\frac{1}{2}$
21 0	1	3 $\frac{1}{2}$	1	2 $\frac{1}{2}$	1	1 $\frac{1}{2}$	1	0	0	11	0	10	0	8 $\frac{3}{4}$	6 $\frac{1}{2}$	3 $\frac{1}{2}$
6	1	3 $\frac{1}{2}$	1	2 $\frac{1}{2}$	1	1 $\frac{1}{2}$	1	0 $\frac{1}{2}$	0	11 $\frac{1}{2}$	0	10	0	9	6 $\frac{1}{2}$	3 $\frac{1}{2}$
22 0	1	3 $\frac{3}{4}$	1	2 $\frac{3}{4}$	1	1 $\frac{3}{4}$	1	0 $\frac{1}{2}$	0	11 $\frac{1}{2}$	0	10 $\frac{1}{2}$	0	9	6 $\frac{1}{2}$	3 $\frac{1}{2}$
6	1	4	1	3	1	1 $\frac{3}{4}$	1	0 $\frac{1}{2}$	0	11 $\frac{1}{2}$	0	10 $\frac{1}{2}$	0	9	6 $\frac{1}{2}$	3 $\frac{1}{2}$
23 0	1	4 $\frac{1}{2}$	1	3 $\frac{1}{2}$	1	2	1	0 $\frac{1}{2}$	0	11 $\frac{1}{2}$	0	10 $\frac{1}{2}$	0	9 $\frac{1}{2}$	6 $\frac{1}{2}$	3 $\frac{1}{2}$
6	1	4 $\frac{1}{2}$	1	3 $\frac{1}{2}$	1	2 $\frac{1}{2}$	1	1	0	11 $\frac{1}{2}$	0	10 $\frac{1}{2}$	0	9 $\frac{1}{2}$	7	3 $\frac{1}{2}$
24 0	1	4 $\frac{3}{4}$	1	3 $\frac{3}{4}$	1	2 $\frac{3}{4}$	1	1	1	0	0	10 $\frac{3}{4}$	0	9 $\frac{3}{4}$	7 $\frac{1}{2}$	3 $\frac{1}{2}$
6	1	5	1	3 $\frac{3}{4}$	1	2 $\frac{3}{4}$	1	1 $\frac{1}{2}$	1	0	0	10 $\frac{3}{4}$	0	9 $\frac{3}{4}$	7 $\frac{1}{2}$	3 $\frac{1}{2}$
25 0	1	5	1	4	1	3 $\frac{1}{2}$	1	1 $\frac{1}{2}$	1	0 $\frac{1}{2}$	0	11	0	10	7 $\frac{1}{2}$	3 $\frac{1}{2}$
6	1	5 $\frac{1}{2}$	1	4 $\frac{1}{2}$	1	3	1	1 $\frac{1}{2}$	1	0 $\frac{1}{2}$	0	11 $\frac{1}{2}$	0	10	7 $\frac{1}{2}$	3 $\frac{1}{2}$
26 0	1	5 $\frac{3}{4}$	1	4 $\frac{3}{4}$	1	3	1	2	1	0 $\frac{1}{2}$	0	11 $\frac{1}{2}$	0	10 $\frac{1}{2}$	7 $\frac{1}{2}$	3 $\frac{1}{2}$
6	1	5 $\frac{3}{4}$	1	4 $\frac{3}{4}$	1	3 $\frac{1}{2}$	1	2	1	0 $\frac{1}{2}$	0	11 $\frac{1}{2}$	0	10 $\frac{1}{2}$	7 $\frac{1}{2}$	3 $\frac{1}{2}$
27 0	1	6	1	4 $\frac{1}{2}$	1	3 $\frac{1}{2}$	1	2 $\frac{1}{2}$	1	1	0	11 $\frac{1}{2}$	0	10 $\frac{1}{2}$	7 $\frac{1}{2}$	3 $\frac{1}{2}$
6	1	6 $\frac{1}{2}$	1	5	1	3 $\frac{1}{2}$	1	2 $\frac{1}{2}$	1	1	0	11 $\frac{1}{2}$	0	10 $\frac{1}{2}$	7 $\frac{1}{2}$	3 $\frac{1}{2}$
28 0	1	6 $\frac{1}{2}$	1	5	1	3 $\frac{3}{4}$	1	2 $\frac{1}{2}$	1	1 $\frac{1}{2}$	0	11 $\frac{1}{2}$	0	10 $\frac{1}{2}$	7 $\frac{1}{2}$	3 $\frac{1}{2}$
6	1	6 $\frac{1}{2}$	1	5 $\frac{1}{2}$	1	4	1	2 $\frac{1}{2}$	1	1 $\frac{1}{2}$	1	0	0	10 $\frac{1}{2}$	8	4
29 0	1	6 $\frac{3}{4}$	1	5 $\frac{3}{4}$	1	4	1	2 $\frac{3}{4}$	1	1 $\frac{1}{2}$	1	0	0	10 $\frac{3}{4}$	8	4
6	1	6 $\frac{3}{4}$	1	5 $\frac{3}{4}$	1	4 $\frac{1}{2}$	1	2 $\frac{3}{4}$	1	1 $\frac{1}{2}$	1	0	0	10 $\frac{3}{4}$	8	4
30 0	1	7	1	5 $\frac{1}{2}$	1	4 $\frac{1}{2}$	1	3	1	1 $\frac{1}{2}$	1	0 $\frac{1}{2}$	0	10 $\frac{1}{2}$	8	4
6	1	7	1	5 $\frac{1}{2}$	1	4 $\frac{1}{2}$	1	3	1	1 $\frac{1}{2}$	1	0 $\frac{1}{2}$	0	11	8 $\frac{1}{2}$	4
31 0	1	7 $\frac{1}{2}$	1	5 $\frac{3}{4}$	1	4 $\frac{3}{4}$	1	3 $\frac{1}{2}$	1	1 $\frac{1}{2}$	1	0 $\frac{1}{2}$	0	11	8 $\frac{1}{2}$	4
6	1	7 $\frac{1}{2}$	1	6	1	4 $\frac{3}{4}$	1	3 $\frac{1}{2}$	1	2	1	0 $\frac{1}{2}$	0	11 $\frac{1}{2}$	8 $\frac{1}{2}$	4
32 0	1	7 $\frac{3}{4}$	1	6 $\frac{1}{4}$	1	4 $\frac{3}{4}$	1	3 $\frac{3}{4}$	1	2	1	0 $\frac{1}{2}$	0	11 $\frac{3}{4}$	8 $\frac{1}{2}$	4
6	1	7 $\frac{3}{4}$	1	6 $\frac{1}{4}$	1	5	1	3 $\frac{3}{4}$	1	2	1	0 $\frac{1}{2}$	0	11 $\frac{3}{4}$	8 $\frac{1}{2}$	4
33 0	1	7 $\frac{3}{4}$	1	6 $\frac{1}{2}$	1	5	1	3 $\frac{3}{4}$	1	2 $\frac{1}{2}$	1	0 $\frac{1}{2}$	0	11 $\frac{3}{4}$	8 $\frac{1}{2}$	4 $\frac{1}{2}$
6	1	8	1	6 $\frac{1}{2}$	1	5	1	3 $\frac{3}{4}$	1	2 $\frac{1}{2}$	1	1	0	11 $\frac{3}{4}$	8 $\frac{1}{2}$	4 $\frac{1}{2}$
34 0	1	8	1	6 $\frac{3}{4}$	1	5 $\frac{3}{4}$	1	3 $\frac{3}{4}$	1	2 $\frac{3}{4}$	1	1	0	11 $\frac{3}{4}$	8 $\frac{1}{2}$	4 $\frac{1}{2}$
6	1	8 $\frac{1}{4}$	1	6 $\frac{3}{4}$	1	5 $\frac{3}{4}$	1	3 $\frac{3}{4}$	1	2 $\frac{3}{4}$	1	1	0	11 $\frac{3}{4}$	8 $\frac{1}{2}$	4 $\frac{1}{2}$
35 0	1	8 $\frac{1}{2}$	1	6 $\frac{3}{4}$	1	5 $\frac{3}{4}$	1	4	1	2 $\frac{3}{4}$	1	1	0	11 $\frac{3}{4}$	8 $\frac{1}{2}$	4 $\frac{1}{2}$
6	1	8 $\frac{1}{2}$	1	7	1	5 $\frac{3}{4}$	1	4	1	2 $\frac{3}{4}$	1	1 $\frac{1}{2}$	0	11 $\frac{3}{4}$	8 $\frac{1}{2}$	4 $\frac{1}{2}$
36 0	1	8 $\frac{3}{4}$	1	7	1	5 $\frac{3}{4}$	1	4	1	2 $\frac{3}{4}$	1	1 $\frac{1}{2}$	0	11 $\frac{3}{4}$	8 $\frac{1}{2}$	4 $\frac{1}{2}$
6	1	8 $\frac{3}{4}$	1	7	1	5 $\frac{3}{4}$	1	4	1	2 $\frac{3}{4}$	1	1 $\frac{1}{2}$	0	11 $\frac{3}{4}$	9	4 $\frac{1}{2}$
37 0	1	8 $\frac{3}{4}$	1	7 $\frac{1}{2}$	1	5 $\frac{3}{4}$	1	4 $\frac{1}{2}$	1	2 $\frac{3}{4}$	1	1 $\frac{1}{2}$	0	11 $\frac{3}{4}$	9	4 $\frac{1}{2}$
6	1	9	1	7 $\frac{1}{2}$	1	5 $\frac{3}{4}$	1	4 $\frac{1}{2}$	1	2 $\frac{3}{4}$	1	1 $\frac{1}{2}$	0	11 $\frac{3}{4}$	9	4 $\frac{1}{2}$
38 0	1	9	1	7 $\frac{3}{4}$	1	5 $\frac{3}{4}$	1	4 $\frac{3}{4}$	1	2 $\frac{3}{4}$	1	1 $\frac{1}{2}$	1	0	9	4 $\frac{1}{2}$
6	1	9	1	7 $\frac{3}{4}$	1	6	1	4 $\frac{3}{4}$	1	3	1	1 $\frac{1}{2}$	1	0	9	4 $\frac{1}{2}$
39 0	1	9	1	7 $\frac{3}{4}$	1	6	1	4 $\frac{3}{4}$	1	3	1	1 $\frac{1}{2}$	1	0	9	4 $\frac{1}{2}$
6	1	9	1	7 $\frac{3}{4}$	1	6 $\frac{1}{2}$	1	4 $\frac{3}{4}$	1	3	1	1 $\frac{1}{2}$	1	0	9	4 $\frac{1}{2}$
40 0	1	9 $\frac{1}{2}$	1	7 $\frac{3}{4}$	1	6 $\frac{1}{2}$	1	4 $\frac{3}{4}$	1	3	1	1 $\frac{1}{2}$	1	0	9	4 $\frac{1}{2}$
6	1	9 $\frac{1}{2}$	1	7 $\frac{3}{4}$	1	6 $\frac{1}{2}$	1	4 $\frac{3}{4}$	1	3	1	1 $\frac{1}{2}$	1	0	9	4 $\frac{1}{2}$

(S10w.)

II.—THATCHED HOUSES,

BRICK OR STONE WALLS, BUILT WITH LIME MORTAR.

Height.	A+	A	A—	B+	B	B—	C+	C	C—
Ft Inch		<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>
6 0	—	4½	3¾	3½	3½	2¾	2½	1½	1
3	—	4½	4	3½	3½	2¾	2½	1½	1
6	—	4½	4	3¾	3½	3	2¾	1½	1½
9	—	4½	4½	4	3½	3	2¾	1½	1½
7 0	—	5	4½	4	3¾	3½	2¾	2	1½
3	—	5	4½	4½	3¾	3½	3	2	1½
6	—	5½	4½	4½	4	3½	3	2	1½
9	—	5½	5	4½	4	3½	3½	2	1½
8 0	—	5½	5	4¾	4½	3½	3½	2	1½
3	—	5½	5½	4¾	4½	3½	3½	2½	1½
6	—	5½	5½	5	4½	3½	3½	2½	1½
9	—	6	5½	5	4½	4	3½	2½	1½
9 0	—	6	5½	5	4¾	4	3½	2½	1½
3	—	6½	5½	5½	4¾	4	3½	2½	1½
6	—	6½	5½	5½	4¾	4	3½	2½	1½
9	—	6½	6	5½	5	4½	3½	2½	1½
10 0	—	6¾	6½	5½	5	4½	4	2½	1½
3	—	6¾	6½	5½	5½	4½	4	2½	1½
6	—	7	6½	6	5½	4½	4	2½	2
9	—	7½	6½	6	5½	4½	4½	2½	2
11 0	—	7½	6¾	6½	5½	4¾	4½	2½	2
3	—	7½	6¾	6½	5½	5	4½	2½	2
6	—	7½	7	6½	5½	5	4½	2½	2
9	—	7½	7½	6½	6	5	4½	3	2
12 0	—	7¾	7½	6½	6	5½	4¾	3	2½
6	—	8	7½	6¾	6½	5½	4¾	3½	2½
13 0	—	8½	7¾	7	6½	5½	5	3½	2½
6	—	8½	7¾	7½	6½	5½	5	3½	2½
14 0	—	8¾	8	7½	6¾	5½	5½	3½	2½
6	—	9	8½	7¾	7	6	6½	3½	2½
15 0	—	9½	8½	8	7½	6	6½	3½	2½
6	—	9½	8½	8½	7½	6½	6½	3½	2½
16 0	—	10	9	8½	7¾	6½	5½	3½	2½
6	—	10½	9½	8½	7¾	6½	6	3½	2½
17 0	—	10½	9½	8¾	8	6½	6	4	2½
6	—	10½	9¾	9	8	6½	6½	4	2½
18 0	—	10¾	10	9½	8½	7	6½	4	3
6	—	11	10½	9½	8½	7½	6½	4½	3
19 0	—	11½	10¾	9½	8½	7½	6½	4½	3
6	—	11½	10¾	9¾	8½	7½	6½	4½	3½
20 0	—	11½	10¾	10	9	7½	6½	4½	3½

(810z.)

III.—THATCHED HOUSES,

PUDDLE MORTAR WALLS,—DRY WALLS, POINTED,—MUD WALLS OF A GOOD KIND.

Height.	A+	A	A—	B+	B	B—	C+	C	C—
<i>Ft. Inch.</i>			<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>
6 0	—	—	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	2	1 $\frac{1}{2}$	2 $\frac{3}{4}$
3	—	—	3 $\frac{1}{2}$	3	2 $\frac{3}{4}$	2 $\frac{3}{4}$	2	1 $\frac{1}{2}$	2 $\frac{3}{4}$
6	—	—	3 $\frac{1}{2}$	3	2 $\frac{3}{4}$	2 $\frac{3}{4}$	2	1 $\frac{1}{2}$	2 $\frac{3}{4}$
9	—	—	3 $\frac{1}{2}$	3 $\frac{1}{2}$	3	2 $\frac{3}{4}$	2	1 $\frac{1}{2}$	2 $\frac{3}{4}$
7 0	—	—	3 $\frac{1}{2}$	3 $\frac{1}{2}$	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	1 $\frac{1}{2}$	2 $\frac{3}{4}$
3	—	—	3 $\frac{3}{4}$	3 $\frac{3}{4}$	3 $\frac{1}{2}$	2 $\frac{3}{4}$	2 $\frac{1}{2}$	1 $\frac{1}{2}$	2 $\frac{3}{4}$
6	—	—	3 $\frac{3}{4}$	3 $\frac{3}{4}$	3 $\frac{1}{2}$	2 $\frac{3}{4}$	2 $\frac{1}{2}$	1 $\frac{1}{2}$	1
9	—	—	3 $\frac{3}{4}$	3 $\frac{3}{4}$	3 $\frac{1}{2}$	2 $\frac{3}{4}$	2 $\frac{1}{2}$	1 $\frac{1}{2}$	1
8 0	—	—	4	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3	2 $\frac{3}{4}$	1 $\frac{3}{4}$	1
3	—	—	4	3 $\frac{3}{4}$	3 $\frac{3}{4}$	3	2 $\frac{3}{4}$	1 $\frac{3}{4}$	1
6	—	—	4 $\frac{1}{2}$	4	3 $\frac{3}{4}$	3 $\frac{1}{2}$	2 $\frac{3}{4}$	1 $\frac{3}{4}$	1
9	—	—	4 $\frac{1}{2}$	4	3 $\frac{3}{4}$	3 $\frac{1}{2}$	2 $\frac{3}{4}$	2	1
9 0	—	—	4 $\frac{1}{2}$	4	3 $\frac{3}{4}$	3 $\frac{1}{2}$	2 $\frac{3}{4}$	2	1
3	—	—	4 $\frac{1}{2}$	4 $\frac{1}{2}$	4	3 $\frac{1}{2}$	2 $\frac{3}{4}$	2	1
6	—	—	4 $\frac{3}{4}$	4 $\frac{1}{2}$	4	3 $\frac{3}{4}$	2 $\frac{3}{4}$	2	1
9	—	—	4 $\frac{3}{4}$	4 $\frac{3}{4}$	4	3 $\frac{3}{4}$	2 $\frac{3}{4}$	2	1 $\frac{1}{2}$
10 0	—	—	4 $\frac{3}{4}$	4 $\frac{1}{2}$	4 $\frac{1}{2}$	3 $\frac{3}{4}$	3	2	1 $\frac{1}{2}$
3	—	—	5	4 $\frac{3}{4}$	4 $\frac{1}{2}$	3 $\frac{3}{4}$	3	2 $\frac{1}{2}$	1 $\frac{1}{2}$
6	—	—	5	4 $\frac{3}{4}$	4 $\frac{3}{4}$	3 $\frac{3}{4}$	3	2 $\frac{1}{2}$	1 $\frac{1}{2}$
9	—	—	5 $\frac{1}{2}$	4 $\frac{3}{4}$	4 $\frac{3}{4}$	3 $\frac{3}{4}$	3 $\frac{1}{2}$	2 $\frac{1}{2}$	1 $\frac{1}{2}$
11 0	—	—	5 $\frac{1}{2}$	5	4 $\frac{3}{4}$	4	3 $\frac{1}{2}$	2 $\frac{1}{2}$	1 $\frac{1}{2}$
3	—	—	5 $\frac{1}{2}$	5	4 $\frac{3}{4}$	4	3 $\frac{1}{2}$	2 $\frac{1}{2}$	1 $\frac{1}{2}$
6	—	—	5 $\frac{1}{2}$	5 $\frac{1}{2}$	5	4	3 $\frac{1}{2}$	2 $\frac{1}{2}$	1 $\frac{1}{2}$
9	—	—	5 $\frac{1}{2}$	5 $\frac{1}{2}$	5	4	3 $\frac{1}{2}$	2 $\frac{1}{2}$	1 $\frac{1}{2}$
12 0	—	—	5 $\frac{3}{4}$	5 $\frac{1}{2}$	5 $\frac{1}{2}$	4 $\frac{1}{2}$	3 $\frac{1}{2}$	2 $\frac{1}{2}$	1 $\frac{1}{2}$
6	—	—	6	5 $\frac{3}{4}$	5 $\frac{1}{2}$	4 $\frac{1}{2}$	3 $\frac{1}{2}$	2 $\frac{1}{2}$	1 $\frac{1}{2}$
13 0	—	—	6	5 $\frac{3}{4}$	5 $\frac{3}{4}$	4 $\frac{1}{2}$	3 $\frac{1}{2}$	2 $\frac{1}{2}$	1 $\frac{1}{2}$
6	—	—	6 $\frac{1}{2}$	6	5 $\frac{3}{4}$	4 $\frac{1}{2}$	3 $\frac{1}{2}$	2 $\frac{1}{2}$	1 $\frac{1}{2}$
14 0	—	—	6 $\frac{1}{2}$	6	5 $\frac{3}{4}$	4 $\frac{3}{4}$	4	2 $\frac{3}{4}$	1 $\frac{1}{2}$
6	—	—	6 $\frac{1}{2}$	6 $\frac{1}{2}$	6	5	4	2 $\frac{3}{4}$	1 $\frac{1}{2}$
15 0	—	—	6 $\frac{1}{2}$	6 $\frac{1}{2}$	6	5	4 $\frac{1}{2}$	3	1 $\frac{1}{2}$
6	—	—	7	6 $\frac{1}{2}$	6 $\frac{1}{2}$	5 $\frac{1}{2}$	4 $\frac{1}{2}$	3	1 $\frac{1}{2}$
16 0	—	—	7 $\frac{1}{2}$	6 $\frac{3}{4}$	6 $\frac{1}{2}$	5 $\frac{1}{2}$	4 $\frac{1}{2}$	3	1 $\frac{3}{4}$
6	—	—	7 $\frac{1}{2}$	6 $\frac{3}{4}$	6 $\frac{1}{2}$	5 $\frac{1}{2}$	4 $\frac{1}{2}$	3 $\frac{1}{2}$	1 $\frac{3}{4}$
17 0	—	—	7 $\frac{1}{2}$	7	6 $\frac{3}{4}$	5 $\frac{1}{2}$	4 $\frac{3}{4}$	3 $\frac{1}{2}$	2
6	—	—	7 $\frac{1}{2}$	7 $\frac{1}{2}$	6 $\frac{3}{4}$	5 $\frac{1}{2}$	4 $\frac{3}{4}$	3 $\frac{1}{2}$	2
18 0	—	—	7 $\frac{3}{4}$	7 $\frac{1}{2}$	7	6	5	3 $\frac{1}{2}$	2
6	—	—	8	7 $\frac{3}{4}$	7	6	5	3 $\frac{3}{4}$	2
19 0	—	—	8 $\frac{1}{2}$	7 $\frac{3}{4}$	7 $\frac{1}{2}$	6 $\frac{1}{2}$	5 $\frac{1}{2}$	3 $\frac{3}{4}$	2
6	—	—	8 $\frac{1}{2}$	7 $\frac{3}{4}$	7 $\frac{1}{2}$	6 $\frac{1}{2}$	5 $\frac{1}{2}$	3 $\frac{3}{4}$	2
20 0	—	—	8 $\frac{3}{4}$	8	7 $\frac{3}{4}$	6 $\frac{1}{2}$	5 $\frac{1}{2}$	3 $\frac{3}{4}$	2

310y.

IV.—BASEMENT STORIES,

OF DWELLING HOUSES, OR CELLARS, USED AS DWELLINGS.

Height.	A+	A	A—	B+	B	B—	C+	C	C—
Ft. Inch	d.	d.	d.	d.	d.	d.	d.	d.	d.
6 0	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	2	1 $\frac{3}{4}$	1 $\frac{1}{2}$	1 $\frac{1}{2}$	1 $\frac{1}{4}$
3	2 $\frac{1}{4}$	2 $\frac{1}{4}$	2 $\frac{1}{4}$	2 $\frac{1}{4}$	2	1 $\frac{3}{4}$	1 $\frac{1}{4}$	1 $\frac{1}{4}$	1 $\frac{1}{4}$
6	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{2}$	2 $\frac{1}{4}$	2	1 $\frac{3}{4}$	1 $\frac{1}{4}$	1 $\frac{1}{4}$
9	3	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{4}$	2	1 $\frac{3}{4}$	1 $\frac{1}{4}$	1
7 0	3 $\frac{1}{4}$	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{4}$	2	1 $\frac{3}{4}$	1 $\frac{1}{2}$	1
3	3 $\frac{1}{4}$	3	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{4}$	2	1 $\frac{1}{2}$	1
6	3 $\frac{1}{2}$	3 $\frac{1}{4}$	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{4}$	2	1 $\frac{1}{2}$	1
9	3 $\frac{1}{2}$	3 $\frac{1}{4}$	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{4}$	2	1 $\frac{1}{2}$	1
8 0	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3 $\frac{1}{4}$	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{4}$	1 $\frac{3}{4}$	1
3	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3 $\frac{1}{4}$	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{4}$	1 $\frac{3}{4}$	1
6	4	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3	2 $\frac{3}{4}$	2 $\frac{1}{2}$	2 $\frac{1}{4}$	1 $\frac{3}{4}$	1 $\frac{1}{4}$
9	4	3 $\frac{3}{4}$	3 $\frac{1}{2}$	3 $\frac{1}{4}$	3	2 $\frac{3}{4}$	2 $\frac{1}{4}$	1 $\frac{3}{4}$	1 $\frac{1}{4}$
9 0	4 $\frac{1}{4}$	4	2 $\frac{3}{4}$	3 $\frac{1}{4}$	3	2 $\frac{3}{4}$	2 $\frac{1}{4}$	1 $\frac{3}{4}$	1 $\frac{1}{4}$
3	4 $\frac{1}{4}$	4	3 $\frac{3}{4}$	3 $\frac{1}{4}$	3	2 $\frac{3}{4}$	2 $\frac{1}{4}$	1 $\frac{3}{4}$	1 $\frac{1}{4}$
6	4 $\frac{1}{2}$	4	3 $\frac{3}{4}$	3 $\frac{1}{4}$	3 $\frac{1}{4}$	2 $\frac{3}{4}$	2 $\frac{1}{4}$	2	1 $\frac{1}{4}$
9	4 $\frac{1}{2}$	4 $\frac{1}{4}$	4	3 $\frac{1}{4}$	3 $\frac{1}{4}$	3	2 $\frac{1}{2}$	2	1 $\frac{1}{4}$
10 0	4 $\frac{3}{4}$	4 $\frac{1}{4}$	4	3 $\frac{3}{4}$	3 $\frac{1}{4}$	3	2 $\frac{3}{4}$	2	1 $\frac{1}{4}$
3	4 $\frac{3}{4}$	4 $\frac{1}{4}$	4	3 $\frac{3}{4}$	3 $\frac{1}{4}$	3	2 $\frac{3}{4}$	2	1 $\frac{1}{4}$
6	5	4 $\frac{1}{4}$	4 $\frac{1}{4}$	3 $\frac{3}{4}$	3 $\frac{1}{4}$	3 $\frac{1}{4}$	2 $\frac{3}{4}$	2	1 $\frac{1}{4}$
9	5	4 $\frac{1}{4}$	4 $\frac{1}{4}$	4	3 $\frac{3}{4}$	3 $\frac{1}{4}$	2 $\frac{3}{4}$	2 $\frac{1}{4}$	1 $\frac{1}{4}$
11 0	5	4 $\frac{3}{4}$	4 $\frac{1}{2}$	4	3 $\frac{3}{4}$	3 $\frac{1}{4}$	3	2 $\frac{1}{2}$	1 $\frac{1}{4}$

Where houses are built of wood, as in America, we deduct 10 per cent. from the value of a brick house of the same size and location, where the winters are cold. In the Southern States, where the winters are warm, we deduct 20 per cent. from the value of a brick house similarly situated. We value a first-class frame or wooden house as if it was built of brick, and then make the above deductions, or that which local modifying circumstances will point out, such as climate, scarcity of timber, brick, lime, etc.

OFFICES.

The rate per square for offices of the I., II., III. and IV. Classes, is half that supplied in the foregoing Tables; Offices of the V. Class have the rate per square as follows:

310s.

V.—OFFICES THATCHED,

WITH DRY STONE WALLS.

Height.	A+	A	A—	B+	B	B—	C+	C	C—
<i>Ft. Inch.</i>			<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>	<i>d.</i>
5 0	—	—	1½	1¼	1	1	¾	½	¼
3	—	—	1½	1¼	1	1	¾	½	¼
6	—	—	1½	1¼	1½	1	¾	½	¼
9	—	—	1½	1½	1½	1	¾	½	¼
6 0	—	—	1½	1½	1½	1	¾	½	¼
3	—	—	1½	1½	1½	1½	¾	½	¼
6	—	—	1½	1½	1½	1	1	½	¼
9	—	—	2	1½	1½	1½	1	½	¼
7 0	—	—	2	1½	1½	1½	1	¾	½
3	—	—	2	1½	1½	1½	1	¾	½
6	—	—	2	1½	1½	1½	1½	¾	½
9	—	—	2½	2	1½	1½	1½	¾	½
8 0	—	—	2½	2	1½	1½	1½	¾	½
3	—	—	2½	2	1½	1½	1½	¾	½
6	—	—	2½	2	1½	1½	1½	¾	½
9	—	—	2½	2½	2	1½	1½	¾	½
9 0	—	—	2½	2½	2	1½	1½	¾	½
3	—	—	2½	2½	2	1½	1½	¾	½
6	—	—	2½	2½	2	1½	1½	¾	½
9	—	—	2½	2½	2	1½	1½	1	½
10 0	—	—	2½	2½	2	2	1½	1	¾
6	—	—	2½	2½	2½	2	1½	1	¾
11 0	—	—	2½	2½	2½	2	1½	1	¾
6	—	—	3	2½	2½	2	1½	1	¾
12 0	—	—	3	2½	2½	2	1½	1	¾
6	—	—	3½	3	2½	2½	1½	1	¾
13 0	—	—	3½	3	2½	2½	1½	1	¾
6	—	—	3½	3	2½	2½	2	1	¾
14 0	—	—	3½	3½	2½	2½	2	1	¾
6	—	—	3½	3½	3	2½	2	1	¾
15 0	—	—	3½	3½	3	2½	2	1½	¾

310A.

HOUSES IN TOWNS.

TABLES for ascertaining, by inspection, the relative value of any portion of a Building (nine square feet, or one yard,) and of any height, from I to V stories.

SIGNIFICATION OF THE LETTERS.

1ST CLASS.	A+	Build or ornamented with cut stone, of superior solidity and finish.
	A	Very substantial building and finish, without cut stone ornament.
	A—	Ordinary building and finish, or either of the above, when built 25 or 30 years.
2ND CLASS.	B+	Medium, in sound order, and in good repair.
	B	Medium, slightly decayed, but in repair.
	B—	Medium, deteriorated by age, and not in good repair.
3RD CLASS.	C+	Old, but in repair.
	C	Old, and out of repair.
	C—	Old, and dilapidated—scarcely habitable.

TABLE PRICES FOR HOUSES, AS DWELLINGS, SLATED.

Stories	FIRST CLASS.			SECOND CLASS.			THIRD CLASS.		
	A+	A	A—	B+	B	B—	C+	C	C—
	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.
I	1 6	1 5	1 4	1 2	1 0	0 10	0 8	0 6	0 4
II	2 6	2 4	2 2	2 0	1 9	1 6	1 3	1 0	0 8
III	3 0	2 10	2 8	2 6	2 3	2 0	1 9	1 4	0 10
IV	3 4	3 3	3 0	2 9	2 6	2 4	2 0	1 7	1 0
V	3 7	3 6	3 3	2 9	2 9	2 6	2 2	1 9	1 1

BASEMENTS AS DWELLINGS.

0 10	0 9	0 8	0 7	0 6	0 5	0 4	0 3	0 2
------	-----	-----	-----	-----	-----	-----	-----	-----

TABLE PRICES FOR OFFICES, SLATED.

Stories	FIRST CLASS.			SECOND CLASS.			THIRD CLASS.		
	A+	A	A—	B+	B	B—	C+	C	C—
	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.
I	0 9	0 8½	0 8	0 7	0 6	0 5	0 4	0 3	0 2
II	1 3	1 2	1 1	1 0	0 10	0 8	0 6	0 5	0 4
III	1 6	1 5	1 4	1 3	1 0	0 10	0 8	0 6	0 5
IV	1 8	1 7	1 6	1 4	1 2	1 0	0 9	0 7	0 5½
V	1 9	1 8	1 7	1 6	1 4	1 1	0 10	0 8	0 6

CELLARS AS OFFICES.

0 6	0 5	0 5	0 4	0 3½	0 3	0 2	0 1½	0 1
-----	-----	-----	-----	------	-----	-----	------	-----

GEOLOGICAL FORMATION OF THE EARTH.

§102. *Rocks*, originally horizontal, are now, by subsequent changes, inclined to the horizon: some are found contorted and vertical; often inclined both ways from a summit, and forming basins, which God has ordained to be great reservoirs for water, coal and oil, from which man draws water by artesian wells, to fertilize the sandy soil of Algiers, and to supply him with fuel and light, on the almost woodless prairies of Illinois.

Unstratified rocks, are those which do not lie in beds, as granite.

Stratified rocks, lie in beds, as limestones, etc.

Dykes, are where fissures in the rocks are filled with igneous rocks, such as lava, trap rocks. Dykes seldom have branches; they cross one another, and are sometimes several yards wide, and extend from sixty to seventy miles in England and Ireland.

Veins, feeders or lodes, are fissures in the rocks, and are of various thicknesses; are parallel to one another in alternate bands, or, cross one another as net work.

Metallic veins, are principally found in the primary rocks in parallel bands, and seldom isolated, as several veins or lodes are in the same locality. Those lodes or veins which intersect others, contain a different mineral.

Gangue or matrix, is the stony mineral which separates the metal from the adjoining rock.

Metallic indications, are the gangue and numerous cavities in the ground, or holes on the surface, corresponding to those formed underneath by the action of the water.

The crust of the earth, is supposed to be four and one-fourth miles, and arranged as follows by Regnault and others:

Forman's Group.		
Forman's Group.	I.	Late Formation. Vegetable soil. Alluvial deposits filling estuaries.
	II.	Upper Tertiary or Pliocene and Miocene. Modern volcanoes, both extinct and burning. Strata of ancient sand, alluvium. Boulders, drift, tufa, containing fossil bones.
	III.	Middle Tertiary. Freshwater limestones, burrstones, sometimes containing lignites. Sandstone of Fontainebleau.
	IV.	Lower Tertiary. Marls with gypsum, fossils of the mammifera. Coarse limestone. Plastic clay with lignite.
	V.	Upper Cretaceous. Extensive limestone stratum called chalk, with interposing layers of silex.
	VI.	Lower Cretaceous. Tuffaceous chalk of Touraine sand, or sandstone, generally green. Ferruginous sands.
	VII.	Oolitic or Jurassic and Lias. Calcareous strata, more or less compact and marly, alternating with layers of clay. The upper strata of this group is termed Oolitic, and the other, Lias.
	VIII.	Trias. Variegated marls, often containing masses of gypsum and rock salt. Limestone very fossiliferous. Sandstone of various colors.
	IX.	Sandstone. Conglomerate and sandstone.
	X.	Permian. Limestone mixed with slate.
		" Limestone conglomerate and sandstone, termed the new red sandstone.
	XI.	Carboniferous Sandstone, slates with seams of coal and carbonate of iron, (clay iron stone.) Carboniferous or mountain limestone, with seams of coal.
	XII.	Devonian. Heavy beds of old red sandstone, with small seams of anthracite (or hard coal.)
	XIII.	Silurian. Limestone, roofing slate, coarse grained sandstone called greywacke.
	XIV.	Cambrian. Compact limestone, argillaceous shale or slate rocks having often a crystalline texture.
	XV.	Primary rocks. Granite and gneiss forming the principal base of the interior of the globe, accessible to our observations.

810c. Quartz, silica or silicic acid, is of various forms, color and transparency, and is generally colorless, but often reddish, brownish, yellowish and black. It is the principal constituent in flint, sea and lake shore gravel, and sandstones. It scratches glass; is insoluble, infusible, and not acted on by acids. If fused with caustic potash or soda, it melts into a glass.

Vitreous quartz, in its purest state, is rock-crystal, which is transparent and colorless.

Calcedonic quartz, resembles rock-crystal, but if calcined it becomes white. It is more tenacious than vitreous quartz, and has a conchoidal fracture.

Sand, is quartz in minute grains, generally colored reddish or yellowish brown, by oxyde of iron, but often found white.

Sandstone, is where the grains of quartz are cemented together with calcareous, siliceous or argillaceous matter.

Alumina. Pure alumina is rarely found in nature. It is composed of two equivalents of the metal aluminum and three of oxygen, and is often found of brilliant colors and used by jewellers as precious stones. The sapphire is blue, the ruby is red, topaz when yellow, emerald when green, amethyst when violet, and adamantinite when brown. On account of its hardness, it is used as *emery* in polishing precious stones and glass. It is infusible before the blowpipe with soda.

Potash or *Potassa*, is the protoxide of the metal potassium, and when pure = $K O$ or one equivalent of each.

Soda = $Na O$ = protoxide of the metal sodium.

Lime = $Ca O$ = protoxide of the metal calcium.

Magnesia = $Mg O$ = protoxide of the metal magnesium.

Felspar, is widely distributed and of various colors and crystallization. In granite, it has a perfect crystalline structure. As the base of porphyries, it is compact, of a close even texture. In granite felspar, the crystals of it is found in groups, cavities or veins, often with other substances. In porphyry, the crystals are embedded separately, as in a paste. It has a clear edge in two directions, and is nearly as hard as quartz. It is composed of silica, alumina and potash.

Common Felspar, is composed of silica, alumina and potassa. (See table of analysis of rocks.)

Albite—soda felspar, differs from felspar in having about eleven per cent. of soda in place of the potash, and in its crystallization, which belongs to the sixth series of solids, the three cleavages all meeting at oblique angles; yet the appearance of felspar and albite are very similar, and difficult to distinguish one from another. Their hardness and chemical characters are the same except the albite, which tinges the blowpipe-flame yellow. It forms the basis of granite in many countries: especially in North America, and is characterized by its almost constant whiteness.

Labradorite, a kind of felspar, contains lime, and about four per cent. of soda. It reflects brilliant colors in certain positions, particularly shades of green and blue; but its general color is dark grey. It is less infusible than felspar or albite, and may be dissolved in hydrochloric acid. It is abundant in Labrador and the State of New York.

Mica. It cleaves into very thin transparent, tough, elastic plates, commonly whitish, like transparent horn, sometimes brown or black. It

is principally composed of silica and alumina, combined with potassa, lime, magnesia, or oxyde of iron.

Quartz or silica, has no cleavage—glassy lustre.

Felspar, has a cleavage, but more opaque than silica.

Mica, is transparent and easily cleaved.

Granite, is of various shades and colors, and composed of quartz, (silica) felspar and mica. It forms the greater portion of the primary rocks. In the common granite, the felspar is lamellar or in plates, and the texture granular.

Porphyritic, is where crystals of felspar is imbedded in fine grained granite. It is red, green, brownish and sometimes gray.

Hornblende, is of various colors. That which forms a part of the basalts and syenites, is of a dark green or brownish color. It does not split in layers like mica when heated in the flame of a candle. Its color distinguishes it from quartz and felspar. It has no cleavage, and is composed of silica, lime, magnesia and protoxide of iron.

Augite, is nearly the same as hornblende, but is more compact. When found in the trap-rocks, it is of a dark green, approaching to black.

Gneiss, resembles granite; the mica is more abundant, and arranged in lines producing a lamellar or schistose appearance; the felspar also lamellar. It has a banded appearance on the face of fracture, the bands being black when the color of rock is dark gray. It breaks easily into slabs which are sometimes used for flagging.

Porphyritic gneiss, is where crystals of felspar appear in the rock, so as to give it a spotted appearance.

Protogine, is where talc takes the place of mica in gneiss.

Serpentine, is chiefly found with the older stratified rocks, but also found in the secondary and trap-rocks. It is mottled, of a massive green color, intermixed with black, and sometimes with red or brown; has a fine grained texture lighter than hornblende; may be cut with a knife, sometimes in a brittle, foliated mass. It is composed of about silica 44, magnesia 43, and water 13. Sometimes protoxide of iron, amounting to ten per cent., replaces the same amount of magnesia.

Syenite, resembles granite, excepting that hornblende, which takes the place of mica. It is not so cleavable as mica, and its laminae are more brittle. It is composed of felspar, quartz and hornblende. The felspar is lamellar and predominates. There are various kinds of syenites, as the

Porphyritic, where large crystals of felspar are imbedded in fine grained syenites.

Granitoid, is where small quantities of mica occur.

Talc, has a soft, greasy feeling, often in foliated plates, like mica, but the leaves or plates are not elastic. The color is usually pale green, sometimes greenish white, translucent, and in slaty masses. The last description from the township of Patton in Canada, and analyzed by Dr. Hunt, for Sir William Logan, Director of the Geological Survey of Canada, gives in the report for 1853 to 1856, the following:

Silica, 59.50; magnesia, 29.15; protoxide of iron, 4.5; oxyde of nickel, traces; alumina, 0.40; and loss by ignition, 4.40; total = 97.95.

A soft silvery white talcose schist from the same township, gave silica, 51.50; magnesia, 22.36; protoxide of iron, 7.88; oxyde of nickel, traces; lime, 1.25; alumina, 3.50; water, 3.60; total = 99.59.

Soapstone or *steatite*, is a granular, whitish or grayish talc.

Chlorite, is a dark or blackish green mineral, and is abundant in the altered silurian rocks, sometimes intermingled with grains of quartz and fespatic matters, forming chlorite sand, stones and schists or slates, which frequently contains epidote, magnetic and specular iron ores. Massive beds of chlorite or potstone, are met with, which, being free from harder minerals, may be sawed and wrought with great facility. A specimen from the above named township (Patton) was of a pale greenish, gray color, oily to the touch, and composed of lamellæ of chlorite in such a way as to give a schistose structure to the mass. Dr. Hunt, in the above report, gives its analysis: silica, 39.60; magnesia, 25.95; protoxide of iron, 14.49; alumina, 19.70; water, 11.30; total = 101.04.

Green sand, has a brighter color than chlorite, without any crystallization.

Limestones, are of various colors and hardness, from the friable chalk to the compact marble, and from being earthy and opaque, to the vitreous and transparent.

Carbonate of lime, when pure, is *calc spar*, and is composed of lime, 56.3; and carbonic acid, 43.7.

Impure carbonate of lime, is lime, carbonic acid, silica, alumina, iron, bitumen, etc.

Fontainbleau limestone, contains a large portion of sand.

Tufa, is lime deposited from lime water.

Stalactite, resembles long cones or icicles found in caverns.

Satin spar, is fibrous, and has a satin lustre.

Carbonate of magnesia or *dolomite*, is of a yellowish color, and contains lime, magnesia and carbonic acid, and makes good building and mortar stone.

Carbonate of magnesia, (pure) is composed of carbonic acid, 51.7, and magnesia, 48.3. Magnesian limestone, dolomite, (pure) is composed of carbonate of lime, 54.2, and carbonate of magnesia, 45.8. The following is the analysis from Sir W. Logan's report above quoted, of six specimens from different parts of Canada.

No. I. From Loughborough, is made up of large, cleavable grains, weathers reddish, with small disseminated particles, probably serpentine, and which, when the rock is dissolved in hydrochloric acid, remains undissolved, intermingled with quartz.

No. II. Is from a different place of said township. It is a coarse, crystalline limestone, but very coherent, snow-white, vitreous and translucent, in an unusual degree. It holds small grains disseminated, tremolite, quartz and sometimes rose-colored, bluish and greenish apatite and yellowish-brown mica, but all in small quantities.

No. III. From Sheffield, is nearly pure dolomite. It is pure, white in color, coarsely crystalline.

No. IV. From Madoc, is grayish-white, fine grained veins of quartz, which intersect the rock.

No. V. From Madoc, fine grained, grayish-white, silicious, magnesian limestone.

No. VI. From the village of Madoc, is a reddish, granular dolomite.

The following table shows the analysis of these specimens:

	I.	III.	IV.	V.	VI.
Specific gravity.....		7.863	2.849	2.757	2.834
Carbonate of Lime.....	55.79	52.57	46.47	51.90	57.87
“ Magnesia.....	37.11	45.97	40.17	11.89	34.06
“ Iron.....				4.71	
Peroxyde of Iron.....		0.21	1.21		1.32
Oxyde of Iron and Phosphates (traces).....					
Quartz and Mica.....		0.60			
Insoluble Quartz.....	7.10				
Quartz.....			12.16	82.00	7.10

MAGNESIAN MORTARS.

Limestones, containing 10 to 25 per cent. of clay, are more and more hydraulic. That which contains 33 per cent. of clay, hardens or sets immediately. Good cement mixed with two parts of clear sand and made into small balls as large as a hen's egg, should set in from one and a half to two hours. If the ball crumbles in water, too much quick-lime is present. Where the ground is wet, it is usually mixed—one part of sand to one of cement, but where the work is submerged in water, then the best cement is required and used in equal parts, and often more, as in the case of Roman cement.

By taking carbonate of lime and clay in the required proportions and calcining them, we have an artificial cement. Example: Let the carbonate of lime produce 45 per cent. of lime, then is it evident that by adding 15 lbs. of pure dry clay to every 100 lbs. of carbonate of lime, and laying the materials in alternate layers and calcining that, we produce a cement of the required strength. The limestones should be broken as small as possible; the whole, when calcined, to be ground together.

Cement used in Paris, is made by mixing fat lime and clay in proper proportions.

Artificial cement, is made in France, by mixing 4 parts of chalk with one of clay. The whole is ground into a pulp, and when nearly dry, it is made into bricks, which are dried in the air and then calcined in furnaces at a proper degree of heat. The temperature must not be too elevated. (See Regnault's Chemistry, Vol. I, p. 617.)

Plaster of Paris, is composed of lime, 26.5, sulphuric acid, 37.5, and water, 17. It is granular, sulphate of lime, slakes without swelling, sets hard in a short time, but being partially soluble in water, should be only used for outside or dry work.

Water lime, is composed of carbonate of lime, alumina, silica and oxyde of iron. It sets under water.

Water cements, differ from water lime in having more silica and alumina. It must be finely reduced. The English engineers use this and fine sharp sand in equal parts.

Building stones. Felspathic rocks, such as green stone, porphyry and syenite, in which the felspar is uniformly disseminated, are well adapted for structures requiring durability and strength. *Syenite*, in which potash abounds, is not fit for structures exposed to the weather. *Granite*, in which quartz is in excess, is brittle and hard, and difficult to work. An excess of mica makes it friable. The best granite is that in which all its constituents are uniformly disseminated, and is free from oxides of iron. Gneiss makes good building and flag stones. *Limestones*, should be free from clay and oxides of iron, and have a fine, granular appearance.

Sand, is quartz, frequently mixed with felspar.

Coarse sand, is that whose grains are from one-eighth to one-sixteenth of an inch in diameter.

Fine sand, is where the diameter of the grains are from one-sixteenth to one twenty-fourth of an inch.

Mixed sand, is where the fine and coarse are together.

Pit sand, is more angular than sea or river sand, and is therefore preferred by many builders in France and America, for making mortar; but in England and Ireland, river sand, when it can be procured, is generally used. Pit sand should be so well washed as not to soil the fingers. By these means, any clay or dirt present in it is removed.

Sand for casting, must be free from lime, be of a fine, siliceous quality, and contain a little clay to enable the mould to keep its form.

Sand for polishing, has about 80 per cent. of silica; is white or grayish, and has a hard feeling.

Sand for glass, must be pure silica, free from iron. Its purity is known by its white color or the clearness of the grains, when viewed through a magnifying glass.

Fuller's earth, has a soapy feeling, and is white, greenish-white or grayish. It crumbles in water, and does not become *plastic*. Its composition is, silica, 44; alumina, 23; lime, 4; magnesia, 2; protoxide of iron, 2; specific gravity, about two and one-half.

Clay, is plastic earth, and generally composed of one part of alumina and two parts of quartz or silica.

Clay for bricks, should be free or nearly so from lime, slightly plastic, and when moulded and spread out, to have an even appearance, smooth and free from pebbles. Clay free from iron, burns white, but that which contains iron, has a reddish color, the protoxide of iron in the clay becoming peroxidized by burning.

Pipe and potters' clay, has no iron, and therefore burns white.

Fire brick clay, should contain no iron, lime or magnesia.

Marl, is an unctuous, clayey, chalky or sandy earth, of calcareous nature, containing clay or sand and lime, in variable proportions.

Clay marl, resembles ordinary soil, but is more unctuous. It contains potash, and is therefore the best kind for agricultural purposes.

Chalk marl, is of a dull, white or yellowish color, and resembles impure chalk; is found in powder or friable masses.

Shelly marl, consists of the remains of infusorial animals, mixed with the broken shells of small fish. It resembles Fuller's earth, usually of a bluish or whitish color, feels soft, and readily crumbles under the fingers. It is found in the bottom of morasses, drained ponds, etc.

Slaty or stony marl, is generally red or brown, owing to the oxyde of iron it contains; some have a gravelly appearance, but generally resembles hard clay.

TABLE SHOWING THE COMPOSITION OF GRASSES AND TREES.

NAME OF SUBJECT.	EARTHS.			OXIDES.		ALKALIES.		ACIDS.		PHOSPHATES.		CHLORIDES.		Miscellaneous.
	Silica.	Lime.	Magnesia.	Iron.	Potash.	Soda.	Sulphuric.	Carbonic.	Phosphoric.	Iron.	Magnesia.	Chlorine.	Chloride of Sodium.	
Lucerne.....	8.46	50.57	3.64	0.63	14.03	6.44	4.23		13.63				8.8	
Upright brome grass.....	38.48	10.38	4.99	0.26	20.33		6.46	0.55	7.53				8.11	Ash, 6.11.
Crested dogstail.....	40.11	10.16	2.43	0.18	24.99		3.20		7.24					" 6.88.
Orchard grass, seed ripe.....	32.18	8.14	3.47	0.23	32.06		3.96	2.88	6.41					" 6.51.
Meadow soft grass.....	28.31	8.31	3.41	0.31	34.83		4.41	1.82	8.02					" 6.37.
Meadow barley grass.....	56.28	5.04	2.42	0.66	20.26	3.40	4.29	4.02	6.04					" 5.67.
Timothy grass.....	31.09	14.94	5.30	0.27	24.25		4.86		11.29					" 5.29.
Yellow oat grass.....	35.20	7.98	3.07	2.40	36.06	0.78	4		9.31					" 5.28.
June grass.....	32.93	5.63	2.71	0.28	31.17		4.26	0.40	10.02					" 5.94.
Hard Fescue.....	28.53	10.31	2.83	0.78	31.84		3.41	1.88	12.07					" 5.42.
Downy oat grass.....	36.28	4.72	3.17	0.72	31.21		3.37		10.82					" 5.22.
White Clover.....	8.68	26.41	8.15	1.96	14.83	3.72	7.21	18.08	11.53					
Red Clover.....	7	33	8	0.4	86		3	15.20	8.02					Ash, 8.
Sanfoin, in seed.....	8.49	29.67	4.59	0.58	20.61	1.25	2.33		7.97					" 6.50.
Italian rye grass, in flower.....	59.18	9.95	2.28	0.78	12.45	3.98	2.82		6.34					" 6.97.
Meadow hay.....	38	23	7	1	19		2		6					" 6.
Apple tree, root.....	1.46	12.13	0.16		15.07	21.99	1.84		0.42	18.96	31.85		0.11	Org. mat. 1.20
" bark.....	1.26	1.86	2.56		0.44	1.53	38.39	49.56		3.60				do & coal 4.61
" outside wood.....	0.45	15.56	3.52		3.29	3.33	12.21	15.79		24.4				Do 3.55
" heart wood.....	0.20	2.06	2.98		2.75	1.62	22.17	38.98						Do 3.61
Cherry tree, wood.....	2.06	28.69	9.19	0.07	20.78	8.40	3.29		7.73					
" bark.....	19.98	41.95	5.10	0.20	7.46	14.53	0.80		3.26					
Peach tree, bark of trunk.....	4.15	42.17	2.16		0.40	14.53	4.19							Org. mat. 3.30
" wood of trunk.....	1.85	28.25	6.40		7.11	11.15	1.51							Do 5.20
Pear tree, bark.....	0.40	30.36	9.40		6.20		1.80	87.29						Loss, 3.81
" aspswood.....	0.30	12.64	3.00	0.31	22.25	1.84	0.50	37.80						

NAMES.	Silica.	Alumina.	Lime.	Magnesia.	Oxide of Iron.	Manganese.	Potash.	Soda.
Plum tree, outside wood.	.45	...	15.56	8.52	...	...	3.29	3.33
“ heart “	.20	...	2.66	2.93	...	...	2.75	1.66
“ root “	1.46	...	11.64	.16	...	...	15.07	22.00
Chestnut, bark	1.20	...	51.60	.60	...	...	1.36	.33
“ outside wood..	1.43	...	40.76	5.77	...	...	4.56	1.41
“ inside “	1.78	...	38.20	.51	...	...	2.73	1.98
Beech, red, bark.....	3.30	2.28	52.29	.82	...	3.85	.18	...
“ outside wood.....	1.45	...	31.56	5.44	...	...	12.13	15.58
“ heart “	1.60	...	31.82	1.44	...	...	4.04	25.53
Butternut, bark.....	.30	...	37.68	10.08	...	...	1.00	11.27
“ outside wood.	4.80	...	38.98	3.52	...	...	4.42	5.61
“ heart “	5.40	...	43.02	4	...	...	1.00	11.82
Basswood, bark	4.60	...	41.92	2.24	...	...	1.26	12.77
“ outside wood..	2.10	...	38.86	7.86	...	...	10.12	2.88
“ heart “	1.40	...	45.24	7.44	...	...	4.05	10.41
Elm, (white) bark	1.75	...	27.46	18.10	...	...	3.79	1.65
“ outside wood	...	...	...	...	...	...	...	...
Maple, bark.....	.15	...	49.33	3.64	...	...	.88	7.75
“ outside wood.....	.50	...	31.86	8.40	...	...	.87	...
“ heart “	.55	...	43.14	7.24	...	...	4.21	...
Oak, (white) sapwood....	1.01	...	30.35	.86	...	...	13.41	...
“ heart wood	1.18	...	43.21	.25	...	...	9.68	...
“ twigs “	1.15	...	34.10	.50	...	...	9.74	6.89
“ (white swamp) bark	2.00	...	52.26	.25	...	...	.46	trace
“ outside wood	1.50	...	30.22	.50	...	...	20.49	3.15
“ heart “	.50	...	35.57	.51	...	...	14.79	3.69
Hickory, outside wood ...	4.48	...	...	6.20	...	...	7.40	.08
“ inside “ ...	6.15	...	...	8.60	...	...	20.19	.09
“ heart “ ...	1.30	...	...	4	...	...	12.21	.06
Pine, pitch.....	7.50	...	13.60	4.35	...	...	14.10	20.75
“ scotch fir.....	6.59	...	23.18	5.02	...	...	2.20	2.22
Rose bush, bark.....	3.30	...	22.56	2.86	...	...	5.12	3.62
Birch, soluble compound	1.00	...	...	...	...	...	...	...
“ insoluble “	5.50	...	52.2	...	.5	3.5	...	...
Lime tree, bark.....	2.27	...	60.81	...	1.24	...	16.14	4.53
“ wood.....	5.26	...	29.93	...	7.97	...	35.80	5.23
Mulberry, (white) soluble	...	...	...	...	...	...	52	11.5
“ insoluble	2.9	...	4.6	...	.5	1.3	...	...
“ chinese, soluble	1.0	...	...	...	...	...	...	68
“ insoluble	13.1	...	7.2	...	...	...	...	...
<i>Datura stramonium</i>	5.21	...	4.11	17.56	pr 3.94	...	20.22	14.24
Sweet Flag.....	2.39	...	7.70	...	pr 1.91	...	6.90	32.93
Common Chamomile.....	6.80	...	3.67	...	pr 3.23	...	9.66	30.58
Cockle.....	2.39	...	6.14	...	pr 1.21	...	13.20	22.86
Foxglove.....	12.78	...	6.53	8.70	8.19	...	10.89	43.53
Hemlock	2.62	...	3.39	9.64	2.40	...	12.80	21.69
Blue Bottle.....	3.29	...	...	4.56	1.61	...	7.32	36.54
Strawberries	12.05	...	14.21	trace	11.12	...	21.07	27.01
Poppy.....	1.41	...	5.06	...	1.21	...	6.85	33.11

AND WEEDS, ETC.

Phosphate of Lime.	Phosphate of Iron.	Phosphate of Magnesia.	Sulphuric Acid.	Carbonic Acid.	Phosphoric Acid.	Chlorine.	Chloride of Sodium.	MISCELLANEOUS
...	...	...	12.21	15.79	...	...	.83	Org. mat. 3.20, coal, .35
trace	trace	trace	22.17	38.98	...	...	.51	" 3.60.
"	"	"	1.84	...	...	...	.11	" 1.20.
2.90	.20	"	.81	39.90	...	...	...	" 5.
17.44	1.80	"	.50	28.84	...	...	...	" 1.74.
8.60	.80	"	...	29.52	...	...	...	" 3.20.
phos-	1.96	phates	...	40.41	5.62	...	...	" 1.50, coal 1.50
17.23	.85	.93	.47	24.89	...	.05	...	" 1.86.
22.04	.40	.02	.62	24.59	...	.24	...	" 2.80.
2.25	.30	.15	.74	32.12	...	...	.15	" 2.80.
2.20	3.40	.06	18.78	20.02	...	...	.16	" 3.40.
.59	8.41	.28	21.48	4.48	...	...	.18	" 3.20.
8.50	.20	.80	.72	25.88	8.50	...	.24	" 1.70.
17.95	1.20	2.60	.88	16.64	17.95	...	.50	" 2.58.
8.96	1.30	.04	.27	17.96	8.96	...	.52	" 2.
...	...	...	.14	39.44	...	...	...	" 2.
...	...	...	12.02	...	...	...	...	" ...
3.13	.32	.02	1.50	37.12	...	...	.08	" 1.50.
5.70	.78	1.80	1.17	87.25	...	...	.08	" 2.40.
5.09	1.34	.22	1.08	33.33	...	...	2.78	" 1.98.
Phosphates.	32.25	...	4.24	8.95	...	...	.89	" 5.70.
	13.30	...	.47	19.29	...	...	.16	" 7.10.
	28.60	...	.25	17.55	...	...	.08	" 5.90.
	...	...	.80	40.84	...	...	...	" 2.18.
	...	...	...	32.92	...	...	...	" ...
	...	...	...	34.41	...	...	...	" 2.70.
	14.44	...	.89	29.57	...	.10	...	...
	11.45	...	4.64	21.41	...	.09	...	...
	6.34	...	5.26	33.63	...	.07	...	...
	11.10	.90	8.45	17.50	...	2.30	...	...
...	17.03	2.75	2.23	36.48	...	...	...	...
...	15.30	...	5.00	28.70	...	...	8.20	Water, 4.
...	...	...	2.00	17.00	...	...	...	...
...	...	...	...	31	4	...	...	...
...	...	...	.75	...	4.02	...	2.21	...
...	...	...	5.30	...	4.85	...	1.49	...
...	...	...	8.00	2.3	...	...	...	Hydrochloric acid, 4.
...	...	...	...	...	...	...	...	" " 2.04.
...	...	...	8.00	22.6	...	...	...	...
...	...	...	...	18.7	5.4	...	...	...
...	...	...	...	...	34.72	...	...	Iodide of Sodium, 34.
...	...	...	5.06	...	11.48	...	2.84	Chlor. of Potass'm 14.66.
...	...	...	4.60	...	16.01	...	...	" " 7.15.
...	...	...	2.89	...	29.27	...	...	" " 7.55.
...	...	...	3.91	...	15.65	...	9.08	...
...	...	...	8.48	...	24.96	...	16.61	...
...	...	...	2.69	...	15.49	...	...	" " 11.88.
...	...	...	8.15	6	8.59	...	2.78	" " ...
...	...	...	2.26	...	28.87	...	...	" " 3.40.

NAMES.	Silica.	Lime.	Magnesia.	Oxide of Iron.	Potash.	Soda.
Barley, grain, mean of 10	28.49	2.58	8.55	1.43	19.77	3.93
“ straw, mean of 8	54.66	8.08	4.13	1.33	18.40	.68
“ grain, at Cleves	21.99	3.86	10.05	1.93	8.91	16.79
“ grain, at Leipsic	29.10	1.67	6.91	2.10	20.91	...
Buckwheat, grain	.69	6.66	10.38	1.06	8.74	30.10
“ grain	7.06	.14	2.66	...	28.33	2.04
“ straw	4.37	.22	40.37	1	10.27	1.94
Maize or Indian Corn, grain	1.44	1.44	16.22	.80	32.48	...
“ straw, mean of 2	26.97	7.97	6.64	.81	9.62	26.30
Millet, grain, (Giessen)	59.63	.86	7.66	.68	9.58	1.31
Oats, grain, mean of 7	47.08	3.92	7.70	.64	16.76	2.49
“ straw, mean of 2	48.42	8.07	8.78	1.83	19.14	9.69
“ potato, grain,	50.08	1.31	8.25	.27	19.70	1.35
Rice, grain	3.85	1.27	11.69	.45	18.48	10.67
“ straw	74.09	.78	4.49	.67	10.27	3.82
Rye, grain, by Way and Ogden	9.22	2.61	12.81	1.04	33.33	.39
“ grain, mean of 3	3.86	4.19	11.17	1.25	26	7.91
“ grain, by Liebig	.69	9.06	2.41	.40	11.43	18.89
“ straw, “	64.50	6	10	2	17.19	...
Wheat, grain, mean of 32	3.85	3.40	12.30	.79	29.97	3.90
“ straw, mean of 10	67.88	6.23	2.74	.74	12.14	.60
Flax, whole plant in Ireland	21.35	12.83	7.79	6.08	9.78	9.82
“ best in Belgium	2.68	18.52	3.93	1.10	22.30	14.11
Hemp, whole plant, mean of 4	8.20	42.91	5.47	2.71	9.93	.50
Linseed	.92	25.98	.22	3.67	25.85	.71
Rape, seed	1.11	12.91	11.39	2.56	25.18	...
“ straw	.89	20.95	.62	...	8.13	19.83
Beet, Mangel Wurzel, (yellow)	2.22	1.78	1.78	3	23.54	19.08
“ “ long red	1.40	1.90	1.79	.52	21.68	3.13
“ mean of 4	4.44	3.65	2.97	1.24	30.80	12.19
“ long blood root	1.85	1.50	1.15	...	13.10	53.65
“ tops	1.99	8.65	8.66	.96	21.36	7.01
Carrot, (white Belgian root,)	1.19	8.83	3.96	1.10	32.44	13.52
“ tops	4.66	32.64	2.92	2.40	7.12	10.97
“ fresh root, (New York report.)	.65	3.65	1.60	...	8.50	40.25
Artichoke, Jerusalem	15.97	2.82	2.81	6.39	54.67	...
Cauliflower, heart	1.92	2.96	2.38	1.69	34.39	14.77
Parsnip	4.10	11.43	9.94	...	36.12	3.12
Potato, mean of	4.23	2.07	5.28	.52	57.75	1.86
“ tops	3.85	16.96	7.09	1.05	28.02	16.24
Tomato	.01	trace	0.10	...	.07	.09
Turnip, white globe	1	8.69	4.56	1.44	42.33	2.66
“ swede	.28	10.67	4.65	.38	47.46	3.93
“ mean of 10	3.43	11.14	3.61	1.09	36.98	6.76
“ tops	.86	23.27	3.09	.86	28.65	5.41
Beans, mean of 6	2.55	19.30	5.91	2	28.87	6.64
“ straw	7.05	19.99	6.69	.22	58.08	1.60
Peas, mean of 4	.52	5.36	8.54	.98	36.30	7.11
“ straw	20.03	54.91	6.88	.40	4.73	...
Lentils	1.07	5.07	1.98	1.61	27.84	6.65
Vetch or tare	2.01	4.79	8.49	.75	30.57	9.56
“ “ straw	8.66	38.33	6.36	.17	35.49	1.02

VEGETABLE AND LEGUMINOUS PLANTS.

Sulphuric Acid.	Carbonic Acid.	Phosphoric Acid.	Chlorine.	Chloride of Sodium.	MISCELLANEOUS.
1.08	...	35.20	...	.47	
2.13	...	8.26	...	6.95	
.26	...	40.63	...	...	
...	...	33.48	...	...	
2.16	...	50.07	...	...	
7.30	...	57.60	.20	...	
6.78	...	9	2.99	...	Oxyde Mang. and Alumina, .8.
2.77	...	44.87	...	...	
1.19	...	17.08	...	3.42	
.35	...	18.19	...	1.43	
1.29	...	18.19	...	.20	Chloride potassium, .14.
3.26	...	2.56	...	...	
.10	...	18.87	...	.07	
...	...	53.86	...	...	
3.56	...	1.09	...	...	
.17	...	39.92	...	...	
.71	...	46.34	...	...	
.51	...	51.81	...	...	
.88	...	3.82	...	.57	Chloride potassium, .26.
.33	...	46	...	.09	
3.88	...	5.43	...	.22	
2.65	...	10.84	...	6	
6.88	...	8.81	4.58	...	
1.28	...	5.26	...	1.41	
.91	...	40.11	...	1.65	
.58	2.20	45.96	...	...	
7.60	16.31	4.76	...	...	
3.68	18.14	4.49	...	24.55	
3.14	...	1.65	...	49.51	
3.03	...	4.19	...	24.55	
1.65	16.27	9.85	.81	...	Phosphate of Iron, 1.15.
6.80	...	5.15	...	33.96	
6.55	17.30	8.55	...	6.50	
6.20	17.82	1.67	...	13.67	
4.30	28.2	10.55	...	...	" " .70.
2.70	...	18.27	...	3.3	Carbonic acid deducted.
11.16	...	27.85	...	2.86	
6.50	...	18.66	...	5.54	Phosphate of Iron, 3.71.
13.64	deduct	12.57	...	7.10	
6.88	...	7.62	12.33	...	
.01	.04	.08	.01	...	
12.6	...	8.61	...	...	
12.16	...	13.07	...	...	
12.43	...	9.74	...	7.85	Chloride potassium, .59.
12.52	...	9.29	16.05	...	
1.91	...	21.66	...	1.35	" " .36.
1.09	...	7.24	...	4.26	
4.39	...	33.52	...	2.16	
6.77	...	4.33	...	...	
...	...	29.07	...	6.13	
4.10	...	33.06	...	2	
2.39	...	5.43	...	2.75	

ANALYSIS OF THE ASHES OF MISCELLANEOUS ARTICLES.

NAME OF SUBJECT.	EARTHS.			OXIDE.		ALKALIES.		ACIDS.		CHLORIDES.		MISCELLANEOUS.
	Silica.	Line.	Magnesia.	Iron.	Potash.	Soda.	Sulphuric.	Carbonic.	Phosphoric.	Chloride.	Sodium.	
Onion, bulb.....	8.04	12.66	2.70	*12.29	32.35	8.04	8.84	...	15.09	...	4.49	Chloride of Iron & signifies Phosphate.
" stalk.....	10.77	25.10	trace	*10.61	13.98	14.48	10.50	...	...	...	trace	
Coffee, bean.....	2.95	3.58	9.01	*5.56	42.11	11.07	...	...	11.24	...	1.67	
" Cabbage, leaves.....	.75	20.97	5.94	.60	11.70	20.42	21.48	...	12.87	5.77	...	U. S. Patent Office.
" cow.....	1.66	15.01	2.89	.70	40.86	2.43	7.27	16.68	12.52	...	...	Cameron.
" stalk.....	1.04	10.61	8.85	.41	40.93	4.05	11.11	6.83	19.57	...	2.08	Chloride potassium, 6.22.
Brocoli, heart.....	.69	4.70	3.93	*2.12	47.16	...	10.85	...	24.53	...	trace	
" leaves.....	1.83	26.44	3.43	6.21	22.10	7.55	16.10	...	16.62	...	...	
Cucumber.....	7.12	6.31	4.26	*2.06	47.42	4.60	...	...	14.97	...	9.06	"
Radish.....	8.17	8.78	3.53	*2.19	21.16	...	7.71	...	40.09	...	7.07	"
Tobacco, leaves, mean of 10.....	...	41.80	12.18	4.41	17.42	.25	4.06	...	2.28	...	5.11	"
Hops.....	21.50	15.98	5.77	5.12	25.18	...	6.41	...	12.13	...	7.24	"
" whole plant.....	17.88	14.15	5.34	2.71	19.41	.70	8.28	11.01	14.64	...	8	"
Vine, grape, mean of 8.....	.88	36.26	6.91	2.12	27.88	8.96	2.70	...	13.18	...	1.39	"
Sugar-cane, stalk.....	17.64	2.34	3.93	...	82.93	...	7.97	...	7.87	...	17.12	"
" whole plant, mean of 2.....	45.68	7.61	9.87	...	16.12	.77	7.76	...	6.34	...	4.40	"
Sorgho, Bagasse.....	14.40	11.80	9.80	trace	8.10	9.60	28.70	trace	13.42	8.70	...	Jackson.
Orange, whole fruit.....	5.24	22.99	6.55	1.74	38.72	7.64	2.95	...	14.17	...	...	
Cherry, whole fruit.....	9.04	7.47	5.46	3.74	51.85	1.12	5.09	...	14.21	...	2.02	
Apple.....	4.82	4.08	8.75	2.65	35.68	26.09	6.09	...	12.84	...	...	
Gooseberry.....	2.58	12.20	5.85	2.65	38.65	9.27	5.89	...	15.58	...	1.28	
Pear.....	1.40	7.98	5.22	*1.96	54.69	8.52	5.69	...	14.28	...	...	
Chestnut.....	2.82	7.84	7.84	1.95	39.86	19.18	3.88	...	7.83	...	4.82	
Celery.....	8.86	13.11	13.11	*2.06	43.17	...	5.58	...	11.58	1.58	...	Oxide manganese, 1.92

810r. PERCENTAGE VALUE OF MANURES.

SUBSTANCES.	Water.	Nitrogen dry state	Nitrogen natural state	ph's acid dry state	MISCELLANEOUS.
Farm yard manure.....	68.2	1.96	.61	1.08	Bechelburn.
" "	70.5	2.45	.72	2.00	Grignon, France.
Wheat straw	12.8	.41	.86	.22	
Rye straw.....	12.4	.35	.80		Alsace.
Oat straw.....	21.0	.36	.28	.21	"
Barley straw.....	11.0	.26	.28	.20	"
Pea straw	8.5	1.95	1.79		"
Buckwheat straw.....	11.6	.54	.48		"
Leaves of rape.....	12.8	.86	.75	.30	
" potato.....	76.0	2.80	.55		
" carrot.....	70.9	2.94	.85		Recently collected.
" oak.....	25.0	1.57	1.18		"
" beech.....	39.3	1.91	1.18		
Saw dust fir.....	24.0	.31	.23	.03	"
" oak.....	26.0	.72	.54	.04	
Malt dust.....	6.0	4.90	4.51		
Apple refuse.....	6.4	.63	.59		
Hop ".....	73.0	2.23	.56		Air dried.
Beet root refuse.....	70.0	1.26	.38	.40	
Linseed cake.....	13.4	6.	5.20	3.88	
Rape cake	10.5	5.50	4.92	4.84	
Hempseed cake.....	5.0	4.78	4.21	1.08	
Cotton seed cake.....	11.0	4.52	4.02		
Cow dung.....	85.9	2.30	.32	.74	Solid excrements.
" urine.....	88.8	3.80	.44		"
" excrements.....	84.3	2.59	.41	.55	Solid and liquid.
Horse excrements.....	75.3	2.21	.55	1.22	"
" urine.....	85.0	14.47	2.04		"
" excrements.....	75.4	3.02	.74	1.12	"
Pigs' urine.....	97.9	11.	.23	2.09	"
Pigs' excrements.....	91.4	5.17	.54	3.65	
Sheeps' excrements.....	57.6	1.70	.72	1.52	
" urine.....	86.5	9.70	1.31	.08	
" dung.....	67.1	2.79	.91	1.82	"
Pigeons' dung.....	61.8	9.12	3.48	5.88	Fresh excrements.
Human urine.....	93.8	21.64	1.46	3.88	
" excrements.....	91.0	14.67	1.33	2.85	Solid and liquid.
Flemish manure.....			.20		Liquid manure.
Poudrette from Belloni	12.5	4.40	3.85		Sauburan.
Do. from Berry in 1847	13.6	2.29	1.98	2.55	"
Do. from Montfaucon..	41.4	2.67	1.56	1.08	"
Do. in 1847.....	28.0	2.47	1.78	4.80	"
Blood, liquid.....	81.0	15.58	2.95	1.63	Slaughter house.
" dry.....	21.4	15.50	12.18	1.68	Commercial.
" coag. & pressed	73.5	17.	4.51		From the press.
Blood, steamed.....		5.59			Wahl's, Chicago.
Bones boiled.....	7.5	7.58	7.02	24.	
" unboiled.....	8.	8.89	6.22	22.20	
" dust.....		7.92		24.	
Glue refuse.....	37.8	8.27	2.13		
Sugar refineries.....		2.44		26.	
Ox hairs	8.9	15.12	13.78		
Woolen rags.....	11.3	20.26	17.98		
Guano, Peruvian.....	25.6	6.31	4.71	18.98	
" African.....	25.	8.25	6.19	17.	
Soot of wood.....	5.6	1.31	1.15	1.	
" coal.....	15.6	1.59	1.35		
Oyster shells.....	17.9	0.40	.32	.65	

SEWAGE MANURE.

310j. The value of this manure is now fully established. Dr. Cameron, Professor to the Dublin Chemical Society, has recently shown that "100 tons of the sewage water of Dublin contain—

Nitrogen,	16 lbs.,	worth 10s. 8d.
Phosphoric Acid,	4.2	" 1s. 4½d.
Salts of Potash,	5.1	" 7½d.
Salts of Soda,	14.2	" 2½d.
Organic matter,	75	" 4d.

Taking the population of Dublin at 300,000, the value of the sewage is worth more than £100,000, or two-thirds of the local taxation of the city."

He calculates the value of the night soil at £3000, and the urine at £85,000, showing one to be thirty times as valuable as the other.

Those who have seen the river Thames or the Chicago river made the receptacle of city sewage, will admit that God never intended that liquid manure should pass into these streams causing disease and death, but that they should be made available in fertilizing the neighboring fields, as in Edinburgh and various other places.

We recommended a plan of intercepting sewers for Chicago in 1854, by which the sewage could be collected at certain places, and from thence wasted into Lake Michigan far from the city, or used for irrigating the adjacent level prairies. The plan was rejected, but the consequence has been that an Act passed the Legislature of Illinois in 1865, creating a commission for cleansing the Chicago river, at an expense of two MILLIONS OF DOLLARS. The commissioners have now (30th June, 1865,) commenced their preparatory survey. In Chicago the people are obliged to connect their water-closets with the main sewers, thereby making the sewers gas generators on a large scale. Public water-closets are built at the crossings of some of the bridges, and private ones without traps or syphons are built under the sidewalks. This system of sewerage begins to show its bad effects, and will have to be abandoned at some future day.

To any person who has spent one hour in a chemical laboratory, it will appear that noxious gases will soon saturate any amount of water that can be held in a trap or syphon, and that no contrivance can be adopted to exclude permanently the poisonous effluvia of sulphide of ammonium and sulphuretted hydrogen.

It will cost London thirty millions of dollars to build the intercepting sewers commenced in 1858. Paris commenced a similar work in 1857, and Dublin is now about to do the same. About April, 1865, an Act passed the English House of Lords for the utilization of town sewage, which was supported by the first vote of the PRINCE OF WALES. The great LIEBIG has commenced operation on the London sewage. He has it free of charge for ten years; so that in a few years the value of sewage will be as well known to the Americans and Europeans as it is now to the Chinese. Then there will not be a scientific engineer who will advocate the converting of currentless streams and neighboring waters into cess-pools. The sanitary and agricultural conditions of the world will forbid it. (*See also sections on Drainage and Irrigation.*)

DESCRIPTION OF MINERALS.

310K. ANTIMONY. *Stibnite*, or gray sulphuret of antimony. Comp. Sb73, S27. Found chiefly in granite, gneiss and mica, with galena, blende, iron, copper, silver, zinc and arsenic. Found columnar, massive, granular, and in delicate threads. Fusible. Gravity, 4.5. Lustre, shining. Fracture, perfect and brittle. Color, lead to steel gray; tarnishes when exposed.

White Antimony. Contains antimony, 84. Found in rectangular crystals, whose color is white, grayish and reddish, of a pearly lustre. H—2.5. Gravity, 5 to 6.

Sulphuret of Antimony and Lead. Found rhombic, fibrous and columnar. Color, lead to steel gray. H = 2 to 4. Specific gravity, 5 to 6.

ARSENIC, White. Sometimes found in primary rocks with Co. Cu. Ag. and Pb. Color, tin white. Is soluble. G., 8.7. Fracture, conchoidal. Lustre, vitreous.

Native Arsenic. Found in Hungary, Bohemia, and in New Hampshire with lead and silver. Color, tin white to dark gray. A = 8.5. Gravity, 5.7. F = imperfect.

Orpiment or Yellow Sulphuret of Antimony. Found in Europe, Asia and New York. Foliated masses and prismatic crystals. Color, fine yellow. H = 1.5 to 2. Gr., 8 to 8.5. F = perfect. Lustre, pearly.

Realger or Red Sulphuret of. Found in Europe, with Cu. and Pb. Color, red to orange. H = 1.5 to 2. Gr., 3 to 4. Lustre, resinous. F = imperfect. Massive and acicular.

BISMUTH. Native. Found in quartz, gneiss, mica, with Co. As., Ag. and Fe. Color, silver white. Found amorphous, crystallized, lamellar. H = 2 to 2.5. Gr. = 9. F = perfect. Lustre. Metallic.

Sulphuret of Bismuth. Comp., Bi. 81, S19. Found as above. Massive acicular crystals. H = 2.3. Gr., 6.6. Color, lead gray.

COBALT. Smaltine. Found in primary rocks, with As. Ag. and Fe. Massive, cubes and octohedrons. H = 5. Gr., 6 to 7. Color, tin white to steel gray. L = metallic. Fracture uneven.

Arsenical Cobalt. Found, as in the latter, massive, staelectical and denticral. Comp., Co. + As. + S. Color, tinge of copper red. Gr., 7.3. F = brittle.

Bloom or Peach Cobalt. Found in oblique crystals. Foliated like mica. Color, red, gray, greenish. H = 1.5 to 2. Gr., 3. Lustre, pearly. Fracture, like mica.

COPPER. Native. Nearly pure. Found in veins in primary rocks, and as high as the new red sandstone, in masses or plates. Aborescent, filiform. Color, copper red. H = 2.5 to 3. Gr., 8.6.

Sulphuret of. Comp., Cu. 78.5, S22 + Fe. 50. Found in great rocks, especially the primary and secondary ones. In double, six-sided pyramids, lamellar, tissular, long tabular, six-sided prisms. Color, blackish steel gray. Gr., 5.5. Fracture, brittle and brilliant.

Sulphuret of Copper and Iron. (Copper pyrites.) Comp., Cu. 36, S32, Fe. 32. Found in veins in granite and allied rocks, graywacks, and with iron pyrites, carbonates of Cu. blende, galena. Color, brass yellow when hammered. H = 3 to 4. Gr., 4. Found in various shapes. Tetrahedral, octohedral, massive, like native and iron pyrites.

Gray Sulphuret of Cu. and Iron. Comp., Cu. 52., Fe. 23. The same location and associates as the last. It is not magnetic like oxide of iron, nor so hard as arsenate of iron. Color, steel gray to black. Lustre, metallic. F = brittle. Found amorphous, disseminated, crystallized in small tetrahedral crystals.

Copper Pyrites, most prevalent. Comp., Cu. 76.5, S22, Fe. .5. Found similar to sulphuret of copper. Color, brass yellow. Found in small, imperfect crystals in concretion and crystallized lamellar. F = uneven. Lustre, metallic. Gr., 4.8.

Red Oxide of Copper. Contains 88 to 91 of copper. Found with other copper ores. It is fusible and effervesces with nitric acid, but not with hydrochloric acid. Color, red. F = generally uneven. H = soft. Found amorphous, crystallized, in cubes and octohedrons.

Blue Carbonate of Cu. Comp., Cu. 70, CO₂ 24, H₂O₆. Found in primary and secondary rocks. Is infusible without a flux, and gives a green bead with borax in the blow pipe flame. It is massive, incrusting and stalactical. Color, blue. F = imperfectly foliated.

Green Carbonate of Copper. Found with other copper ores, in incrustations and other forms. Color, light green. L = adamantine. H = 3 to 4. Gr., 4.

NICKEL. *Arsenical.* Comp., As. 54, Ni. 4.4. Found in secondary rocks, as gneiss, with cobalt, arsenic, Fe., sulphur and lead, and is massive, reticulated, botryoidal. Gives out garlic odor when heated. Color, copper red, which tarnishes in air. H = 5. Gr., 7 to 8. L = metallic.

Nickel Glance. Found with arsenic and sulphur, massive and in cubes. Comp., Ni. 28 to 38. Color, silver white to steel gray. H = 5. Gr., 6.

White Nickel. Comp., Ni. 20 to 28, As. 70 to 78. Color, tin white, found as cubic crystals.

Placodine. Ni. 57. Color, bronze yellow. Found tabular, oblique and in rhombic prisms. H = 5 to 6. Gr., 8.

Antimonial Nickel. Ni. 29. Found in hexagonal crystals. Color, pale copper red, inclined to violet.

Nickel Pyrites. Contain Ni. 64. Color, brass yellow to light bronze. Found capillary and in rhombohedral crystals.

Green Nickel. Contain 86 per cent. of oxide of nickel. Found with copper and other ores of nickel. Color, apple green.

ZINC. *Blende. Mock-lead. Block Jack.* Found in veins in primary and secondary rocks, with Fe. Pb. and Cu. Comp., zinc 67, Pb. 33. Found massive, lamellar, granular and crystallized. It decrepitates if heated, and is infusible. Color, yellow, brown or black. Lustre, shining and adamantine. F = brittle and foliated. Gr., 3 to 4.

Carbonate of Zinc. (Calamine.) Comp., zinc, 64.5, carbonic acid, 35.5. Found in beds or nests in secondary limestones, and in veins, with oxides of iron and sometimes lead. Crystallized, compact, amorphous, cupreous and pseudomorphous. Color, gray, greenish, brown, yellow and whitish. L = vitreous and pearly. F., brittle. Gr., 4 to 4.5.

Red Oxide of Zinc. Comp., zinc 94, protoxide of manganese 6. Found in iron mines and limestones. Massive and disseminated. Cleavage like mica. Color, deep or light red with a streak of orange yellow. Lustre, subadamantine and brilliant.

Sulphate of Zinc. Found in rhombic prisms. Color, white. L = vitreous. F., perfect. Gr., 20.4.

MANGANESE. *Binoxide of.* Comp., $\text{MnO}_2 = \text{Mn } 64 + \text{O } 86$. Found in veins and masses in primary rocks, with iron. Forms a purple glass with borax in the blow pipe flame. Color, dark steel gray, with a black streak. L = metallic. F., conchoidal and earthy. H = 2 to 2.5. Gr., 4 to 5. Found massive, and in fibrous concretions. Crystallized. Infusible alone.

Phosphate of Manganese. (Triplite.) Protoxide Mn. 88, protoxide of Fe. 82, and phosphoric acid 88. Gives a violet gloss with borax. Color, yellowish, streak of gray or black. L = resinous and opaque. H 5 to 5.5. Gr., 8 to 4.

Bog Ore of Mn., or Wad. Found in low places, formed from minerals, containing manganese. Comp., Mn. 80 to 70, protoxide of iron 20 to 25. Color, brownish black. Lustre, dull and earthy. H = 1. Gr., 4.

TIN. *Oxide of.* Comp., tin, 77.5, O 21.5, oxide of iron .25, and silver .75. Found in the crystalline rocks with Cu. and iron pyrites. Found in various places, especially in Cornwall in England. Color, brown or black, with a pale gray streak. Found lamellar, in grains and massive. Decipitates on charcoal. L = adamantine. F., indistinct and brittle. H = 6 to 7. Gr., 6.5 to 7.

Sulphuret of Tin, or Pyrites. Color, steel gray or yellowish. Streak, black. F = brittle. H 4. Gr., 4. Comp., tin 84, S 25, Cu. 86 and Fe. 2.

PLATINUM. Found only in the metallic state, with various metals, such as gold, silver, iron, copper and lead, and disseminated in rocks of igneous origin, as the primary. Often found in syenite with gold, but it is principally found in alluvium or drift. Color, very light steel gray to silver white. Lustre, glistening. It is found in grains and rolled pieces, seldom larger than a pea. Resembles coarse iron filings. It is malleable; infusible, excepting in the flame of the oxyhydrogen blowpipe.

GOLD. Found in granite, quartz, slate, hornstone, sandstone, limestone, clay slate, gneiss, mica slate, and especially in talcose slate, rarely in graywack and tertiary slate, but never in serpentine. Associated with Cu., Zn., Fe., Pb., Baryta., antimony, platinum. Where it is found in primary rocks, it is frequently in schiste. Color, yellow. Seldom found massive; often disseminated, capillary, amorphous, dentritic, crystallized in cubes, octohedrons, rhomboidal, dodecahedron and tetrahedron. Lustre, glistening and metallic. Fracture, hackly and tissular. H = 2.5 to 3. Gr., 19.4. It is malleable and unaltered by exposure, and is easily cut and flattened under the hammer, which distinguishes it from copper and iron pyrites, which crumble under the hammer.

SILVER. *Sulphuret of.* Comp., Ag. 87, S 18. It is soluble in nitric acid. Found in primary and secondary rocks, with other ores of silver. Gives off sulphurous odor when heated in the flame of a blow pipe flame. Found in cubes and octohedrons, reticulated. Imperfect at cleavage, is malleable, amorphous and in plates. Color, blackish, lead gray, with a shining streak. L = metallic, F. flat and conchoidal. H 2.3. Gr., 7.

Silver, native. Usually alloyed with gold, bismuth and copper. Found in primary and secondary rocks, often in penetrating crystals, or amorphous in common quartz, with copper and cobalt. It is fusible into a globule. Color, silver white, but often gray or reddish. It is seldom found massive, but often in plates and spangles, dentiform, filiform and

aborescent. Crystallized in cubes, octohedrons, lamellar and ramose, with no cleavage. L=splendent to shining. F., fine hackly. H2.5 to 3. Gr., 10.4

Sulphuret of Silver and Antimony. Comp. S16, Sb. 14.7, Ag. 68.5, Cu. 6. Found in the primary rocks, such as granite and clay slate, with native silver and copper. It is found massive and in compound crystals, having an imperfect cleavage. Color, iron black. L=metallic. F., conchoidal. H2.2. Gr., 6.8.

Chloride of Silver. Comp., Ag. 75, chlorine 25. Found in the primary rocks with other ores of silver. Massive, seldom columnar, often incrusting, in cubes, with no distinct cleavage, also reniform and acicular. Color, pearly gray, greenish, blue or reddish, with a shining streak. Lustre, resinous to adamantine.

Mercury, native. Found in Austria, Spain, Peru, Hungary and California. Found in fluid globules. Color, tin white. Gr., 13.6.

Sulphuret of Mercury, or Cinnabar. Comp., mercury s. 14.75. Found chiefly in the new red sandstone, sometimes in mica slate, limestone, gneiss, graywack, beds of bituminous shale of coal formation. In California, at the Almaden mines, it is found in greenish talcose rock. Color, brownish black to bright red, cochineal red, lead gray, sometimes a tinge of yellow. Found massive, six-sided prisms, sometimes fibrous, with a streak of scarlet red. It evaporates before the blow pipe and does not give off allicaceous fumes. L=metallic to unmetallic. Fracture, perfect, fibrous, granular or in thin plates. H2.3. Gr., 7 to 8.

LEAD. *Native.* Rarely met. It has been found in the County of Kerry in Ireland, Carthagena in Spain, and Alston moor, in the County of Cumberland, England.

Sulphuret of Lead, or Galena. Comp., Pb. 86.5, S18.8. Found in veins, beds and imbedded masses, in primary and secondary mountains, but more frequently in the latter, particularly in limestone. The indications are calc spar, mineral-blossom, red color of the soil, crumbling of magnesian limestone and sink-hole appearance of the surface. Color, leaden or blackish gray. Found amorphous, reticulated and crystallized in cubes and octohedrons, with a perfect cleavage, parallel to the planes of the cubes. L=metallic. F., lamellar and brittle. Gr., 7.6.

Sulphate of Lead. Comp., Pb. 78, sulphuric acid, 27. It is produced from the decomposition of galena, and found associated with galena. Color, white, sometimes green or light gray. Found massive, granular, lamellar, and often in slender crystals. L=vitreous or resinous. F., brittle. H2.8 to 3. Gr., 6.8 to 6.5.

Minium or Red Lead. Found with galena in pulverulent state. Color, bright red and yellow. Gr., 4.6.

Phosphate of Lead. Comp., Pb. 78.6, phosphoric acid 19.7, hydrochloric acid 1.7. Color, bright green or orange brown. Found in hexagonal prisms, reniform, globular and radiated. Streak, white. H3.8 to 4. Gr. 6.5 to 7.

Chromate of Lead. Found in gneiss. Color, bright red, with a streak of orange yellow. Found massive and in oblique rhombic prisms.

Black Lead, Plumbago, or Graphite. Found in gneiss, mica, granular limestone, clay slate, and generally in the coal formation. Color, iron

black. Lustre, metallic. In six-sided prisms, foliated and massive. $H = 1$ to 2 . Gr., 2 .

IRON. Native. Is found in meteorites, alloyed with nickel. It is massive, magnetic, malleable and ductile. $F =$ hackley. $H4.5$. Gr. 7.3 to 7.8 . A specimen in Yale College contains $Fe. 9.1$ and $Ni. 9$.

Iron Pyrites, or Bisulphuret of Iron. Occurs in rocks of all ages and in lavas. Found usually in cubes, pentagonal, dodecahedrons or octohedrons. Also massive. Color, bronze yellow, with a brownish streak. Lustre, metallic and splendid. Brittle. $H = 6$ to 6.5 . Gr. 4.8 to 5.1 . Strikes fire with steel, and is not magnetic. Comp., $Fe. 45.74$, $S54.26$.

Auriferous Iron Pyrites. Is that which contains gold.

Magnetic Pyrites, or Sulphuret of Iron. Found massive, and sometimes in hexagonal, tabular prisms. Color, bronze yellow to copper red, with a dark streak. $F =$ brittle. $H3.5$ to 4.5 . Gr. 4.6 to 4.65 . Slightly magnetic. Comp., $Fe. 59.6$, $S40.4$. This ore is not so hard as the bisulphuret of iron, and is of a paler color than copper pyrites.

Magnetic Iron Ore. Found in granular masses, octohedrons, dodecahedrons, granite, gneiss, mica, clay slate, hornblende, syenite, chlorite, slate and limestone. Color, iron black, with a black streak. $F =$ brittle. $H5.5$ to 6.5 . Gr., 5 to 5.1 . Highly magnetic. Comp., $Fe. 71.8$, oxygen 28.2 . This is the most useful and diffused iron ore.

Specular Iron Ore, Peroxide of Iron. Found massive, granular, micaceous, sometimes in thin, tabular prisms. Color, dark steel gray or iron black. Lustre, often splendid, passing into an earthy ore of a red color, yielding a deep red color without lustre. $H = 5.5$ to 6.5 . Gr., 4.5 to 5.3 . Slightly magnetic.

The Specular Variety. Has a highly, metallic lustre.

Micaceous, Specular Iron Ore. Has a foliated structure.

Red Ochre. Often contains clay, is soft and earthy. It is more compact than red chalk.

Bog Iron Ore. Occurs in low ground; is loose and earthy; of a brownish, black color.

Clay Iron Stone. Has a brownish red, jaspery and compact appearance. Comp. of specular iron are $Fe. 69.3$, oxygen 30.7 . The celebrated iron mountains of Missouri are composed of specular iron ore. One of the mountains is 700 feet high. There, the massive, micaceous and ochreous varieties are combined.

Chromate of Iron. Found massive and octohedral crystals, in serpentine rocks, imbedded in veins or masses. Color, iron and brownish black, with a dark streak. $L =$ sub-metallic. $H5.5$. Gr., 4.3 to 4.5 . When reduced to small fragments, it is magnetic. Comp., chromium 60 , protoxide of iron 20.1 , alumina 11.8 , and magnesia 7.5 .

Carbonate of Iron. Found principally in gneiss and graywack, also in rocks of all ages. Found massive, with a foliated structure, in rhombohedrons and hexagonal prisms. Color, light gray to dark brown red; blackens by exposure. $L =$ pearly to vitreous. $H3$ to 4.5 . Gr., 3.7 to 3.8 . Comp., protoxide of iron 61.4 , carbonic acid 38.6 . This ore is extensively used in the manufacture of iron and steel. These, with the magnetic, specular, bog ore and clay ironstone, are the principal sources of the iron commerce.

EXAMINATION OF A SOLID BODY.

310L. Note its state of aggregate, hardness, specific gravity, fracture, lustre, color, locality and associates. Heat a portion of the substance, (reduced to a fine powder) in a test tube; if no change of color appears, it is free from *organic matter*.

It is free from water, if there is no change of weight.

If organic matter is present, it blackens first, then reddens.

No organic matter is present, if it entirely volatilizes.

It is a compound of two or more substances, when only a portion volatilizes.

It is an alkali or alkaline earth, if it fuses without any other change.

Is it soluble, insoluble, or partially soluble in water?

Is it soluble with boiling dilute hydrochloric acid?

Take two portions of the substance, burn one part, and to the other, add dilute hydrochloric acid; if no effervescence takes place until we put dilute acid on the burnt substance, it shows the presence of an *organic acid*.

The substance may be either a borate, carbonate, chlorate, nitrate, phosphate or sulphate.

Borates. The alkaline borates are soluble in water, the others are nearly insoluble. They are decomposed in the wet way by sulphuric, nitric and hydrochloric acids, thus liberating boracic acid. If the mixture of any borate and fluorspar be heated with sulphuric acid, fluoride of boron is disengaged, recognized by the dense, white fumes it gives off in the air, and its mode of decomposition by contact with water.—*Reynolds*.

Otherwise. From moderately, dilute solutions of borates. Mineral acids separate boracic acid, which crystallizes in scales.

Otherwise. Heat the solution of a borate with one-half its volume of concentrated sulphuric acid and the same of alcohol. Kindle the latter. The boracic acid imparts a fine green color to the flame. Stir the mixture whilst burning. Melt the borate with two parts of fluorspar and one of bisulphuret of potash in a dark place; the flame at the instant of fusion is tinged green.

Carbonates. Dissolved in cold or heated acids, disengage carbonic acid with a lively effervescence, which, if conducted through a tube into lime water, gives the latter a milk-white appearance. This gas will also slightly redden blue litmus paper previously moistened; but heat restores the blue color. If the gas is collected in a tube, and a small lighted taper let down into it, it will be extinguished.

An *engineer* constructing tunnels or subterraneous works, will find the above tests sufficient to warn him of approaching danger from "foul air" or "choke damp." Water absorbs an equal bulk of this gas, hence the benefit of workmen throwing down a few buckets of water into a well, previous to going down into it after recess. Although the above tests will detect the presence of carbonic acid in subterraneous work, where the air may be impure, it requires the greatest caution on the part of the engineer to preserve the health of the workmen.

Carbonic acid, is inodorous and tasteless. Sulphuretted hydrogen has the odor of rotten eggs, and is often found in subterraneous works.

Alkaline carbonates are soluble, the other carbonates are not.

Nitrates. All nitrates, excepting a few sub-nitrates, are soluble in water.

A solid nitrate, heated with concentrated sulphuric acid, evolves fumes of nitrous acid, sometimes accompanied by red-brown vapors of peroxide of nitrogen.

Otherwise, heat the nitrate with concentrated sulphuric acid, then put in a slip of clean metallic copper, red vapors of peroxide of nitrogen are evolved.

Otherwise, to a solution of a nitrate, add its bulk of concentrated sulphuric acid. When cool, suspend a crystal of protosulphate of iron, (green copperas.) After sometime, a brown ring will appear about the crystal. The liquid in this case must not be stirred or heated.

Phosphates. Generally dissolve in nitric and hydrochloric acids. Sulphuric acid does not give any reaction, but generally decomposes them. With phosphates soluble in water, nitrate of silver gives a lemon-yellow phosphate of silver. Is soluble, with difficulty, in acetic acid.

Phosphates. Insoluble in water, are dissolved in nitric acid, then this solution is neutralized by ammonia; to this neutral mixture, the nitrate of silver test gives the above yellow color.

Sesquioxide of Iron. In an alkaline solution of a phosphate, gives an almost white gelatinous precipitate of phosphate of sesquioxide of iron. Insoluble in acetic acid.

Molybdate of Ammonia, added to any phosphate solution, and then nitric or hydrochloric acid added in excess, a yellow color soon appears, and subsequently a yellow precipitate.

This is a very characteristic test. The substance ought to be first dissolved in nitric acid, and then nearly neutralized before adding the molybdate of ammonia.

Sulphates. Nearly all the sulphates are soluble in water. They do not effervesce with acids. This distinguishes them from carbonates. The sulphates of baryta, strontia and lead, are nearly insoluble; that of lime is slightly soluble.

From all the soluble sulphates, nitrate of baryta or chloride of barium, throws down a white precipitate insoluble in nitric acid, which is a characteristic property of the sulphates. In applying this test, the solution ought to be neutral or nearly so. This can be done by adding Magnesia to the solution so as to render it equal to sulphate of magnesia, MgO, SO_2 .

BLOW PIPE EXAMINATIONS.

310x. Heat a portion of the substance on charcoal, in the inner flame of the blow pipe.

If potash or soda, the flame is tinged yellow.

If an alkaline earth, (barium, calcium, strontium, magnesium,) it will radiate a white light, and is infusible. Now moisten this infusible mass with nitrate of cobalt and heat again.

If the flame becomes blue, alumina is present.

If green, oxide of zinc.

If pale pink, magnesia; but if silica, it will fuse into a colorless bead, on the addition of carbonate of soda.

If a bead, or colored infusible residue is formed, mix it with carbonate of soda, and heat on charcoal in the inner flame of the blow pipe.

If tin, copper, silver or gold, are present, a bead of the metal will be formed, without any incrustation on the charcoal.

If iron, cobalt or nickel, are present, the metal will be mixed up with the carbonate of soda, giving the bead a gray opaque appearance.

If zinc or antimony, it will give a white deposit around the bead.

If lead, bismuth or cadmium, a yellow or brown deposit.

QUALITATIVE ANALYSES OF METALLIC SUBSTANCES.

310x. Let M = equal the mass or substance to be analyzed. We reduce it to a fine powder and boil with hydrochloric acid, so as to reduce it to a chloride, but if we suspect the presence of a metal not soluble by the above, we boil it with aqua regia (= nitro-hydrochloric acid) until it is dissolved; then we evaporate and boil again with dilute hydrochloric acid and evaporate to dryness, and so continue till every trace of nitric acid disappears. We have the metals now reduced to chlorides, which are soluble in distilled water. The solution is now set aside for analysis, which is to be acid, neutral or alkaline, as the nature of the reagent may require.

The solution is acid if it changes blue litmus paper red, and alkaline, if it changes red litmus paper blue, or turmeric paper brown.

Taylor gives nitro-prusside of sodium as a very delicate test for alkali. He "passes a little hydrosulphuric acid into the solution to be examined, and then adds the solution of the nitro-prusside of sodium, which gives a magnificent rose, purple, blue or crimson color, according to the strength of the alkaline. This will indicate an alkali in borates, phosphates, carbonates, and in the least oxideable oxides, as lime and magnesia."

The metals are divided into groups or classes.

CLASS I. Potash = KO, soda = NaO, and ammonia NH₃. None of these, in an acidified solution, gives a precipitate with hydrosulphuric acid, hydrosulphate of ammonia, or carbonate of soda.

CLASS II. Magnesia, MgO. Lime, CaO. Baryta, BaO. Strontia, SrO. None of these gives a precipitate with hydrosulphuric acid, or hydrosulphate of ammonia.

Carbonate, or phosphate of soda, with either of this class, gives a copious white precipitate insoluble in excess.

CLASS III. Alumina = Al ₂ O ₃ .	Oxide of nickel NiO.
Oxide of zinc ZnO.	Oxide of cobalt CoO.
Oxide of chromium.	Protoxide of iron FeO.
Protoxide of manganese MnO.	Per oxide of iron Fe ₂ O ₃ .

In neutral solutions these metals are precipitated by hydrosulphate of ammonia.

In a slightly acid solution, hydrosulphuric acid gives no precipitate excepting with peroxide of iron, with which it gives a yellowish white precip.

CLASS IV. Arsenious acid AsO₃, arsenic acid AsO₅, teroxide of antimony SbO₃, oxide of mercury HgO, peroxide of mercury HgO₂, oxides of lead, copper, silver, tin, bismuth, gold and platinum.

All of this class are precipitated from their acid solution by hydrosul-

phuric acid. We can thus determine to which of the four classes of metals the substance under examination belongs.

POTASH, in a solution of chloride of potassium.

* *Bichloride of platinum*, in a neutral or slightly acid solution, gives a fine yellow crystalline prec., = $\text{KCl} \cdot \text{Pt} \cdot \text{Cl}_2$, slightly soluble in water, but insoluble in alcohol; somewhat soluble in dilute acids. When the solution is dilute, evaporate it with the reagent on a water bath, and then digest the residue with alcohol, when the above yellow crystals will appear.

Tartaric acid. Let the solution be concentrated, then add the reagent, and agitate the mixture with a glass rod for some time, and let it remain, when a white prec, slightly soluble in water, will appear, the prec = $\text{KO} \cdot \text{HO} \cdot \text{C}^8 \text{H}_4 \text{O}_{10}$.

Blow Pipe flame. Wash the platinum wire in distilled water, then place a piece of the salt to be examined on the wire, which will give a violet color to the outer flame.

Alcohol flame, having a potash salt in solution, gives the same reaction as the last.

SODA, in a solution of sulphate of soda.

Bichloride of platinum, added as for potassa, then evaporated, will give yellow needle-shaped crystals different from that by potassa. The prec. is readily soluble in water and alcohol.

Antimoniate of potash. Let the solution and the reagent be concentrated, and the solution under examination slightly alkaline or neutral; then apply the reagent, which, if soda is present, will produce a white crystalline prec. of antimoniate of soda.

Blow Pipe. Hold the salt on the platinum wire in the inner or reducing flame, it will impart a golden yellow color to the outer, or oxidizing flame.

OXIDE OF AMMONIUM, NH_4O , in a solution of chloride of ammonium.

Bichloride of platinum gives the same reaction as for potassa. If we have a doubt whether it is potassa or ammonia, ignite the precipitate and digest the residue with water, then, if nitrate of silver be added, and gives a precipitate, it shows the presence of potassa. In this case we must take care that all traces of hydrochloric acid are removed.

Heated in a test tube. If the substance be heated in a test tube with some hydrate of lime, or caustic potassa or soda, it will give off the peculiar odor of ammonia, and changes moistened turmeric paper brown and red litmus paper blue. If this does not happen, we say ammonia is absent.

BARYTA, = BaO , in a solution of chloride of barium.

Sulphuric acid. White prec. in very dilute solution, insoluble in dilute acids.

Sulphate of lime, in solution, gives an immediate prec., requiring 500 times its weight of water to dissolve it.

Oxalate of ammonia. White prec. readily sol. in free acids. This is the same reaction as for lime, but it requires a stronger solution of baryta than of lime.

Flame of alcohol, containing baryta, is yellowish, and is different from that of lime, which has a reddish tinge, and strontia, which is carmine.

Blow Pipe, in the inner flame, the substance strongly heated on plati-

* Those marked with an asterisk are the most delicate tests.

num wire, imparts a light green color to the outer flame. If the substance be insoluble, first moisten it with dilute hydrochloric acid.

LIME, = CaO , in a solution of chloride of calcium.

Oxalate of ammonia. Let the solution be neutralized with muriate of ammonia; then add the reagent, which will give a copious white prec. of oxalate of lime, soluble in hydrochloric acid, but insoluble in acetic acid. This detects lime in a highly diluted solution.

Sulphuric acid, dilute. In concentrated solution gives an immediate prec. soluble in much water, which is not the case with baryta.

Blow Pipe. Heated in the inner flame, gives an orange red color to the outer flame. Moisten an insoluble compound with dilute hydrochloric acid before this test.

Burnt with alcohol, the flame will be a reddish tint, but not so red as that given by strontia.

STRONTIA = SrO . In a solution of chloride of strontium.

Oxalate of ammonia, in concentrated solution, a white prec., but not in dilute solution. This distinguishes strontia from lime.

Sulphate of lime. The prec. will be formed after some time even in a concentrated solution. This distinguishes strontia from baryta. (See above.)

Sulphuric acid gives an immediate prec. in a concentrated solution, but only after some time in a dilute one, where the prec. will be minute crystals.

In the flame of alcohol, stir the mixture, and a beautiful carmine color is produced.

Blow Pipe, in the inner flame, an intense carmine red. Moisten the insoluble compound with dilute H.Cl as above for lime and baryta.

NOTE. *Sulphuric acid* gives, with a weak solution of lime, no precipitate; with chloride of barium, an immediate white p.; with a weak solution of strontia, a prec. after some time. The prec. from baryta and strontia are insol. in nitric acid, but that from lime is sol.

MAGNESIA MgO. , in a solution of sulphate of magnesia MgO. SO_3 .

Phosphate of soda, a white, highly crystalline prec. of phosphate of magnesia = 2MgO. HO. PO_5 . In this case the solution must not be very dilute. By boiling the solution and reagent together the prec. is more easily produced.

Phosphate of soda and ammonia. In using this reagent, add ammonia or its carbonate, which makes the prec. less soluble. Agitate with a glass rod, which, if it touches the side of the test tube, will cause the prec. there to appear first. The prec. is crystalline, slightly soluble in water, less in ammonia, but readily in dilute acids; $\therefore$ the solution must be ammoniacal. Ignite this prec., the ammonia is driven off, and the residue = phosphate of magnesia = 2MgO. PO_5 .

Blow Pipe. Moisten the substance with nitrate of cobalt, and heat in the blow-pipe, the compound assumes a pale flesh or rose color.

NOTE. Sulphate of lime gives a prec. with baryta and strontia.

Oxalate of ammonia gives a prec. with a very dilute solution of lime, but only with a concentrated solution of magnesia and strontia, and in a much stronger sol. of baryta than lime.

Phosphate of soda, with lime, a gelatinous precipitate.

do do with magnesia.

Hydrofluosilic acid, ~~the~~ a solution of baryta, gives a white, transparent prec. By evaporating the prec. to dryness, and washing the residue with alcohol, we obtain all of the silico-fluoride of barium undissolved. If the sol. is dilute, the prec. will be after some time.

ALUMINA, (Al_2O_3 .) in a sol. of sulphate of alumina.

Caustic Ammonia, (NH_3 .) gives a semi-transparent, gelatinous, bulky prec. nearly insol. in excess of the ammonia.

Caustic Potash, (KO .) gives a similar prec. soluble in an excess of the reagent, but if we add chlorate of ammonia to the solution, the alumina is again precipitated.

Hydrosulphate of Ammonia, added to a neutral solution, gives a white prec. of hydrate of alumina, (Al_2O_3 , HO) and hydrosulphuric acid is liberated.

Phosphate of Soda, white prec., sol. in mineral acids; nearly insol. in acetic acid.

Lime Water, precipitates alumina.

NOTE. Ammonia in excess precipitates alumina, but not magnesia or the other alkaline earths.

CHROMIUM, (Cr_2O_3 .) in a sol. of sulphate of chrom.

Hydrosulph. Acid, in neither acid or neutral solutions, gives no prec.

Hydrosulphate of Ammonia, in a neutral solution, gives a dark green prec. insol. in excess of the reagent.

Caustic Ammonia, if boiled with the solution, will produce the same as the last. If not boiled, a portion of the prec. will re-dissolve, giving the liquid a pink color.

Blow Pipe. Reduce the substance to a sesquioxide of chromium, which will give in the inner flame a yellowish green glass, and in the outer flame a bright emerald green.

Heat with a mixture of nitrate of potash and carbonate of soda; a yellow bead is formed. Dissolve this bead in water acidulated with nitric acid, and add acetate of lead; a bright yellow prec. of chromate of lead is formed.

PEROXIDE OF IRON. In a solution of sulphate of iron, FeO . SO_3 . The compound is boiled with nitric acid to oxidize the metal, and then evaporated to dryness.

Hydrosulphuric Acid, gives no precipitate.

Sulphide of Ammonium, precipitates the iron completely as a black precipitate of sulphide of iron, FeS , which is insoluble in an excess of the precipitant.

The above precipitate when exposed for some time to the air, becomes brown sesquioxide of iron.

Ferrocyanide of Potassium, (prussiate of potassm,) light blue precipitate of $\text{KFe}(\text{C}_6\text{F}_5)_2$. The precipitate is insoluble in dilute acids. *This is the most delicate test for iron.*

Sulphocyanide of Potassium. A red solution, but no precipitate.

Tincture of Galls. Bluish black in the most dilute solution.

Caustic Potash. Reddish prec. sol. in excess.

Caustic Ammonia the same, insol. in excess.

Blow Pipe, heated on a platinum wire with borax in the outer flame, gives a brownish red glass, which assumes a dirty green color in the inner or reducing flame.

OXIDE OF COBALT. CoO , in a solution of nitrate or chloride of cobalt.

Ammonia, when the solution does not contain free acid, or much ammoniacal salt, the metal is partially precipitated as a bluish precipitate, readily soluble in excess of the reagent, giving a reddish brown solution.

Sulphide of Ammonium. A black precipitate of sulphide of cobalt, CoS , soluble in nitric acid, but sparingly in hydrochloric acid.

Sesquicarbonate of Ammonia. A pink prec. CoO , CO_2 readily soluble in excess, giving a red solution.

Solution of Potassa. Blue prec. changing by heat to violet and red.

Ferrocyanide of Potassium. A grayish green prec.

Blow Pipe. In both flames with borax, a beautiful blue glass whose color is scarcely affected by other oxides. In this reaction the cobalt must be used in a small quantity.

OXIDE OF NICKEL, NiO in a sol. of sulphate of nickel, NiO , $\text{SO}_3 + 7\text{H}_2\text{O}$.

Hydrosulphate of Ammonia. Black prec. from neutral solution, slightly sol. in excess of the reagent, if the ammonia is yellow. The prec. is sol. in NO_5 and sparingly in HCl .

Hydrosulphuric Acid in acidified sol., no prec., but in neutral sol., it gives a partial prec.

* *Caustic Ammonia.* A light green prec. sol. in excess, giving a purplish blue solution. In this case any salt of ammonia must be absent.

Caustic Potash. Apple green prec. insol. in excess.

Ferrocyanide of Potassium, greenish white prec. Cyanide of potassium, yellowish green prec. sol. in excess, forming a dull yellow sol. From this last sol., SO_3 precipitates the nickel.

Blow Pipe. Any compound of nickel with carbonate of soda or borax in the inner flame, is reduced to the metallic state, forming a dusky gray or brown beads. In the outer flame the bead is violet while hot, becoming brown or yellow on cooling.

OXIDE OF MANGANESE = MnO in a solution of sulphate of manganese = MnO , $\text{SO}_3 + 7\text{H}_2\text{O}$.

* *Hydrosulphate of Ammonia* in neutral sol. gives a bright flesh colored gelatinous prec. becoming dark on exposure to the air. It is insoluble in excess of the reagent, but sol. in HCl and NO_5 .

* *Caustic Ammonia*, if free from muriate of ammonia, gives a white or pale flesh colored = MnO , HO , becomes brown in air.

* *Caustic Potash*, the same as the last, but muriate of ammonia does not entirely prevent the precipitate.

Carbonate of Potash, or Ammonia, white prec. which does not darken so readily as the above. It is slightly soluble in chloride of ammonium.

Blow Pipe. Mix the substance with carbonate of soda and a little nitrate or potash, and heat in the outer flame; it will give a green color, and produce manganate of soda, which will color water green.

If the substance is heated with borax in the outer flame, it will produce a bead of a purple color; this if heated in the inner flame will cause the color to disappear.

OXIDE OF ZINC, ZnO in a solution of sulphate of zinc, Zn , $\text{SO} + 7\text{H}_2\text{O}$.

* *Hydrosulphate of Ammonia*, in neutral or alkaline solution, gives a copious white curdy prec. if the zinc is pure. If iron is present it will be colored in proportion to the iron present in the sol.

Hydrosulphuric Acid in acid sol. no prec.

Caustic Ammonia, or Potash, a white gelatinous prec. soluble in excess. From either solution in excess, hyd. sulph. acid (HS) throws down the white prec. of sulphide of zinc.

Carbonate of Potash, when no other salt of potash is present, gives a white prec. = $3(\text{ZnO}, \text{HO}) + 2(\text{ZnO}, \text{CO}_2)$ insol. in excess of the reagent.

Blow Pipe, moistened with nitrate of cobalt and heated in the outer flame, gives a pale green color which is a delicate test to distinguish it from manganese, alumina and cobalt.

ARSENIC ACID = AsO_6 . Boil the compound with HCl, and at the boiling point, add nitric acid as long as red flames of nitrous vapor appear, then evaporate slowly so as not to redden the powder, and expel the acid; then dissolve in distilled water for examination. HS, added to the above sol. slightly acidified with HCl, gives no immediate prec., but if allowed to stand for some time, or if heated to boiling point, a yellow prec. is obtained. Apply the gas several times, always heating to boiling point each time.

Hyd. Sulph. of Ammonia, as in the above solution, but a little more acid gives the same prec. but of a lighter color:

Ammonia nitrate of Silver. In a neutral solution as first made, add nitrate of silver which gives but a faint cloudy appearance; now add ammonia drop by drop till it gives a yellow prec. of arsenite of silver, which is very soluble in alkali.

NOTE. The same prec. is obtained from the presence of phosphate of soda.

Reineck's test, in a solution acidified by adding a few drops of hydrochloric acid is a very delicate test, and considered nearly as delicate as Marsh's.

Boil with the acidified liquid in a test tube, a clean strip of copper foil; the arsenic will be prec. on the copper as a metallic deposit. Antimony, bismuth, mercury and silver, give the same reduction as arsenic.

In order to determine which is present, take out the copper foil and dry it between folds of filtering paper, or before a gentle heat; place it in a dry test tube and apply heat; the arsenic being volatile, will be deposited in the upper end of the tube as a crystalline deposit, using but gentle heat. If it were antimony it would not be volatile, and would be deposited as a white sublimate, insol. in water, amorphous, and requiring more heat than arsenic. If it were mercury, it would be in small metallic globules.

Marsh's test, is dangerous, excepting in the hands of an experienced chemist. Those who wish to apply it, will find the method of using it in Sir Robert Kane's Chemistry, or in those of Graham, Fowne, Bowman, and others.

TROXIDE OF ANTIMONY = SbO_3 , in a solution of chloride of antimony = SbCl_3 . This solution is made by dissolving the gray ore, or bisulphide of antimony in hydrochloric acid; the solution then diluted with water, acidified with HCl, is examined.

Hydrosulphuric Acid, gives an orange red prec. of SbS_3 , insol. in cold dilute acids, soluble in potassa and sulphide of ammonia.

Hydrosulphate of Ammonia. Add the reagent in small quantities; it will give an orange prec. of SbS_3 , soluble in excess.

Caustic Ammonia, or Potassa. Add slowly, and it will give a white prec. of teroxide of antimony = SbO_3 , soluble in excess.

Water in excess. A white prec. which crystallises after some time, and is sol. in tartaric acid.

NOTE. The same reaction is had with bismuth, but the prec. is not soluble in tartaric acid.

A piece of zinc, in a dilute solution made with *aqua regia*, precipitates both antimony and tin.

A piece of tin, in the above sol., prec. the antimony.

TEROXIDE OF BISMUTH, in a solution of nitrate of teroxide of bismuth = BiO_3 , 2NO_5 .

Hyd. Sulph. Acid. A black prec. insol. in cold dilute acids, but sol. in hot dilute nitric acid.

Chromate, or Bichromate of Potash, yellow prec. very sol. in dilute nitric acid.

Water in excess, added to a solution of sesquichloride of bismuth, slightly acidified with hydrochloric acid, produces a white prec. insol. in tartaric acid, which distinguishes it from antimony.

Heat a salt of Bismuth. It turns yellow, but on cooling off, becomes again colorless.

Blow Pipe. In the inner flame with carbonate of soda, it forms small metallic globules, easily broken.

Blow Pipe. In the outer flame with borax, gives a yellowish bead, becoming nearly colorless when cool.

OXIDE OF TIN = SnO , in a sol. of chloride of tin, SnCl .

Hydrosulphuric Acid, dark brown prec. in neutral or acid solutions. Insol. in cold dilute acids. If the prec. is boiled with nitric acid, it is converted into the insoluble binoxide of tin.

Hydrosulphate of Ammonia, brown prec. sol. in excess if the reagent is yellow.

Chloride of Mercury. First a white prec. then a gray prec. of metallic mercury, even in very dilute solution and in the presence of much HCl .

Caustic Ammonia, white bulky prec. insol. in excess.

Caustic Potash, do. = SnOHO , sol. in excess.

Terchloride of Gold = (AuCl_3) very dilute. In dilute solutions, gives a dark purple prec. known as the purple of Cassius. If this mixture is now heated, it is resolved into metallic gold and binoxide of tin.

PEROXIDE OF TIN = SnO_2 , in a sol. of bichloride of tin = SnCl_2 .

Hyd. Sulph. Acid, bright yellow prec. insol. in dilute. SO_3 , or HCl , made insoluble by boiling with NC_5 , soluble in HCl added to a little NO_5 . Sol. in alkalies.

Caustic Potassa, or Ammonia, white bulky prec. sol. in excess, especially with potassa. The prec. with ammonia is SnO_2HO , and with potassa = KO , SnO_2 .

Blow Pipe. In the outer flame with borax, it will give a colorless bead, but if there is much tin, the bead will be opaque.

A piece of clean zinc, in a sol. of perchloride of tin, will precipitate the tin in the metallic state in beautiful feathery crystals; which under the microscope appear as brilliant crystalline tufts.

OXIDE OF MERCURY = HgO , in a solution of bichloride of mercury, (corrosive sublimate) = HgCl_2 .

Hydrosulphuric Acid, added slowly, gives a white or yellow prec. If added in excess, it gives a black prec. of HgS , insol. in dilute SO_3 , HCl or NO_5 . It is soluble in aqua regia with the aid of heat. If the precipitate be dried and heated in a test tube, metallic mercury is produced.

Chloride of Tin, add slowly, a white prec. of Hg_2Cl_2 = subchloride of mercury will appear, this prec. becomes gray with an excess of the reagent. If we boil this precipitate in its solution, the mercury is reduced to the metallic state.

* *Iodide of Potassium*, add drop by drop, gives a beautiful red prec. soluble in an excess of either the solution or reagent.

Heat a strip of copper; the mercury will be deposited on it which when rubbed will appear like silver. If the strip be heated in a test tube, the mercury will appear in minute globules in the cool part of the tube.

OXIDE OF LEAD = PbO , in a solution of nitrate of lead, = PbO , NO_5 , made by dissolving the substance in nitric acid, and allowing it to crystallise. We may also use a solution of acetate of lead. Acetate of lead is formed by dissolving oxide of lead in an excess of acetic acid, then evaporate to dryness, the salt is acetate, or sugar of lead.

The following reactions take place with *nitrate of lead*:

Hydrosulphuric acid, in neutral or slightly acid solution, gives a black prec. of sulphide of lead = PbS , but if boiled with nitric acid, it becomes $\text{PbO} + \text{SO}_3$.

Caustic Ammonia, a white prec. insol. in excess. Other ammoniacal salts must not be present.

Dilute, SO_3 , a white heavy prec. nearly insol. in acids, but soluble in potassa. Now collect the prec. and moisten it with a little hydrosulphate of ammonia, it will become black. This distinguishes lead from baryta and strontia, which are insoluble.

Carbonate of Potassa, white prec. insol. in excess. Prec. = PbO , CO_2 .

Iodide of Potassium, beautiful yellow prec. If this is boiled with water and allowed to cool, beautiful yellow scales are formed.

Chromate of Potassa, fine yellow prec. insol. in dilute acids, but sol. in potassa.

Hydrochloric Acid, a white prec. Boil the solution and let it cool, then needle-shaped crystals will be formed.

OXIDE OF SILVER, AgO , in a solution of nitrate of silver.

Hydrochloric Acid, or any soluble chloride, a white curdy prec. of chloride of silver, insol. in water and nitric acid, sol. in ammonia. This becomes violet on exposure to light, and is sparingly sol. in HCl .

Common Table Salt, gives the same prec.

Hyd. Sulph. Acid, and *Hyd. Sulphate of Ammonia*, gives a black prec. insol. in dilute acids, but sol. in boiling nitric acid.

Caustic Ammonia, brown prec. sol. in excess.

Caustic Potassa, brown prec. insol. in excess.

Phosphate of Soda, a pale yellow prec. sol. in NO_5 and ammonia.

Chromate of Potassa, dark crimson prec.

NOTE. With lead, the prec. would be yellow.

Strip of clean copper, iron or zinc, suspended in the liquid, precipitates the silver in the metallic state.

NOTE. Silver is precipitated by other metals more electro-negative, such as tin, lead, manganese, mercury, bismuth, antimony, and arsenic.

OXIDE OF COPPER. CuO , in a solution of sulphate of copper = CuO , $\text{SO}_3 + 5\text{H}_2\text{O}$.

Hyd. Sulph. Acid, in a neutral, acid or alkaline solution, gives a black prec. = CuS , insol. in dilute SO_3 , or HCl , but sol. in moderately dilute nitric acid. Insol. in excess of the reagent.

Hyd. Sulphate of Ammonia. The same as the last, excepting that the reagent in excess dissolves the prec.

Caustic Ammonia, added slowly, precipitates any iron as a greenish or red brown mud, and the supernatant liquid is of a fine blue color. With nickel, ammonia gives a blue but of a pale sapphire color, whilst that of copper gives a deep ultramarine.

Caustic Potassa, blue prec. insol. in excess. If the potassa be added in excess and then boiled, the prec. will be black oxide of copper = CuO .

Ferrocyanide of Potassium = *Prussiate of Potassa*, gives a chocolate colored prec. = Cu^2 , FeCy_3 , insoluble in dilute acids. This is a very delicate test. The prec. is soluble in ammonia. Potassa decomposes it. Before adding this test, acidify the solution with acetic acid or acetate of potassa.

If but a small quantity of copper is present, no prec. will be produced, but the solution will have a pink color.

Iron or Steel perfectly cleansed in a neutral sol. or one slightly acidified with SO_3 , will become coated with metallic copper, thus enabling us to detect a minute quantity of copper, which is sometimes entirely precipitated from its solution. This detects 1 of copper in 180,000 of solution.

Blow Pipe. In the outer flame with borax while hot, the copper salt is green, but becomes blue on cooling.

TEROXIDE OF GOLD = AuO_3 in a solution of terchloride of gold.

Hydrosulphuric acid, black prec. of tersulphide of gold = AuS_3 , insol. in mineral acids, but sol. in *aqua regia*.

Sulphate of Iron, bluish black prec. becomes yellow when burnished.

Oxalic acid, if boiled, a prec. of a purple powder, which will afterwards cohere in yellow flakes of metallic gold when burnished.

Chloride of Tin, with a little bichloride of tin, gives a purple tint, whose color varies with the quantity of gold in the solution, and is insol. in dilute acids. In using this test, first add the golden solution to the chloride of tin, and then add the solution of bichloride of tin, drop by drop. If only a small quantity of gold is present, the solution will have but a pink tinge.

Tin-iron Solution. This reagent is made by adding sesquichloride of iron to chloride of tin, till a permanent yellow is formed.

Pour the golden solution, much diluted in a beaker, and set it on white paper. Now dilute the tin-iron reagent, and dip a glass rod into it, which remove and put into the gold solution, when, if a trace of gold is present, a purple or bluish streak will be in the track of the rod. This may be used in acid solutions.

BINOXIDE OF PLATINUM = PtO_2 , in a solution of bichloride of platinum.

Hyd. Sulphuric Acid, black prec. when boiled. Insol. in dilute acids.

Chloride of Ammonium. After several hours, a yellow crystalline prec. is lightly sol. in water, but insol. in alcohol.

Chloride of Tin, in the presence of hydrochloric acid, is a dark brown color; but if the solution is dilute, the color is yellow.

310o.

QUANTITATIVE ANALYSES.

The mineral is finely pulverized, in an agate or steel mortar. The pestle is to have a rotary motion so as not to waste any part of the mineral. When pulverized, wash and decant the fine part held in the solution, and again pulverize the coarse part remaining after decantation.

If the mineral is malleable, we file off enough for analysis.

Digesting the mineral, is to keep it in contact with water or acid in a beaker, and kept for some time at a gentle heat. If the mineral is insol. in water or HCl, we use aqua regia, (nitro-hydrochloric acid) composed of four parts of hydrochloric acid and one part of nitric acid. Aqua regia will dissolve all the metals but silica and alumina.

Filtering papers, are made of a uniform size, and the weight of the ash of one of them marked on the back of the parcel.

Filtering.—One of the filtering papers is placed in a glass funnel which is put into a large test tube or beaker, and then the above solution poured gently on the side of the filtering paper, wash the filter with distilled water. The filter now holds silica and alumina. Burn the filter and precipitate or insoluble residue, the increase of weight will be the siliceous matter in the amount analyzed, which may be twenty-five, fifty or one hundred grains, perhaps fifty grains will be the most convenient; therefore, the increase of weight found for siliceous matter if multiplied by two, will give the amount per cent.

Decanting, is to remove the supernatant liquid from vessel A to vessel B, and may be easily done by rubbing a little tallow on the outside of the edge of A, over which the liquid is to pass, and holding a glass rod in B, and bringing the oiled lip of A to the rod, then decant the liquid.

The Engineer is supposed to have seen some elementary work on Chemistry or Pharmacy. Fowne's, Bowman's and Lieber's are very good ones; from either of which he can learn the first rudiments.

The following table shows the substances treated of in this work, showing their symbols, equivalents or atomic weights and compounds.

310p.

TABLE OF SYMBOLS AND EQUIVALENTS.

Name.	Sym- bol.	Equi- val't.	Compound.	Equi- val't.
Aluminum	Al	14	Al ₂ O ₃ , Alumina.....	52
“	“	14	Al ₂ Cl ₃ , Chloride of Aluminum.....	136
“	“	14	Al ₂ O ₃ , 3SO ₃ , Sulphate of Alumina...	172
Antimony	Sb	129	SbO ₃ , Oxide of Antimony.....	153
Arsenic	As	75	AsO ₃ , Arsenious Acid	99
“	“	75	AsO ₅ , Arsenic Acid.....	116
Barium.....	Ba	69	BaO, Baryta.....	77
“	“	69	BaCl, Chloride of Barium.....	105
Bismuth	Bi	107	Bi ₂ O ₃ , Sesquioxide of Bismuth.....	238
“	“	107	Bi ₂ O ₃ , 3NO ₅ , Nitrate of Bismuth....	400
“	“	107	Bi ₂ , Cl ₃ , Sesquichloride of Bismuth.	322
Cadmium	Cd	56	CdO, Oxide of Cadmium.....	64
“	“	56	CdS, Sulphide of Cadmium.....	72
Calcium	Ca	20	CaO, Lime.....	28
“	“	20	CaCl, Chloride of Lime.....	56
Carbon	C	6	CO ₂ , Carbonic Acid.....	22
“	“	6	CO, Carbonic-Oxide.....	14
“	“	6	CS ₂ , Sulphide of Carbon.....	38
Chlorine	Cl	36	ClO ₅ , Chloric Acid.....	76
“	“	36	HCl, Hydrochloric Acid.....	37

Name.	Sym- bol.	Equi- val't.	Compound.	Equi- val't.
Chromium	Cr	28	Cr ₂ O ₃ , Sesquichloride of Chromium.	80
“	“	28	Cr ₂ O ₃ , 8SO ₃ , Sulphate of Chromium.	200
Cobalt	Co	30	CoO, Protoxide of Cobalt.....	88
“	“	30	Co ₂ O ₃ , Sesquioxide of Cobalt.....	84
Copper, (Cuprum)...	Cu	32	Cu ₂ O, Suboxide of Copper.....	72
“	“	32	CuO, Black Oxide of Copper.....	40
“	“	32	CuO, SO ₃ , Sulphate of Copper.....	80
Fluorine	F	18	HF, Hydrofluoric Acid.....	19
Gold, (Aurum).....	Au	200	AuO, Oxide of Gold.....	208
“	“	200	Au ₂ O ₃ , Ter oxide of Gold.....	224
“	“	200	AuCl ₃ , Ter chloride of Gold.....	308
Hydrogen	H	1	HO, Water.....	9
“	“	1	HO ₂ , Binoxide of Hydrogen.....	17
Iodine	I	126	IO ₅ , Iodic Acid.....	166
“	“	126	HI, Hydriodic Acid.....	127
Iron, (Ferum).....	Fe	28	FeO, Protoxide of Iron.....	36
“	“	28	Fe ₂ O ₃ , Sesquioxide of Iron.....	80
Lead, (Plumbum)...	Pb	104	PbO, Protoxide of Lead.....	112
“	“	104	Pb ₂ O ₄ , Red Oxide of Lead.....	344
“	“	104	PbCl, Chloride of Lead.....	140
Magnesium	Mg	12	MgO, Magnesia.....	20
“	“	12	MgCl, Chloride of Magnesium.....	48
Manganese.....	Mn	28	MnO, Protoxide of Manganese.....	36
“	“	28	MnO ₂ , Binoxide or Black Oxide of Manganese.....	44
“	“	28	MnO ₃ , Manganic Acid.....	52
“	“	28	Mn ₂ O ₇ , Permanganic acid.....	112
Mercury.	Hg	202	HgO, Protoxide of Mercury.....	210
“	“	202	HgO ₂ , Red or Binoxide of Mercury.	218
“	“	202	HgCl, Chloride of Mercury.....	238
“	“	202	HgCl ₂ , Perchloride of Mercury.....	274
Nickel.....	Ni	80	NiO, Oxide of Nickel.....	38
“	“	80	Ni ₂ O ₃ , Sesquioxide of Nickel.....	84
Nitrogen	N	14	NO ₅ , Nitric Acid.....	54
“	“	14	NO ₂ , Binoxide of Nitrogen.....	40
“	“	14	NH ₃ , Ammonia.....	17
Oxygen	O	8	Air = 23.10 of O, and 76.9 per cent of N.....	72
Phosphorous	P	32	PO ₅ , Phosphoric Acid.....	56
“	“	32	PO ₃ , Phosphorous Acid.....	
“	“	32	PH ₃ , Phosphoretted Hydrogen.....	35
Platinum	Pt	99	PtO, Protoxide of Platinum.....	107
“	“	99	PtO ₂ , Binoxide of Platinum.....	115
Potassium, (Rorium)	K	40	KO, Potash.....	48
“	“	40	KCl, Chloride of Potassium.....	76
Silicon	Si	22	SiO ₂ , Silicic Acid or Silica.....	46
Silver, (Argentum)..	Ag	108	AgO, Oxide of Silver.....	116
“	“	108	AgCl, Chloride of Silver.....	144
Sodium, (Natronium)	Na	24	NaO, Soda.	82
“	“	24	NaCl, Chloride of Sodium.....	60
Strontium	Sr	44	SrO, Strontia.....	52
“	“	44	SrCl,.....	80
Sulphur	S	16	SO ₃ , Sulphuric Acid.....	40
“	“	16	HS, Hydrosulphuric Acid.....	17
Tin, (Stannum).....	Sn	59	SnO, Protoxide of Tin.....	67
“	“	59	SnO ₂ , Peroxide of Tin.....	75
Zinc	Zn	32	ZnO, Oxide of Zinc.....	40
“	“	32	ZnCl, Chloride of Zinc.....	68

TABLE, SHOWING THE ACTION OF REAGENTS ON METALLIC OXIDES.

Name of Base of Metal.	Hyd. Sulph. Acid, H ₂ , in acidified sol.	Hyd. Sulph. of Ammonia, NH ₄ HS.	Carbonate of Soda, Na ₂ O, CO ₂ .	Carb. of Ammonia, 2NH ₄ O, 3CO ₂ .	Caustic Potash, KO.	Caustic Ammonia, NH ₃ .	Ferrosy of Potassium, K ₂ FeC ₂ O ₄ .
Alumina.....	No reaction.....	White, insol. in xs.	W. insol xs.....	W. insol.....	W. sol. in xs.....	W. insol xs.....	No reaction.
Antimony.....	Orange red SbS ₃	Or red sol. xs.....	W. sparingly sol.....	W. spar. sol.....	W. spar. sol.....	W. insol.....	White. 0
Baryta.....	0	0	W. insol xs.....	W. insol.....	0	0	White. 0
Bismuth.....	Brown to blk BiS ₃	Brown to black.....	W. insol xs.....	W. insol.....	W. insol.....	W. insol.....	White.
Cadmium.....	Bright yellow CdS.....	Br't yel. insol. xs.	W. insol. xs.....	W. insol.....	W. insol.....	White sol. xs.....	White.
Calcium, (lime).....	0	0	W. insol. xs.....	W. insol.....	0	0	0
Chromium.....	0	Greenish blue.....	Green.....	Green.....	Green sol xs.....	Gr'n'h blue sp. sol.	0
Cobalt.....	0	Black, CoS.....	Pink insol.....	Pink spar sol.....	Blue insol. xs.....	Blue sol. xs.....	Pale green
".....	0	0	Bluish if boiled.....	In excess.....	Boiled, a dirty red	Brownish red.....	Or gray.
Copper, oxide of.....	Black CuS.....	Black.....	Greenish blue.....	Greenish blue.....	Dark br'n if boiled	Pale to fine blue.....	Chocolate
".....	0	0	Dark br'n if boiled	Sol. in xs.....	In excess of KO.....	Sol. in xs.....	Color.
Gold.....	Black AuCl ₃	Br'n-black sol. xs.	0	Yellow insol.....	Yellowish brown.....	Yellow insol.....	0
Iron, peroxide of.....	Yellow white.....	Black.....	Rust colored.....	Rust col. insol.....	Rust col insol.....	Rust colored insol.	Deep blue.
Lead, oxide of.....	Black, PbS.....	Black insol.....	White insol.....	White insol.....	White sol.....	White insol.....	White.
Magnesia.....	0	0	White insol.....	0	White insol.....	White insol.....	0
Mercury, per oxide.....	W becomes blk.....	W. becomes blk.....	Reddish br'n insol.	W. insol.....	Yellow insol.....	White insol.....	White.
Manganese.....	0	Flesh col. insol.....	White insol.....	White insol.....	White to brown.....	White to brown.....	White.
Nickel.....	0	Black NiS.....	Pale green insol.....	Pale green sol.....	Pale green insol.....	Pale green sol.....	Pale green.
Platinum.....	Br'h blk PtCl ₂	Br'o'h blk sol.....	Yellow white.....	Yellow.....	Yellow.....	Yellow.....	0
".....	".....	".....	With carb potassa	".....	".....	".....	".....
Silver.....	Black.....	Black insol.....	White insol.....	White sol.....	Pale br'n insol.....	Pale brown sol.....	White. 0
Strontia.....	0	0	White insol.....	White insol.....	0	0	0
Tin, per oxide.....	Yellow.....	Yellow sol.....	White insol.....	White insol.....	White sol.....	White sol.....	White.
Zinc, oxide of.....	0	White.....	White insol.....	White sol.....	White sol.....	White sol.....	White.

For Potassa, Soda and Ammonia, see p. 72B75.

xs. = in excess.

ANALYSIS OF VARIOUS SOILS.

Name of Soil.	Water.	EARTHS.			Mag- nesia.	OXIDES.		Potash & Soda.	ACIDS.			Chloride Sodium.	Humic acid.	Organic matter.		
		Alu- mina.	Silica.	Lime.		Iron.	Manga- nese.		Sul- phuric.	Car- bonic.	Phos- phoric.					
Clay, potters'.....	11.	27.	61.			1.										
" " porcelain.....	15.	27.	55.	2.		1.										
" " Stourbridge.....	12.6	30.4	57.													
" " pure.....		40.	60.													
" " for tiles.....		38 to 34	62 to 66													
" " loam.....		34 to 28	66 to 72													
Loamy soil.....		28 to 16	72 to 84													
Sandy loam.....		16 to 4	84 to 96													
Sandy soil.....		4.	96.													
Marly soil.....				5 to 20												
Fertile pasture, Hanover.....	1.	9.35	71.85	.99	.25	5.41	.93	.01	.17	.08	3.33	.13		8.82	2.	
" " arable, Ohio.....		5.67	87.14	.56	.31	2.22	.86	.15	.08			.06		2.37		
Barren soil.....		.45	61.58	.32	.13	.52								38.01		
Calcareous soil.....				20.												
Vegetable mould.....																
Fertile alluvial pasture.....	.03	6.44	84.51	.74	.53	5.41	.45	.01	.06					.01	3.78	.96
Good red clover land.....		1.74	98.72	.12	.70	2.06	.32	.17	.01			.10	.05	.89	.12	
Bad ".....		2.65	92.01	.24	.70	3.19	.48	.16	trace			.08	trace	.34	.15	
Very fertile arable in Moravia.....		8.51	77.21	.96	1.96	6.59	1.52	.78	.11				.01	1.52	1.11	
Good cotton soil, Mississippi.....		7.82*	80.55		1.81	1.45	.66	.60	.21			.04	.11		4.74	
Cotton soil exhausted.....		24.*	85.92		.76	.67		.39	.09			trace			6.29	
Green sand.....	11.5	33.93	48.03		1.30			5.61								
Shell marl.....		.60	8.10	45.75		in alumina					35.95				14.6	
Clay marl.....	1.40	8.20	84.9	4.7							3.70				2.8	

NOTE.—Those marked * contain iron not determined.

* Green soil, includes insol. humus.

ANALYSIS OF SOILS.

310s. The fertility of soils is composed of their siliceous matter, phosphoric acid and alkalis. The latter ought to be abundant. The surveyor may judge of the soil by the crops—as follows :

If the straw or stalks lodge, it shows a want of silica, or that it is in an insoluble condition, and requires lime and potash to render it soluble.

If the seeds or heads does not fill, it shows the want of phosphoric acid.

If the leaves are green, it shows the presence of ammonia; but if the leaves are brown, it shows the want of it.

Chemical analysis. By qualitative analysis, we determine the simple bodies which form any compound substance, and in what state or combination.

Quantitative analysis, points out in what proportion these simple bodies are combined.

A body is organic, inorganic, or both.

The body is organic, if when heated on a platinum foil, or clean sheet of iron over a spirit lamp, it blackens and takes fire. And if by continuing the heat the whole is burnt away, we conclude that the substance was entirely organic, or some salt of ammonia.

Soluble in water.—The substance is reduced to powder, and a few grains of it is put with distilled water in a test tube or porcelain capsule; if it does not dissolve on stirring with a glass rod, apply gentle heat. If there is a doubt whether any part of it dissolved, evaporate a portion of the solution on platinum foil; if it leaves a residue, it proves that the substance is partially soluble in water. Hence we determine if it is soluble, insoluble or partially so in distilled water.

Substances soluble in water, are as follows :

Potassa. All the salts of potassa.

Soda. Do. do. do.

Ammonia. (Caustic,) and all the ordinary salts of it.

Lime. Nitrate, muriate, (chloride of calcium.)

Magnesia. Sulphate and muriate.

Alumina. Sulphate.

Iron. Sulphates and muriates of both oxides.

Substances, insoluble, or slightly soluble in water, are as follows :

• *Lime.* Carbonate, phosphate and sulphate of.

Magnesian. Phosphate of ammonia and magnesian.

Magnesia. Carbonate, phosphate of.

Alumina, and its phosphate.

Iron, oxides, carbonate, phosphate of.

Inorganic substances found in plants, as bases, are—alumina, lime, magnesia, potash, soda, oxide of iron, oxides of manganese.

As acids—sulphuric, phosphoric, chlorine, fluorine, and iodine and bromine in sea plants.

Take a wheelbarrowful of the soil from various parts of the field, to the depth of one foot. Mix the whole, and take a portion to analyze.

Proportion of clay and sand in a soil. Take two hundred grains of well dried soil, and boil it in distilled water, until the sand appears to be divided. Let it stand for some time, and decant the liquid. Add a fresh supply of water, and boil, and decant as above, and so continue until the

clay is entirely carried off. The sand is then collected, dried and weighed. For the relative proportion of sand in fertile soils, (see sec. 8091.)

Organic matter in the soil. Take two hundred grains of the dry soil, and heat it in a platinum crucible over a spirit lamp, until the black color first produced is destroyed; the soil will then appear reddish, the difference or loss in weight, will be the organic matter.

Estimation of ammonia. Put one thousand grains of the unburnt soil in a retort, cover it with caustic potash. Let the neck of the retort dip into a receiver containing dilute hydrochloric acid, (one part of pure hydrochloric acid to three parts of distilled water;) bring the neck of the retort near the liquid in the receiver, and distill off about a fourth part; then evaporate the contents of the receiver in a water bath; the salt produced will be sal ammoniac, or muriate of ammonia, of which every one hundred grains contains 82.22 grains of ammonia.

Estimation of silica, alumina, peroxide of iron, lime and magnesia.

Put two hundred grains of the dry soil in a Florence flask or beaker, then add of dilute hydrochloric acid four ounces, and gently boil for two hours, adding some of the dilute acid from time to time as may be required, on account of the evaporation. Filter the liquid and wash the undissolved soil, and add the water of this washing to the above filtrate. Collect the undissolved in a filter, heat to redness and weigh; this will give clay and siliceous sand insoluble in hydrochloric acid.

Estimation of silica. Evaporate the above solution to dryness, then add dilute hydrochloric acid, the white gritty substance remaining insoluble is silica, which collect on a weighed filter, burn and weigh.

Estimation of alumina and peroxide of iron. The solution filtered from the silica is divided into two parts. One part is neutralized by ammonia, the precipitate contains alumina and peroxide of iron, and possibly phosphoric acid. It is thrown on a filter and washed, strongly dried, (not burnt) and weighed; it is now dissolved in hydrochloric acid, and the oxide of iron is precipitated by caustic potash in excess; the precipitate is washed, dried and burnt, its weight gives the oxide of iron, which taken from the above united weight of iron and alumina, will give the weight of the alumina.

The phosphoric acid here is considered too small and is neglected.

Estimation of lime. The liquid filtered from the precipitate by the ammonia, contains lime and magnesia. The lime may be entirely precipitated by oxalate of ammonia. Collect the precipitate and burn it gently and weigh. In every one hundred grains of the weight, there will be 56.29 grains of lime.

Estimation of magnesia. Take the filtered liquid from the oxalate of ammonia, and evaporate to a concentrated liquid, and when cold, add phosphate of soda and stir the solution. Let it stand for some time. Phosphate of magnesia and ammonia will separate as a white crystalline powder. Collect on a filter, and wash with cold water, and burn. In one hundred grains, there are 36.67 grains of magnesia.

Estimation of potash and soda. Take the half of the liquid. Set aside in examining for silica, (see above,) and render it alkaline to test paper by adding caustic barytes, and separate the precipitate. Again, add carbonate of ammonia, and separate this second precipitate, and evapor-

ate the liquid to dryness in a weighed platinum dish; heat the residue gently to expel the ammoniacal salts. Weigh the vessel with its contents; the excess will be the alkaline chlorides, which may be separated if required, by bi-chloride of platinum, which precipitates the potassa as chloride of potassium; one hundred parts of which contain 68.26 of potassa, and one hundred parts of chloride of sodium contain 58.29 of soda.

Estimation of Phosphoric Acid. For this we will use Berthier's method, which is founded on the strong affinity which phosphoric acid has for iron. Let the fluid to be examined contain, at the same time, phosphoric acid, lime, alumina, magnesia, and peroxide of iron. Let the oxide of iron be in excess—to the fluid add ammonia, the precipitate will contain the whole of the phosphoric acid, and principally combined as phosphate of iron. Collect the precipitate, and wash, and then treat with dilute acetic acid, which will dissolve the lime, magnesia, and excess of iron, and alumina, and there will remain the phosphate of iron or phosphate of alumina, because alumina is as insoluble as the iron in acetic acid. Collect the residue and calcine them. In every one hundred grains of the calcined matter, fifty will be phosphoric acid.

Estimation of Chlorine and Sulphuric Acid. These are found but in small quantities in soils, unless gypsum or common salt has been previously applied. Boil four hundred grains of the burnt soil in half a pint of water, filter the solution, and wash the insoluble residue with hot water, then burn, dry, weigh, and compare it with the former weight; this will give an approximate value of the constituents soluble in water. Now acidulate the filtered liquid with nitric acid, and add nitrate of silver; if chlorine is present, it will give a white curdy precipitate, which collect on a filter, wash, dry and burn in a porcelain crucible; the resulting salt, chloride of silver, contains 24.67 grains, in one hundred of chlorine.

Estimation of Sulphuric Acid. To the filtered solution, add nitrate of barytes; a white cloudiness will be produced, showing the presence of sulphuric acid. The precipitate will be sulphate of barytes, which collect, wash, and weigh as above. In one hundred grains of this precipitate, there will be 34.87 of sulphuric acid.

Estimation of Manganese. Heat the solution to near boiling, then mix with excess of carbonate of soda. Apply heat for some time. Filter the precipitate, and wash it with hot water, dry, and strongly ignite with care. The resulting salt, carbonate of manganese = MnO, CO_2 . In every one hundred grains of this salt, there are 62.07 of protoxide of manganese.

Analysis of Magnesian Limestone.

810r. Supposed to contain carbonate of lime, carbonate of magnesia, silica, carbonic acid, iron and moisture.

Weigh one hundred grains of the mineral finely powdered, and dry it in a dish on a sand-bath or stove. Weigh it every fifteen minutes until the weight becomes constant, the loss in weight will be the hygroscopic moisture.

Otherwise. Pulverize the mineral, and calcine it in a platinum or porcelain crucible, to drive off the carbonic acid and moisture.

To determine the Silica. Take one hundred grains. Moisten it with water, and then gradually with dilute hydrochloric acid. When it

appears to be dissolved, add some of the acid and heat it, which will dissolve everything but the silica, which is filtered, washed and weighed.

To determine the Iron. Take the filtrate last used for silica. Neutralize it with ammonia, then add sulphide of ammonium, which precipitates the iron as sulphide of iron, FeS . The solution is boiled with sulphate of soda to reduce the iron to the state of protoxide. Boil so long as any odor is perceptible; then pass a current of HS , which will precipitate the metals of class IV. Collect the filtrate and Boil it to expel the hydrosulphuric acid gas, then boil with caustic soda in excess, until the precipitate is converted into a powder.

Collect the precipitate and reduce it to the state of peroxide, by adding dilute nitric acid; then add caustic ammonia, which precipitates the iron as Fe_2O_3 , then collect and dry at a moderate heat. In every 100 parts of the dried precipitate, there are 70 of metallic iron.

To determine the Lime. Boil the last filtrate from the iron, having made it slightly acid with hydrochloric acid. When the smell of sulphide of ammonium is entirely removed, filter the solution and neutralize the clear solution with ammonia, then add oxalate of ammonia in solution, as long as it will give a white precipitate. We now have all the lime as an oxalate. Boil this solution, and filter the precipitate, and ignite; when cool, add a solution of carbonate of ammonia, and again gently heat to expel the excess of carbonate of ammonia. We now have the whole of the lime converted into carbonate of lime, which has 56 per cent. of lime. Or, dry the oxalate at 212° . When dry, it contains 88.4 per cent. of lime.

NOTE. If we have not oxalate of ammonia, we use a solution of oxalic acid, and add caustic ammonia to the liquid containing the lime and reagent till it smells strong of the ammonia; then we have the lime precipitated as an oxalate, as above.

If we suspect Alumina, the liquid is boiled with NO_5 to reduce the iron to a sesquioxide, (peroxide.) Then boil it with caustic potassa for some time, which will precipitate the iron as Fe_2O_3 , which collect as above.

To determine the Alumina, supersaturate the last filtrate with HCl , and add carbonate of ammonia in excess, which will precipitate the alumina as hydrate of alumina, which collect, dry and ignite; the result is Al_2O_3 = sesquioxide of alumina, which has 53.85 per cent. of alumina.

To determine the Magnesia. In determining the lime, we had in the solution, hydrochloric acid and ammonia, which held the magnesia in solution; we now concentrate the solution by evaporation, and then add caustic ammonia in excess. Phosphate of soda is then added as long as it gives a precipitate. Stir the liquid frequently with a glass rod, and let it rest for some hours. The precipitate is the double phosphate of ammonia and magnesia. Wash the precipitate with water, containing a little free ammonia, because the double phosphate is slightly soluble in water. When the prec. is dried, ignite it in a porcelain crucible, and then weigh it as phosphate of magnesia = 2MgO , PO_5 . By igniting as above, the water and ammonia are driven off, and the double phosphate is reduced to phosphate of magnesia. In every 100 grains are 17.86 of magnesia. (**NOTE.** This simple method is from Bowman's Chemistry.)

To determine the Carbonic Acid. Take 100 grains and put them into a bottle with about 4 ounces of water. Put about 60 grains of hydro-

chloric acid into a small test tube and suspend it by a hair through the cork in the bottle, and so arranged that the mouth of the test tube will be above the water. Let a quill glass tube pass through the cork to near the surface of the liquid in the bottle. Weigh the whole apparatus, and then let the test tube and acid be upset, so that the acid will be mixed with the water and mineral. The carbonic acid will now pass off; but as it is heavier than air, a portion will remain in the bottle, which has to be drawn out by an India-rubber tube applied to the mouth, when effervescence ceases. The whole apparatus is again weighed; the difference of the weights will be the carbonic acid.

Analysis of Iron Pyrites.

810u. This may contain gold, copper, nickel, arsenic, besides its principal ingredients, sulphur and iron, and sometimes manganese.

To determine the Arsenic. Reduce a portion of the pyrites to fine powder; heat it in a test tube in the flame of a spirit lamp. The sulphur first appears as a white amorphous powder, which becomes gradually a lemon yellow, then to tulip red, if arsenic is present.

To determine the Sulphur. One hundred grains of the pyrites are digested in nitric acid, to convert the sulphur into sulphuric acid; dilute the solution, and decant it from the insoluble residue, which consists in part of gold. If any is in the mineral, it is readily seen through a lens.

This decanted solution will contain the iron, together with oxides of copper, if any is present, and the sulphur as sulphuric acid. Evaporate the solution to expel the greater part of the nitric acid, now dilute with three volumes of water, and add chloride of barium as long as it causes a precipitate. Boil the mixture; filter, wash and ignite the precipitate, which is now sulphate of baryta, in every 100 parts of which there are 18.67 of sulphur. To this sulphur, must be added the sulphur that was found on top of the liquid as a yellow porous lump when digested with the nitric acid.

To determine the Iron. Add sulphide of ammonium as long as it will cause a precipitate of sulphide of iron = FeS , whose equivalent is 44; that is, iron 28 and sulphur 16; therefore every one hundred parts of FeS contain 68.63 of iron. But heat to redness and weigh as per oxide of iron = Fe_2O_3 . In every 100 grains there are 70 of iron.

NOTE. Sulphide of ammonia precipitates manganese.

To determine the Manganese and Iron separately. Take a weighed portion and dissolve it in aqua regia as above, evaporate most of the acid, and then dilute, leaving the solution slightly acid; pass H_2S through it, which will precipitate the gold, copper and arsenic, and leave the iron and manganese in solution. Collect the filtrate, to which add chlorate of potassa to peroxide of iron; now add acetate of soda, and then heat to a boiling point; this precipitates the iron, and that alone as peroxide of iron, which collect, wash, dry, weigh, and heat to redness; the result is Fe_2O_3 , having 70 per cent. of iron.

To find the Manganese, neutralize the last filtrate, and add hypochlorite of soda, let it stand for one day, then the manganese will be precipitated as binoxide of manganese = MnO_2 ; collect, dry, etc. In every 100 grains of it, there are 68.68 of manganese.

Analysis of Copper Pyrites.

810v. *The moisture* is determined as in sec. 810r.

To determine the Sulphur. Proceed as in sec. 810v, by reducing 100 grains to powder, then boil in aqua regia until the sulphur that remains insoluble collects into a yellowish porous lump. Dilute the acid with three volumes of water, filter and wash the insoluble residue (consisting of sulphur and silica) until the whole of the soluble matter is separated from it. Reserve the insoluble residue for further examination.

Now evaporate the filtered solution so as to expel the nitric acid, and add some hydrochloric acid from time to time, so as to have HCl in a slight excess. From this solution precipitate the sulphur, as sulphuric acid, by chloride of barium, (as in 810v.) Collect the precipitate, wash, dry and weigh, as has been done for iron pyrites.

To determine the Copper. To the filtered solution add hydrosulphuric acid, which precipitates the copper as sulphide of copper = CuS . This precipitate is washed with water, saturated with HS. The precipitate and ash of the filter is poured into a test tube or beaker, and a little aqua regia added to oxidize the copper. Then boil and add caustic potassa, which will precipitate the copper, as black oxide of copper, CuO , having 79.84 per cent. of copper.

To determine the sulphur and siliceous matter in the above residue. Let the residue be well dried and weighed, then ignited to expel the sulphur; now weighed, the difference in weight will be the sulphur, which, added to the weight of sulphur found from the sulphate of baryta, will give the whole of the sulphur.

The siliceous matter is equal to the weight of the above residue after being ignited.

To determine the Iron. The solution filtered from the sulphide of copper is now boiled to expel the hydrosulphuric acid, filtered, and then heated with a little nitric acid to reduce the iron to a state of peroxide. To this add ammonia in slight excess, which precipitates the iron as a peroxide. This filtered, dried and weighed, will contain, in every 100 grains, 70 grains of iron; because $40 : 28 :: 100$. Here 28 is the atomic weight of iron, and 40 that of sesquioxide of iron = $\text{Fe}_2\text{O}_3 = 56 + 24 = 80$; but 80 and 56 are to one another as 40 is to 28.

Those marked with an asterisk (*) are the most delicate tests.

810w. *Sulphuret of Zinc*, (blende) may contain Iron, Cadmium, Lead, Copper, Cobalt and Nickel.

The mineral is dissolved in aqua regia. Collect the sulphur as in sec. 810v, and expel the NO_5 by adding HCl and evaporating the solution, which dilute with water, and again render slightly acid by HCl. To this acid solution (free from nitric acid) add HS, which precipitates all the copper, lead and cadmium, and leaves the iron, manganese and zinc in solution. Let the precipitate = A.

To determine the Iron, neutralize the solution with ammonia, and precipitate the iron by caustic ammonia, or better by succinate of ammonia. Collect the precipitate, and heat to redness in the open air, which will give peroxide of iron = Fe_2O_3 , which has 70 per cent. of iron.

To determine the Zinc. The last filtrate is to be made neutral, to which add sulphide of ammonium, which precipitates the zinc from magnesia,

lime, strontia or baryta, as sulphide of zinc. Pour the filtrate first on the filter, then the precipitate. Collect, dry and heat to redness, gives oxide of zinc = ZnO , having 80.26 per cent. of zinc.

We may have in the reserved precipitate A, copper, lead and cadmium.

To determine the Cadmium. Dissolve A, in NO_6 , and add carbonate of ammonia in excess, which will precipitate the cadmium. Collect the precipitate and call it B. To the filtrate add a little carbonate of ammonia, and heat the solution when any cadmium will be precipitated, which collect and add to B, and heat the whole to redness to obtain oxide of cadmium, which has 87.45 per cent. of cadmium.

To determine the Copper, make the last filtrate slightly acid. Boil the solution now left with caustic ammonia, collect and heat to redness, the result will be oxide of copper CuO , having 80 per cent. of Cu.

To determine the Lead. The lead is now held in solution, render it slightly acid and pass a current of HS , which will precipitate black sulphide of lead; if any = PbS , which collect and heat to redness to determine as oxide of lead = PbO , which has 92.85 per cent. of lead.

To separate Zinc from Cobalt and Nickel. The mineral is oxidized as above, and then precipitated from the acid solution by carbonate of soda. The precipitate is collected and washed with the same reagent, so as to remove all inorganic acids. The oxides are now dissolved in acetic acid, from which HS will precipitate the zinc as sulphide of zinc = ZnS , which oxidize as above and weigh.

To separate the oxides of Nickel and Cobalt. Let the oxides of nickel and cobalt be dissolved in HCl , and let the solution be highly diluted with water; about a pound of water to every 15 grains of the oxide. Let this be kept in a large vessel, and let it be filled permanently with chlorine gas for several hours, then add carbonate of baryta in excess; let it stand for 18 hours, and be shaken from time to time. Collect the precipitate and wash with cold water; this contains the cobalt as a sesquioxide, and the baryta as carbonate. Reserve the filtrate B. Boil the precipitate with HCl , and add SO_3 , which will precipitate the baryta and leave the cobalt in solution, which precipitate by caustic potassa, which dry and collect as oxide of nickel.

The nickel is precipitated from the filtrate B, by caustic potassa, as oxide of nickel, which wash, dry and collect as usual.

To separate Gold, Silver, Copper, Lead and Antimony.

310x. The mineral is pulverized and dissolved in aqua regia, composed of one part of nitric acid and four parts of hydrochloric acid. Decant the liquid to remove any siliceous matter. Heat the solution and add hydrochloric acid which will precipitate the silver as a chloride, which wash with much water, dry and put in a porcelain crucible. Now add the ash of the filter to the above chloride of silver, on which pour a few drops of NO_5 , then warm the solution and add a very few drops of HCl to convert the nitrate of silver into chloride of silver. Expel the acid by evaporation. Melt the chloride of silver and weigh when cooled. When washed with water any chloride of lead is dissolved; but if we suspect lead, we make a concentrated solution, and precipitate both lead and silver as chlorides by HCl ; then dissolve in NO_5 and precipitate the lead by caustic potassa as oxide of lead, leaving the silver in solution, which if acidified,

and HS passed through it, will precipitate the silver as sulphide of silver which heat to redness, and weigh as oxide of silver.

To determine the Gold. We suppose that every trace of NO_5 is removed from the last filtrate and that it is diluted. Then boil it with oxalic acid, and let it remain warm for two days, when the gold will be precipitated, which collect and wash with a little ammonia to remove any oxalate of copper that may adhere to the gold. Heat the dried precipitate with the ash of the filter to redness, and weigh as oxide of gold AuO , which has 96.15 per cent. of gold.

To determine the Copper. To the last filtrate diluted, add caustic potassa at the boiling point, which will precipitate the copper. Wash the precip. with boiling water, dry, heat to redness, and weigh as protoxide of copper = CuO . In every 100 grains there are 79.84 grains of copper.

To separate Lead and Bismuth.

The mineral is first dissolved in NO_5 , then add SO_3 in excess, and evaporate until the NO_5 is expelled. Add water, then the lead is precipitated as sulphate of lead, which collect, etc. In every 100 grains there are 68.28 of lead.

The bismuth is precipitated from the filtrate by carbonate of ammonia. The precipitate is peroxide of bismuth = Bi_2O_3 , which collect, etc. This precip. has 89.91 per cent. of bismuth.

To determine the Antimony. Let a weighed portion be dissolved in NO_5 . Add much water and evaporate to remove the acid, leaving the solution neutral. Now add sulphide of ammonium, which precipitates the alumina, cobalt, nickel, copper, iron and lead. Collect the filtrate, to which add the solution used in washing the precipitate. Concentrate the amount by evaporation and render it slightly acid. Then add hydrochloric acid, which precipitates the silver as a chloride, leaving the antimony in solution, which is precipitated by caustic ammonia as a white insoluble precip. SbO_3 , which, when dried, etc., contains 84.31 per cent. of antimony.

NOTE. The caustic ammonia must be added gradually.

For the difference between antimony and arsenic, see p. 72879.

To determine Mercury.

810x. Mercury is determined in the metallic state as follows: There is a combustion furnace made of sheet iron about 8 inches long, 5 inches deep, and 4 inches wide. There is an aperture in one end from top to within 2 inches of the bottom, and a rest corresponding within 1 inch of the other end. A tube of Bohemian glass is opened at one end, and bent and drawn out nearly to a point at the other. The bent part is to be of such length as to reach half the depth of a glass or tumbler full of water and ice, into which the fine point of the reducing tube must be kept immersed during the distillation of the mercury. Fill the next inch to the bottom or thick end with pulverized limestone and bicarbonate of soda; then put in the mineral or mercury. Next 2 inches of quick or caustic lime, then a plug of abestoes. The tube is now in the sheet-iron box and heated with charcoal, first heating the quick lime, next the mineral, and lastly the limestone and soda. Allow the process to go on some time, until the mercury will be found condensed in the glass of water, which collect, dry on blotting paper, and weigh.—*Graham's Chemistry.*

Otherwise. Dissolve the mineral in HCl. Add a solution of protochloride of tin in Cl in excess, and boil the mixture. The mercury is now reduced to the metallic state, which collect as above.

To determine Tin.

Dissolve in HCl and precipitate with HS in excess, letting it remain warm for some hours. Collect the precipitate and roast it in an open crucible, adding a little NO_5 so as to oxidize the tin and the other metals that may be present. To a solution of the last oxide, add ammonia and then sulphide of ammonium, which will hold the tin in solution and precipitate the other metals of class 8. See p. 72874.

If we suspect antimony in the solution, the reagent last used must be added slowly, as antimony is soluble in excess of the reagent.

W A T E R.

810z. Distilled water is chemically pure. Ice and rain water are nearly pure. Distilled water at a temperature of 60° has a specific gravity of 1000. That is, one cubic foot weighs 1000 ounces = $62\frac{1}{2}$ lbs., containing 6.282 imperial gallons = 7.48 United States gallons.

NOTE. Engineers in estimating for public works, take one cubic foot of water = $6\frac{1}{4}$ imperial gallons, and one cubic foot of steam for every inch of water.

Water, at the boiling point, generates a volume of steam = to 1689 times the volume of water used. The volume of steam generated from one inch of water will fill a vessel holding 7 gallons.

Water presses in all directions. Its greatest pressure is at two-thirds of the depth of the reservoir, measured from the top. The same point is that of percussion.

Greatest density of water is at $89^\circ 30'$, from which point it expands both ways. Ice has a specific gravity of 0.918 to 0.950. The water of the Atlantic Ocean has a specific gravity of 1.027; the Pacific Ocean = 1.026; the Mediterranean (mean) = 1.0285; Red Sea, at the Gulf of Suez = 1.089.

MINERAL WATERS, are carbonated, saline, sulphurous and chalybeate.

Carbonated, is that which contains an abundance of carbonic acid, with some of the alkalies. This water reddens blue litmus, and is sparkling.

Saline, is that in which chloride of sodium predominates, and contains soda, potassa and magnesia.

Sulphurous, is known by its odor of rotten eggs, or sulphuretted hydrogen, and is caused by the decomposition of iron pyrites, through which the water passes. The vegetation near sulphur springs has a purple color.

Chalybeate, is that which holds iron in solution, and is called *carbonated* when there is but a small quantity of saline matter. It has an inky taste, and gives with tincture of galls, a pink or purple color. It is called *sulphated* when the iron held in solution is derived from iron pyrites, and is found in abundance with the smell of sulphuretted hydrogen. The chalybeate waters of Tunbridge and Bath in England, derive their strong chalybeate taste from one part of iron in 35,000 parts of water, or two grains of iron in one gallon of the water. Water travers-

ing a mineral country, is found to contain arsenic, to which, when found in chalybeate, chemists attribute the tonic properties of this water. Hoffman finds one grain of arsenic per gallon in the chalybeate well of Weisbaden. Mr. Church finds one grain of arsenic in 250 gallons of the river Whitbeck in Cumberland, England, which water is made to supply a large town.

Arsenic has been found in 46 rivers in France. The springs of Vichy, of Mont d'Or and Plombiers, contain the 125th part of a grain of arsenic in the gallon.

If lime is present, oxalate of ammonia gives a white precip.

If chloride of sodium, nitrate of silver gives a precip. not entirely dissolved in nitric acid.

If an alkaline carbonate, such as bicarbonate of lime.

Arsenic nitrate of silver gives a primrose yellow precip.

An alkaline solution of logwood, gives a violet color to the water if lime is present. The solution of logwood gives the same reaction with bicarbonate of potassa and soda. To distinguish whether lime or potassa and soda are present, we add a solution of chloride of calcium, which gives no precipitate with bicarbonate of lime.

Sulphuric acid, is present, if, after sometime, nitrate of baryta gives a precip. insol. in nitric acid.

Carbonate of lime is present, if the water when boiled appears milky. Lime water as a test, gives it a milky appearance.

Organic matter is precipitated by terchloride of gold, or a solution of acetate of copper, having twenty grains to one ounce of water. After applying the acetate of copper, let it rest for 12 hours; at the end of which time all the organic matter will be precipitated.

Organic matter may be determined by adding a solution of permanganate of potassa, which will remain colored if no organic matter is present; but when any organic substance is held in solution, the permanganate solution is immediately discolored. We make a permanganate solution by adding some permanganate of potassa to distilled water, till it has a deep amethyst red tint. We now can compare one water with another by the measures of the test, sufficient to be discolored by equal volumes of the waters thus compared.

Carbonates of lime and magnesia, also sulphate of lime, act injuriously on boilers by forming incrustations.

The presence of chloride of sodium and carbonate of lime in small quantities, as generally found in rivers, is not unhealthy.

M. Boussingault has proved that calcareous salts of potable water, in conjunction with those contained in food, aid in the development of the bony skeleton of animals. *Taylor* says that the search for noncalcareous water is a fallacy, and that if lime were not freely taken in our daily food, either in solids or liquids, the bones would be destitute of the proper amount of mineral matter for their normal development.

Where the water is pure, lead pipes should not be used, as the purest water acts the most on lead. Let there be a slip of clean lead about six to eight inches square immersed in the water for 48 hours, and exposed to the air. Let the weight before and after immersion be determined, and then a stream of sulphuretted hydrogen made to pass through the

water and then into the supposed lead solution, which will precipitate the lead as a black sulphide of lead.

Taylor says, that water containing nitrates or chlorides in unusual quantity, generally acts upon lead.

Water in passing through an iron pipe, loses some if not all of its carbonic acid, thereby forming a bulky prec. of iron, which is carried on to meet the lead where it yields up its oxygen to the lead, forming oxide of lead, to be carried over and supplied with the water, producing lead disease.

It is to be hoped that iron supply pipes or some others not oxidizable, will be used.

HYDRAULICS.

SUPPLY OF TOWNS WITH WATER.

310z. Water is brought from large lakes, rivers or wells. That from small lakes is found to be impure, also that from many rivers. A supply from a large lake taken from a point beyond the possibility of being rendered impure is preferable, provided it is not deficient in the mineral matter required to render it fit for culinary purposes. The water must be free from an excess of mineral, or organic matter, and be such as not to oxidize lead.

Solid matter in grains per gallon, are as follows in some of the principal places:

Loch Katrine in Scotland,	2	Danube at Vienna,	10.15
Loch Ness in Annandale,	2	Scheldt, Belgium,	20.88
River Thames at London,	28.86	Schuylkill, Philadelphia.	4.49
" " Greenwich,	27.79	Croton, N. Y.,	4.16
" " Hampton,	15	Chicago river,	20.75
Mean of 4 English rivers,	20.75	Lake Michigan 2 miles out,	8.01
Rhone at Lyons, France,	12.88	Cochituate at Boston,	8.12
Seine at Paris,	20	St. Lawrence, near Montreal,	11.04
Garonne at Toulon,	9.56	Ottawa, " "	4.21
Rhine at Basle,	11.97	Hydrant at Quebec,	2.5

Water drawn from wells contains variable quantities of mineral matter, which, according to *Taylor*, is from 180 to 140 grains in wells from 40 to 60 feet deep. The artesian wells which penetrate the London clay, contain from 50 to 70 grains in the imperial gallon.

Catch basin, or water shed, is that district area whose water can be impounded and made available for water supply. One-half the rain-fall may be taken as an approximate quantity to be impounded, which is to be modified for the nature of the soil and local evaporation.

Mr. Hawkesly in England collects 48 per cent. of the rain-fall.

Mr. Stirrat in Scotland, finds 67 " "

In Albany, U. States, 40 to 60 per cent. may be annually collected.

The engineer will consult the nearest meteorological observations.

ANNUAL RAIN-FALL.

310a*. The following table of mean annual rain-fall is compiled from authentic sources. That for the United States is from the Army Meteorological Register for 1855.

Penzance, England,	48.1	Santa Fe, New Mexico,	19.8
Plymouth, "	35.7	Ft. Deroloe, "	16.6
Greenwich, "	23.9	Ft. Yuma, "	10.4
Manchester, "	27.3	San Diego, "	12.2
Keswick, Westmoreland,	60	Monterey, "	24.5
Applegate, Scotland,	33.8	San Francisco, California,	23.5
Glasgow, "	33.6	Hancock Barracks, Maine,	37
Edinburgh, "	25.6	Ft. Independence, Mass.,	35.3
Glencose, Pentlands, Scotland,	36.1	Ft. Adams, Rhode Island,	52.5
Dublin, Ireland,	30.9	Ft. Trumbull, Connecticut,	45.6
Belfast, "	35	Ft. Hamilton, N. Y.,	43.7
Cork, "	36	West Point, "	54.2
Derry, "	31.1	Plattsburgh, "	38.4
St. Petersburg, Russia,	16	Ft. Ontario, "	30.9
Rome, Italy,	36	Ft. Niagara, "	31.8
Pisa, "	37	Buffalo, "	38.9
Zurich, Switzerland,	32.4	Ft. Mifflin, Penn.,	45.3
Paris, France,	21	Ft. McHenry, Maryland,	42
Grenada, Central America,	126	Washington City,	41.2
Calcutta, E. Indies,	77	Ft. Monroe, Virginia,	50.9
Detroit, Michigan,	30.1	Ft. Johnston, N. Carolina,	46
Ft. Gratiot, "	32.6	Ft. Moultrie, South Carolina,	44.9
Ft. Mackinaw, Michigan,	23.9	Oglethorp, Georgia,	53.3
Milwaukee, Wis.,	30.8	Key West, Florida,	47.7
Ft. Atkinson, Iowa,	39.7	Ft. Pierce, "	68
Ft. Des Moines, "	26.6	Mt. Vernon, Alabama,	63.5
Ft. Snelling, Minnesota,	25.4	Ft. Wood, Louisiana,	60
Ft. Dodge, "	27.3	Ft. Pike, "	71.9
Ft. Kearney, Nebraska,	28	New Orleans, "	50.9
Ft. Laramie, "	35	Ft. Jessup, "	45.9
Ft. Belknap, Texas,	22	Ft. Town, Indian Territory,	51.1
Brazos Fork, "	17.2	Ft. Gibson, "	36.5
Ft. Graham, "	40.6	Ft. Smith, Arkansas,	42.1
Ft. Croghan, "	36.6	Ft. Scott, Kansas,	42.1
Corpus Christi, Texas,	41.1	Ft. Leavenworth, Kansas,	30.3
Ft. McIntosh, "	18.7	Jefferson, Missouri,	37.8
Ft. Filmore, New Mexico,	9.2	St. Louis, "	42
Ft. Webster, "	14.6		

Daily supply of water to each person in the following cities :

New York, 52 gallons. Boston, 97. Philadelphia, 86. Baltimore, 25. St. Louis, 40. Cincinnati, 80. Chicago, 43. Buffalo, 48. Albany, 69. Jersey City, 59. Detroit, 31. Washington, 19. London, 30.

Reservoirs. The following is a list of some of the principal reservoirs with their contents in cubic feet and days' supply :

Rivington Pike, near Liverpool, 504,960,000 cubic feet, holds 150 days' supply.

Bolton, 21 millions cubic feet = 146 days' supply.

Belmont, 75 million cubic feet = 136 days' supply.

Bateman's Compensation, near Manchester, has 155 million cubic feet.

Bateman's Crowdon, near Manchester, 18,493,600 cubic feet.

Bateman's Armfield, near Manchester, 38,755,556 cubic feet.

Longendale, 292 million cubic feet = 74 days' supply.

Preston, 4 reservoirs, 26,720,000 cubic feet = 180 days' supply.

Compensation, Glasgow, 12 millions cubic feet.

Oroton, New York, 2 divisions, 24 millions cubic feet.

Chicago, Illinois, the water will be, in 1867, taken from a point two miles from the shore of Lake Michigan, in a five-foot tunnel, thirty-two feet under the bottom of the Lake, thus giving an exhaustless supply of

pure water. The water now supplied is taken from a point forty-five feet from the shore, and half a mile north of where the Chicago River enters Lake Michigan, consequently the supply is a mixture of sewage, animal matter and decomposed fish, with myriads of small fish as unwelcome visitors.

CONDUITS OR SUPPLY MAINS.

310b*. Best forms for open conduits, are semi-circle, half a square, or a rectangle whose width = twice the depth, half a hexagon, and parabolic when intended for sewerage. (See sec. 183.)

Covered conduits ought not to be less than 8 feet wide and $3\frac{1}{2}$ high, so as to allow a workman to make any repairs. A conduit 4 feet square with a fall of 2 feet per mile, will discharge 660,000 imperial gallons in one hour. The conduit may be a combination of masonry on the elevated grounds, and iron pipes in the valleys; the pipes to be used as syphons.

The ancients carried their aqueducts over valleys, on arches, and sometimes on tiers of arches. They sometimes had one part covered and others open. Open ones are objectionable, owing to frost, evaporation and surface drainage.

DISCHARGE THROUGH PIPES AND ORIFICES.

310c*. *Pipes under pressure.* Pipes of potter's clay, can bear but a light pressure, and therefore are not adapted for conveying water.

Wooden Pipes, bear great pressure, but being liable to decay, are not to be recommended.

Cast Iron Pipes, should have a thickness as follows: $t = 0.08289 + 0.015 D$. Here d = diameter, and t = thickness of the metal. *D'Aubisson's Hydraulics.* $t = 0.0238, d + 0.33$. According to Weisbach.

Claudel gives the following, which agrees well with Beardmore's table of weight and strength of pipes. $t = 0.00025 h d$ for French metres. $t = 0.00008 h d$ for English feet. Here t = thickness, h = total height due to the velocity, and d = diameter.

Lead Pipes, will not bear but about one-ninth the pressure of cast iron, and are so dangerous to health, as to render them unfit to be used for drawing off rain water, or that which is deficient in mineral matter.

The pressure on the pipe at any given point, is equal to the weight of a column of water whose height is equal to that of the effective height, which is the height, h diminished by the height due to the velocity in the pipe.

$Pressure = h - 0.15,536 v^2$. Here v is the theoretical velocity.

Torricellis' Fundamental Formula, is

$V = \sqrt{2 g h}$ for theoretical velocity.

$v = m \sqrt{2 g h}$ for practical or effective velocity.

The value of $2g$ is taken at 64.408 as a mean from which it varies with the latitude and altitude.

The value of g can be found for latitude L , and altitude A , assuming the earth's radius = R .

$$g = 32.17 (1.0029 \cos. 2 L) \times \left(1 - \frac{2 A}{R}\right)$$

$$g = 20887600 \text{ (1.0016 Cos. 2 L)}$$

$$v = m \sqrt{2gh} = 8.025 \text{ m } \sqrt{h} = \text{mean velocity.}$$

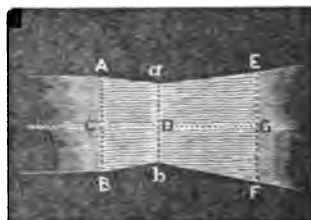
$$Q = 8.025 \text{ A m } \sqrt{h} = \text{discharge in cubic feet per second.}$$

$$A = \frac{Q}{8.025 \text{ m } \sqrt{h}} = \text{sectional area.}$$

$$\sqrt{h} = \frac{Q}{8.025 \text{ m A}} \text{ from which } h \text{ is found.}$$

The value of m , the coefficient of efflux is due to the *vena contracta*. Its value has been sought for by eminent philosophers with the following result: As the prism of water approaches an outlet, it forms a contracted vein, (*vena contracta*) making the diameter of the prism discharge less than that of the orifice, and the quantity discharged consequently less by a multiplier or coefficient, m . The value of m is variable according to the orifice and head, or charge on its centre.

Vena Contracta. The annexed figure shows the proportions of the contracted vein for circular orifices, as found by Michellotti's latest experiments. A B is the entrance, and $a b$ the corresponding diameter at outlet; that is the theoretical orifice, A B, is reduced to the practical or actual one, $a b$. When A B = 1, then C D = 0.50, and $a b = 0.787$; therefore the area of the orifice at the side A B = $1 \times .785$ and that at $a b = .787^2 \times 0.7854$; that is the theoretical is to the actual as 1 is to 0.619 $\therefore m = 0.619$.



The values of m have been given by the following:

Dr. Bryan Robinson, Ireland, in 1739,	gives $m = 0.774$.
Dr. Mathew Young, do. 1788,	" .628.
Venturi, Italy,	" .622.
Abbe Bossuet, France,	" .618.
Michellotti, Italy,	" .616.
Eytelwein, Germany,	" .618.
Castel, France, 1836,	" .644.
Marriot, do	" .692.
Rennie, England,	" .625.
Xavier, France,	" .615.

NOTE. It is supposed that Dr. Robinson used thick plates, chamfered or rounded on the inside, thereby making it approach the *vena contracta*, and consequently increasing the value of m or coefficient of discharge.

Rejecting Robinson and Marriot's, we have a mean value of $m = 0.622$, which is frequently used by Engineers.

Taking a mean of Bossuet, Michellotti, Eytelwein and Xavier, we find the value of $m = 0.617$, which appears to have been that used by Neville in the following formulas, where A = sectional area of orifice, r = radius, Q discharge in cubic feet per second, h = height of water on the centre of the orifice, and $m = 0.617$ = coefficient of discharge.

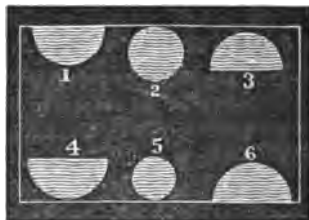
When $h = r$,	then $Q = 8.025 m \sqrt{H} \times .960 A$.
Do. 1.25 r ,	do. do. .978 A.
Do. 1.5 r ,	do. do. .978 A.
Do. 1.75 r ,	do. do. .989 A.
Do. 2 r ,	do. do. .992 A.
Do. 3 r ,	do. do. .996 A.
Do. 4 r ,	do. do. .998 A.
Do. 5 r ,	do. do. .9987 A.
Do. 6 r ,	do. do. .9991 A.

Hence it appears, that when $h = r$, the top of the orifice comes to the surface, and that when h becomes greater or equal to $8r$, that the general equation $Q = 8.08 m \sqrt{H} \times A$, requires no modification.

The following 6 formulas are compiled from Neville's Hydraulics.

In the annexed figure, 1, 3, 4 and 6 are semi-circular, and 2 and 5 are circular orifices.

The value of Q may be found from the following simple formulas, where A is the area of each orifice, and $m = 0.617 =$ the coefficient of efflux.



1. $Q = 8.0218 A \sqrt{r}$.
2. $Q = 4.7553 A \sqrt{r}$.
3. $Q = 3.6264 A \sqrt{r}$.
4. $Q = 4.9514 \sqrt{h} \times A \cdot \left(1 + \frac{1}{4.712} \cdot \frac{r}{h} - \frac{1}{82} \cdot \frac{r^2}{h^2}\right)$
5. $Q = 4.9514 \sqrt{h} \times A \cdot \left(1 - \frac{1}{32} \cdot \frac{r^2}{h^2} - \frac{5}{1024} \cdot \frac{r^4}{h^4}\right)$
6. $Q = 4.9514 \sqrt{h} \times A \cdot \left(1 - \frac{1}{4712} \cdot \frac{r}{h} - \frac{1}{82} \cdot \frac{r^2}{h^2}\right)$

Adjutages, with cylindrical tubes, whose lengths = $2\frac{1}{2}$ times their diameters, give $m = 0.815$.

Michellotti, with tubes $\frac{1}{2}$ an inch to 8 inches diameter and head over centre of 8 to 20 feet, found $m = 0.813$.

The same result has been found by Bidone, Eytelwein and D'Aubisson.

Weisbach, from his experiments, gives $m = 0.815$. Hence it appears that cylindrical tubes will give 1.325 times as much as orifices of the same diameter in a thin plate.

For tubes in the form of the contracted vein, $m = 1.00$.

For conical tubes converging on the exterior, making a converging $<$ of $13\frac{1}{2}^\circ$, $m = 0.95$.

For conical diverging the narrow end toward the reservoir and making the diverging $< = 5^\circ 6'$, $m = 1.46$, and the inner diameter to the outer as 1 is to 1.27.

NOTE. The *adjutage* or tube, must exceed half the diameter (that length being due to the contracted vein) so as to exceed the quantity discharged through a thin plate.

Circular Orifices. $Q = 3.908 d^2 \sqrt{h}$.

Cylindrical adjutage as above. $Q = 5.168 d^2 \sqrt{h}$.

Tube in the form of vena contracta. $Q = 5.673 d^2 \sqrt{h}$.

In a compound tube, (see fig., sec. 310c*) the part A a b B is in the form of the contracted vein, and a b E F a truncated cone in which $DG = 9$ times $a b$ and $EF = 1.8$ times $a b$. This will make the discharge 2.4 times greater than that through the simple orifice. (See Byrne's Modern Calculator, p. 321.)

Orifices Accompanied by Cylindrical Adjutages.

When the length of the adjutage is not more than the diameter of the orifice, then $m = 0.62$.

Length 2 to 3 times the diameter, $m = 0.82$.	36 times $m = 68$.
Do. 12 do. $m = .77$.	48 " $m = 63$.
Do. 24 do. $m = .73$.	60 " $m = 60$.

810D*. *Orifices Accompanied with Conical Converging Adjutages.*

When the adjutage converges towards the extremity, we find the area of the orifice at the extremity of the adjutage the height h of the water in the reservoir above the same orifice. Then multiply the theoretical discharge by the following tabular coefficients or values of m :

Let A = sectional area, then $Q = m A \sqrt{2gh} = 8.03 m A \sqrt{H}$.

Angle of Convergence	Coefficients of the		Angle of Convergence	Coefficients of the	
	Discharge.	Velocity.		Discharge.	Velocity.
0° 0'	.829	.880	13° 24'	.946	.962
1 36	.866	.866	14 28	.941	.966
3 10	.895	.894	16 36	.938	.971
4 10	.912	.910	19 28	.924	.970
5 26	.924	.920	21 00	.918	.974
7 52	.929	.931	23 00	.913	.974
8 58	.934	.942	29 58	.896	.975
10 20	.938	.950	40 20	.869	.980
12 40	.942	.955	48 50	.847	.984

The above is Castel's table derived from experiments made with conical adjutages or tubes, whose length was 2.6 times the diameter at the extremity or outlet. In the annexed figure A C D B represents Castel's tube where $m n$ is 2.6 times $C D$ and angle $A O B = \angle$ of convergence.

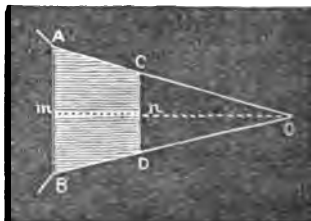
NOTE. It appears that when the angle at O is $13\frac{1}{2}$ degrees the coefficient of discharge will be the greatest.

The discharge may be increased by making $m n$ equal to $C D$, $A B = 1.2$ times $C D$, and rounding or chamfering the sides at A and B .

In the next two tables, we have reduced Blackwell's coefficient from minutes to seconds, and call $C = m$. $Q = 8.03 m A \sqrt{h}$ or $Q = C A \sqrt{h}$, where C is the value of $8.03 m$ in the last column. h is always taken back from the overfall at a point where the water appears to be still.

Experiments 1 to 12, by Blackwell, on the Kennet and Avon Canal.

Experiment 13, by Blackwell and Simpson, at Chew Magna, England.



310*. OVERFALL WEIRS, COEFFICIENT OF DISCHARGE.

No.	Description of Overfall.	Head in inches.	Value of m	Value of $8.08 m = C$.
1	Thin plate 8 feet long.	1 to 8	.440	8.588
	" " "	8 to 6	.402	8.228
2	" 10 feet long.	1 to 8	.501	4.028
	" " "	8 to 6	.485	8.498
	" " "	6 to 9	.370	2.971
3	Plank 2 inches thick with a notch 8 feet long.	1 to 8	.842	2.746
	" " "	8 to 6	.884	3.088
	" " "	6 to 10	.406	3.260
4	Plank 2 in. thick, notch 6 ft.	1 to 8	.859	2.888
	" " "	8 to 6	.896	3.179
	" " "	6 to 9	.892	3.148
	" " "	9 to 14	.858	2.878
5	Pl'k 2 in. thick, notch 10 ft.	1 to 8	.846	2.778
	" " "	8 to 6	.897	3.191
	" " "	6 to 9	.874	3.008
	" " "	9 to 14	.886	2.698
6	Same as 5, with wing walls	1 to 2	.476	3.822
	" " "	4 to 5	.442	3.549
7	Overfall with crest 8 feet. Wide sloping 1 in 12—8 ft. Long like a weir.	1 to 8	.842	2.746
	" " "	8 to 6	.828	2.684
	" " "	6 to 9	.811	2.497
8	Same as 7, but slopes 1 in 18	1 to 8	.862	2.907
	" " "	8 to 6	.845	2.787
	" " "	6 to 9	.832	2.666
9	Same as 7 & 8 but 10 ft long	1 to 4	.828	2.634
	" " "	4 to 8	.850	2.810
10	Level crest 8 ft w. & 6 long	1 to 8	.805	2.449
	" " "	8 to 6	.811	2.497
	" " "	6 to 9	.818	2.553
11	" " "	8 to 7	.830	2.649
	" " "	7 to 12	.810	2.489
12	Same as 11 but 10 ft. long.	1 to 5	.806	2.457
	" " "	5 to 8	.827	2.626
	" " "	8 to 10	.818	2.518
13	Overfall bar 10 feet long And 2 inches thick.	1 to 8	.487	3.509
	" " "	8 to 6	.499	4.007
	" " "	6 to 9	.505	4.055

BLACKWELL'S SECOND EXPERIMENTS.

Overfall of cast iron, 2 inches thick, 10 ft. long, square top. Canal had wing walls, making an angle of 45 degrees.

Head in feet.	Coeff. m	Head in ft.	Coeff. m	Head in ft.	Coeff. m
.083 to .078	.591	.844	.743	.500	.749
.083 to .088	.626	.859	.760	.516	.748
.182 to .187	.682	.865	.741	.521	.747
.229	.665	.861	.750	.578	.772
.244	.670	.875	.725	.689	.717
.240	.655	.416	.780	.667	.802
.242	.653	.423	.781	.784	.737
.245	.654	.451	.749	.745	.750
.250 to .252	.725	.453	.751	.750	.781
.838	.745	.495	.728		

From the above we have a mean value of $m = 0.723$.

The reservoir used on the Avon and Kennet canal, in England, contained 106,200 square feet, and was not kept at the same level, but the quantity discharged for the experiment was not more than 444 cubic feet, which would reduce the head but .05 inch. In the Chew Magna we have an area of 5717 square feet kept constantly full by a pipe 2 inches in diameter from a head of 19 feet. The inlet of the pipe to the overfall being 100 feet, consequently the water approaches the fall with a certain degree of velocity, which partially accounts for the difference in value of m , in experiments 18 and 5.

Poncelet and Lebros' experiments on notches, 8 inches long, open at top:

Size of Notches.	Coefficient m .	Size of Notches.	Coefficient m .
8 $\times$ 0.4	.686	8 $\times$ 8.2	.595
8 $\times$ 0.8	.625	8 $\times$ 4.	.592
8 $\times$ 1.2	.618	8 $\times$ 6.	.590
8 $\times$ 1.6	.611	8 $\times$ 8.	.585
8 $\times$ 2.4	.601	8 $\times$ 9.	.577

From these small notches we have a mean value of $m = .603$.

Du Buat's experiments on notches 18.4 long, give a mean coefficient $m = .682$.

Smeaton and Brindley, for notches 6 inches wide and 1 to $6\frac{1}{2}$ high, give $m = .637$.

Rennie, for small rectangular orifices, gives as follows:

Head 1 to 4 feet, orifice 1 inch square, mean value of $m = .618$.

" " " 2 inches long and $\frac{1}{2}$ high, $m = .618$.

" " " 2 inches long and $\frac{1}{2}$ deep, $m = .682$.

The following table is from *Poncelet and Lebros' experiments on covered orifices in thin plates*. Width of the orifice .20 metre (about 8 inches) $l =$ length, and $h =$ height of the orifice.

810r*.

HEIGHT OF THE ORIFICES.

Head on centre of orifice.	0.20 m $l = h$.	0.01 m $l = 2h$	0.05 m $l = 4h$	0.08 m $l = 6.7h$	0.02 m $l = 10h$	0.01 m $l = 20h$
m	m	m	m	m	m	m
0.02					.660	.698
.03				.688	.660	.691
.04			.612	.640	.659	.685
.05			.617	.640	.659	.682
.06		.590	.622	.644	.658	.678
.08		.600	.626	.639	.657	.671
.10		.605	.628	.638	.655	.667
.12	.572	.609	.630	.637	.654	.664
.15	.585	.611	.631	.635	.653	.660
.20	.592	.613	.634	.634	.650	.655
.30	.598	.616	.632	.632	.645	.650
.40	.600	.617	.631	.631	.642	.647
.50	.602	.617	.631	.630	.640	.643
.70	.604	.616	.629	.629	.637	.638
1.00	.605	.615	.627	.627	.632	.627
1.80	.604	.613	.623	.623	.625	.621
1.60	.602	.611	.619	.619	.618	.616
2.00	.601	.607	.613	.613	.613	.613
3.00	.601	.603	.606	.607	.608	.609

Here the water takes the form of the hydraulic curve, nearly that of a parabolic, and its sectional area = $\frac{2}{3} l h$. The co-efficient increases as the orifice approaches the sides or bottom.

Let C = coeff. of perfect contraction, and C' = coeff. of partial contraction, then $C' = C + 0.01$.—Neville.

The presence of a *courseir*, mill-race, or channel, has no sensible effect on the discharge, when the head on its centre is not below .50 to .60 metres, for orifice of .20 to .15 metres high, .30 to .40 for .10 metres high, and .20 for .05 metres high.

The charge on the centre is seldom below the above.—*Morin's Aide Memoire*, p. 27.

310f. *Example 10: From Neville's Hydraulics*, p. 7.—What is the discharge in cubic feet per minute from an orifice 2 ft. 6 in. long, and 7 in. deep; the upper edge being 3 in. under the surface of apparent still water in the reservoir.

$l h = 2.5 \text{ ft.} \times 7'' = \text{area, } S \text{ of orifice} = 1.458 \text{ square feet.}$

$H = \text{half of } 7'' + 3 = 6.5 \text{ in.} = 0.541666 \text{ ft.} = \text{surface of the water in the reservoir above the centre of the orifice. The square root of}$

$0.541666 = \sqrt{H} = 0.736$. Head on centre of orifice = 6.5 in. = 165 metres. Ratio of length of orifice to its height = 4. Then opposite, 165 metres, and under $l = 4 h$, find $m = 0.616$

$Q = 8.03 \times 0.616 \times 1.458 \times 0.736 = \text{cubic ft. per second.}$

$Q = 481.8 \times 0.616 \times 1.458 \times 0.736 = \text{cubic ft. per minute.}$

Neville makes $m = 0.628$, and $Q = 320.4$ cubic feet.

M. Boileau, in his *Traite de la Mesure des eaux courantes*, (Paris, 1854,) recommends Poncelet and Lebros' value of m in the general formula.

$Q = m A \sqrt{2 g h}$ or $Q = m l h \sqrt{2 g h}$

Complete contraction is when the orifice is removed 1.5 in. to twice its lesser diameter of the fluid vein.

The French make $m = .625$ for sluices near the bottom, discharges either above or under the water.

Castel has found that 3 sluices in a gate did not vary the value of m .

310g. Let $R = h y d$, mean depth; V = surface velocity, by Sec. 312; D = diam.; r = radius of circular orifices; v = mean, and w = bottom velocities; Q = discharge in cubic feet per second; T = time in seconds; A = area of section of conduit; I = the head; per unit = height divided by the horizontal distance between the reservoir and out-let.

$v = 0.90 V$ for rectangular canals, and $v = 0.903 V$ for those with earthen slopes.—Boileau.

$v = .80 V$ for large channels, by Prony.

$v = 0.835 V$ for large channels, by Ximes, Funk, and Bruning.

V = surface, W = bottom velocities.

$v = 0.80 V$, and $W = .60 V$, by Conference on Drainage and Irrigation at Paris in 1849 and 1850.

$Q = 8.025 m A \sqrt{h}$ is the general formula where A = sectional area.

Q = quantity in cubic feet; h = height of reservoir; m = co-efft. of efflux.

$Q = 8.025 m A T \sqrt{h}$ in time T .

$R I = 0.00002427 V + 0.000111416 V^2$ all in feet, Eytelwein; from which he gives $V = \frac{10}{11} \sqrt{R f}$ in which formulas he puts $R = h y d$, mean depth, f = twice the fall in feet per mile, and I = inclination, = head divided by the length.

$V = \frac{10}{11} \sqrt{Rf}$ is used by Beardmore and many Engineers.

310g. For clear, straight rivers, with average velocities of 1.5, *Neville* gives $V = 92.3 \sqrt{R 1}$, and for large velocities $V = 93.3 \sqrt{R 1}$. He says that co-effs. decrease rapidly when velocities are below 1.5 ft. per second. In his second edition of his valuable treatise on hydraulics, he states that the best formula proved by experiments for discharges over weirs is,

$$Q = 1.06 (3h + V a^2)^{2/3} - V a^3. \quad \text{Here } V a = \text{velocity of approach.}$$

310h. *M. Boileau*, in his *Traité de la Mesure des eaux courantes*, p. 345: For discharge through orifices,

O = sectional area of reservoir at still water, h = diff. of level between the summit of the section O and that of the section (*remous d'aval*), where the ripple begins.

$$Q = A \sqrt{2g \frac{h}{1 - \frac{A^2}{O^2}}} = 8.025 A \sqrt{\frac{h O^2}{O^2 - A^2}}$$

In his tables he makes the value of m , coefft. of contraction for short *remous*, or eddy, = 0.622, 0.600 when it attains the summit, and 0.688 when the orifice is surrounded by the *remous*.

310ii. Let Q = the quantity in feet per second.

$Q = 8.025 m \sqrt{h}$ = effective discharge in cubic ft. per second, m = variable.

$Q = 4.879 A \sqrt{h}$ orifice surrounded on all sides, $m = 0.608$

$Q = 5.048 A \sqrt{h}$ orifice surrounded on three sides, $m = 0.629$

$Q = 5.489 A \sqrt{h}$ orifice coincides with sides and bottom, $m = 0.684$

$Q = 5.939 A \sqrt{h}$ as last sluice makes angle 60° against stream, $m = 0.740$

$Q = 6.420 A \sqrt{h}$ as last but. sluice makes the angle 45° , $m = 0.800$

$Q = 5.016 A \sqrt{h}$ sluice vertical, orifice near the bottom, $m = 0.625$

$Q = 4.253 A \sqrt{h}$ 2 sluices, or orifices, within 10 ft. of each other, $m = 0.530$

$Q = 6.019 A \sqrt{h}$ the flood gates make 160° with the current, and $m = 0.750$ that there are 3 sluices guarded to conduct the water into the buckets of a water wheel = sum of the areas.

$T v = 5.35 m \sqrt{h}$ = mean vel. for regular orifices, open at top, and is the time required to empty a given vessel when there is no efflux, and is double the time required to empty the same when the vessel or reservoir is kept full.

$T = \frac{A \sqrt{h}}{4.013 m S}$ Where S = sectional area of orifice, and A = that of the reservoir.

$t = A \left\{ \frac{\sqrt{H} - \sqrt{h}}{4.013 m S} \right\}$ = time required to fall a given depth, $H - h$

$Q = 8.025 t m S \cdot \left\{ \frac{8.025 t m S}{4 A} + \sqrt{h} \right\}$ = discharge in time t .

$Q = 8.025 m S \sqrt{H - h}$ when reservoir A discharges into A' under water.

$T = \frac{A \sqrt{H}}{4.013 m S}$ = time required to fill the inferior A' .

$t = \frac{A \cdot A' \sqrt{H - h}}{4.013 m S \sqrt{A - A'}}$ time to bring both to the same level in canal locks.

$V = 5.35 \sqrt{(h + 0.0349410 w^2)}$. Here the water comes to the reservoir with a given velocity, w .

310i. *For D'Arcy's Formula, see p. 264.*

He has given for $\frac{1}{2}$ inch, pipes $m = 65.5$ and $v = 65.5 \sqrt{rs}$

For 1" diameter $v 80.3 \sqrt{rs} = m \sqrt{rs}$

2", $m = 94.8$, 4" $m = 101.7$, 6" $m = 105.3$

for 9", $m = 107.8$, 12" $m = 109.3$, 18" $m = 110.7$

24" diam. $v = 111.5 \sqrt{rs} = m \sqrt{rs}$

for large pipes $v = \left\{ \frac{rs}{0.00007726} \right\}^{\frac{1}{2}} = 118 \sqrt{rs}$

310i. Neville's general formula for pipes and rivers:

$v = 140 (ri)^{\frac{1}{2}} - (ri)^{\frac{1}{2}}$ here $r = h y d$, mean depth, and $i =$ inclination.

Frances, in Lowell, Mass., has found for over falls, $m = .623$. (See his valuable experiments made in Lowell.)

Thompson, of Belfast College, Ireland, has found from actual experiments that for triangular notches, $m = 0.618$, and $Q = 0.317 h^{5.3} =$ cubic feet per minute, and $h =$ head in inches.

M. Girard says it is indispensable to introduce 1.7 as a co-efficient, due aquatic plants and irregularities in the bottom and sides of rivers. Then the hydraulic mean depth (see Sec. 77,) is found by multiplying the wetted peremeter by 1.7 and dividing the product into the sectional area.

A velocity of $2\frac{1}{2}$ feet per second in sewers prevents deposits.—*London Sewerage.*

310j. *Spouting Fluids.*—Let T = top of edge of vessel, and B = bottom, O = orifice in the side, and BS = horizontal distance of the point where the water is thrown. (See fig. 60.)

$BS = 2 \sqrt{TO} \cdot OB = 2 OE$, by putting OE for the ordinate through O, making a semi-circle described on FB.

310k. *On the application of water as a motive power:* $Q =$ cubic ft. per minute, $h =$ height of reservoir above where the water falls on the wheel, $P =$ theoretical horse-power.

$$P = 0.00189 Q h, \text{ and } Q = \frac{528 P}{h}$$

Available horse-power = 12 cubic ft., falling 1 ft. per second, and is generally found = to 66 to 73 per cent. of the power of water expended. Assume the theoretical horse-power as 1, the effective power will be as follows:

Over-shot wheels = .68	For turbine wheels, .70
Under-shot wheels, .35	For hydraulic rams in raising water, .80
Breast wheels, .55	Water pressure engines, .80
Poncelet's under-shot .60	High breast wheels, .60

Let $P =$ pressure, in lbs., per square inch.

$$P = Q, 4333 h \text{ and } h = 2.31 p$$

$P = .00128 Q h$ for over-shot wheels, and $Q = 777 P$ divided by h

$P = .00113 Q h$ for high-breast wheels, and $Q = 882 P$ divided by h

$P = .00101 Q h$ for low-breast wheels, and $Q = 962 P$ divided by h

$P = .00066 Q h$ for under-shot wheels, and $Q = 1511$ divided by h

$P = .00113 Q h$ for Poncelet's undershot wheels, and $Q = 822$ divided by h

For under-shot wheels, velocity due to the head $\times 0.57$ will be equal to the velocity of the periphery, and for Poncelet's, 0.57 will be the multiplier.

310j. *High-pressure turbines for every 10-horse power.*

$h =$	30	40	50	60	70	80	90	100
$Q =$	4.2	3.1	2.5	2.1	1.8	1.6	1.4	1.25
$V =$	36	42	47	51	55	59	63	66

We have seen, S.-E. of Dedham, in Essex, England, a small stream collected for a few days, in a reservoir, thence passed on an over-shot wheel, and again on an undershot wheel. If possible, let the reservoirs be surrounded by shade trees, to prevent evaporation.

310k. *Artesian Wells* may be sunk and the water raised into tanks to be used for household purposes, irrigating lands, driving small machinery, and extinguishing fires.

310L. *Reservoirs* are collected from springs, rivers, wells, and rain-falls, impounded on the highest available ground, from whence it may be forced to a higher reservoir, from which, by gravitation, to supply inhabitants with water.

310p. *Land and City Drainage.*

In draining a hilly district.—A main drain, not less than 5 ft. deep, is made along the base of the hill to receive the water coming from it and the adjacent land; secondary drains are made to enter obliquely into the main, these ought to be 4 to 5 ft. deep, filled with broken stones to a certain height; tiles and soles, or pipes. The first form is termed French draining; the last two mentioned are now generally used. In 1838 to 1842 we have seen, near Ipswich, England, drains made by digging 4 feet deep, the bottom scooped 2 to 3 inches and filled with straw made in a rope form, over this was laid some brushwood, then the sod, and then carefully filled.

The French drains were sometimes 15 inches deep, 5 inches at bottom and 8 inches at top, all filled with stone, then covered with straw and filled to the top with earth.

In tile draining the sole is about 7 inches wide, always $\frac{3}{4}$ in. on each side of the tile, and is about 12 to 15 inches long, its height is to be one-fourth its diameter. The egg shape is preferable. Never omit to use the tile, let the ground be ever so hard.

Pipe Drains.—Pipes of the egg shape are the best; pipes 2 to 4 in. diameter have a 4 in. collar. In retentive land put 4 feet deep and 27 feet apart; when $3\frac{1}{2}$ feet deep, put 33 feet apart.

From the best English sources we find the comparative cost. $2\frac{1}{2}$ ft. deep cost $3\frac{1}{2}$ pence, add $1\frac{1}{2}$ pence for every additional 6 inches in depth. Profit by thorough drainage is 15 to 20 per cent. See Parliamentary Report.

310q. *In draining Cities and Towns* our first care is to find an outlet where the sewage can be used for manure, and to avoid discharging it into sluggish streams. The result of draining into the river Thames, and the Chicago river with its far-famed Healy slough ought to be sufficient warning to Engineers to beware of like results. (See Sec. 310j.) Where the city or town authorities are not prepared to use the sewage as a fertilizer, and that there is a river near, or through it, let there be intercepting sewers, egg-shaped, with sufficient fall to insure $2\frac{1}{2}$ feet per second, which in London is found sufficient to prevent deposit; should not exceed $4\frac{1}{2}$ feet per second. When these main sewers get to a considerable depth, the sewage is lifted from these into small, covered res-

ervoirs, thence to be conveyed to another deep level, and so on until brought far enough to be discharged into the river, or some outlet from which it cannot return. But we hope it will not be wasted; the supply of Guano will fail in a few years, then the people will have to depend on the home supply.

Sewers under 15 inches diameter are made of earthenware pipes, with collars, laid in cement; 2 foot diameter are 4 inches, or half a brick, thick; 3 to 5 feet, 8 inches thick; 6 to 8 feet, 12 inches thick, according to the nature of the earth. Where the soil is quick-sand, the bottom ought to be sheeted, to prevent the sinking of the sewer.

As the sewers are made, connecting pipes are laid for house drainage at about every 20 feet, and man-holes at proper intervals to allow cleansing, flushing, and repairing. A plat is on record, showing the location of each sewer, with its connections, man-holes, and grade of bottom, to guide house and yard drains or pipes, whose fall is one-quarter inch per foot, in Chicago.

310Q. *Irrigation of Land.*

In wet districts the land is cut up in about 10-acre tracts; the ditches deep; ponds made at some points to collect some of the water, these ponds to be surrounded by a fence and shade trees, such as willow and poplar, a place on the North side of it may be sloped, and its entrance well guarded with rails, so that cattle may drink from, but not wade in, the pond, which may be of value in raising fish.

$v = 55 \sqrt{2af}$ and $Q = va$. Here v = vel. in feet, a = area, and f = fall in feet per mile.

In irrigating, the land is laid off and levelled so that the water may pass from one field to another, and may be overflowed from sluices in canals fed from a reservoir or river. The water from a higher level, as reservoir, may be brought in pipes to a hydrant, where the pressure will be great enough to discharge, through a hose and pipe, the required quantity in a given time. Water or sewage can be thus applied to 10 acres in 12 hours by one man and two boys.

The profit by irrigation is very great,—witness the barren lands near Edinburgh, in Scotland, and elsewhere.

In England, on irrigated land, they grow 50 to 70 tons of Italian rye grass per acre. Allowing 25 gallons of water to each individual will not leave the sewage too much diluted, and 60 to 70 persons will be *sufficient for one acre*, applied 8 times a year. At the meeting of the Social Science Association in England, in 1870, it was decided that the sewage must be taken from the fountain head, as they found it too much diluted, and that alum and lime had been used to precipitate the fertilizing matter, but had failed. They estimated the value due to each person at $8\frac{1}{3}$ shillings, but in practice realized but 4 to 5 shillings.

Mr. Rawlinson recommended its application diluted; others advocated the dry earth closet system, which in small towns is very applicable, owing to the facility of getting the dry earth and a market for the soil.

310R. The supply of guano will, in a few years, be exhausted, then necessity will oblige nations to collect the valuable matter that now is wasted. See Sec. 310J.

310s. *On the Steam Engine.*

H = horse-power capable of raising 33000 pounds 1 ft. high in 1 minute.

P = pressure in pounds per square inch.

D = diameter of cylinder piston in inches.

A = area of cylinder or its piston.

S = length of stroke, and 2 S = total length travelled.

R = number of revolutions per minute.

V = mean vel. of piston in feet per minute.

Q = total gallons (Imperial) raised in 24 hours.

q = quantity raised by each stroke of the piston.

C = pounds of coal required by each indicated horse-power.

$$H = \frac{2 S A P R}{33000} = \text{indicated horse-power.}$$

$$H = \frac{D^2 \sqrt[3]{S}}{15.6} \text{ indicated horse-power for high-pressure engines.}$$

$$D = \frac{15.6 H}{\sqrt[3]{S}} \text{ and } V = 128 \sqrt[3]{S}$$

$$H = \frac{D^2 \sqrt[3]{S}}{47} \text{ for condensing engines, from which we have}$$

$$D = \frac{\sqrt[3]{47 H}}{\sqrt[3]{S}} \text{ and } V = 128 \sqrt[3]{S}$$

$$\text{Admiralty Rule. } H = \frac{D^2 V}{6000} = \text{nominal horse-power.}$$

The American Engineers add one-third for friction and leakage.

Example. The required gallons in 12 hours = 3,000,000; Stroke, 10 feet; number of strokes per minute = 12; time in minutes = 1440. From the above, we find $q = 173.6$ Imperial gallons; $d = 22.6$ inches—the diameter of the pump, as taken by the American engineers; $d = 22$, as taken by the English.

For much valuable information on the steam engine, see Appleton's (Byrne's) Dictionary of Mechanics, and Haswells' tables.

Average duty of a Cornish engine is 70 million lbs., raised one foot high, with 112 lbs. of bituminous coal.

Example. From Pole on the Cornish Engine, as quoted by Hann on the Steam Engine.

Cylinder, 70 inches diameter; stroke, 10 feet; pressure per square inch, 45 lbs. during one-sixth the stroke, and during the remainder the steam is allowed to expand.

$$70 \times 70 \times 0.7854 = \text{area of piston} = 3858 \text{ square inches.}$$

$$3848 \times 45 \times \frac{10}{6} = \text{pounds raised one foot high} = 288,600.$$

This is the work performed before the steam is cut off.

To find the work done by expansion.—Find from a table of Hyperbolic Logarithms for $C = 1.7916$, which, multiplied by the work done before the steam is cut off, will give the work required, that is, $1,7916 \times 288600$

$$\text{Work done after the steam is cut off, } 517102$$

310T. *Pressure of Fluids and Retaining Walls.*

(DEF.—Retaining Wall is that which sustains a fluid, or that which is liable to slide.)

310. *The Centre of Pressure* is that point in the surface pressed by any fluid, to which, if the whole pressure could be applied, the pressure would be the same as if diffused over the whole surface.

If to this centre a force equal to the whole pressure be applied, it will keep it in equilibrium.

Against a rectangular wall the centre of pressure is at two-thirds of the height from the top, and the

Pressure $P = \frac{h^2}{2} \cdot l w$. Here w = specific gravity of the fluid, and l the length pressed.

In a cylindrical vessel or reservoir the same formula will hold good, by substituting the circumference for the length, l , of the plane.

Example.—For a lock-gate 10 ft. long, 8 ft. deep, the pressure

$$P = \frac{64}{2} \times 10 \times 62.5 = 20,000 \text{ pounds.}$$

Example.—For a circular reservoir, diameter 20 ft., depth 10 ft., filled with water, we have

$$P = \frac{10 \times 10 \times 20 \times 3.1416 \times 62.5}{2} = 196,350 \text{ lbs., the pressure on the sides of the reservoir.}$$

The pressure on the bottom = $20 \times 20 \times .7854 \times 62.5 = 19,635 \text{ lbs.}$

Total pressure, 215,985 lbs.

Dams are built at right angles to the stream entering the reservoir. All places of a porous nature are made impervious to water by clay or masonry laid in cement; *top* to be 4 ft. above the water; *width*, in ordinary cases, equal to one-third the height; the *inner* slope, next the water, to be 3 to 1; the outer slope 2 to 1. *In low Dams*, width at top equal to the height.

Dams, in Masonry, by the French Engineers, *Morin* and *Rondelet*, at bottom $0.7 h$, at middle, $0.5 h$, and top, $0.3 h$.

310U. Thickness of rectangular walls is found from

$$t = 0.865 (H - h) \cdot \sqrt{\frac{1000}{w}} \quad \text{Here } 1000 = \text{weight of a cubic ft. of water.}$$

w = weight of 1 cubic foot of masonry, and t = required thickness,
 H = total height, and h = height from top of dam to water.

Foundations of Basins and Dams are to rest on solid clay, sometimes on concrete, laid with puddled clay. The side next the water is laid with stones 12 inches deep, laid edgewise; sometimes they are laid with brick in cement, the outer face covered with sod. A puddled wall is brought up the middle whose base = one-third the height, and top = one-sixth the height; the top is made to curve, to carry off the rain water.

Waste-weir is regulated with a waste-gate, and made so as to carry off the surplus water; the sluice or gate may be made self-acting. *Bywash* receives the surface water from the waste-weir, and from the supply streams when not required to enter the reservoir in times of heavy rains and when the water becomes muddy.

310M. *Cascade.* Let f = fall from crest of weir, h , as usual, the height of still water above the crest of the weir, $v = 5.35 \sqrt{h}$ and $x = \frac{4}{3} \sqrt{h f}$ = distance to which the water will leap; this distance is to be covered with large stones, to break the fall of the water.

310c. *Retaining Walls* are sometimes built along the base of the dam.

St. Ferrel Reservoir, destined to feed the Languidoc Canal, in France, contains 1541 million gallons of water; the dam at its highest part is 106.2 feet.

One reservoir in *Ancient Egypt* contains 33,200 million cubic feet of water. Some are in Spain holding 35 to 40 million cubic feet—similar ones are found in France. The *Chinese* collect water into large reservoirs for the supply of towns and cities, and the irrigation of their lands.

The *Hindoos* have built immense reservoirs to meet the periodical scarcity of rain, which happens once in about five years. One of their reservoirs, the Veranum, contains an area of 35 square miles, made by a dam 12 miles long. The evaporation in India for 8 months is $\frac{1}{2}$ inch in depth per day. One-fourth of an inch may be a safe calculation in milder or colder climates.

In *Dams of Masonry*, buttresses are made at every 18 to 20 feet. Depth = the thickness of the wall, and length = double the thickness. Mahan and Barlow, in their Treatise on Engineering, say, "It is better to put the material uniformly into the wall."

310U. To find the thickness of a rectangular wall, *AB*, to resist its being turned over on the point *D*. (See Fig. 70.) Let the perpendicular, *EF*, pass through the centre of the rectangle; by Sec. 313 it passes through the centre of gravity *G*, makes *CP* = one-third of *BC*. We have the vertical pressure = weight of the wall, and the lateral pressure equal to that of the pressing fluid or mass. Let *w* = specific gravity of the water, and *W* that of the wall. We have the pressure of the fluid represented by *HD* = *CP*, and that of the wall by *DF*, and *TDH* is a bent lever of the first order.

By Section 319c, $P : W :: \frac{DC}{2} : \frac{BC}{3}$

and $\frac{P \times BC}{3} = \frac{DC \times W}{2}$ clear of fractions.

then $P = \frac{3 DC \times W}{2 BC}$

and $DC = AB = \frac{P \times 2 BC}{3 W}$

We have the value of $P \times 2 BC$ per lineal foot, and find the value of $3 W$ for height, *BC*, and one foot thick, which, divided into $P \times 2 BC$, will give the value of *AB* or *DC* when on the point of turning over.

Let *w* = weight of material, and *S* = weight of water; *h* = height of wall = that of the water, and *b* = breadth of wall required, then we have

$P = \frac{h^2}{2} \cdot 62\frac{1}{2}$ lbs. = pressure of water against the wall, and

$P = \frac{3b \times hbw}{2h} = \frac{3b^2w}{2}$

$62\frac{1}{2} \cdot \frac{h}{2} = \frac{3b^2w}{2}$

$$62.5 h^2 = 3 b^2 W$$

$$b = \left\{ \frac{62.5 h^2}{3 W} \right\}^{\frac{1}{2}} = h \sqrt{\frac{62.5}{3 W}}$$

$$h = b \sqrt{\frac{3}{62.5}}$$

Example.—Height of dam and water = 20 ft.; specific gravity of water = $62\frac{1}{2}$ lbs., and that of the masonry 120 lbs.—to find thickness b .

$$b = \left\{ \frac{62.5 \times 20 \times 20}{3 \times 120} \right\}^{\frac{1}{2}} = 8.33 \text{ feet.}$$

As this formula gives but the thickness, to form an equilibrium, add one foot to the thickness, for safety.

Rondelet recommends, to find the required thickness of 1.8 times the calculated pressure, which in this case would be 28800, which divided by 263, gives b^2 79.33088, whose square root = 8.91 feet. We prefer to use *Rondelet's* formula for safety.

310u*.

REVETMENT WALLS.

In retaining walls we have to support water, but in revetment walls we have to support moveable matter, such as sand, earth, etc. (See fig. 71) Let C = tangent squared of half the angle of repose, which may be taken at $22\frac{1}{2}$ deg., which angle is called the angle of rupture, as shown by *Coulomb* and others. The angle of $V D W$ is the angle of repose, and the angle $W D S$ being half the angle, $W D S$ is the angle of rupture, and the line $D S$ = line of rupture. Assume the angle $W D S = 22\frac{1}{2}^\circ$ whose tangent squared equals $.41421 \times .41421 = 0.1715699$, nearly 0.1716, which we take for the coefficient of c in the following formula: b = width at top

$$b = h \times \left\{ \frac{c w}{3 W} \right\}^{\frac{1}{2}} \quad \text{And } h = b \left\{ \frac{3 W}{c w} \right\}^{\frac{1}{2}} \quad \text{And } P = \frac{h^2 W c}{2} \times \frac{1}{2}$$

$$b = h \left\{ \frac{0.1716 w}{3 W} \right\}^{\frac{1}{2}} \quad \text{And } h = b \left\{ \frac{3 W}{0.1716 c} \right\}^{\frac{1}{2}} \quad \text{And } P = \frac{0.1716 h^2 w}{2}$$

Here w = specific gravity of the material to be sustained, and W = that of the wall $C = 0.625$ for water. 0.410 for fine dry sand. 0.350 earth in its natural state; and for earth and water mixed, 0.40 to 0.65. To the value of b thus found the English engineers add for safety about one-sixth of it.

310ul. When the wall has an external batter. Let t equal the mean thickness; then we have:

$$t = ch \sqrt{\frac{w}{W}} = ch \sqrt{\frac{w}{W}} \text{ for a vertical wall.}$$

$$t = 0.95 ch \sqrt{\frac{w}{W}} = ch \sqrt{\frac{w}{W}} \text{ batter 1 in 16.}$$

$$t = 0.90 \sqrt{\frac{w}{W}} \quad \text{" 1 in 14.}$$

$$t = 0.86 \sqrt{\frac{w}{W}} \quad \text{" 1 in 12.}$$

$$\begin{aligned}
 t &= 0.83 \sqrt{\frac{w}{W}} \quad \text{" 1 in 10.} \\
 t &= 0.80 \sqrt{\frac{w}{W}} \quad \text{" 1 in 8.} \\
 t &= 0.76 \text{ ch } \sqrt{\frac{w}{W}} \quad \text{" 1 in 6.}
 \end{aligned}$$

From the mean thickness t , take half the total batter, and it will give the thickness at top; and to t add the half batter it will give the thickness at the base.

310w2. Where there is a surcharge running back from the walls at a slope of $1\frac{1}{2}$ to 1. Column A for hewn stone or rubble laid in mortar. B for well scabbled rubble in mortar, or brick. Col. C, well scabbled dry rubble. Col. D the same as A. Col. E the same as B. Columns A, B, and C are from the English. Cols. D and E are from *Poncelet*. H = total height of the walls and surcharge. h = that of a rectangular wall above the water. *Poncelet* has the surcharge:—

WHEN.	A	B	C	D	E
$H = h$	$0.35h$	$.40h$	$.50h$	$.35h$	$.45h$
$H = 1.2h$	$.46h$	$.51h$	$.61h$	$.44h$	$.55h$
$H = 1.4h$	$.51h$	$.56h$	$.66h$	$.53h$	$.67h$
$H = 1.6h$	$.54h$	$.59h$	$.69h$	$.62h$	$.78h$
$H = 1.8h$	$.56h$	$.61h$	$.71h$	$.67h$	$.85h$
$H = 2.h$	$.58h$	$.63h$	$.73h$	$.71h$	$.93h$

WALLS OF DAMS.

310w3. *Morin* in his *Aide Memoire*, gives for thickness at base

$t = 0.865 (H-h) \sqrt{\frac{1000}{p}}$. Here H = height of the wall and h = height from the surface of the water to the top of the wall. 1000 = specific weight of one kilogramme of water, and p = specific weight of one kilogramme of the masonry.

Example wall four metres high. $h = 0.50$ m. $p = 2000$,
 $t = 0.865 \times (4.0 \text{ met} - 0.50) \sqrt{\frac{1000}{2000}} = 2.04$ metres.

310w4. DRY WALLS are made one-fourth greater than those laid in mortar.

310w5. *Line of resistance in a wall or pier.* (See fig. 71.)

Let PQ = the direction of the pressure P , which is supported by the wall. The line EF passing through the centre of gravity meet PQ at G . Make GL = the pressure P , and GH = pressure by the weight of the wall $ABCD$. Complete the parallelogram $GHLK$. Join GK and produce it to meet the base CD at M . Then M is a point in the line of resistance.

310w6. *The celebrated Vaubani* in his walls of fortifications, makes $MF = \frac{4}{9}$ of CF . F being where the line through the vertical of the centre of gravity of the wall intersects the base.

Let w = weight of the wall. h = BD . b = AB . a = angle PGE . d = SE and x = MF .

$$x = P \cdot \frac{h \sin a - d \cos a}{wbh + P \cos a} -$$

310u6a. *The greatest height to which a pier can be built*, is when the line of resistance intersects the base at C, that is, when H is a maximum, $x = \frac{1}{2}b$ MF must not exceed from 0.3 to 0.375 the thickness of CD.

Vauban in his walls of fortifications makes the base 0.7h. At the middle 0.5h, and at the top 0.3h.

310u6b. In fig. 72. Let CE = nat. slope. G = centre of gravity of the triangular piece to be supported. Draw FGR parallel to CE, then the triangular wall BCR will be a *maximum* in strength. And by making BA = 1.5 to 2 ft. and producing EB to O, making AO = OR and describing the curve AKR the figure ABCRK will be a strong and graceful wall.

310u7. (See fig. 72.) *Rondelet's Rules*.—Assume the nat. slope to be 45 degrees. In the parallelogram BCDE draw the diagonal CE. When the wall is rectangular, then BA = CR = one-sixth of CE.

When the wall batters 2 inches per foot AB = one-ninth do.

do do do 1 1-2 inches per foot AB = one-eighth do.

The English Engineers, make their walls less than the French. They put 1-15 1-10 respectively where *Rondelet* has 1-8 and 1-9. When the batter is one inch per foot, the English make AB = one-eleventh of CE.

For dry walls, make AB = 2-3 of CE, never less than one-half; and in order to insure good drainage, ought to be built of large stones, and batter three inches per foot.

310u8. *Colonel Wurmb* in his *Military Architecture*, gives

$$T = 0.845 h \tan. \frac{a}{2} \sqrt{\frac{w}{W}}, \text{ and } t = T + \frac{nh}{10}.$$

Here T = thickness of a rectangular wall and t = that of a sloping one at the base, n = ratio of batter to h and $\frac{a}{2}$ = half the complement of the angle of repose = WDS. (fig. 71.)

310u9. *Safety pressure per square foot*. White marble 83,000 lbs.; variegated do. 129,000 lbs.; veined white do. 17,400 lbs.; Portland stone 30,000 lbs.; Bath stone 17,000 lbs.

Pressure on—The Key of the Bridge of Neuilly, Paris, 18,000 lbs. Pillars of the dome of the Invalides, Paris, 39,000 lbs. Piers of the dome of St. Paul, London, 39,000 lbs. Do. of St. Peter's, in Rome, 33,000 lbs.; of the Pantheon, in Paris, 60,000 lbs. All Saints, Angiers, 80,000 lbs.

Rankine gives on firm earth 25,000 to 35,000.

do on rock a pressure equal to one-eighth of the weight that would crush the rock.

Fox on the Victoria R. R., London, clay under the Thames 11,200 lbs., and for cast iron cylinders filled with concrete and brickwork 8,960 lbs.

Brunlee on the Leven and Kent viaduct, gravel under cast iron 11,200 lbs.

Blyth—On Loch Kent viaduct, gravel under the lake 14,000 lbs.

Hawkshaw.—Charing Cross R. R., London, clay 17,920 lbs.

Built on cast iron cylinders 14 ft. diameter below the ground and 10 ft. dia. above it, sunk 50 to 70 ft. below high water mark, filled with Portland cement, concrete, and brickwork.

General Morin, of France, recommends for Ashlar one-twentieth of the crushing weight, for a permanent safe weight.

Vicat says that *sometimes* we may load a column equal to one-tenth of the crushing weight, but it is safer to follow *Morin*.

OUTLINES OF SOME IMPORTANT WALLS.

310u11. (Fig. 72 a.) Wall built at the India Docks, London. Radius 72 ft. = DB = DE. Wall is 6 ft. uniform thickness. Counterforts $3' \times 3'$, 18 ft. apart. AE = h = 29 ft.

The wall at East India Dock, built by Walker, is 22 ft. high, 7 1-2 ft. thick at base and 3 1-2 ft. at top. Radius 28 ft. Counterforts $2\frac{1}{4}$ ft. wide, 7 1-2 ft. at bottom and 1 1-2 at top. Lines of the two walls are on the same line with the top. Their backs vertical.

Fig. 73. *Liverpool Sea Wall*, built in 1806, base 15', top 7 1-2. Front slope 1 in 12. Counterforts 15 wide and 36' from centre to centre. Height 30 ft.

Fig. 73 a. *Dam at Poona*, near Bombay, in the East Indies. Top of dam is 3 ft. above water. 60 1-2 ft. thick at base and 13 1-2 at top. 100 ft high.

(Fig. 74.) *The Toolsee Dam*, near Bombay, is built of Basalt, rubble masonry. Mortar of lime and Roman cement. Height 80 ft., thickness at base 50 ft., at top 19 ft.

(Fig. 75.) Dublin Quay Wall, 30 ft. high. Counterforts 7 ft. long and 4 1-2 ft. deep, and 17 1-2 ft. from side to side. A puddle wall at the back, built on piles. Sheeted on top to receive the masonry.

(Fig. 76.) Wall of Sunderland Docks, England.

(Fig. 77.) Bristol Docks.

(Fig. 78.) Revetment wall on the Dublin and Kingston R. R. This is in face of a cut and is surcharged.

(Fig. 79.) Chicago street revetment walls.

Blue Island Avenue viaduct in Chicago.

Steepest grade on the streets crossing is 1 in 30, rather too steep for traffic. On the avenue it is but 1 in 40.

310u12. *Blue Island dam* on the Calumet feeder taken away in 1874. Timber of Oak and Elm. Built in compartments, well connected and the spaces filled with stones. It was down 27 years and did not show the slightest decay in the timber used.

Jones' Falls dam, on the Rideau canal, is 61 feet high, built of sand stone, with puddle embankments behind it. Several other dams made similar to that at Blue Island, are between Kingston and Ottawa (formerly Bytown), in Canada.

PILE-DRIVING, COFFER-DAMS, AND FOUNDATIONS.

Pile driving machines are of various powers and forms. A simple portable machine may be 12 to 16 feet high, hammer 350 to 400 pounds weight, without nippers or claws, and worked by about 10 men.

A Crab may be placed and worked, but where a small engine can be placed it is preferable. The locality and ground will control which to use. The site is bored to find the under lining stratas, both sides of the banks, (if for a bridge,) to be brought to the same level.

It is an old rule that a pile that will not yield to an *impact* of a ton, will bear a constant pressure of $1\frac{1}{2}$ tons.

The power of a pile driver may be determined from the following formulas:

310v1. *Screw Piles* 6 1-2 ft. in dia. have been driven in India and elsewhere. 4 levers are attached to a capstan, each lever moved by oxen,

Hollow Cast Iron Piles.—When these are driven, a wooden punch is put on top to receive the blows and protect the piles from breaking.

m = velocity in feet acquired at the time of impact.

h = height fallen through in time s , in seconds.

s = time of descent in seconds, w = weight of hammer.

$$s = \sqrt{\frac{h}{16.083}} = \sqrt{\frac{h}{4.01}} \quad v = 8.82 \sqrt{h}$$

$$m = 2w \sqrt{\frac{16.083 h}{w}} \quad \text{Let } h = 10 \text{ feet, } w = 2 \text{ tons;}$$

$$\text{Then } m = 4 \sqrt{160.83} = 30.4 \text{ tons.}$$

$$v = 25.2 \text{ feet.}$$

Otherwise We determine the safe load to be borne by each pile, and in driving find the depth driven by the last blow = d . W = weight of the hammer in cwts., H = height fallen, and L = safe load in cwts. of 112 lbs.

$$L = \frac{WH}{8D} \quad \text{and} \quad D = \frac{WH}{8L}$$

Example.—Hammer 2000 lbs., fall 35 feet. Safe load $L = 44,000$ lbs.,

$$\text{then } D = \frac{2000 \times 35}{8 \times 40,000} = 0.22 \text{ inches, nearly the length to be driven by the last blow.}$$

Let w = safe weight that a pile will bear where there is no scouring or vibration caused by rolling pressure on the superstructure.

R = weight of ram in pounds. f = fall in feet and d equal depth driven by the last blow.

$$w = \frac{Rh}{8d} \quad \text{this is the same as Major Sander's, U.S. Engineers.}$$

$$w = \frac{80}{3} \cdot (R + 0.228 \sqrt{h} - 1) \quad \text{The same as Mr. McAlpine's formula assuming } w = \text{one-third of the extreme weight supported.}$$

$w = 1,500$ lbs. $\times$ by the number of square inches in the head of the pile. This agrees with the late Mahan and Rankine's formulas for piles driven to the firm ground.

$w = 460$ lbs. (mean safe working load) per inch, by *Rondelet*.

$w = 990$ lbs. per square inch for piles 12 in. dia., by *Perronet*.

$w = 880$ lbs. do. do. do. 9 do. do.

$w = 0.45$ tons in firm ground. According to English Engineers.

$w = 0.09$ tons in soft ground. do. do. do.

Piles near, or in, salt water deteriorate rapidly and must be filled with masonry or concrete.

Lime stone exposed to sea air also suffers, and ought not to be used, as granite laid in cement can alone remain permanent.

Piles are driven, according to the French standard, until 120,000 lbs. pressure equal to 800 lbs. falling 5 ft. 30 times will penetrate but one-fifth of an inch. The most useful fall is 30 feet—should not exceed 40 ft.

Where there is no vibration of the pile the friction of the sand and clay in contact with it increases its strength, and is greater under water where there is no scouring, than in dry land.

The Nasmith Steam Hammer strikes in rapid succession, so as to prevent the material being displaced at each blow to settle about the pile. The blows are given about every second.

When men are used as a force, there is one man to every 60 lbs. of the weight. Piles driven in hard material are shod with iron and an iron hoop put on top, to prevent splitting.

For much valuable information, see a paper by Mr. McAlpine, in the *Franklin Journal*, vol. 55, pp. 98 and 170.

It sometimes happens that below a hard strata there is one in which the pile could be driven easier, therefore boring must be first used to find the stratas, and observations made on the last three or four blows.

310v. Mr. McAlpine's formula, from observations made at the Brooklyn Navy Yard, gives as follows:

$x = W + .0228 \sqrt{F-1}$. Here x = supporting weight of the pile. W = weight of the ram in tons. F = fall in feet.

He says that only 1-3 of the value of x should be used for safety weights.

These piles were driven until a ram 2,200 lbs. falling 30 ft. would not drive the piles but 1-2 an inch. They were made to bear 100 tons per square foot.

Piles in firm ground will bear 0.45 tons per square inch, and in wet ground 0.09 tons. The greatest load ranges from .9 to 1.35, tons per square inch.

310v1. *Cast iron cylinders* were first used in building the railway bridge across the Shannon, in Athlone, Ireland; next at Theis, in Austria, and now generally used. Those used in the bridge of Omaha, United States, are in cylinders 10 ft. long, 8' inner diameter; thickness $1\frac{1}{2}$ inches. Flanges on the inside 2". These when down are filled with concrete. The lower ends of those sunk in Athlone were bevelled, and sunk by Potts' method of using atmospheric pressure—that is, by exhausting the air in the cylinder, which caused the semifluid to rise and pass off. The pipe of the air pump was attached to the cap of the cylinder.

310v2. *Foundations of Timber*.—Where timber can be always in water, several layers of oak or elm planks are pinned together. We have seen the Calumet dam, on the Illinois and Michigan Canal, removed, in 1874, after being built 27 years. The foundation was of oak logs, pinned together, and in compartments filled with stones. The lumber did not show the least sign of decay.

Timbers 10 to 12 in. square are laid $2\frac{1}{2}$ to 3 feet apart, and another layer is laid across these, and the spaces between them filled with concrete, the whole floored with 3-inch plank.

Pile Foundations.—Piles ought to have a diameter of not less than one-twentieth of their length, to be $2\frac{1}{2}$ to 3 feet apart, and the load for them to bear, in soft ground, 200 lbs. and in hard, firm ground, 1000 lbs. per square inch of area of head. Piles ought to be driven as they grew—with butt end downwards—all deprived of their bark; a ring is sometimes put on top, to prevent their splitting and riving.

Pile-Driving Engine.—When worked by men, there is one man to every 40 lbs. weight of the ram or hammer used. A pile is generally said to be deep enough when 120,000 foot lbs. will not drive it more than one-fifth of an inch. 120,000 foot lbs. pressure is a hammer of 1000 lbs. weight falling 6 feet 20 times.

Let W = weight of ram, h = height of fall, x = depth driven by the last blow, P = greatest load to be supported, S = sectional area of the pile, l = its length, E = its modulus of elasticity.

$$P = \sqrt{\left\{ \frac{4 E S p^2 l}{4 E S l} + \frac{4 E^2 S^2 x^2}{l^2} \right\} - \frac{2 E S x}{d}}$$

By this formula P is to be 2000 to 3000 lbs. per square inch of S , and the working load is taken at 200 to 1000 lbs.

COFFER-DAMS.

310v3. In building the Victoria bridge, in Montreal, the coffer-dam was 188 ft. long, width 90, pointed against the stream, and flat at the other end. Double sides made to be removable. Depth of rapid water 5 to 15 ft. On the outside at intervals of 20 ft., strong piles were driven, in which steel pointed bars, 2 in. dia. were made to drill to a depth of two feet in the rock, to keep the dam in position. When the pier was built these bars, etc., were removed as required. In floating it to its required place the dam drew 18" of water.

For building cofferdams in deep water, see Mr. Chanute's treatise on the Kansas City bridge, on the Missouri.

Cofferdam of earth, where it is feasible, is the cheapest. It has to be built slowly. There are two rows of piles driven, then braced and sheeted, and filled with clay of a superior quality.

The Thames embankment reclaimed a strip of land 110 to 270-ft. wide. Depth of water in front 2 ft. Rise of tide $18\frac{1}{2}'$. Strata, gravel and sand resting on London clay at a depth of 21 to 27 ft. Depth of wall 14 ft. below low water mark. Dams were $11\frac{1}{2}$ ft. long and 25 broad inside, made of two rows of piles 40 to 48 ft. long, 13 in. square, shod with cast iron shoes 70 lbs. each, and driven 6 ft. apart. The sheeting driven 6 ft. in the clay. At intervals of 20 ft., other piles were driven as buttresses and supported by walling at every $6\frac{1}{2}$ ft. horizontally, and connected with two other piles bolted with iron bolts $2\frac{1}{2}$ in. dia., with washers 9" dia. and $2\frac{3}{4}"$ thick. An iron cylinder 8 ft. dia. sunk in each dam as pump wells.

WOOD PRESERVING.

310v4. Trees ought to be cut down when they arrive at maturity, which, for oak, is about 100 years, fir, 80 to 90, elm, ash, and larch, 75. Should be cut when the sap is not circulating, which, in temperate climates, is in winter, and in tropical climates in the dry season—the bark taken off the previous spring. When cut, make into square timber, which, if too large, ought to be sawed into smaller timbers.

310v4a. *Natural Seasoning*.—By having it in a dry place, sheltered from the sun, rain, and high winds, supported on cast-iron bearers, in a yard thoroughly drained and paved, this requires two years to fit it for the carpenter's shop, and for joiners, four years. Timber steeped in water about two weeks after felling, takes part of the sap away. Thus, the American timber, rafted down stream to the sea-board, affords a good opportunity for this natural process.

310v4b. *Artificial Seasoning*, is exposing it to a current of hot air, produced by a fan blowing 100 feet per second. The fan air-passages and chambers are so arranged that one-third the air in the chamber is expelled per minute. The best temperature is, for oak, 105° Fahr., pine in thick pieces, 120° , pine in boards, 180° to 200° , bay mahogany, 280° to 300° .

Thickness in inches,	1	2	3	4	6	8
Time required in days,	1	2	3	4	7	10

each day, only twelve hours at a time.

310v4c. *Robert Napier's Process* is by a current of hot air through the chamber, and thence into a chimney, is found very successful. The air admitted at 240° , requires 1 lb. of coke to every 3 lbs. moisture evaporated.

The short duration of wooden bridges, ties, etc., calls for a method for preventing the dry rot in timber. The following brief account will be sufficient to inform our readers of the means used to this time:

Tanks are made to hold the required cubic feet, and sunk in the ground level with the surface.—*Kyan's Process, patented March, 1832.*

On the Great Western Railway, England, the tank was 84 feet long, 19 feet wide at top, 60 feet long and 12 feet 8 in. wide at bottom, and 9 feet deep.

Corrosive sublimate (bichlorate of mercury) was used at the rate of 1 lb. to 5 gallons of water. Cost per load of 50 cubic feet, 20 shillings, sterling; of this sum, one-fourth was for the mercury, one-fourth for labor, and one-half for license, risk, and profit. The solution is generally made of 1 lb. of the mercury to 9 to 15 lbs. of water. Time of immersion, eight days; timber to be stacked three weeks before using. Experiments are reported against Kyan's method.

Sir William Burnet's Method—Patented in England, March, 1840. He uses *chloride of zinc* (muriate of zinc). Timber prepared with this was kept in the fungus-pit at Woolwich dock-yard for five years, and was found perfectly sound. The specimens experimented on were English oak, English elm, and Dantzic fir. Cost—one pound at one shilling is sufficient for ten gallons of water, a load of 50 cubic feet thus prepared in tanks costs, for landing, 2 shillings, preparation, labor, etc., 14 shillings, total, 16 shillings.

Bethell's Method.—Close iron tanks are provided, into which the wood is put, also coal-tar, free from ammonia and other bituminous substances. The air is exhausted by air-pumps under a maximum pressure of 200 lbs. per square inch during 6 or 7 hours, during which time the wood becomes thoroughly impregnated with the tar oil, and will be found to weigh from 8 to 12 lbs. per cubic foot heavier than before. The ammonia must be taken away from the tar oil by distillation.

Payne's Method—Patented 1841.—The timber is enclosed in an iron tank, in which a vacuum is formed by the condensation of steam, and air-pumps. A solution of sulphate of iron is then let into the tank, which immediately impregnates all the pores of the wood. The iron solution is now withdrawn, and replaced with a solution of chloride of lime, which enters the wood. There are then two ingredients in the wood—sulphate of iron and muriate (chloride) of lime. The timber thus prepared has the additional quality of being incombustible.

Boucher's Method.—Use a solution of 1 lb. of sulphate of copper to 12½ gallons of water. Into this solution the timber is put endwise, and a pressure of 15 lbs. per square inch applied.

W. H. Hyett, in Scotland, impregnated timber standing,—found the month of May to be the best season. From his experiments on beech, larch, elm, and lime, we find that prussiate of potash is the best for beech—½ lb. per gallon—chloride of calcium the best for larch. Time applied, 17 to 19 days. For further information, see Parnell's Applied Chemistry.

A. Lege and Fleury Peronnet, in France, in 1859, used sulphate of copper, which they found to be better and cheaper than Boucher's method.

310v5. *By exhausted steam.*—In Chicago, at Harvey's extensive lumber yard and planing mill, the following process is found very cheap and effective:—

The machinery is driven by a 100-horse power engine, the fuel used is exclusively shavings; the exhausted steam is conducted from the engine house to the kiln, where it is conveyed along its east side in a live steam

coil of 20 pipes, 2 inches in diameter. The heat thereof passes up and through the timber, separated by inch strips and loaded on cars. The heat passes to the west through the lumber cars, and thence to the north-west corner of the kiln, where it escapes. Connected with the last main pipe (8 inches in diameter,) are condensing pipes, 2 inches in diameter, laid within 4 inches of one another, and connected with a main exhaust pipe 4 inches into a chimney—one of which is over each car.

There are five tracks, or places for ten cars in each, about 80 by 60 feet; each car is 16 feet long, 6 feet wide, and 7 feet high, and is moved in and out on a railway; the whole, when filled, contains 200,000 feet of lumber. The temperature is kept, day and night, at 160° Fahr., and the whole dried in 7 days, losing about half its weight, and selling at about one dollar more per thousand. This makes a great saving in the transportation of lumber from the yard to various places in the west, as the freight is charged per ton.

MORTAR, CONCRETE, AND CEMENT.

From experiments made by the Royal Engineers, they find that 1120 bu. *gravel*, 160 bu. lime, and 9 of coals, made 1440 cubic feet in foundation; 4522 bu. gravel, 296 lime, and 30½ coal, made 2325 feet in abutments; 3591 bu. gravel, 354 lime, and 30 bu. coal, made 2180 cubic feet in arches. Cost per cubic foot—in foundations, 3½d, abutments, 4½d, arches, 5¼d; specific gravity, 2,2035; 16 cubic feet = 1 ton = 2240 lbs. Breaking weight of concrete to that of brick-work, as 1 to 13.

At Woolwich that concrete in foundations cost one-third, and in arches one-half that of brickwork.

Stoney, in his *Theory of Strains*, p. 234, edition of 1873, says Rondelet states that plaster of Paris adheres to brick or stone about two-thirds of its tensile strength; is greater for mill-stones and brick than for limestone, and diminishes with age; lime mortar, its adhesion to stone or brick exceeds its tensile strength, and increases with time.

On the Croton Water Works. Stone backing. 1 cement to 3 of sand. Brick work, inside lining 1 c to 2 s.

At Fort Warren, Boston Harbor, the proportions for the stone masonry were stiff lime paste 1 part, hydraulic cement 0.9, loose damp sand 4.8.

At Fort Richmond, hyd. cement 1.00, loose damp sand 3.2.

Vicat, a well-known French Engineer, recommends pure limepaste 1, sand 2.4, and hyd. lime paste 1, sand 1.8.

Cement for water work. Friessart recommends hyd. lime 30 parts, Terras of Andrenach 30 parts, sand 20, and broken stones 40.

Grouting. Smeaton, who built the Eddystone light house, recommends 4 parts of sand, one of lime made liquid. For Terras mortar he substitutes iron scales 2 parts, lime 2 and sand 1 part. This makes a good cement.

Iron cement. Gravel 17 parts by weight, iron filings or turnings 1 part, spread in alternate layers. Used in sea work, forms a hard cement in two months.

310v6. Stoney at Sec. 304, edit. 1873, gives the crushing weight per square inch at 3, 6, and 9 months, as follows:

Specimens acted on were made into bricks $9 \times 4\frac{1}{2} \times 2\frac{3}{4}$ inches. They began to fail at five-eighths of the ultimate load.

At Sec. 688 of Stoney on strains, the working load is taken at one-sixth of the crushing weight.

Vicat gives tenacity (one year after mixture) of hydraulic cement 190 lbs. to 160, and common mortar 50 to 20.

Cement for moist climates. Lime one bushel, $\frac{1}{2}$ bu. fine gravel sand, $2\frac{1}{2}$ lbs. copperas, 15 gallons of hot water. Kept stirred while incorporating.

CONCRETE.

310v7. In London, architects use one part of ground lime and 6 parts of good gravel and sand together. Broken bricks or stones are often added. Strong hydraulic concrete, is made of 2 parts of stone and 1 of cement.

In the United States, 1 of cement to 3 of broken stone and sand is frequently the proportions.

The stones and sand are spread in a box to a depth of 8 inches, the proportion of cement is then spread on the whole and sufficiently wetted. Four men with shovels and hoes mix up the ingredients from the sides to the centre, and mix one time in one direction and again in the opposite one. It is then taken on wheel-barrows and thrown from a height where it is spread and well rammed. One part of the materials before made makes $\frac{3}{8}$ in foundation. Lime must not be mixed when used in sea-walls.

Concrete is made into domes and arches.

The central arch of Ponte d'Alma, 161 ft. span and 28 ft. rise is made of concrete. Also the dome of the Pantheon at Rome, 142 ft. diameter.

Beton is concrete where cement takes the place of lime. In building the harbor at Cherbourg, in France, Beton blocks 52 tons weight, dimensions $12 \times 9 \times 6$ 1-2ft., 712 cubic feet, built of stone and cement, mortar made of sand 3 and cement $\frac{1}{4}$. These blocks at nine months old bore a compressive strength of 113 tons, nearly equal to that of Portland stone.

The Mole, at Algiers, Africa, built by French Engineers, is made of blocks of Beton, not less than 353 cubic feet each. All the blocks are of the same form, 11' long, $6\frac{1}{2}$ ft. wide and 4 ft. 11" high. *Composition of Beton Mortar* is made of lime 1, Pozzuolana 2, makes two parts of mortar. Beton is composed of mortar 1, stone 2. The stones are broken into pieces of about $1\frac{1}{2}$ cubic ft. each. Weight per cubic foot of this Beton = 137 lbs.

An adjustable frame is made so as to be removable when the block is dry, the bottom is covered with two inches of sand and the sides of the frame lined with canvass to prevent their being washed. They are cast in making a slope on the outside 1 to 1, and on the land side $\frac{1}{2}$ to 1. The blocks are put on small wheeled trucks and moved on a tramway to an inclined float, where it is lowered to a depth in water of 3 ft. 3 inches, and placed by chains between two pontoons and floated to the required place in the Mole.

PRESERVATION OF IRON.

310v8. The iron is heated to the temperature of melting lead (630° Fahr.), then boiled in coal tar.

Where the iron is to be painted with other parts of the structure, the iron is heated as above, and brushed over with linseed oil—this forms a good priming coat for future coats of paint. Galvanizing with zinc is not successful, being acted on by the acid impurities found in cities, towns, and places exposed to the sea, or sea air.

Steel hardened in oil is increased in strength.—*Kirkaldy*.

VICTORIA ARTIFICIAL STONE.

310v9. Rev. H. Heighton, England, uses at his works, Mount Sorrel and Guernsey granite, refuse of quarries, broken into small fragments and mixed with one-fourth its bulk of granite and water, to make the whole into a thick paste, which is put into well-oiled moulds, where it is allowed to stand for four or five days, or until the mass is solidified. After this, it is placed in a solution of silicate of soda for two days, after which it is ready for use. He keeps the silicate of soda in tanks which are to receive the concrete materials, the silica is ground up and mixed with the bath. The lime removes the silica, forming silicate of lime. The caustic soda is set free, which again dissolves fresh silica from the materials containing it. This, in flags of 2 inches thick, serves for flagging. It is made into blocks for paving, is impervious to rain and frost. Mr. Kirkaldy has found the crushing weight to be 6441 lbs. per square inch—Aberdeen granite being 7770, Bath stone, 1244, Portland stone, 2426.

310v10. *Ransom's Method* to prevent the decay of stone, and when dried then apply a solution of phosphate of lime, then a solution of baryta, and lastly, a solution of silicate of potash, rendered neutral by Graham's system of dialysis—this is Frederick Ransom's process. With Mr. Ransom, of Ipswich, England, in 1840 and 1841, we have spent many happy hours in constructing equations, etc. The above process, by Mr. Ransom sets the opposing elements at defiance. Ransom dissolves flint in caustic soda, adds dry silicious sand and lime-stone in powder, forms the paste into the desired forms, and hardens it in a bath of a solution of chloride of calcium, or wash it by means of a hose.

Make blocks of concrete with hydraulic cement. When well dried, immerse in a bath of silicate of potash or soda, in which bath let there be silica free or in excess. Here the lime in the block takes the alkali, leaving the latter free to act again on the excess of silica, and so proceed till the block is an insoluble silicate of lime, known as the silicated concrete, or Victoria stone, of which pavements have been made and laid in the busiest part of London; also, as above stated, enormous buildings, such as the new warehouses, 27 South Mary Ave., London.

Silicate of Potash is composed of 45 lbs. quartz, 30 lbs. potash, and 3 lbs. of charcoal in powder.

Silicate of Soda—Quartz 45, soda 23, charcoal 3. These are fused, pulverized, and dissolved in water.

This silica absorbs carbonic acid, therefore it must be kept closely stopped from air. The strength is estimated by the quantity of dry powder—40 degrees means 40 of dry powder and 60 of water.

In applying this, begin with a weak solution, make the second stronger. One pound of the silica to five pounds of water will answer well. It is not to be applied to newly-painted surfaces.

Mortar and lime stones ultimately produce silicate of lime.

If the surface is coated with a solution of chloride of calcium, the chlorine will combine with the soda, making the soluble salt, chloride of sodium, and there remains on the surface silicate of lime, which is highly insoluble. The surface is washed with cold water, to remove the chloride of sodium.

When applied to stone or brick, add 3 parts of rain-water to a silicate of 33 degrees. A final coating of paint, rubbed up with silicate of soda, will render the surface so as to be easily cleaned with soap and water.

This silicate adheres to iron, brass, zinc, sodium, etc. Enormous buildings have been built and repaired by this means. The best colors to be used with it are Prussian blue, chromate of lead and of zinc, and blue-green sulphide of cadmium.

BEAMS AND PILLARS.

310v11. The strongest rectangular beam that can be cut out of a log is that whose breadth = d divided by 1,732, where d = diameter of the log. (See Fig. 80.)

In the figure, $a e$ = diameter, make $a f$ = one-third of d , erect the perpendicular $f b$, join $b c$ and $a b$, make $c d$ parallel to $a b$, join $a d$, then the rectangle, $a b c d$, is the required beam. See Sections 21, 22.

A beam supported at one end and loaded at the other will bear a given load, = w , at the other end.

When the load is uniformly distributed, it will bear $2w$.

Beam supported at both ends and loaded at the middle = $4w$.

Beam supported at both ends and the weight distributed = $8w$.

When both ends are firmly fixed in the walls, the beam will support fifty per cent. more.

The following table are the breaking weights for different timbers and iron—the safe load is to be taken at *one-fourth to one-sixth of these*:—one-sixth is safer.

310v12.

TABLE.

SPECIFIC GRAVITIES, BREAKING WEIGHTS, AND TRANSVERSE STRAINS OF BEAMS SUPPORTED AT BOTH ENDS AND LOADED IN THE MIDDLE.

KIND OF WOOD.	Specific Gravity	Brk'g Weight W	Transv Strain. S	AUTHORITY.
Ash, English, - -	760		2022	Barlow.
" African, - - -	985	1701	2484	Nelson.
" American, - - -	611	274	1550	"
" White, " seasoned,	642		2041	Lieut. Denison.
" Black, " - - -	533		8861	Moore.
Elm, English, - - -	605		551	Nelson.
" Canada, - - -	703	1377	1966	"
" " - - -	685	1265	1819	Denison.
" Rock, seasoned, -	752		2312	"
" " green, - - -	746		2049	Nelson.
Hickory, American, -	838	1857	1332	"
Iron-wood, American, -	879		1800	"
Butternut, green, -	772		1387	"
Oak, American, mean of 11,	1034	1000+	1806	"
" Live, - - -	1120	1041	1513	"
Pine, White, mean of 6, -	453	966	1456	"
" North of Europe, -	587		1387	Moore.
" Red, West Indies, -			1799	Young.
" " American, mean 3,	621	1292	1944	Nelson.
Hemlock, - - -	911		1142	Chatham, England.
Larch, Scotch, - - -	480		1193	" "
Coudie, New Zealand, -	550		1873	" "
Bullet-tree, West Indies, -	1075		2733	Young.
Green-heart, " - - -	1006		2471	"
Kakarally, " - - -	1223		2379	"
Yellow-wood, mean of 3,	926	1364	2103	"
Wallabia, - - -	1147		1643	"
Lancewood, South African,				
mean of 4, - - -	1066	1167	2305	Nelson.
Teak, mean of 9, - -	719	1292	1898	"

Let l = length, b = breadth, d = depth, W = breaking weight, loaded at the centre, S = transverse strain acting perpendicularly to the fibres. l , b , and d in inches— W and S in pounds.

$$S = \frac{l W}{4 b d^2} \quad W = \frac{4 b d^2 S}{l}$$

$$b = \frac{W l}{4 d^2 S} \quad d = \frac{W l}{4 b S}$$

TIMBER PILLARS. BY RONDELET.

310v13. Let w = the weight which would crush a cube of fir or oak. When height = 12 times the thickness of the shorter side, the face = $0.833w$

"	24	"	"	"	"	"	0.500w
"	36	"	"	"	"	"	0.334w
"	48	"	"	"	"	"	0.166w
"	60	"	"	"	"	"	0.083w
"	72	"	"	"	"	"	0.042w

1. Example. A white pine pillar 24 ft. long, 12 inches wide and 6 inches thick. Required the breaking weight.

From Sec. 3107. The crushing weight of white deal = 7293

$$\begin{aligned} \text{Length} &= 48 \text{ times the shorter side.} & \frac{525096}{.166} &= \frac{72 = 12 \times 6}{\%} \\ & & 87,516 \text{ lbs.} & \\ & & \text{Rondelet} &= 39.07 \text{ tons.} \end{aligned}$$

310v14. Hodgkinson's formula for long square pillars more than thirty times the side—

W = breaking weight in tons, l = length in feet, d = breadth in inches.

Note. With the same materials a square column is the strongest, the timber in all cases being dry.

$$W = 10.95 \frac{d^4}{l^2} \text{ for Dantzic oak.}$$

$$W = 6.2 \frac{d^4}{l^2} \text{ for American red oak.}^*$$

$$W = 6.8 \frac{d^4}{l^2} \text{ for red pine.}$$

$$W = 6.9 \frac{d^4}{l^2} \text{ French oak.}$$

$$W = 12.4 \frac{d^4}{l^2} \text{ for Teak.}^*$$

Note. These marked * are put in from the values of C . Sec. 319v6.

310v15. Brereton's experiments on pine timber. For pieces 12 inches square and 20 feet long, he finds the breaking weight in tons 120, for 20, 30 and 40 ft., he finds 115, 90, and 80 tons respectively. Stoney says "this is the most useful rule published," and gives a table calculated from Brereton's curve to every five feet.

Ratio of length to the least breadth, 10, 15, 20, 25, 30, 35, 40, 45, 50.

Corresponding breaking wt. in tons per sq. ft. of section, 120, 118, 115, 120, 90, 89, 80, 77, 75.

2. Example. White pine pillar 24' ft. by 12" $\times$ 16".

Ratio 24 ft. to 6 in. = 1.48 tabular number for 50 = 75 and for 65 = 77 $\therefore$ or therefore for 48 = 75.8.

$$\frac{12' \times 6'' \times 75.8}{12 \times 12} = 37.9 \text{ tons. Brereton.}$$

By Hodgkinson least side 6" in the fourth power 1296
which multiply by the coefft for red deal 7.8

$$10108.8$$

Divide by the square of the length in feet 576 and the quotient will be for red pine and 6 inches square 17.55 tons.

As 6":17.55:12" = for 12" x 6" = 35.10 tons.

The crushing weight of white deal = 7293 lbs. and of red deal 6586, that is white deal is 1.11 times that of red = $35.1 \times 1.11 = 38.96$ tons. Hodgkinson's.

Safe load in structures, includes weight of structure.

Stone and brick one-eighth the crushing weight.

Wood one-tenth. Cast iron columns, wrought iron structures and cast iron girders for tanks each one-fourth, and for bridges and floors one-sixth. A dense crowd, 120 lbs. per sq. ft. For flooring $1\frac{1}{2}$ to 2 cwt. per sq. ft., exclusive of the weight of the floor.

310v16. The strength of cast iron beams are to one another as the areas of their bottom flanges, and nearly in proportion to their depths.

$W = \frac{cad}{l}$ = theoretical weight, which is from 4 to 6 times the weight to be sustained. Here W = breaking weight in tons placed on the middle of the beam. c and a constant multipliers derived from experiments. One-sixth the breaking weight where there is rolling or vibration and one-fourth where stationary and quiet, generally taken at 26. a = sectional area of the bottom flange, taken in the middle. d = depth of beam = g a (fig. 81) b = length between the supports.

The strongest form, according to Hodgkinson, is where the area of the lower flange is six times that of the upper flange.

Fairbairn's form is shown in fig. 81, where $e d = 1$, $a b = 2.5$, $ag = 4$, $a h = 0.42$, $ef = 0.20$ and $ik = 0.25$.

Area of bottom flange = 1.05 and of top one = 0.20. Here we have the bottom flange area = $5\frac{1}{4}$ times that of the top.

Mr. Fairbairn says, at page 32 of his treatise, that "a beam made in the above form, will be safer without truss, bars, or rods than with them."

At page 65, he shows that the advantage of a truss beam is but two-thirds of that of the simple beam as determined by experiments.

310v17. To calculate the strength of a truss beam, dimensions in inches.

$$W = \frac{(26a + 3a_1) \cdot d}{3l} \text{ tons. Here } w = \text{safe weight, } a = \text{area of bottom}$$

flange, and b = area of the truss rods, l = the distance between the points of support, and d = depth of the cast metal beam. At p. 51, he states that when the broad flange is uppermost its strength is 100, and when undermost its strength is 173.

Note A. There are various causes which render cast iron beams unsafe for bridges, ware-houses, and factories. The wrought iron beams are lighter, easier handled in building, stronger, and cheaper than cast iron, and are only about two-fifths the weight of cast iron beams of the same strength.

Note B. By comparing thirty principal American trussed bridges, we find that their depth is about one-eighth their span, ranging from one-fifth to one-tenth.

310v18. *Wrought iron beams.*

Note C. The *box-beam* (fig. 82) is the strongest form, weight for weight, best beam (fig. 83) on account of its simple construction, facility of painting; it is recommended by Fairbarn, who says that "taking the strength of a box beam (fig. 82) at 1, that in the form of Fig. 83 would be 0.93, each of equal weight. Beams like Fig. 83 can be made for buildings 60 ft. wide without columns, and with one row of columns they may be 22 inches deep and 5-16 inches thick, with angle iron rivetted.

Let W = breaking weight in tons, $d = 22''$ = depth of beam, a area of the bottom flange, l distances between the supports in inches = 360

$$W = \frac{adc}{l} \quad \text{Here} = \text{constant} = 75 \text{ and } a = 6''$$

$$\text{that is } W = \frac{6 \times 22 \times 75}{360} = 27.5 \text{ tons in the middle, or 55}$$

tons distributed. Fairbarn gives the weight of this beam equal to 40 cwt, and that of wrought iron, having the same strength, equal to 16 cwt. 1 qr. and 14 lbs.

CAST IRON PILLARS.

310v19. $W = m \frac{D^{3.5}}{l^{1.63}}$ tons. W = breaking weight in tons. D = external and d = internal diameters in inches, and b = length in feet.

Hodgkinson gives a mean value of 13 irons = 4.6.

To find D in the power $3\frac{1}{2}$. Find the logarithm of D . Multiply it by $3\frac{1}{2}$ and find the natural number corresponding to it.

$W = 42.6 \frac{D^{3.5}}{l^{1.63}}$ tons. The thickness of metal in a hollow pillar is usually taken at one-twelfth its diameter. Assuming the strength of a round pillar at 100, then a square pillar with the same amount of material = 93, a triangular pillar with the same amount of material = 110.

310v20. *Gordon's rule* is considered the best formula.

$P = \frac{fS}{1+a} \frac{l^2}{h^2}$ Here P = breaking weight in lbs., S = sectional area, l = length, and h = the least external diameter on the least side of a rectangular pillar, f and a = constants. (All in inches.)

For Wrought iron, $f = 36,000$ and $a = .00033$.

" Cast iron, $f = 80,000$ and $a = .0025$

" Timber, $f = 7,200$ and $a = .004$.

Example 1. Let length = $l = 14$. Diameter = $h = 8$ inches of a timber pillar or column.

Sectional area = 50,265 multiplied by the value of $f = 72,000$ gives 361908 = fS .

$\frac{14 \times 12 \times 14 \times 12}{8 \times 8} = 336 = \frac{l^2}{h^2}$. This multiplied by .004 = 1,344 and $1 + 1.344 = 2.344$ = the denominator in the formula, which divided into 361908, gives the value of $P = 154,397$ lbs.

The safe weight to be taken at one-sixth to one-eighth for permanent loads and one-third to one-fourth for temporary loads.

310w. We are to find the weight of the proposed wall with the pressure of the roof thereon, and prepare a foundation to support eight times this weight on rock foundation, and in hard clay the safe load may be taken from 17 to 23 lbs. per square inch. In Chicago, on blue clay the weight is

for two stories, 12 inches thick. If more than two stories above the basement, the basement and first story shall not be less than 16 inches, and the walls above the second story not less than 12 inches, and with the exception of the front walls the others will extend at least 12 inches above the roof. For dwelling-houses the above may be reduced 4 inches.

In all buildings over 25 ft. in width, and having no partitions or girders supported by columns or piers, 4 inches must be added for every additional 10 ft. in width of said building.

CHIMNEYS.

310w9. *Chimneys.* That of the *Refining Company*, in Chicago, is 151 ft. high, 12 ft. square at base. Two courses of heavy dimension stones. Base 16 ft. square. Mortar made of cement and roofing gravel. Pressure on foundation 34 lbs. per square inch.

The McCormick Reaper, 160 ft. high, 14 ft. square at base of column, which is circular. It is 25 ft. square at the ground. Flue 6' 8" diameter. Covers an area of 625 square ft. Weighing 1100 tons. Pressure $24\frac{1}{2}$ lbs. per square inch. Built on hard, dry clay.

The Tobacco Chimney, in Paris, is 29 metres high. Outer diameter at the bottom 3.45 metres. Inner dia. at the same = 2.15 metres. At the top the outer dia. = 1.30 and the inner = 1.03 met. This burns 700 kilogrammes of coal, which gives about 150 horse power. Chimneys in France are built from 20 to 30 metres high, rarely 40 metres.

M. Darcel recommends one square decimeter to every 3 kilogrammes of coal consumed per hour = 0.64 horse power.

The surface for the grating three times greater than that for the chimney.

Best forms in order are, circular, octagonal, or many sided.

Thickness at top usually one brick in length.

Thickness at top 0.11 to 0.22 metres, and putting d = inner diameter and d_1 = outer diameter at top, we have $d = d_1 + 0.22$ or $d + 0.44$. And putting D = bottom diameter and D_1 = outer dia. at do., we have H = height and m = a tabular constant for slant or inclination. Then $D_1 = D + 2 Hm$, where m for the interior is taken bet. 0.012 and 0.018 and for the outer dia. m is taken at 0.025 to 0.035.

Example. H = 60 metres. d = 0.60 = interior diam.

Then $D = d + 2Hm = 0.60 + 120 \times 0.015 = 2.40$ metres mean

$I' = 0.60 + 0.33 + 120 \times 0.30 = 4.53$ took mean thickness and constants.

Chimney at St. Rollox, Glasgow, 455½ ft. high, and 435½ ft. above bottom of the base, which is of concrete, 6 ft. thick. Load per square ft. 6720 lbs. or 3 tons on concrete.

Chimney at Muspratt's Chemical Works, St. Helen's, near Liverpool, is 406 ft. high.

Chimney at West Cumberland, Harmetide Iron Works. Base of concrete 3 ft. thick, made with hydraulic lime. Height above ground, 250; total height, 267 ft. Pressure at a section 2 ft. above the ground. Working load on concrete 3 tons, and on brickwork 8 tons.

Chimney at Atkins' Soap Works, near Birmingham, England. Total height, 312 ft. Brickwork. Pressure per square ft. at base, 6 tons.

Shot-Tower, Baltimore. Brickwork, 244 ft. high. Working load on base, $6\frac{1}{2}$ tons. The crushing weight = 4.8 times the working load.

United States Light House, 160 ft. high. Brick. Pressure, 3.7 tons. Crushing weight at base = $8\frac{1}{2}$ times the working or safety load.

Pressure on roof for weight of roof, snow, and pressure of the wind, 40 lbs. per square, weather side, and 20 lbs. on the other, under 150 foot span; add 1 lb. for every additional 10 feet, stoney, 524.

Greatest pressure of wind observed in Great Britain was 55 lbs. per square foot = 0.382 lbs. per square inch.

TUNNELS.

310w1. *Tunnels*.—Within brick work, for single track, 20 feet high, 15 feet wide. Double track, 24 feet high, and 24 to 30 feet wide. For canals, 14 to 30 feet high and 14 to 30 feet wide.

Minimum tunnels, 4½ by 3 feet. The form of a horse-shoe, or elliptical, is generally adopted, resting on an inverted arch, excepting that directly under the side walls, which is horizontal. Shafts are sunk to facilitate the lifting of materials and supplying fresh air; they are about 8 feet in diameter. The Mount Ceniz tunnel is 7.59 miles long, has no shaft, owing to its great depth below the top of the surface. At each end there is a sump or well, to collect the water of the tunnel, and from which it is lifted.

Some are made by sinking cast-iron cylinders, others stined with brick, those of iron ought to be preserved from oxidation. See Sec. 310v8.

In sinking a shaft lined with brickwork, a drum curb, made of oak timber, mounted with bevelled iron to enable it to sink. The outer diameter is the same as that of the brickwork. The excavation is made greater than the intended shaft. The drum is lowered horizontally as low as the excavation will permit, and the brickwork laid in single courses or rings. There are temporary supports to keep the drum in position until another length is excavated, then these are removed. The sharp edge of the drum penetrates deeper, carrying its already built load of masonry, and giving an additional length on top to be built, etc. At the London Thames tunnel, the Wapping-Street shaft was sunk on a drum to 72 feet deep.

Tunnels in soft ground must be timbered in length. The cost of brickwork in tunnels is about double of what it is over ground.

The Mount Ceniz tunnel was made by driving jumpers by atmospheric pressure, using the pressure of five atmospheres, each jumper driven by its own air-cylinders, holes 13-16 in. diameter, 80 holes driven in the surface of the tunnel, making a progress of 1½ yards per day.

Thames tunnel, built in soft earth by Brunel, 37½ feet wide, 22 feet high, being in two parallel elliptical archways, each 14 feet wide and 17 feet high; thickness at invert, 2½ feet, at the crown 2½ feet, sides 3 feet, central wall 3½ feet, the whole laid on a flooring of elm planks 3 inches thick. In constructing this, Brunel used a *shield*—a machine resembling a man, moving forward step by step and arm after arm. Cost £12 5s per cubic foot of its bulk, and £1137 per lineal foot. For a full account of this great feat of engineering, see Weal's Quarterly Papers on Engineering.

We believe the mortar made by the ancients was mixed with blood and iron filings.

From the Professional Papers of the Royal English Engineers.

310w2. *Concrete of lime and stones* will not concrete.

Godwin gives stones, 2, sand, 1, lime sufficient to mix well, then add boiling water; use while hot, and ram; the stones not larger than a hen's

egg. Gravel means coarse gravel 5, sand, 3. $3\frac{1}{2}$ buckets of gravel, $\frac{1}{2}$ bucket of lime, and - bucket of boiling water—ready for use in $2\frac{1}{2}$ minutes.

An arch of concrete, 4 feet thick, was found to be bomb-proof, at Woolwich, England.

TUNNELS.

310w3. *Hoosac Tunnel*, (fig. 83c), has shafts, the central one of which is 1030 ft. deep, of an elliptical form. The conjugate diameter across the roadway is 15 ft., and the transverse along the road 27 ft. There are other shafts, some $6' \times 6'$, $10' \times 8'$, and $13' \times 8'$. Where the shaft is not in rock, it is lined on one side $2' 8''$ to $2' 2''$, and on the other side, $2' 4''$ to $1' 8''$. The work was carried on the same as Mount Ceniz, using the Burleigh rock drill, mounted on two carriages; each carrying five drills, standing on the same cross section, 6 ft. asunder. The explosives used, were nitroglycerine in hard rock; and powder in other places. The compressed air, at the time of the application, was 63 lbs. per square inch, which was 2 lb. less, due to its passage through two cast-iron pipes, each 8 inch. in diameter, through which fresh air was supplied to the workmen. Three gangs of men worked each eight hours per day, excepting Sundays.

Average shafts, 26 ft. high and 26 ft. at widest part, sunk 25 feet per month, and in rock, about 9 ft. per month.

Tunnel for one track is 19 ft. from the top of the rail to the intrados of the crown, and widest width = $18\frac{1}{2}$ ft. Thickness of the arch = $1' 10''$, horse shoe form.

310w4. *The Box Tunnel*, (Fig. 83a), on the Great Western Railroad, England, (horse shoe form), is 28 ft. wide at the top of the rail and $24\frac{1}{2}$ ft. high. Thickness of arch $2' 3''$.

At 13 ft. above the rail, width is 30 ft. At 20 ft. above the rail, width is 20 ft. At $24\frac{1}{2}$ ft., width is 0. Length 9600 ft. in clay and lime stone. Shafts at about every 1200 ft.

310w5. *The Sydenham Tunnel*, (Fig. 83b). On the London and Chatham Railroad, England. Length 6300 ft. Five shafts, each 9 ft. diameter. Thickness of arch 3 ft. Width at level of rail $22\frac{1}{2}$ ft. At 5 ft. above rail 24 ft. At 10 = 23 ft. At 16 = 18 ft. At $20\frac{1}{2}$ ft. met under part of the crown.

310w6. *Tunnel for one track*. (Fig. 83e.)

310w7.

BLASTING ROCK.

Let P = lbs. of powder required when l = the length of line of least resistance, that is, to the nearest distance to the surface of the rock in feet, which should not exceed half the depth of the hole.

$P = \frac{l^3}{32}$. One pound of powder will loosen about 10,000 lbs. of rock.

Nitroglycerine is ten times as powerful as powder, but extremely dangerous.

Dualine is ten times as powerful as powder. *Gun-cotton* is about five times that of powder. Giant, Rendrock, Herculan, and Neptune, about the same as nitroglycerine. Giant powder is preferable, but is more expensive.

In small blasts, 1 pound of powder loosens $4\frac{1}{2}$ tons of rock; and in large blasts, it loosens 2 3-5ths. tons.

It is usual to use $\frac{1}{2}$ to $\frac{3}{4}$ lb. of powder for ton weight of stone to be removed, taking advantage of the veins and fissures of the rock in sinking.

A man in one day will drill in granite, by hammering,	100 to 200 in.
" " " " " churning,	200 "
" " " " " lime stone,	500 to 700 "

310208. The bottom of the hole may be widened by the action. Of one part nitric acid added to three parts of water. See Fig. 85, which represents a copper funnel of the same size as the hole. Inside of this is a lead pipe an inch in diameter, reaching to within one inch of the bottom. About the outside of the funnel is made air-tight at the surface with clay around it. At g, above the neck, is a filling of hemp. The acid acting on the limestone in a bore of $2\frac{1}{2}$ inches, will remove 55 lbs. of stone in four hours. The frothy substance of the dissolved rock will pass through the copper tube. And after a few hours, the hole is cleaned and dried, and made ready to receive the powder.

One lb. of powder occupies 30 cubic inches of space, fills a hole 1 inch in diameter and 38 inches deep.

As the square of 1 inch diameter filled with 1 lb. of powder is to 38 inches in depth, so is the square of any other diameter to the depth filled with 1 lb. of powder. See *Sir John Burgoyne's Treatise on Blasting*.

When the several holes are charged they are connected by copper wires with a battery and then discharged.

The blowing up of Hell Gate, by Mr. Newton, is the greatest case of blasting on record.

At the Chalk Cliff, near Dover, England, 400,000 cubic yards were removed by one blast. Length of face removed, 300 feet. Total pounds of powder, 18,500.

ARCHES, PIERS, AND ABUTMENTS.

310209. Next page is a table showing several bridges built by eminent engineers, giving their thickness at the crown or key of each, as actually existing, and the calculated thickness, by Levell's formulas. We also give Trautwine. Rankine & Hurst's formulas. M. Levelle, in 1855, and since, has been chief engineer of Roads and Bridges in France. We believe that all surveyors and engineers are familiar with the names and works of Trautwine. Rankine & Hurst.

C = thickness of the crown. r = radius of the intrados. h = height of the arch. s = half span. v = height of the arch to the intrados, and r = the radius of the circle. Then,

$$r = \frac{s^2 - v^2}{2v}$$

See Euclid, Book III, prop. 35.*

By *Levelle*. $C = \frac{S+10}{30}$ for French meters, $= \frac{S+32.809}{30}$ for English ft.

By *Prof. Rankine*. $C = \sqrt{0.12r}$ for a single arch and $\sqrt{0.17r}$ for a series of arches.

By *Trautwine*. $C = \sqrt{\frac{r+\frac{1}{2}S}{4}} + 0.2$ feet for first-class work.

To this add one-eighth for second-class work, and one-fourth for brick or fair rubble work.

By *Hurst*. $C = 0.3 \sqrt{r}$ for block stone work.

" " $C = 0.4 \sqrt{r}$ for brickwork and $0.45 \sqrt{r}$ for rubble work.

" " $C = 0.45 \sqrt{S} + \frac{S}{12}$ for straight arch of brick, with radiating joints.

Mr. Levelle finds his formula to agree with a large number of arches now built from spans of 5 to 43 meters, including circular, segments of circles, semicircular, and elliptical.

* If two lines intersect one another in a circle, the product of the segments of one = the product or rectangle of the others.

BRIDGES, WITH THEIR ACTUAL AND CALCULATED DIMENSIONS.

310w10. THE CALCULATED ARE BY LEVELLE'S FORMULA.

NAMES OF BRIDGES.	Span.	Rise to intrados.	Actual thickness of the key.	Calculated thickness of the key	Height of the Abutment.	Actual thickness of abutment.	Calculated thickness of abut.	Function of $\frac{1}{2}$ in formula.
SEGMENTS OF CIRCLES.								
Pont de la Concorde, Paris.....	23.40	1.93	0.97	1.11				
" de Pasta, ".....	5.00	.80	.52	.50	2.0	1.70	1.95	.160
" de Courcelles du Nord.....	9.80	.90	.65	.66				
" des Abbatoirs, Paris.....	16.05	1.55	.90	.87	3.93	10.	7.24	.097
" de Ecole Militaire, ".....	28.	2.99	1.14	1.29				
" de Melisey.....	11.40	1.50	.60	.71	3.55	5.20	4.68	.132
" sur le salat.....	14.	1.90	1.10	.80	6.21	5.80	6.06	.136
" de Marbre, Florence, Italy.....	42.23	9.10	1.62	1.74				
" on the Forth, at Stirling. Scot'd.....	16.30	3.12	.84	.88	6.32	4.88	5.15	.192
" de Bourdeaux, France.....	26.49	8.83	1.20	1.23				
" Saint Maxence Sur la Oise, ".....	23.40	1.95	1.46	1.11	8.45	11.8	12.2	.083
" de la Boucherie, Nuremberg.....	29.60	3.90	1.22	1.32				
" de Dorlaston.....	26.37	4.11	1.07	1.21	5.03	9.76	9.00	.156
" du Rempart, R. R. Orleans to Tours.....	1.20		.45	.37	1.20	.55	.74	1.70
" de Saint Hylarion, R. R. Paris to Chartres.....	2.0		.40	.40	3.80	1.20	1.09	4.40
" de la Tuilerie, ".....	4.0		.50	.47	3.40	1.40	1.58	4.10
" des Voisins, ".....	5.0		.55	.50	2.50	1.50	1.73	5.15
" y Prydd, Wales.....	140	35.	1.5	5.76				
Cabin John, Washington Aqueduct.....	220	57.	4.16	8.42				
Ballochmyle, Ayr, Scotland.....	181	90.5	4.5	7.16				
Dean, Edinburgh, ".....	90	30	3.0	4.09				
Ordinary over a double R. R. track.....	30	7.5	1.83	2.09				
Grovenor, on the Dee.....	200	42.	4.	7.76				
Turin, Italy.....	147.6	18.	4.90	6.01				
Mersey Grand Junction.....	75.	14.5	3.	3.59				
Philadelphia & Reading R. R.....	14	8.	2.50	2.56				
SEMICIRCLES.								
Pont des Tetes, on the Durance.....	38.0	19.	1.62	1.60				
" de Sucres.....	18	9.	1.	0.93				
" de Corbeil.....	16.82	8.41	0.75	0.89				
" de Franconville.....	7.40	3.70	.60	.58				
" du Crochet.....	4.	2.	.50	.47				
" des Chevres.....	1.50	.75	.35	.38				
" de Orleans A'Tours.....	20.	10.	1.	1.	1.	4.50	4.49	
ELLIPTICAL.								
Pont de Neuilly, Paris.....	38.98	9.74	1.62	1.63	2.30	1080	1080	.250
" de Vissile Sur le Romanche B.....	41.90	11.09	1.95	1.73				
" du Canal Saint Denis.....	12.	4.50	.90	.73	3.10	3.75	3.40	.375
" de Moielins A' Nojent.....	18.	5.13	1.	.93				
" du Saint du-Rhone.....	34.	9.74	1.80	1.47				
" de Wellesly a' Limerick, Ireland.....	21.34	5.33	.61	1.04	3.66	5.03	6.47	.250
" Sur le Loir.....	24.26	8.	1.20	1.14				
" de Trilport.....	25.61	8.77	1.95	1.19				
" ".....	24.50	8.44	1.36	1.15	1.95	5.85	6.21	.344
" Royal, Paris.....	23.52	9.30	1.10	1.12				
" Gignac sur le Herault.....	48.72	13.30	1.95	1.96				
" Alma sur le Seine.....	43	8.60	1.50	1.76				
" de Vieille Brioude sur le Allier.....	54.20	21.	1.30	2.14				
" Auss, on the Vienna R. R.....	20.	6.67	1.10	1.00				

The Line of Rupture in a semicircle arch, with a horizontal extrados, is where the line of 60 degrees from the vertical line through the crown meets the arch.

This has been established by Mr. Mery, and Mr. Petit, of France, the latter a Captain of Engineers.

Mr. Lavelle, from Petit, gives for semicircular arches, where d = diameter, t = thickness of the arch or key at the crown.

When the diameter = 2,000, 5,000, 10,000, 20,000, then

$t = \left(\frac{1 + 0.1d}{3} \right) = 0.40, 0.50, 0.67, 1.00$, whose corresponding angles of rupture are 59°. 63°. 64°. and 65°, from the vertical line CD.

Lavelle adopts 60°.

310x. TELFORD'S TABLE.—HIGHLAND BRIDGES.

Span, in feet.	Versed Sine.	Depth of key-stone.	Height of Abutment to Springing.	Mean Thickness of Abutment.	Thickness of Spandries.	Thickness of necessary inverted arches.
6	2.0"	1.0"	2.6"	2.0"	1.6"	1.0"
8	1.6	1.2	2.6	2.0	2.0	1.0
10	3	1.3	3.0	2.6	2.0	1.0
12	3.6	1.4	3.0	3.0	2.6	1.0
18	4	1.6	3.0	4.6	2.9	1.4
24	6	1.9	4.0	5.0	2.9	1.4
30	8	2.0	4.0	5.6	3.0	1.6
50	..	2.6	6.0	6.0	3.6	1.6

310x0. SEGMENT ARCHES.

BATTER OF PIERS $\frac{1}{4}$ -INCH IN ONE FOOT.

Span.	Depth of Crown.	Height of Piers.	Thickness of Piers.	Thickness of Pier at Top Battered.	Thickness of Piers for Flat Girder Trusses.	Thickness of Piers for Flat Girders with Batter.	Rise of Arch to the Intrados.
10 ⁶	1.2"	5' to 20'	3' to 3'. 9"	3'. 0'	1'. 3' to 2'. 7 $\frac{1}{2}$ '	2. 3	3'.0'
15	1.6	5 " 20	" " "	3. 0	2. 7 $\frac{1}{2}$ " 3. 4 $\frac{1}{2}$ "	2. 7 $\frac{1}{2}$	4.6
20	1.6	5 " 40	3 " 4. 6	3. 0	2. 7 $\frac{1}{2}$ " 3. 4 $\frac{1}{2}$ "	"	6.0
25	1.6	5 " 40	3 " 4. 10 $\frac{1}{2}$	3. 9	3. 0 " 4. 1 $\frac{1}{2}$	3. 4 $\frac{1}{2}$	7.3
30	1.10 $\frac{1}{2}$	5 " 40	4. 1 $\frac{1}{2}$ " 6.	4. 1	4. 1 $\frac{1}{2}$ " 6.0	4. 1 $\frac{1}{2}$	9.0
35	2.3	10 " 40	4. 10 $\frac{1}{2}$ " 6. 4 $\frac{1}{2}$	4. 10	5. 3 " 6. 4 $\frac{1}{2}$	4. 6	10.6
40	2.3	" " "	5. 77 " 7. 1 $\frac{1}{2}$	5. 3	4. 10 $\frac{1}{2}$ " 6.	4. 10 $\frac{1}{2}$	11.3
45	2.7	" " "	6. 47 " 7. 6	6. 0	5. 7 $\frac{1}{2}$	"	13.0
50	3.0	" " "	7. 1 " 8. 3	7. 1	6. 9	"	14.6
55	3.0	" " "	7. 10 " 9. 4	7. 10	7. 1 $\frac{1}{2}$	"	16.0
60	3.0	" " "	8. 7 " 9. 8	8. 3	7. 10 $\frac{1}{2}$	"	17.3

310x1. RADIUS OF CURVATURE. Fig. 86—Let ABCD be a curve of hard substance. Wind a cord on it from D to A. Take hold of the cord at A and unwind it, describing the oscillatory curve a, b, c, d. When the cord is unwound as far as B and C, etc., the point or end A will arrive at B, C, etc., and the line BC will be the radius of curvature to the point B, and the line Cc will be the radius of curvature to the point C.

The curve ABCD may be made on thick pasteboard, and drawn on a large scale, by which mechanical means the radius of curvature can be found sufficiently near.

The radius of curvature of a circle is constant at every point.

310x2. *Tension* is the radius of curvature at the crown.

310x3. *Piers*. L. B. Alberti says piers ought not to be more than one-fourth or less than one-fifth the span.

The pier of Blackfriar's Bridge, London, is about one-fifth the span. The pier of Westminster Bridge, London, is about one-fourth the span. The pier at Vicenza, over the Bacchilione, Palladio, makes one-fifth the span.

Piers generally are found from one-fourth to one-seventh of the span. The end of the pier against the current is pointed and sloped on top, to

break the current and floating ice, if any. When the angle against the current is ninety degrees, the action of the water is the least possible, and half the force is taken off.

310x4. *The horizontal thrust of any semi-arc.* Fig. 87, AEKD. By section 313, find G, the centre of gravity of said arc, or by having the plan drawn on a large scale—about four feet to one inch—the point G can be found sufficiently near.

Draw OGM at right angles to AQ, and draw DO parallel to AQ. We find the area A, of AEKD. We have AM from construction, and OM = QD = rise at the arch, and AQ = one-half the span, and the height of the pier, XY, to find the thickness of FE = BL. We have OM : AM :: A : T, equal to its thrust in direction of AH on the pier. We have taken the area A to be in proportion to the weight, and make the pier to resist three times the thrust, T. This fourth term F, will be the surface of the pier BEFL, whose height, XY, is given. Therefore,

$$\text{Thickness of the pier out of water.} = \frac{3T}{XY}$$

Let AQ = 28, MO = 18, AM = 9, A = 270, and XY = 30.

18 : 9 :: 270 : 135 = T = thrust on the pier at B.

The pier 30 feet high is to sustain for safety three times 135 = 405
 $\frac{405}{30} = 13.5$ ft. = BL, the required thickness.

310x5. The thrust to overturn the pier about the point I,

$$= \frac{AM \times A \times CB}{OM} \text{ which must be } = EB \times BL.$$

$$\therefore BL = \left(\frac{2AM \times A \times CB}{OM \times EB} \right)^{\frac{1}{2}} = \text{thickness of a dry pier.}$$

BL = $\left(\frac{7AM \times A \times CB}{OM(3.5EB - AB)} \right)^{\frac{1}{2}}$ thickness *w*, when in water. Here we take A, as before, three times the area of AEKD.

In circular and elliptical arches, we take AB = diameter for circular, and transverse axis for elliptical; CD for rise or versed sine in the circular, and the conjugate diameter in elliptical, and DQ for the generating circle of the cycloid. DP = abscissa, and PC its corresponding ordinate to any point, C, in the curves.

Having determined on the span and rise of the arch, and the thickness, DK, at the crown, we find the height, CI, at the point C, corresponding to the horizontal line, PC, an ordinate to the abscissa DP. See the above figure.

$$CI = \frac{DK \times DQ^3}{PQ^3} \quad \text{For the circle.}$$

$$CI = \frac{DK \times DQ^3}{PQ^3} \quad \text{For the ellipse - same as for the circle.}$$

$$CI = \frac{DK \times DQ^2}{(DQ - DP)^2} \quad \text{For the cycloid.}$$

$$CI = \frac{DK \times (C + DP)}{C} \quad \text{For the catenary.}$$

Here C is the tension or radius of curvature at D.

The above three forms are practicable. Sometimes for single arches the parabolic arch is used.

CI = DK for every point, C, in a parabola.

In all cases, CI is at right angles to the line AB.

Gwilt, in his work on the equilibrium of arches, says: "The parabola may be used with advantage where great weights are required to be discharged from the weakest part of an edifice, as in warehouses, but the scantiness of the haunches renders them unfit for bridges."

310x6. THE CATENARIAN is correctly represented by driving two nails in the side of a wall or upright scantlings, at a distance equal to the required span BA. From the centre, drop a line marking the distance DQ equal to the rise of the arch, and let a light chain pass through the point to ADB, and we have the required curve. Let DP and CP be any abscissa and corresponding ordinate, to find CI from the intrados to the extrados.

TO FIND THE TENSION AT D.

310x7. Let c = tension constantly at the vertex.

KD = thickness of the arch at crown = a .

DP = any abscissa x , and PC = y , its corresponding ordinate.

$$C = \frac{x}{2} \times \left(\frac{y^2}{x^2} + 0.3333 - \frac{8x^2}{45y^2} + \frac{691x^4}{3780y^4} - \frac{23851x^6}{453600y^6} \&c. \right)$$
 This is Dr. Hutton's formula, excepting that the parenthesis, is erroneously omitted.

$$C = \frac{x}{2} \times \left(\frac{y^2}{x^2} + 0.3333 - 0.1778 \frac{x^2}{y^2} + 0.1828 \frac{x^4}{y^4} - 0.0526 \frac{x^6}{y^6} \&c. \right)$$

Example given by Hutton. Let DQ = 40 = x , and one-half the span AQ = 50 = y .

Here the tension $C = 20 \times (1.5625 + 0.3333 - 0.1137 + 0.0749 - 0.0138, \&c.)$ That is $C = 20 \times 1.8432 = 36.864$, as given by Hutton.

TO FIND THE RADIUS OF CURVATURE AND TANGENT TO ANY POINT C OF THE CATENARIAN. Fig. 90.

310x8. Produce QD to P making $OP = CO \times \sqrt{2c + DO + DO^2}$.

Join PC, which will be the tangent to the point C. From the point C, draw CW at right angles to AP. And make As $c : c + DO :: c + DO : CR$ = Rad. of curvature.

When the abscissa $DO = 0 : C : c :: c : CR = c$. Hence the tension at the lowest point D is equal to the radius of curvature.

Let the span = 100 and rise = 40 feet, then radius of curvature for a segment of a circle = 51.25 = radius of curvature.

" Parabola,	= 30.125	"
" Ellipsis,	= 62.5	"
" Catenary,	= 36.864	"

The strength of the Parabola at the crown is to the above figures as the rad. of curvature of the other figures, to that of the parabola; hence the strength of the parabola is 2.1 times that of the ellipsis, and $P : C :: 36.864 : 30.129$.

Parabola is 1.22 as strong as the Catenarian.

To find the extrados to the point C. Whose abscissa $DO = x$ and ordinate $CO = y$ are given. Fig. 90.

Let KD = a and $DO = x$ and $CO = y$ as above. Then from Hutton:

$$CI = \frac{ac + ax}{c} = a + \frac{ax}{c}$$

$$KV = \frac{c - a}{c} \times x = x - \frac{ax}{c}$$

DO : KV :: always as $c : c - a$.

The extrados will be a straight line when $a = c$, the tension at K.

In the above example, where we have found $c = 36,864$ feet to have the extrados a straight line, would require $a = KD$, to be nearly 37 feet.

Assuming the same span 100, rise = 40, and putting $DK = 6$ feet, the extrados and the arch will be as figure 91. This arch is only proper for a single arch, where the extrados rises considerably from the springing to the top.

$AC = CB$ is given = $a = \frac{1}{2}$ -span. $CD = h$ = height. Figure 92. DE = distance of chain to the lower part of the roadway parameter. K and M any points in the curve, from which we are to find the suspension rods KD and MP , etc.

$$DK = \frac{CD-DE}{AC} \times HK^2 + \text{and } \frac{CD-DE}{AC} \times DM^2 + DE = MP$$

We have $\frac{CD-DE}{AC}$, a constant quantity; Let it = c , and divide EG into any parts as Q, P, D, R , etc. Then the length of the rod at $R = RS = c \times ER^2$ and rod $QT = c \times EQ^2$.

310x9. To find the sectional area in inches of any rod, as DK , and the strain in pounds on it, at K .

Let W = weight of one lineal foot of the roadway when loaded with the maximum weight.

Strain on K.—Let $2 \frac{h-t}{a^2} - 0.0003$ be divided into W , it will give the strain in pounds on K . Let this strain be represented by S .

Sectional area of the rod $DK = S + 0.0000893$ lbs.

$$DK = \frac{CD-DE}{AC^2} \times HK^2 + DE = \text{length of the rod } DK.$$

Let W = weight of every lineal foot of roadway and its maximum load thereon. *Strain* = $2 \frac{CD-DE}{AC^2} - 0.0003$, this divided into W , gives the strain on the lowest point D of the chain.

Sectional area of chain at D is found by multiplying the last, by .0000893.

Example. Half span $AC = 200$. $DE = 2$ feet, wt. of one lineal foot of road = 500. Horizontal distance $HK = 100$ ft. $CD = 40$ ft.

$$40 - 2 = \frac{38 \times 100^2}{200 \times 200} = \frac{380000}{200 \times 200} = 9.5 = IID. \text{ And } 9.5 + 2 = 11.5 = \text{rod } KD.$$

$$2 \frac{(40-2)}{200 \times 200} = \frac{38 \times 2}{40000} = \frac{76}{40000} = 0.0019, \text{ and } .0019 - 0.0003 = 0.0016.$$

$$\text{And } \frac{500}{.0016} = 3125.000 \text{ lb. lb.} = \text{strain on the lowest point of chain at } B.$$

$$\text{And } 31250000 \times 0.0000893 = 279 \text{ square inches} = \text{sec. area at } B.$$

$$\frac{2 \times 9\frac{1}{2}}{100} = \frac{19}{100} = .19, \text{ this squared} + 0.0261 + 1 = 1.0262, \text{ whose}$$

square root = 1.013, which $\times$ by 3125000 = 3165625 lbs. strain on the point K , which $\times$ by 0.0000893 = sectional area = 283 square inches of chain at K .

Basis here. Took one-sixth the load for coefficient of safety.

A bar of iron 12 feet long and 1 inch square weighs 3.3 lb.

The tensile strain to break a square inch of wrought-iron is taken at 6720 lb., the iron loaded with one-sixth its breaking weight.

On bridges, the load should not exceed one-twentieth of the weight which would crush the materials in the arch stones; and where there is a heavy travel, should not exceed one-thirtieth.

PIERS AND ABUTMENTS.

310x10. When the angle at the point of an abutment against the stream is 90 degrees, then the pressure on the pier is but one-half what it would be on the square end. The longer the side of the triangular end of the pier is made the less will be the pressure. Let ABC represent the triangular end against the stream, and C the furthest point or vertex. *Gwill* says "that the pressure on the pier is inversely proportional to the square of the side AC, or BC, and that the angle at C ought not to be made too acute, lest it should injure navigation, or form an eddy toward the pier. *Abutments.* In a list of the best bridges, we find the abutment at the top from one-third to one-fifth the radius of curvature at the crown of the arch:

Molesworth gives the following concise formula:

$$T = \text{thickness of abutments} = \left(6R + \left(\frac{3R}{2H} \right)^2 \right)^{\frac{1}{2}} - \frac{3R}{2H}$$

Here R = rad. at crown in feet, H height of the abutment to springing in feet, for arches whose key does not exceed three feet in depth.

Example. R = 20 +. H = 10.

$(120 + 9)^{\frac{1}{2}} = 11.36$ from which take 3, will give the abutments without wing walls or counterforts.

Abutments.—To counteract the tendency to overturn an abutment, let the arch be continued through the abutment to the solid foundation, or by building, so as to form a horizontal arch, the thrust being thrown on the wing walls, which act as buttresses.

ad.—By joggling the courses together with bed dowel joggles so as to render the whole abutment one solid mass.

310x0. The depth of the voussoirs must be sufficient to include the curve of equilibrium between the intrados and extrados.

The voussoirs to increase in depth from the key to the spangling, their joints to be at right angles to the tangents of their respective intersections and curve of equilibrium.

The curve of equilibrium varies with the span and height of the arch stones, the load and depth of voussoirs, and has the horizontal thrust the same at any point in it.

The pressure on the arch stones increase from the crown to the haunches.

310x1.

SKEW ARCHES.

In an ordinary rectangular arch, each course is parallel to the abutments, and the inclination of any bed-joint with the horizon will be the same at every part of it. In a skew arch this is not possible. The courses must be laid as nearly as possible at right angles to the front of the arch and at an angle with the abutments. The two ends of any course will then be at different heights, and the inclination of each bed-joint with the horizon will increase from the springing to the crown, causing the beds to be winding surfaces instead of a series of planes, as in the rectangular arch. The variation in the inclination of the bed-joints is called the thrust of the beds, and leads to many different problems in the cutting. See *Buck on Skew Bridges*.

EAST RIVER BRIDGE, NEW YORK.

310x2. Brooklyn tower, 316 feet high, base of caisson, 102 × 168 feet.

New York tower, 349 feet high, base of caisson, 102 × 178 feet.

The Victoria Bridge, at Montreal, 7000 feet long, one span, 330 feet and fourteen of 242 feet, built in six years. Cost, \$6,300,000. Built by Sir Robert Stephenson.

Concrete Bridges.—One of these built by Mr. Jackson in the County of Cork, Ireland, is of cement, one part sand. Clear sharp gravel, six to eight parts. Rammed stones in the piers. He also built skew bridges of the same materials.

Mr. McClure built one 18 feet span, $3\frac{1}{2}$ feet rise, and $1\frac{1}{2}$ feet thick at the key, and $2\frac{1}{2}$ feet at the springing. Built in ten hours, with fifteen laborers and one carpenter. Piers are of stone, centre not removed for two months. Proportions of materials used: Portland cement, 1, sand, 7 to 8, 40 per cent of split stone can be safely used in buildings, and 25 per cent in bridges. Stones used in practice, 4 to 6 inches apart. Cottage walls, 9 inches thick. Chimney walls, 18 inches. Partitions, 4 inches. Walls, sometimes 18 feet high and 12 inches thick. Garden walls, $\frac{1}{4}$ mile long, 11 feet high, and 9 inches thick. Cisterns, 5 feet deep and 6×5 feet 9 inches thick.

Cost of one cubic yard of this concrete wall, 12 to 15 shillings, at 3 to 4 dollars.

310x3. These kind of buildings are common in Sweden, since 1828, and built in many towns of Pomerania, where its durability has been tested. It is applicable to moist climates. Where sand can be had on or near the premises, walls can be built for one-fourth the cost of brickwork. In Sweden, they use as high as 10 parts of sand to 1 of hydraulic lime. The lime is made into a milk of lime, then 3 parts of the sand is added, and mixed in a pug-mill made for that purpose. After thus being thoroughly mixed the remainder of the sand is added. These walls resist the cold of winter, as well as the heat of summer.

The pug-mill is made cylindrical, in which on an axis are stirrers, moved by manual labor, or horse power, as in a threshing machine. One of these, in ordinary cases, will mix 729 cubic feet in one day. Let us suppose a house, 40 feet long, 20 feet wide, and 1 foot thick. This *caisse* will mix 1 to $1\frac{1}{2}$ *toise*, cubic, per day, which will be made into the wall by three men, making the wall all round, 6 feet high, moved upwards between upright scaffolding poles. There is a moveable frame, stayed at proper distances, laid on the wall to receive the beton where two men are employed in spreading it.

310x4.

TO TEST BUILDING STONES.

Take a cubic 2 inches each way, boil it in a solution of sulphate of soda (Glauber salts) for half an hour, suspend it in a cold cellar over a pan of clear sulphate of soda. The deposit will be the comparative impurities.

Rubble work.—The stones not squared.

Coursed work.—Stones hammered and made in courses.

Ashlar.—Each stone dressed and squared to given dimensions.

To prevent sliding.—Bed dowels are sunk one-half inch in each, made of hardwood.

Walls faced with stone and lined with brick are liable to settle on the inside, therefore set the brickwork in cement, or some hard and quick setting mortar. The stones should be sizes that will bond with the brickwork.

Bond in masonry is placing the stones so that no two adjoining joints are above or below a given point will be in the same line. The joints must be broken.

Stones laid lengthways are called stretchers, and those laid crossways, headers.

Brick work.—English bond is where one course is all stretchers and the next all headers.

Flemish bond is where one brick is laid stretcher, the next a header and in every course a header and stretcher alternately.

Tarred hoop-iron is laid in the mortar joints as bonds.

310r5. ANGLES OF ROOFS, WITH THE HORIZONTAL.

CITY.	COUNTRY.	N. Latitude.	Plain tiles.	Hollow tiles.	Roman tiles.	Slates.
Carthagea,	Spain,	37° 32'	24° 12'	16° 12'	19° 12'	22° 12'
Naples,	Italy,	40 52	26 12	18 12	21 12	24 12
Rome,	do	41 58	27 0	19 0	22 0	25 0
Lyons,	France,	45 48	30 0	22 0	25 0	28 0
Munich,	Germany,	48 7	31 48	23 48	26 48	29 48
Viena,	Austria,	48 5	32 0	24 0	27 0	30 0
Paris,	France,	48 52	32 36	24 36	27 36	30 36
Frankfort,	On the Main,	50 8	33 48	25 48	28 48	33 48
Brussels,	Belgium,	50 52	34 36	26 36	29 36	32 36
London,	England,	51 31	35 24	27 24	30 24	33 24
Berlin,	Germany,	52 33	36 36	28 36	31 36	34 36
Dublin,	Ireland,	53 21	37 48	29 48	32 48	35 48
Copenhagen,	Denmark,	55 42	40 48	32 48	35 48	38 48
St Petersburg,	Russia,	59 40	48 24	40 24	43 24	46 24
Edinburgh,	Scotland,	55 57	41 12	33 12	36 12	39 12
Bergen,	Norway,	60 5	48 24	40 24	43 24	46 24

From the above table, we see that the elevation of the roof increases one degree for every $\frac{3}{4}$ th degree of latitude, from Carthagen to Bergen.

Pressure on Roof. For weight of roof, snow, and pressure of the wind, 40 lbs. per square foot, on the weather side, and 20 lbs. on the other, under 150 feet span. Add 1 lb. for every additional 10 feet.—*Stoney*, p. 524.

Greatest pressure of wind observed in Great Britain has been 55 lbs. per square foot = 0.382 lbs. per square inch.

310r6. TRUSSED BEAMS AND ROOFS.

Let $AD = b =$ half the span. $AB =$ tie-beam resting on the wall-plates.
 $CD = h =$ height = king-post. $AC = b =$ length of principal rafters, 10
to 12 feet asunder.

Q = angle BAC = angle of *minimum* pressure on the foot of the rafter.
Secant of the angle Q = l . See fig. 83 A.

When $Q = 35^{\circ} 16'$, the pressure P is a minimum. *Moseley's Mechanics*, Sec. 302, Eq. 395.

Then $h = 0.7072b$ } Here s = distance between each pair of rafters,
 $l = 1.2248b$ } " w = weight of each square foot of roof,
 $W = 1.2248b \cdot s \cdot w$ } including pressure of the wind and snow, as
determined in the locality. W = weight on each rafter.

310x7. *To calculate the parts of a common Roof.* Let a = sectional area of a piece of timber. d = its breadth, and l = its length. s = span of the roof in feet. p = length in feet of that part of the tie-beam supported by the queen-post.

King-post. $a = ls \times 0.12$ for fir, and $a = ls \times 0.13$ for oak.

Queen-post. $a = lp \times 0.27$ for fir, and $a = lp \times 0.32$ for oak.

The Beam. $d = \sqrt[3]{\frac{l}{b}} \times 1.47$ for fir.

Principal rafters with a king-post, $d = \frac{l^2 s}{b^3} \times 0.96$ for fir.

" with two queen-posts, $d = \frac{l^2 s}{h^3}$

Straining Beam. Its depth ought to be to its thickness as 10 to 7.

$$d = \sqrt[3]{l s^{\frac{1}{2}}} \times 0.9 \text{ for fir.}$$

Struts and Braces. $d = \sqrt[3]{l p^{\frac{1}{2}}} \times 0.8$ for fir, and $b = 0.7 d$.

Purlins. $d = \sqrt[3]{b^2 c}$ for fir, or multiply by 1.04 for oak, and $b = 0.6 d$.

Common Rafters. $d = \frac{l}{\sqrt[3]{b}} \times 0.72$ for fir, or 0.74 for oak.

Two inches is the least thickness for common rafters, therefore, in this case, $d = 0.571 l$ for fir.

310x8. *Laminated arched beams* formed of plank bent round a mould to the required curve and bolted are good for heavy travel and great speed.

JETTIES.

310x9. In rivers, at and near their outlets, sand bars are formed where the velocity is less than that of the deep water on either side. The desired channel is marked out, and two rows of piles are driven on the out-sides, to which the mattresses are tied. The space or jetties thus piled are filled with mattresses made of fascines of brushwood, bolted by wooden bolts and boards on the top and bottom of each, sloping from the outside towards the channel.

One in New Orleans, now in progress of construction by Capt. Eads, C. E., is from 35 to 50 at bottom, and 22 to 25 at the top, mattresses 3 feet thick. From 3 to 6 layers are laid on one another. Mud and sand assist to fill the interstices. They are loaded with loose stones, and the top covered with stone. The water thus confined causes a current, which removes the bars. Drags may be attached to a boat and dragged on the bars, which will assist in loosening the sand.

The mattresses are built on frames on launchways on the shore, and then floated and tied to the piles.

Jetties may be from 10 feet upwards, according to the location. Those of the Delta, at the mouth of the Danube, are filled with stones.

See Hartley on the Delta of the Danube, for 1873.

General Gilmore, U. S. Engineer's report on the Jetty System, for 1876.

General Comstock's, U. S. A., report on the New Orleans South Pass.

310x9. *Excavations for Foundation*, measured in cubic yards, pit measurement. Allow 6 inches on each side for stone and brickwork, and no allowance is made where concrete is used. Where excavation is made for water or gas pipes, slopes of 1 to 4 is allowed. State for moving away the earth not required for backfilling, the distance to which it is to be moved, and inclination, and how disposed of, whether used as a filling or put in a water embankment. This done for first proposed estimate.

Filling is measured as embankment measurement, for the allowances for shrinkage add 8 per cent for earth and clay when laid dry. When put in water, add one-third. Bog stuff will shrink one-fourth. See p. 210.

100 cubic feet of stone, broken so as to pass through a ring $1\frac{1}{2}$ inch in diameter, will increase in bulk to..... 190 cubic feet.

Do do to pass through a 2 inch ring, 182 " "

Do do " " $2\frac{1}{2}$ " 170 " "

Rubble Masonry.—One cubic yard requires 1 1-5 cubic yards of stone and 1-4 cubic yard of mortar. Ashlar masonry requires 1-8 its bulk of mortar.

All contractors ought to be informed that when they haul 100 yds from the pit, that it will not measure the same in the "fill" or embankment.

Isolated Peirs are measured solid, to which add 50 per cent.

Brick Walls are measured solid, from which deduct one-half the openings; then reduce to the *standard measurement*, for example: multiply the cubic feet by $2\frac{1}{2}$, and divide by 1000, to find the number of thousands of bricks, as calculated in Chicago, where the brick is 8 by 4 by 2 inches.

NOTE.—One must observe the local customs.

The English standard rod is $16\frac{1}{2}' \times 16\frac{1}{2}' \times 13\frac{1}{4}" = 272$ superficial feet of the standard thickness of $1\frac{1}{2}$ bricks or $11\frac{1}{2}" = 306$ cubic feet. 100 cubic feet brickwork requires 41 imperial gallons of water, or 49 United States to slake the lime and mix the mortar. When the wall is circular and under 25 feet radius, take the outside for the width. Include sills under 6 inch.

Cornices. The English multiply the height by the extreme projection for a rectangular wall.

In various places in America, the height of the cornice is added.

Chimneys, flues, coppers, ovens, and such like, are measured solid, deducting half the opening for ash-pits and fireplaces.

Three inches of the wall-plate is added to the height for the wall; this compensates for the trouble of embedding the wall-plates.

Stone Walls. Measured as above, and take 100 cubic feet per cord of stone mason's measurement. The cord is 8x4 feet by 4 feet, or 12; cubic feet, or it is measured in cubic feet. The surface is measured by the superficial foot, as ashler hammered cut stone, and entered separate.

Chimneys are measured solid, only the fireplaces deducted in England.

Slater's Work. Measured by the square of 100 feet. Measure from the extreme ends. Allow the length by the gauge for the bottom course or eve. Deduct openings; but add 6 inches around them; also 6 inches for valley hips, raking, and irregular angles.

Filling. Measured as above. Add for valleys, 12", eaves, 4". All cutting hip, etc., 3 inches.

A Pantile is $1\frac{1}{2}" \times 1\frac{1}{2}" \times \frac{1}{2}"$ inch, weighs $5\frac{1}{2}$ lb, more or less, 1 sq. = 897 lb.

A Pantile $10\frac{1}{2}" \times 6\frac{1}{2}" \times \frac{3}{8}"$ inch, weighs $2\frac{1}{2}$ lb, " " " = 1680 lb.

Pantile laths, are 1 inch thick and $1\frac{1}{2}$ inch wide and 10 feet long.

Plastering. Render two coats and set. Lime, 0.6 cubic feet; sand, 68; hair, 19 lb; water, 2.7 imperial gallons.

Measure from top of baseboard to one-half the height of the cornice; deduct one-half for openings, or as the custom may be.

Gutters should have a fall of 1 inch in 10 feet.

Painting. 1 lb. of paint will cover 4 superficial yards, the first coat, and about 6 yards each additional coat. About 1 lb of putty for stopping every 20 yards.

1 gallon of tar and 1 lb of pitch cover 12 yards first coat, and about 17 yards the second coat.

1 gallon of priming color will cover 50 superficial yards.

" white paint " 41 " "

Other paints range from 41 to 50 " "

Take where the brush touched. Keep difficult and ornamental work separate. Also, the cleaning and stopping of holes, and other extras.

Joiner's Work. Measured as solid feet or squares of 100 feet superficial.

Flooring by the square of 100 feet superficial.

Skirting, per lineal foot, allowing for passages at the angles.

Sashes and frames. Take out side dimensions, add 1 inch for any middle bar in double sashes.

Engineers and architects ought to discountenance draining and wasting sewage into rivers. The paving of streets with wooden blocks, which is certainly unhealthy, causing malarial fevers. MacAdam stones, heavily rolled, etc., or stone blocks, are better. The French pavement, now used in London, is the best, which is made by putting a coat of asphalt $2\frac{1}{2}$ to 3 inches thick, on a bed of concrete 8 to 10 inches thick.

CHICAGO, Oct. 15, 1878.

M. McDERMOTT.

SANITARY HINTS.

310x10. The surveyors and engineers are frequently obliged to encamp where they encounter mosquitoes and diseases of the bowels.

Oil of pennyroyal around the neck, face, and wrists.

Apply around the neck and face, at the line of hair, and around the wrists, two or three times during the day and once or twice at night. This is a pleasant application to use, but disagreeable to the mosquitoes. We used to use a mixture of turpentine and hog's fat or grease, and at other times, wear a veil; both were but of temporary benefit; the first, was a nuisance, and the latter, by causing too much perspiration, was unhealthy.

Drinking too much water can be avoided by using it with finely ground oatmeal; by using this, the surveyor and engineer, and all their men using it, will not drink one-fifth as much water as if they did not use it.

DIARRHŒA.

The best known remedy is tincture of opium; tinct of camphor; tinct of rhubarb; tinct of capsicum (Cayenne pepper); of each one ounce. Add, for severe griping pains, 5 drops of oil of Anisee to each dose.

Dose.—25 drops in a little sweetened water, every hour or two, till relieved. Sometimes we put a little tannic acid, which is a powerful astringent. Avoid fresh meat, and use soda crackers.

To escape Chills and Fever, use quassi, by pouring some warm water on quassi chips, and letting it stand for the night. Take a cupful every morning. Never allow wet clothes to dry on you, if it can possibly be avoided. Tannic acid and glycerine will heal sore or scald feet.

Wafers applied to your corns, after being well soaked in lye water, will cure them. Apply the wafer after being moistened on the tongue; then apply a piece of linen or lint. Repeat this again when it falls off, in two or three days, and it will remove the corn and the pain together.

To Disinfect Gutters, Sewers, etc. Take one barrel of coarse salt and two of lime; mix them thoroughly, and sprinkle sparingly where required. This acts as chloride of lime.

To Disinfect Rooms in Buildings. Take, for an ordinary room, half an ounce of saltpetre; put on a plate previously heated, on this pour half an ounce of sulphuric acid (oil of vitriol); put the plate and contents on a heated shovel, and walk into the room and set the plate on some bricks previously heated. This destroys instantaneously every smell, enables the nurse to go to the bedside of any putrid body and remove it. Where there is sickness, as now in Memphis, etc., it causes great relief to the sick and protection to those in attendance. This is Dr. Smith's disinfectant, used at Gross Isle, Quarantine Station, below Quebec, Canada, in 1848. We have used it on many occasions, with satisfactory results, since then. Clothes hung in a well-closed room for two days, and subjected to this on three plates, would be rendered harmless.

CHICAGO, 23d Sept., 1878.

M. McDERMOTT.

For Gravity of a liquid. The annexed is a small bottle ttle, which, when filled to the cut or mark a b on

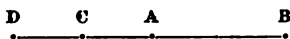
311. *Matter* is any substance. Temperature of 60° Fahrenheit, 1000 grains of *Inertia of Matter* is that vttles have thermometers attached to them; but *Motion* is the constant of accurate to have the bottle and thermometer on the *Force* is a power that ghe heat of the surrounding atmosphere and liquid *Power* is the body that mōain 500 grains. Some have a small hole through *Weight* is the body acted upon. ed, and the surplus water allowed to pass *Momentum* of a body is the product matter in it.

Gravity is the force by which bodies descend to the centre of the earth.

Centrifugal Force is that which causes a body, moving around a centre, to go off in a straight line.

Centripetal Force is that which tends to keep the body moving around the centre.

Let D B represent a straight line; D, C, A and B, given forces.

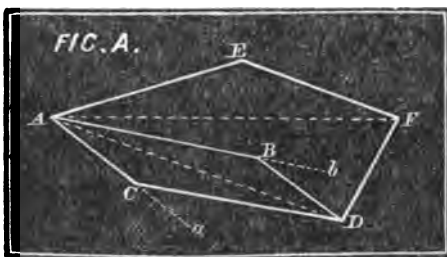


If D and C in the same direction act on A, their force = their sum.

If D and B in the same line act on A, but in different directions, the effect of their force will equal their difference, as $D - B$, where D is supposed the greater.

If D and C act on A in one direction, and B in the other, then the effect = $D + C - B$.

When the forces C and B act on A, making a given $< B A C$, the single force equal to both is called the resultant.



Resultant of the forces B and C acting on A is = D; or by representing forces B, C and D by the lines A B, A C, A D, then the resultant in the above will be the diagonal A D, and A B and A C are its components.

Composant or Component Forces are those producing the resultant, as A B and A C.

Rectangular Ordinates are those in which the $< B A C$ is right angled, or when a force acts perpendicularly to the plane A C or A B.

In the last figure, the force A C forces A in a direct line towards a, and the force A B forces A towards b in the same line; but when both forces act at the same time, A is made to move in A D, the diagonal of the parallelogram made by the forces A C and A B, by making $C D = A B$, and $A C = B D$.

Parallelogram of Forces is that in which A B and A C, the magnitudes of forces applied to the body A, gives the diagonal A D in position and magnitude. The diagonal A D is called the *resultant*, or resulting force.

Example. The force A B = 300 lbs., the force A C = 100 lbs., the angle B A C = right angle. Here we have A C and A B = B D and C D; $\therefore \sqrt{(A C^2 + C D^2)} = A D$; i. e., $\sqrt{(10000 + 90000)} = \sqrt{(100000)} = A D = 316.23$ lbs.

Otherwise, $A D = (A B^2 + A C^2 + 2 A B \times A C \times \cos. < B A C)^{\frac{1}{2}}$

Engineers and architects ought to discountenance them
sewage into rivers. The paving of streets with
certainly unhealthy, causing malarial fevers.

rolled, etc., or stone blocks, are better. The Pav

in London, is the best, which is made by putting $\frac{1}{2}$ = 860.55 lbs.

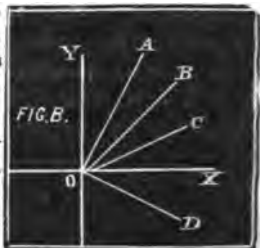
3 inches thick, on a bed of concrete 8 to 10 inches. $A D : A B :: \sin \angle B A C$
CHICAGO, Oct. 15, 1878.

SANITARY ENGINEERING.

THE SURVEYORS and ENGINEERS.

Let $C A$, $B A$, and A be three forces in magnitude. We find the re-
sultant $A D$ of the forces $C A$ and $B A$; then between this resultant and
the force $E A$ find the line $E F$, the required resultant of the three forces;
and so on for any other number of forces. By drawing a plan on a scale
of 100 lbs. to the inch, we will find the required forces.

Or, let $O X$ and $O Y$ be two rectangular axis,
and $A O$, $B O$, $C O$ and $D O$ represent forces,
and a , b , c , d = the angles made by the forces
 A , B , C and D , with the axis $O X$. Let S =
sum of the forces acting in direction of axis $O X$,
and s the sum of the forces acting in the direc-
tion of $O Y$; then we have $S = A O \cdot \cos a$
 $+ B O \cdot \cos b + C O \cdot \cos c + D O \cdot \cos d$.
 $s = A O \cdot \sin a + B O \cdot \sin b + C O \cdot \sin c$
 $- D O \cdot \sin d$. Resultant = $\sqrt{(S^2 + s^2)}$.



In this case, the forces are supposed to move inclined to the axis $O X$,
as well as to $O Y$.

Note. In the first quadrant $X O Y$, the sines and cosines are positive;
but in the fourth quadrant $X O W$, the sines must be negative.

The effect of any force acting on a body is in proportion to the cosine
of its inclination.

If three forces, B , C and D , act on a point A , so as to keep it in equili-
brium, each of these is proportional to the sine of the $\angle$ made by the
other two. (See fig. B.)

Let B and C be the components of the resultant D , then

$$D : C :: \sin \angle B A C : \sin \angle C A D.$$

$$D : B :: \sin \angle B A C : \sin \angle C A D.$$

If we represent the three forces meeting in A , by the three contiguous
edges of a parallelepiped, their resultant will be represented in magni-
tude and direction, by the diagonal drawn from their point of meeting to
the opposite angle of the parallelepiped.

If four forces in different planes act upon a point and keep it in equili-
brium, these four forces will be proportional to the three edges and diag-
onal of a parallelepiped formed on lines respectively parallel to the direc-
tions of the forces.

Polygon of Forces. Let $O A$, $O B$, $O C$ and $O D$ in fig. B. represent
forces in position and magnitude. From A draw $A E$ = and parallel to
 $O B$, $E F$ = and parallel to $O C$, $F G$ = and parallel to $O D$; then the
line $O G$ = resultant in magnitude and direction.

The sum of the moments, of any number of forces acting on a body,
must be equal to sum of the moments of any number of forces acting
in opposite directions, so as to keep the body from being overthrown.

livity bottle, which, when filled to the cut or mark a b on

312. All bodies are in a glass bottle, which, when filled to the cut or mark, is placed on a level surface, and with the surface of the liquid at the temperature of 60° Fahrenheit, 1000 grains of the body are weighed. Some bottles have thermometers attached to them; but the thermometers are not accurate to have the bottle and thermometer on the same level. The bottles are uniformly accelerated by the heat of the surrounding atmosphere and liquid, and the time from its commencement to the fall is noted. Some have a small hole through which the liquid can escape, and the surplus water allowed to pass.

ANALYSIS OF THE MOTION $\dot{e} = \dot{e}_0$ to the empty bottle and stop r.

Time in seconds from a state of rest.	Comparative spaces fallen through in each successive second.	Constant difference.	Comparative heights fallen through from a state of rest — H.
		Velocities acquired at the end of times in second col. — V.	
1	1 h	2 h	1 h
2	3 h	4 h	4 h
3	5 h	6 h	9 h
4	7 h	8 h	16 h
5	9 h	10 h	25 h
6	11 h	12 h	36 h
etc.	etc.	etc.	etc.
n	(2 n — 1) h	2 n h	n ² h

From the above, we find that by putting H = total height, and V = the acquired velocity, $V = 12 \text{ h} = \sqrt{4 \text{ h} \times 86 \text{ h}} = \sqrt{2 g H}$. Here $2 g = 4 \text{ h}$.

Let $V = 10 \text{ h} = \sqrt{(4 \text{ h} \times 25 \text{ h})} = \sqrt{2gH} = 8.02 \sqrt{H}$, etc.
 $V = 2 \text{ n h} = \sqrt{(4 \text{ h} \times \text{n}^2 \text{ h})} = \text{do.} = \text{do.}$

This is the general equation for the velocities of bodies moving in vacuo, from which it appears that

Velocities are to one another as the square roots of their heights.

Heights are to one another as the squares of their velocities. But as bodies do not move in vacuo, the velocities are less by a constant quantity of resistance, which we put $= m$.

Theoretical Velocity = $8.02 \sqrt{H}$, or as now used = $8.03 \sqrt{H}$.

Actual Velocity = $8.03 m \sqrt{H}$, in which m is to be determined by experiments.

To find the velocity of a stream of water. Take a ball of wax, two inches in diameter, or a tin sphere partly filled with water, and then sealed, so that two-thirds of it will be in the water. Find the elapsed time from the ball passing from one given point to another. Repeat the measurements until two of them agree.

Mean velocity is in the middle of the stream and at half its depth.

Let V = surface, and v = mean velocity; then, according to Prony, $v = 0.816 V$ for velocities less than 10 feet per second. (See *Sequel for Water Works.*)

Composition of Motions is like the composition of forces, and the same operations may be performed. If, in fig. A, last page, a body acting on

Engineers and architects ought to discountenance, then and in the direction of the united forces to sewage into rivers. The paving of streets with certainly unhealthy, causing malarial fevers.

rolled, etc., or stone blocks, are better. The Flag in London, is the best, which is made by putting space fallen through. 3 inches thick, on a bed of concrete 8 to 10 inches. CHICAGO, Oct. 15, 1878.

SANITARY ENGINEERING.

Let C A, B A, etc., be the space fallen through in 10 seconds.

Or, $H = \frac{A D^2}{2} = 1610$; or, by the third equation for S,

$$H = \frac{(322)^2}{2 \times 32.2} = \frac{103684}{64.4} = 1610.$$

When the velocity begins with a given acquired velocity $= c$, $V = c + t g$. Here c is constant for all intervals.

$$H = c t + \frac{t^2 g}{2} = \left(\frac{c + V}{2}\right) t = \frac{V^2 - c^2}{2 g} \text{ for accelerated motion.}$$

When the motion is retarded, and begins with velocity c , then $V = c - t g$.

$$H = c t - \frac{t^2 g}{2} = \left(\frac{c - V}{2}\right) \cdot t = \frac{c^2 - V^2}{2 g}$$

From above we have $V = t g$; $\therefore t = \frac{V}{g}$

Also, $H = c t - \frac{t^2 g}{2}$. Substituting the value of t , we have,

$$H = \frac{V^2 g}{2 g^2} = \frac{V^2}{2 g}$$

$V^2 = 2 g H$; but $H = \text{the total height} = H$;

$\therefore V = \sqrt{2 g H} = 8.02 \sqrt{H} = \text{formula for free descent.}$

$H = \frac{V^2}{64.4}$, and by putting $m = \text{coefficient or constant of resistance}$, we

find $V = m \sqrt{2 g H}$, and $H = \frac{V^2}{m^2 \times 2 g}$

Actual velocity $V = (8.08 m \sqrt{H})$ and $H = \left(\frac{V^2}{64.4 \times m^2}\right)$ all in feet.

CENTRE OF GRAVITY.

313. *Centre of Gravity* is that point in a mass which, if applied to a vertical line, would keep the whole body or mass in equilibrium.

In a *Circle*, the centre of gravity is equal to the centre of the circle.

In a *Square or Parallelogram*—where the diagonals intersect one another.

In a *Triangle*—where lines from the angles to the middle of the opposite line cut one another (see annexed figure). Where C H, D G and B P cut one another in the same point F, then G F = one-third of G D, and H F = one-third of C H. Hence, the centre of gravity of a triangle is at one-third of its altitude.

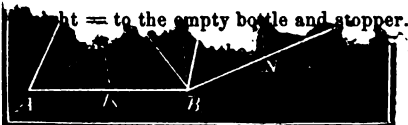
In a *Trapezoid*, A B C D, let E F be perpendicular to A B and C D.

When $E G = \frac{E F}{3} \times \frac{C D + 2 A B}{C D + A B}$, let E F = h, A B = b, and C D = c;

then $E G = \frac{1}{2}$ find the Specific Gravity of a liquid. The annexed is a small bottle

specific gravity bottle, which, when filled to the cut or mark a b on contains, at the temperature of 60° Fahrenheit, 1000 grains of be the given. Some bottles have thermometers attached to them; but B and C; find the centre of gravity of the bottle and thermometer on the gravity E of the bottle. Raise the heat of the surrounding atmosphere and liquid also the centre of the bottle, and contain 500 grains. Some have a small hole through the $\triangle C B D$; The bottle is filled, and the surplus water allowed to pass let $E F = 86$; stop the area of

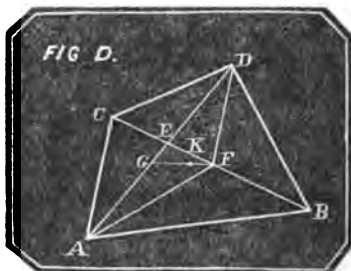
$\triangle A C B = 15$ stop the area of $\triangle C B D = 1500$; Speed, as $1200 + 1500 : 1200 :: 86 : F G = 16$; and in general figure, $A B D C : A C B :: F E : F Y$.



In the annexed figure, $A K = K B$, $C G = G B$, $B H = H D$, and Y is the required centre of gravity of $A B D$.

Let the figure have three triangles, as $A B L D C$. Find the centre of gravity N of the $\triangle B L D$; join Y and N ; then, $A B L D C : \triangle B L D :: Y N : Y S$. Hence, $S =$ required centre of gravity of $A B L D C$. Points E, F, N , are the centres of the inscribed circles. By laying down a plan of the given figure on a large scale, we can find the areas and lines $E Y$ and $Y S$, etc., sufficiently near.

Otherwise, by Construction. Let $A B C D$ be the required figure. Draw the diagonals $A D$ and $C B$; bisect $B C$ in F ; make $D E = A G$; join $F G$, and make $F K =$ one-third of $F G$; then the point K will be the required centre of gravity.



Cone or pyramid has its centre of gravity at one-fourth its height.

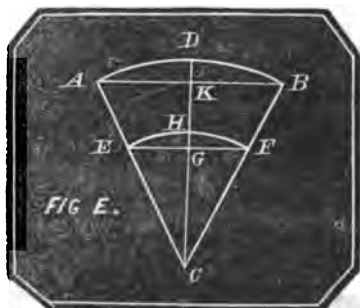
Frustrum of a Cone has its centre of gravity on the axis, measured from the centre of the lesser end, at the distance $\frac{h}{4} \left(\frac{8 R^2 + 2 R r + r^2}{R^2 + R r + r^2} \right)$. Here $R =$ radius of the greater end, and $r =$ that of the lesser; $h =$ height of the frustrum.

Frustrum of a Pyramid, the same as above, putting $S =$ greater side, instead of R , and $s =$ lesser side, instead of r .

In a Circular Segment, having the chord b , height h , and area A , given. Distance from the centre of the circle to the centre of gravity on $h = \frac{1}{12} \left(\frac{b^3}{A} \right)$

In a Circular Sector $C A B$, there are given the arc $A D B$, the angle $A C B$, $A B$ and the arc $A D B$ can be found by tab. 1 and 5, the radius $C D$ bisecting the arc $A D B$, and putting $G =$ centre of gravity, then its distance from the centre

$$= C G = \frac{\text{chord}}{\text{arc}} \times \frac{C}{D}$$



Engineers and architects ought to discountenance the use of G. Here
sewage into rivers. The paving of streets with the unit of A K
certainly unhealthy, causing malarial fevers. By table
rolled, etc., or stone blocks, are better. The Fr² = 84.91.
in London, is the best, which is made by putting 7 = space
3 inches thick, on a bed of concrete 8 to 10 inches thick we 10.4244 r from
CHICAGO, Oct. 15, 1878.

SANITARY ENGINEERING.

Surveyors and engineers use the following formulae:
If A, B, and C are the angles of a triangle, and a, b, and c are the sides opposite to them, then
$$\frac{4}{3} \cdot \frac{\sin. \frac{1}{2} c}{c} \cdot \frac{R^3 - r^3}{(R^2 - r^2)}.$$
 Here c = angle A C B.

Centre of Gravity of Solids.

814. *Triangular Pyramid or Cone.* The point G, or centre of gravity, is at three-fourths of its height measured from the vertex.

Wedge or Prism. The point G is in the middle of the line joining the centres of gravity of both ends.

In a Conic Frustrum, the distance of G from the lesser end is equal to
$$\frac{h}{4} \left(\frac{8R^2 + 2Rr + r^2}{R^2 + Rr + r^2} \right).$$
 Here R = radius of greater base, and r = that of the lesser.

In a Frustrum of a Pyramid, the above formula will answer, by putting R for the greater side and r for the lesser side of the triangular bases. The value will be the length from lesser end.

In any Polyhedron, the centre of gravity is the same as that of its inscribed or circumscribed sphere.

In a Paraboloid, the point G is at $\frac{3}{8}$ height from the vertex.

In a Frustrum of do. The distance of G from lesser end = $\frac{h}{8} \left(\frac{2R^2 + r^2}{R^2 + r^2} \right).$

In a Prismoid or Ungula, the point G is at the same distance from the base as the trapezoid or triangle, which is a right section of them.

In a Hemisphere, the distance of the centre of gravity is three-eighths of the radius from the centre.

In a Spherical Segment, the point G, from the centre of the sphere = $\frac{8.1416 h^2}{S} \left(r - \frac{h}{2} \right).$ Here h = height, and S = solidity.

SPECIFIC GRAVITY AND DENSITY.

815. *Specific Gravity* denotes the weight of a body as compared with an equal bulk of another body, taken as a standard.

Standard weight of solids and liquids is distilled water, at 60° Fahrenheit or 15° Centigrade. At this temperature, one cubic foot of distilled water weighs 1000 ounces avoirdupois.

When 1 cubic foot of water, as above, weighs	1000 ounces,
1 cubic foot of platinum weighs	21500 "
That is, when the specific weight of water =	1,
then the specific weight of platinum =	21.5.
One cubic foot of potassium weighs	865 "
∴ its specific gravity, compared with water, =	0.865.

816. *To find the Specific Gravity of a liquid.* The annexed is a small bottle called specific gravity bottle, which, when filled to the cut or mark a b on the neck, contains, at the temperature of 60° Fahrenheit, 1000 grains of distilled water. Some bottles have thermometers attached to them; but it will be sufficiently accurate to have the bottle and thermometer on the same table, and raise the heat of the surrounding atmosphere and liquid to 60°. Some bottles contain 500 grains. Some have a small hole through the stopper. The bottle is filled, and the surplus water allowed to pass through the stopper.

C is a counterpoise, that is, a weight = to the empty bottle and stopper.

To find the Specific Gravity. Fill the bottle with the liquid up to the mark a b (which appear curved), and put in the stopper. Put the bottle now filled into one scale, and the counterpoise and necessary weight in the other. When the scales are fairly balanced, remove the counterpoise. Let the remaining weight be 1269 grains; then the specific gravity = 1.269, which is that of hydrochloric or muriatic acid.

Density of a body is the mass or quantity compared with a given standard. Thus, platinum is $21\frac{1}{2}$ times more dense than water, and water is more dense than alcohol or wood.

Hydrometer is a simple instrument, invented by Archimedes, of great antiquity (300 B. C.), for finding the specific gravity of liquids. It can be seen in every drug store. See the annexed figure, where A is a long, narrow jar, to contain the liquid; B, a vessel of glass, having a weight in the bulb and the stem graduated from top downward to 100. The weight is such that when the instrument is immersed in distilled water at 60° Fahrenheit, it will sink to the mark or degree 100.

Example. In liquid L the instrument reads 70°. This shows that 70 volumes of the liquid L is = to 100 volumes of the standard, distilled water; $\therefore 70 : 100 :: 1 : 1.428$ = specific gravity of L.

The property of this instrument is, that it sustains a pressure from below upwards = to the weight of the volume of the liquid displaced by such body. Those generally used have a weight in the bulb and the stem graduated, and are named after their makers, as Baume, Carties, Gay Lussac, Twaddle, etc. Syke's and Dica's have moveable weights and graduated scales.

To find the Specific Gravity by Twaddle's Hydrometer. Multiply the degrees of Twaddle by 5; to the product add 1000; from the sum cut off three figures to the right. The result will be the specific gravity.

Example. Let 10° = Twaddle; then $10 \times 5 + 1000 = 1.050$ = specific gravity.

817. *To find the Specific Gravity of a solid, S.* Let S be weighed in air, and its weight = W. Let it be weighed in water, and its weight = w. Then $W - w$ = weight of distilled water displaced by the solid S. Then $\frac{W}{W - w}$ = specific gravity.

Rule. Divide the weight in the air by the difference between the weight in air and in water. The quotient will be the specific gravity.

Let a piece of lead weigh in air = 398 grains,
and suspended by a hair in distilled water = 362.4 "
Difference = 85.8

This difference divided into 398, gives specific gravity = 11.176, because $35.6 : 1 :: 398 : 11.176$ = specific gravity of the lead.

818. *To find the Specific Gravity of a body lighter than water.*

<i>Example.</i> A piece of wax weighs in air = B =	133.7 grains.
Attached to a piece of brass, the whole weight in air = C =	183.7
Immersed in water, the compound weighs = c =	88.8
Weight of water = in bulk to brass and wax = C - c =	144.9
Weight of brass in air = W =	50
" " in water = w =	44.4
Weight of equal bulk of water = W - w =	5.6
Bulk of water = to wax and brass = C - c =	144.9
" " = to brass alone = W - w =	5.6
" " = to wax alone = C - c - (W - w) =	139.3

That is, $C - c + w - W = 139.3$.

B : $C - c + w - W$:: specific gravity of body : specific gravity of water. That is,

W : $C - c + w - W$:: specific gravity of body : 1.

$$\text{Specific gravity of body} = \frac{B}{C - c + w - W} = \frac{133.7}{139.3} = 0.9598.$$

The above example is from Fowne's Chemistry; the formula is ours.

819. *To determine the Specific Gravity of a powder or particles insoluble in water.* Put 100 grains of it into a specific gravity bottle which holds 1000 grains of distilled water; then fill the bottle with water to the established mark, and weigh it; from which weight deduct 100, the weight of the powder. The remainder = weight of water in the bottle. This taken from 1000, leaves a difference = to a volume of water equal to the powder introduced.

<i>Example.</i> In specific gravity bottle put B =	100 grains.
Filled with water, the contents = C =	1060 "
Deduct 100 from 1060, leaves weight of water = C - B =	960 "
This last sum taken from 1000, leaves 1000 - C + B =	40 "

Which is = to a volume of water = to the powder.

$$40 : 1 :: 100 : 2.5 = \text{required specific gravity} = \frac{B}{1000 + B - C}$$

To find the Specific Gravity of a powder soluble in water. Into the specific gravity bottle introduce 100 grains of the substance soluble in water; then fill the bottle with oil of turpentine, olive oil, or spirits of wine, or any other liquid which will not dissolve the powder, and whose specific gravity is given; weigh the contents, from which deduct 100 grains. The remainder = the weight of liquid in the bottle, which taken from 1000, leaves the weight of the liquid = to the bulk of the powder introduced.

<i>Example.</i> In specific gravity bottle put of the powder =	100 grains.
Fill with oil of turpentine, whose specific gravity =	0.874
Found the weight of the contents	890 "
890 - 100 = weight of oil of turpentine in bottle =	790 "

which has not been displaced by the powder.

$$\text{But the bottle holds 874 grains, } \therefore 874 - 790 = 84$$

That is, 84 is the weight of a volume of the oil, which is equal to the volume of powder introduced. Consequently,

84 : 1000 :: 84 : 96.1 = weight of water = to the volume of powder introduced. And again, as 96.1 : 100 :: 1 : 1.04 = required specific gravity.

319a. SPECIFIC GRAVITIES OF BODIES.

SUBSTANCES.	Specific Gravity. ounces.	Weight of one cubic foot in lb.	SUBSTANCES.	Specific Gravity. ounces.	Weight of one cubic foot in lb.
<i>Metals.</i>			Mansfield sandstone.	2888	146.1
Brass, common.....	7820	489.8	Unhewn stones.....		185
Copper wire.....	8878	554.8	Hewn freestone.....		170
“ cast.....	8788	549.2	Coal, bituminous.....	1270	79.8
Iron, cast.....	7207	450.1	Coal, Newcastle.....	1270	79.8
“ bars.....	7788	486.7	“ Scotch.....	1800	81.2
Lead, cast.....	11852	709.5	“ Maryland.....	1855	84.6
Steel, soft.....	7888	489.5	“ Anthracite.....	1486	89.7
Zinc, cast.....	6861	428.8	<i>Granites.</i>		
Silver, not hammer'd	10474	654.6	Granite, mean of 14.		169
“ hammered....	10511	656.9	Granite, Aberdeen...	2625	164
<i>Woods.</i>			“ Cornwall.....	2662	166.4
Ash, English.....	845	52.8	“ Susquehanna.	2704	169
Beech.....	700	48.8	“ Quincy.....	2652	165.8
Ebony, American....	1881	88.1	“ Patapasco.....	2640	165.7
Elm.....	671	41.9	Grindstone.....	2148	188.9
Fir, yellow.....	657	41.1	<i>Limestones.</i>		
“ white.....	669	35.5	Limestone, green....	3180	198.7
Larch, Scotch.....	540	33.8	“ white.....	3156	197.2
Locust.....	950	59.4	Lime, quick.....	804	50.8
Norway spars.....	580	36.8	Marble, common.....	2686	167.9
Lignumvita.....	1888	88.8	“ French.....	2649	165.6
Mahogany.....	1068	66.4	“ Italian white..	2708	169.8
Maple.....	750	46.8	Mill-stone.....	2484	155.3
Oak, live.....	1120	70	Paving do.....	2416	151
“ English.....	982	58.2	Portland do.....	2428	151.7
“ Canadian.....	872	54.5	Sand.....	1800	112.5
“ African.....	980	61.8	Shale.....	2600	162.5
“ Adriatic.....	990	61.9	Slate.....	2672	167
“ Dantsic.....	760	47.5	Bristol stone.....	2510	156.9
Pine, yellow.....	660	41.2	Common do.....	2088	127
“ white.....	554	34.6	<i>Grains and Liquids.</i>		
Walnut.....	671	41.9	Water, distilled.....	1000	62.5
Teak.....	750	46.9	“ Sea.....	1026	64.1
<i>Stones, Earth, etc.</i>			Wheat.....		46.08
Brick.....	1900	118.7	Oats.....		24.58
Chalk.....	2784	174	Barley.....		48.01
Charcoal.....	441	27.6	Indian corn.....		46.08
Clay.....	1980	120.6	Alcohol, commercial.	887	52.8
Common soil.....	1984	124	Beer, pale.....	1028	68.9
Loose earth.....			“ brown.....	1084	64.6
Brick work.....		109	Cider.....	1018	63.6
Sand.....		112.8	Milk, cow's.....	1082	64.5
Craigleith sandstone	2282	189.5	Air, atmospheric....		.075
Dorley Dale do.....	2628	164.2	Steam.....		.087

One ton, or 2240 lbs. of	Average bulk in cubic feet.	Name of Materials used.	Shrink 'ge or incre'se per cent.
Paving Stone,.....	14.885	Light sandy earth,.....	.15 shr.
Brick,.....	18.828	Yellow clayey “.....	.10 “
Granite.....	18.505	Gravelly “.....	.08 “
Marble,.....	18.070	Surface or vegetable soil,....	.15 “
Chalk,.....	12.574	Puddled clay,.....	.25 “
Limestone, filled in pieces,....	14	Earth filled in water,.....	.30 “
“ compact,.....	11.273	Rock broken into small pieces,	¾ to 1 in.
Elm,.....	64.460	Rock broken to pass through	
Mahogany, Honduras,.....	64	an inch and a half ring,....	106 “
“ Spanish,.....	42.006	Do. do. 2 inch ring,.....	90 “
Fir, Mar forest,.....	51.650	Do. do. 2½ do.	70 “
“ Riga,.....	47.762	One cubic yard of the 1½ stone	
Beech,.....	51.494	above weighs.....	2180 lb.
Ash and Dantsic oak,.....	47.158	Do. to pass through 2 inch,	2800 lb.
Oak, English,.....	36.206	Do. to pass through 2½ inch	
Common soil,.....	18.044	ring,.....	2608 lb.
Loose earth.....	20.551		
Clay,.....	18.574		
Sand,.....	30		

MECHANICAL POWERS.

The Mechanical Powers are: the lever, inclined plane, wheel and axle, the wedge, pulley, and the screw.

319c. *Levers* are either straight or bent, and are of three kinds.

LEVERS CONSIDERED WITHOUT WEIGHT.

Lever of the first kind is when the power, P , and weight, W , are on opposite sides of the fulcrum, F . Then $P : W :: AF : BF$, which is true for the three kinds of levers, and from which we find $P \times BF = W \times AF$.

$$P = \frac{W \times AF}{BF}, \text{ and } W = \frac{P \times BF}{AF}. \quad (\text{See Fig. I.})$$

$$BF = \frac{W \times AF}{P}, \text{ and } AF = \frac{P \times BF}{W}.$$

Lever of the second kind is when the weight is between the fulcrum and the power, (Fig. II.) Then $P : W :: AF : BF$, as above.

Lever of the third kind (Fig. III.) is when the power is between the fulcrum and the weight. Then $P : W :: AF : BF$, as above.

Hence, we have the general rule: *The power is to the weight as the distance from the weight to the fulcrum, is to the distance from the power to the fulcrum.*

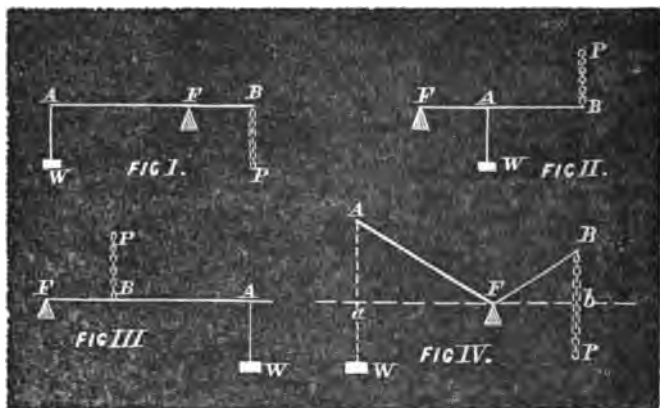
In a bent lever (Fig. IV.), instead of the distances AF and BF , we have to use Fa and Fb . Then $P : W :: Fa : Fb$; or, $P : W :: FA \times \cos. < A Fa : FB \times \cos. < B Fb$.

Let $PABW$ represent a lever (see Fig. V.) Produce PA and WB to meet in C . Now the forces P and W act on C ; their resultant is CB , passing through the fulcrum at F .

Let $AF = a$, $BF = b$, $\angle PAB = n$, and $\angle ABW = m$. Then

$$P : W :: b \sin. < m : a \sin. < n;$$

$$\text{And } P \cdot a \sin. n = W \cdot b \sin. m.$$



LEVERS HAVING WEIGHT.

819d. When the lever is of the same uniform size and weight. Let $AB =$ a lever whose weight is w . (Fig. VI.)

Case 1. Let the centre of gravity, f , be between the fulcrum, F , and power, P ; then we have, by putting $Ff = d$, $W \cdot AF = P \cdot BF + dw$,

$$P = \frac{W \cdot AF - dw}{BF}, \text{ and } W = \frac{P \cdot BF + dw}{AF}.$$

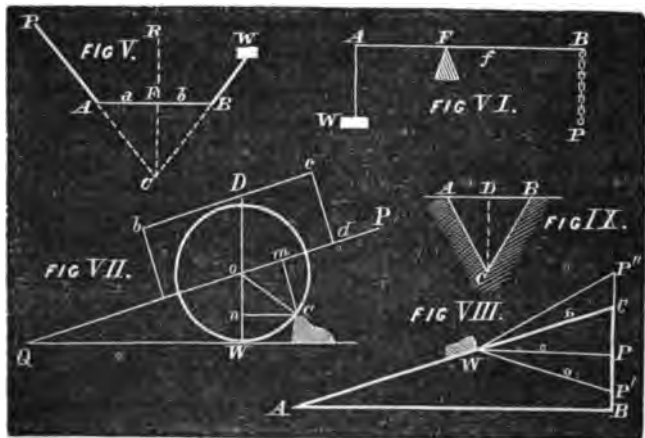
Case 2. When the centre of gravity, f , is between the fulcrum and the weight.

Then $W \cdot AF + dw = P \cdot BF$.

$$W = \frac{P \cdot BF - dw}{AF}, \text{ and } P = \frac{W \cdot AF + dw}{BF}.$$

Example from Baker's Statics. Let the length of the lever = 8 feet, $AF = 8$; $\therefore BF = 5$, its weight = 4 lbs., and W suspended at $A = 100$ lbs. Required the weight P suspended at B , the beam being uniform in all respects. We have the centre of gravity, a , = 4 feet from A , and at 1 foot from F towards P . Then, by case 1,

$$P = \frac{W \cdot AF - dw}{BF} = \frac{100 \cdot 8 - 1 \times 4}{5} = \frac{800 - 4}{5} = 59 \cdot 1 \cdot 5 \text{ lbs.}$$



319a. *Carriage wheel meeting an obstruction* (see Fig. VII.) is a lever of the first kind, where the wheel must move round C .

Let DWC = a wheel whose radius = r , load = $a b c d = W$. The angle of draught, PQW , = a , and C , the obstruction, whose height = h .

Let Cn and Cm be drawn at right angles, to OW and OP . Then Cm represents the power, and Cn the weight; then $P : W :: Cn : Cm$: $\sin \angle CON : \sin \angle COM$.

$DW = 2r$; $\therefore Dn = 2r - h$; and by Euclid, B. 2, prop. 5,

$$(2r - h) \cdot h + ON^2 = CO^2.$$

$$(2rh - h^2)^{\frac{1}{2}} = \sqrt{(CO^2 - ON^2)} = Cn; \therefore$$

$$\sin \angle CON = \frac{Cn}{Co} = \frac{\sqrt{(2rh - h^2)}}{r} = Cm.$$

When the line of draught is parallel to the road, then $Cm = r - h$.

From this we have $P : W :: \sqrt{2rh - h^2} : r - h$.

And $P = W \cdot \frac{\sqrt{(2rh - h^2)}}{r - h}$. A general formula.

Example. A loaded wagon, having a load of 3200 lbs., weight of wagon 800, meets a horse-railroad, whose rails are 8 inches above the street, the diameter of the wheel being 60 inches. Require the resistance or necessary force to overcome this obstacle.

Total weight of wagon and load, 4000 lbs. Weight on one wheel, 2000.

$$\therefore P = 2000 \times \frac{\sqrt{60 \times 8 - 9}}{30 - 8} = 968.9 \text{ lbs., which is about three times}$$

the force of a horse drawing horizontally from a state of rest.

Hence appears the injustice of punishing a man because he cannot leave a horse-railroad track at the sound of a bell, and the necessity of the local authorities obliging the railroad companies to keep their rail level with the street or road.

Of the Inclined Plane.

819f. Let the base, $AB = b$, height, $BC = h$, and length, $AC = l$. The line of traction or draught must be either parallel to the base, AB , as WP' parallel to the slant, or the inclined plane, as WP , or make an angle a with the line CW , W being a point on the plane where the centre of pressure of the load acts.

When the power P' acts parallel to the base, we have—

$$P' : W :: BC : BA :: h : b; \text{ or,}$$

$$P' : W :: \sin < BAC : \sin < ACB.$$

$$P' = \frac{W \cdot h}{b}, \text{ and } W = \frac{P' \cdot b}{h}.$$

$$h = \frac{P' \cdot b}{W}, \text{ and } b = \frac{W \cdot h}{P'}.$$

When the line of traction is parallel to the slant.

$$P : W :: h : l; \text{ hence, we have } Pl = Wh,$$

$$W = \frac{Pl}{h}, \text{ } P = \frac{Wh}{l},$$

$$h = \frac{Pl}{W}, \text{ and } l = \frac{Wh}{P}.$$

When the line of traction makes an angle a with the slant, then

$P'' : W :: \sin < BAP'' : \cos < P''WC$, from which, by alternation and inversion, we can find either quantity.

Example. $W = 20000$ lbs., $< BAC = 6^\circ$, $< P''WC = 4^\circ$. Required the sustaining power, P'' .

$$P'' = \frac{\sin BAP''}{P''WC} = \frac{W \sin BAP}{\cos < P''WC} = W \cdot \frac{\sin 4^\circ}{\cos 6^\circ} = W \cdot \frac{.06976}{.99452}$$

$$= \frac{1395.2}{.99452} = 1418 \text{ lbs.}$$

Of the Wheel and Axle.

819g. When the axle passes through the centre of the wheel at right angles to its plane, and that a weight, W , is applied to the axle, and the power, P , applied to the circumference, there will be an equilibrium, when the power is to the weight as the radius of the axle is to the radius of the wheel. Let $R =$ radius of the wheel, and $r =$ radius of the axle, both including the thickness of the rope; then we have

$$P : W :: r : R; \text{ from which we have}$$

$$PR = Wr, \text{ and } P = \frac{Wr}{R}, \text{ and } W = \frac{PR}{r}. \quad (A.)$$

$$R = \frac{Wr}{P}, \text{ and } r = \frac{PR}{W}.$$

Compound Axle is that which has one part of a less radius than the other. A rope and pulley is so arranged that in raising the weight, W , the rope is made to coil on the thickest part, and to uncoil from the thinner. An equilibrium will take place, when $2P \cdot D = W(R - r)$.

$D =$ distance of power from the centre of motion. $R =$ radius of thicker part of axle, and $r =$ that of the thinner.

819A. *Toothed Wheels and Axles or Pinions.* Let a, b and c be three axes or pinions, and A, B and C , three wheels.

The number of teeth in wheels are to one another as their radii.

$P : W :: abc : ABC$; that is, the power is to the weight as the product of all the radii of the pinions is to the product of all the radii of the wheels. Or, P is to W , as the product of all the teeth in the pinions is to the product of all the teeth in the wheels. (B.)

Example 1. A weight 2000 lbs. is sustained by a rope 2 inches in diameter, going round an axle 6 inches in diameter, the diameter of the wheel being 8 feet.

$$\text{From formula A, } P = \frac{Wr}{R};$$

That is, $P = \frac{2000 \times 4}{49} = 163.26 \text{ lbs.}$

Example 2. In a combination of wheels and axles there are given the radii of three pinions, 4, 6 and 8 inches, and the radii of the corresponding wheels, 20, 30 and 40 inches. What weight will $P = 100 \text{ lbs.}$ sustain at the circumference of the axle or last pinion.

By formula B, $P A B C = W a b c$.

$$W = \frac{P A B C}{W a b c} = \frac{100 \times 20 \times 30 \times 40}{4 \times 6 \times 8} = 12500 \text{ lbs.}$$

Of the Wedge. (Fig. IX.)

819i. The power of the wedge increases as its angle is acute. In tools for splitting wood, the $\angle A C B = 30^\circ$, for cutting iron, 50° , and for brass, 60° .

$P : W :: A B : A C$; or,

$P : W :: 2 \sin A C B : 1$.

Of the Pulley. (See next Fig.)

819j. The pulley is either fixed or moveable.

In a fixed pulley (Fig. I.), the power is equal to the weight.

In a single moveable pulley (Fig. II.), the rope is made to pass under the lower pulley and over the upper fixed one. Then we have $P : W :: 1 : 2$.

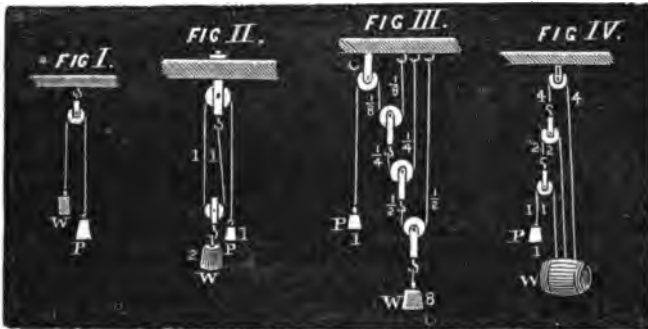
When the upper block or sheave remains fixed, and a single rope is made to pass over several pulleys (Fig. IV.)—for example, n pulleys—then

$P : W :: 1 : n$, and $P n = W$, and $P = \frac{W}{n}$, so that when $n = 6$, the power will be one-sixth of the weight.

When there are several pulleys, each hanging by its own cord, as in Fig. III., $P : W :: 1 : 2^n$.

Here n denotes the number of pulleys.

Example. Let $W = 1600 \text{ lbs.}$, $n = 4$ pulleys. Then $P \times 2^4 = W$; that is, $P \times 16 = 1600$, and $P = 100 \text{ lbs.}$



Of the Screw.

819k. Let $L D$ = distance between the threads, and r = radius of the power from the centre of the screw. Then

$P : W :: d : 6.2832 r$.

$P r \times 6.2832 = W D$.

$$W = \frac{P r \times 6.2832}{d}, \text{ and } P = \frac{W d}{6.2832 r}.$$

Example. Given the distance, 70 inches, from the centre of the screw to a point on an iron bar at which he exerts a power of 200, the distance between the contiguous threads 2 inches, to find the weight which he can raise. Here $r = 70$, $d = 2$, and $P = 200 \text{ lbs.}$

$$W = \frac{200 \times 70 \times 6.2832}{2} = 48982.4 \text{ lbs.}$$

VIRTUAL VELOCITY.

319m. *In the Lever*, $P : W :: \text{velocity of } W : \text{velocity of } P$.

In the Inclined Plane, $\text{vel. } P : \text{vel. } W :: \text{distance drawn on the plane} : \text{the height raised in the same time}$.

Let the weight W be moved from W to a , and raised from o to a ; then $\text{vel. } P : \text{vel. } W :: W a : o a$. (Fig. VIII.)

In the Wheel and Axle, $\text{vel. } P : \text{vel. } W :: \text{radius of axle} : \text{rad. of wheel} : W : P$.

In the single Moveable Pulley, $\text{vel. } P : \text{vel. } W :: 2 : 1 :: W : P$.

In a system of Pulleys, $\text{vel. } P : \text{vel. } W :: n : 1 :: W : P$. Here n = number of ropes.

In the ARCHIMEDEAN Screw, $\text{vel. } P : \text{vel. } W$, as the radius of the power multiplied by 6.2882 is to the distance between two contiguous threads. Let R = radius of power, and d = distance between the threads; then $\text{vel. } P : \text{vel. } W :: 6.2882 R : d :: W : P$.

OF FRICTION.

319n. Friction is the loss due to the resistance of one body to another moving on it. There are two kinds of friction—the sliding and the rolling. The sliding friction, as in the inclined plane and roads; the rolling, as in pulleys, and wheel and axle.

Experiments on Friction have been made by Coulomb, Wood, Rennie, Vince, Morin, and others.

Those of Morin, made for the French Government, are the most extensive, and are adopted by engineers. When no oily substance is interposed between the two bodies, the *friction* is in proportion to their perpendicular pressures, to a certain limit of that pressure. The friction of two bodies pressed with the same weight is nearly the same without regard to the surfaces in contact. Thus, oak rubbing on oak, without unguent, gave a coefficient of friction equal to 0.44 per cent.; and when the surfaces in contact were reduced as much as possible, the coefficient was 0.41½.

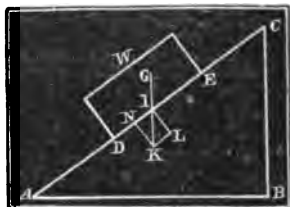
Coulomb has found that oak sliding on oak, without unguent, after a few minutes had a friction of 0.44, under a vertical pressure of 74 lbs.; and that by increasing the pressure from 74 to 2474 lbs., the coefficient of friction remained the same.

Friction is independent of the velocities of the bodies in motion, but is dependent on the unguents used, and the quantity supplied.

Morin has found that hog's lard or olive oil kept continuously on wood moving on wood, metal on metal, or wood on metal, have a coefficient of 0.07 to 0.08; and that tallow gave the same result, except in the case of metals on metals, in which case he found the coefficient 0.10.

Different woods and metals sliding on one another have less friction. Thus, iron on copper has less friction than iron on iron, oak on beach has less than oak on oak, etc.

The angle of friction is $= \angle B A C$, in the annexed figure, where W represents the weight, kept on the inclined plane $A C$ by its friction. Let G = centre of gravity; then the line $I K$ represents the weight W , in direction of the line of gravity, which is perpendicular to $A B$; $I L$ = the pressure perpendicular to $A C$, and $I N = L K$ = the friction or weight sufficient to keep the weight W on the plane. The two triangles, $A B C$ and $I K L$ are similar to one



another; $\therefore KL:LI::BC:AB::$ the altitude to the base. Also, $KL:KI::BC:AC$.

In the first equation, we have the force of friction to the pressure of the weight W , as the height of the inclined plane is to its base.

In the second equation, we have the force of friction to the weight of the body, as the height of the plane is to its length.

Hence it appears that by increasing the height of BC from B to a certain point C , at which the body begins to slide, that the $<$ of friction or resistance is $= < BAC$.

That the *Coefficient of Friction* is the tangent of $< BAC$, and is found by dividing the height BC by the base AB .

Angle of Repose is the same as the angle of friction, or the $< BAC =$ the angle of resistance.

819o. *Friction of Plane Surfaces having been some in Contact.*

Surfaces in Contact.	Disposition of the Fibres.	State of the Surfaces.	Coefficient of Friction.	Angle of Repose.
Oak upon oak.....	Parallel.....	Without unguent	0.62	31° 48'
do.	do.	Rubbed with dry soap.....	0.44	23 45
do.	Perpendicular..	Without unguent	0.54	28 22
do.	End of one on flat of other..	do. do.	0.48	23 16
Oak upon elm.....	Parallel.....	do. do.	0.38	20 49
Elm upon oak.....	do.	With soap.....	0.41	22 18
do.	Perpendicular..	Without unguent	0.57	29 41
Ash, fir or beach on oak.....	Parallel.....	do. do.	0.53	27 56
Tanned leather upon oak.....	Leather lengthways, sideways	do. do.	0.43	23 16
Black strap leather upon oak.....	Parallel.....	do. do.	0.74	36 30
do. do. on rounded oak.....	Perpendicular..	do. do.	0.47	26 11
Hemp cord upon oak.....	Parallel.....	do. do.	0.80	38 40
Iron upon oak.....	do.	Steeped in water	0.65	33 02
Cast-iron upon oak.....	do.	do. do.	0.66	33 02
Copper upon oak.....	do.	Without unguent	0.62	31 43
Bl'k dress'd leather on iron pulley	Flat.....	do. do.	0.23	15 33
Cast iron upon cast iron.....	do.	do. do.	0.16	9 6
Iron upon cast iron.....	do.	do. do.	0.10	10 46
Oak, elm, iron, cast iron and brass, sliding two and two, on one another.....	do.	With tallow.....	0.10	5 43
do. do. do.	do.	Hog's lard.....	0.15	8 22
Common brick on common brick.....	do.	do. do.	0.67	33 50
Hard calcareous stone on the same, well dressed.....	do.	do. do.	0.70	35 00
Soft calcareous stone upon hard calcareous stone.....	do.	do. do.	0.75	36 52
do. do. do. on same, with fresh mortar of fine sand.....	do.	do. do.	0.74	36 30
Smooth free stone on same.....	do.	do. do.	0.71	35 23
do. do. do. with fresh mortar.....	do.	do. do.	0.66	33 26
Hard polished calcareous stone on hard polished calcareous stone.....	do.	do. do.	0.58	30 07
Well dressed granite on rough granite.....	do.	do. do.	0.66	33 26
Do., with fresh mortar.....	do.	do. do.	0.49	26 07

819p. *Friction of Bodies in Motion, one upon another.*

Surfaces in Contact.	Disposition of the Fibres.	State of the Surfaces.	Coefficient of Friction.	Angle of Repose.
Oak upon oak.....	Parallel.....	Without unguent	0.48	25° 39'
do.	do.	Rubbed with soap	0.16	9 06
do.	Perpendicular..	Without unguent	0.43	18 47
Elm upon oak.....	Parallel.....	do. do.	0.43	23 17
do.	Perpendicular..	do. do.	0.45	24 14
Iron upon oak.....	Parallel.....	Rubbed with dry soap.....	0.21	11 52
do.	do.	Without unguent	0.49	26 07
Cast iron upon oak.....	do.	Rubbed with soap	0.19	10 46
Iron upon elm.....	do.	Without unguent	0.25	14 03
Cast iron on elm.....	do.	do. do.	0.20	11 19
Tanned leather upon oak.....	Lengthways and sideways.....	do. do.	0.56	29 15
do. on cast iron and brass.....	do. do.	With oil.....	0.15	8 32

819g. *Friction of Axles in motion on their bearings.*

Cast iron axles in same bearings, greased in the usual way with hog's lard, gives a coefficient of friction of 0.14, but if oiled continuously, it gives about 0.07.

Wrought iron axles in cast iron bearings, gives as above, .07 and .05.

Wrought iron axles in brass bearings, as above, .09 and .00.

MOTIVE POWER.

819r. *Nominal horse power* is that which is capable of raising 33,000 pounds one foot high in one minute. The English and American engineers have adopted this as their standard; but the French engineers have adopted 32,560 lbs. Experiments have proved that both are too high, and that the average power is 22,000 lbs.

The following tables are compiled, and reduced to English measures, from Morin's *Aide Memoire*:

<i>Work done by Man and Horse moving horizontally.</i>	Hours worked per day.	Kilometres carried 1 metre in 1 sec.	Pounds avoirdupois carried 1 foot in 1 minute.
A man unloaded.....	10	97.50	12902
A laborer with a small two-wheel cart, going loaded and returning empty.....	10	50	6617
Do. with a wheelbarrow as above.....	10	30	3970
Do. walking loaded on his back.....	7	30	3970
Do. loaded on his back, but returning unloaded.....	6	32.5	4801
Do. carrying on a handbarrow as above.....	10	16.5	2188
A horse with a cart at a pace continually loaded.....	10	770	101894
Do. do. returning unloaded.....	10	420	55579
Do. with a carriage at a constant trot.....	4.5	770	101894
Do. loaded on the back, going at a pace.....	10	182	17467
Do. do. at a trot.....	7	176	23290

<i>819s. Work done by Man in moving a body vertically.</i>	Hours worked per day.	Kilograms raised 1 met. high per sec.	Lbs. avoirdupois raised 1 foot high per minute.
Man ascending an inclined plane.	8	9.75	1290
Do. raising weight with a cord and pulley, the cord descending empty.....	6	8.60	476
Do. raising weight with his hands.....	6	8.40	450
Do. raising a weight, and carrying it on his back to the top of an easy stairway, and returning empty..	10	1.20	159
Do. shovelling earth to a mean height of 1.60 metres.	10	1.08	148

<i>819t. Action on Machines.</i>	Hours per day.	Force in Kilograms per second.	Force in pounds per minute.
A man acting on a wheel or drum at a point level with the axle.....	8	9	1191
Do. acting at a point below the axle at an $\angle$ of 24° ..	8	8.40	1112
Do. drawing horizontally, or driving before him.....	8	7.20	758
Do. acting on a winch.....	8	6	794
Do. pushing and drawing alternately in vert. position	8	5.50	728
A horse harnessed to a carriage and going at a pace..	10	68	8337
Do. harnessed as a riding horse, going at a pace.....	8	40.50	586
Do. do. going at a trot.....	4.5	60	7940

ROADS AND STREETS.

319u. Roman roads were made to connect distant cities with the Imperial Capital. In low and level grounds, they were elevated above the adjoining lands, and made as follows:

1st. The *Statumen*, or foundation—all soft matter was removed.

2d. The *Rudratio*, composed of broken stones or earthenware, etc., set in cement.

3d. The *Nucleus*, being a bed of mortar.

4th. The *Summa Crusta*, or outer coat, composed of bricks or stones. Near Rome, the upper coat was of granite; in other places, hard lava, so closely jointed, that it was supposed by Palladio that moulds were used for each stone or piece.

The *Curator Viarum*, or superintendent of highways, was an officer of great influence, and generally conferred on men of consular dignity after *Julius Cæsar*, who held that office, assisted by his colleague, *Thernus*, a noble Roman. *Victorius Marcellus*, of the prætorian order, had been selected to this office by the Emperor Domitian. These are but a few instances of the many in which men of the highest position in society became *Curator Viarum*—or, as the Americans call him, commissioner of highways, or path master.

The *Appian Way*, called also Queen of the Roman ways, was made by Censor Appius Cæcus, about 311 years before the Christian era, and built then as far as Capua, 125 miles; but subsequently to Brundisium, about the year B. C. 249. "The Appian Way was of a sufficient width (18 to 22 feet) to allow two carriages to pass; was made of hard stone, squared, and made to fit closely. After 2000 years, but little signs of wear appear."—*Eustace*.

Gravel roads, with small stones, were commonly used by the Romans. Porticoes were built at convenient distances, to afford shelter to the traveler.

Roman Military roads were 36 to 40 feet wide, of which the middle 16 feet were paved. At each side there was a raised path, 2 feet wide, which again separated two sideways, each 8 feet wide.

The breadth of the Roman roads, as prescribed by the laws of the twelve tables, was but 8 feet; the width of the wheel tracks not above 8 feet. There were twenty-nine military roads made, equal in length to 48500 English miles.

The *Carthaginians*, according to Isadore, were the first who paved their public ways.

The *Greeks*, according to Strabo, neglected three objects to which the Romans paid especial attention: the *cloacæ*, or common sewers, the aqueducts, and the public highways. The Greeks made the upper part of their roads with large, square blocks of stone, whilst the Romans mostly used irregular polygons.

The *French roads* are from 30 to 60 feet wide, the middle 16 feet being paved; but once a vehicle leaves the pavement, it becomes a matter of much difficulty to extricate it from the soft surface of the sides. To obviate this difficulty, the system of using broken stones is now generally adopted, and has been used in France, under the direction of M. Turgos, a long time before *McAdam* introduced it into England.

The *German roads* resemble those of France.

The *Belgium roads* have their surfaces composed of thin brick tiles, which answer well for light work.

Sweden has long been famous for her excellent roads of stone or gravel, on which there is not a single tollgate. Each landowner is obliged to keep in repair a certain part of the road, in proportion to his property, whose limit is marked by land marks on each side of the road.

The *English, Irish and Scotch roads* are now generally made of broken stones, or *macadamised*; are 25 to 50 feet wide: well drained—having the centre 12 inches higher than on the sides, in a road 40 feet wide, and in proportion of 3 inches in 10 feet wide; the stones broken so as to pass through an inch-and-half ring. For the purpose of keeping them in repair, there are *depôts*, or heaps of broken stones, at intervals of 600 feet. When a small hole makes its appearance, a man loosens the stones around the spot to be repaired, and then fills it up with new material, which soon becomes as when originally made.

Arthur Young states that it was not until 1660 that England took an interest in her roads. (See *Encyclopædia Britannica*, vol. xii, p. 528.) In his tour through the British Isles in 1779, he states that Ireland then had the best roads in Europe. This is not to be wondered at, when we consider that there, granite, limestone and gravel beds are abundant; that since the beginning of the reign of Charles I, the roads were under the charge of the grand jury. There, good roads must have existed at a very early date, as the stones of which the round towers are built are large, and, in some places, have been brought from a great distance.

Many of the English and Irish highways were turnpike roads; that is, roads having tollgates. Since the introduction of railways, these have been falling off in revenue. In a parliamentary inquiry into turnpike trusts in Ireland, the unanimous testimony of all the witnesses examined were against them, and in favor of having them kept in repair by *presentment*.

Presentment is where the grand jury receives proposals to keep road B, blank miles, from point A to point B, in repair, according to the specification of the county surveyor, during time T, at the rate of sum s per rod, subject to the approval of the county surveyor, who has the general supervision of all the public works, and are gentlemen of integrity and high scientific attainments. The work on hydraulics by Mr. Neville, county surveyor for Louth, and that on roads by my school-fellow, Edmond Leahy, county surveyor for Cork, are generally in the hands of every engineer.

By the parliamentary report for 1839–40, England had 21962 miles of turnpike trusts. The tolls amounted to £1,776,586; the expenditure for repairs and officers, £1,780,849, leaving a deficiency of £3,768. The same deficiency appears to take place on the Irish roads.

In England, the parish roads equal 104772 miles, costing annually for highway rates £1,168,207. The number of surveyors and deputy surveyors, or way-wardens, is 20000, or one way-warden to every $5\frac{1}{4}$ miles of road. It was then shown that the trusts had incurred debts to the enormous amount of £8,577,182.

Under the new system, one man keeping a horse is supposed to take charge of 40 miles of road.

Making and Repairing Macadamised Roads.

§19c. The road bed should have a curved surface of about 1 foot rise for 40 feet wide, be a segment of a circle, and have at least 12 inches of stones on the centre, and 8 to 10 on the sides, both of which are to be on the same level. When the stones are well incorporated with one another, a layer of sand, 1 inch in thickness, is spread on top. The bed must be thoroughly drained, and the water made to flow freely in the adjoining ditches. The overseers should never allow any water to accumulate on the road, and every appearance of a rut or hole immediately checked. Where there is frost, it is liable to disintegrate the road material, unless it is built of very compact stuff. In boggy land, a soling of 12 to 18 inches of stiff clay must be laid under the broken stone. Where the bottom is sandy, and stiff clay hard to be procured, rough pavements or concrete, from 6 to 12 inches thick, under the broken stones, will be the best. In general, where the soil is well drained, broken stones will be sufficient. The road is never to have less than 8 inches on the centre and 4 on the sides. All large stones raked to the sides, and broken, so as to pass through a ring $1\frac{1}{2}$ inches in diameter. The surface always kept uniform. The English and Irish roads are generally 25 feet between the ditches, but in approaches to cities and towns, they are 40 to 50 feet. On the Irish roads, no house is allowed nearer than 80 feet of the centre of the road.

To allow for shrinkage. Mr. Leaby, in his work on roads, p. 100, says: In bog stuff, add *one-fourth* of its intended height; if the road is of clay or earth, add *one-twelfth*.

When the road passes through boggy land, the side ditches, or drains, must be dug to a depth of 4 feet below the surface of the road, and have parallel drains running along in the direction of the road, about 40 feet on each side. In this manner, roads have been made over the softest bogs in Ireland. On the Milwaukee and Mississippi Railroad, near Milwaukee, a part of the road passed over the Menomonee bottoms. After several weeks of filling, the company was about to relinquish that part of the route, for all the work done during the week would disappear during Sunday. The author being employed as city engineer in the neighborhood, saw the respective officers holding a consultation. He came up, and on being asked his opinion, replied: "Imitate nature; first lay on a layer of brushwood, 1 foot thick; then 2 feet of clay, and so on alternately." The plan was adopted, and has succeeded.

Where the road is wet and springy, cross drains filled with stones are to be made, to connect with the side drains or ditches; and if made within 50 or 60 feet of one another, will be sufficient to drain it.

Where the road runs along a sloping ground, catch-water drains should be run parallel with the road, so as to keep off the hill water.

Retaining walls should have a batter or slope of 8 inches to each foot in height, and the back may be parallel to the same. The thickness, $2\frac{1}{4}$ feet for 10 feet in height, and in all other cases, the thickness shall be *one-fourth* of the height. An offset of 8 inches should be left at front of the footing course, and the foundation cut into steps. Where such walls are along water courses, the foundation should be 15 inches below the bottom of the water, and paved along the side to a width of 18 inches or 2 feet. The filling behind is put in in layers, and rammed in.

Parapet walls should be 20 inches thick and 8½ feet high, built of masonry laid in lime mortar, in courses of 12 or 14 inches, the top course or coping to be semicircular, and have a thorough bond at every 3 feet.

Where drains are covered, dry masonry walls, covered with flags, are preferable. Where the width of the drain is not more than 30 inches, these drains will require flags 6 inches thick; those between 18 and 24 inches are to have flags 5 inches thick; and those from 8 to 18 inches, require flags 4 inches thick.

Drainage. When the road runs along a hill, cut a drain parallel to the road, and 3 to 4 feet below the surface; then cut another of smaller dimensions near the road, and sunk below the road-bed. Again, at every 60 or 100 feet, sink cross drains, about 15 to 24 inches below the road-bed; fill with broken stones to within 6 inches of the top, which space of 6 inches is to be filled with small broken stones of the usual size in road making—these cross drains to communicate with a ditch or drain on the lower side of the road, to keep it dry.

Drain holes, about 100 feet apart; 8 inches square, and about 2 inches under the water table of the drain; may be made of 4 flag stones, draining tiles, or pipes.

Road Materials. Granite is the best.

Sienite is granite, in which hornblende is mixed. This is very durable, and resists the action of the atmosphere. This stone has a greenish color when moistened.

Sandstone, if impregnated with silica, is hard, and makes a good material. Some varieties are composed of pure silex, which makes an excellent material; but others are mixed with other substances, which make the stone porous, and unfit to be used by the action of frost, it easily disintegrates.

Limestone has a great affinity for water, which it imbibes in large quantities. If frozen in this condition, it is easily crumbled under the wheels of carriages, and becomes mud. Hence the great necessity of keeping a road made with broken limestone *thoroughly drained*, in all places where frost makes its appearance. There is nothing more injurious to roads than frosts.

Stones having fine granular appearance, and whose specific gravity is considerable, may be considered good road material.

Experiments made by Mr. Walker, civil engineer, during seventeen months of 1830 and 1831, on the Commercial Road, near London, will show the quality of the following stones: (See *Transactions Inst. Civil Engineers*, Vol. 1.)

Description of Stone.	Where procured.	Absolute wear in 17 months.	Time in which 1 inch would wear down.
Granite.....	Dartmoor.....	.207 inches.	6.8 years.
".....	Guernsey.....	.060	22.5
".....	Herm, near Guernsey..	.075	19.
Blue Granite.....	Peterhead.....	.131.	10.8
Granite.....	Heyton.....	.141	10.
Red Granite.....	Aberdeen.....	.159	9.
Blue Granite.....	".....	.225	6.88
Whinstone.....	Budle.....	.082	17.88

COMPRESSION.

	Lbs. avoirdupois to crush a cube of 1½ inches.		Lbs. avoirdupois to crush a cube of 1½ inches.
Chalk.....	1127	Cornish granite.....	14302
Brick, pale red color.....	1265	Dundee sandstone.....	14918
Red brick, mean.....	1817	Craighleith gritstone, with the	
Yellow-faced paviors.....	2254	. strata.....	15560
Fire brick.....	3864	Devonshire red marble, vari-	
Whitby gritstone.....	5328	egated.....	16712
Derby " and friable		Compact limestone.....	17354
sandstone.....	7070	Penryn granite.....	17400
Do. from another quarry.....	9776	Peterhead " close grained...	18686
White freestone, not stratified.	10264	Black compact Limerick lime-	
Portland stone.....	10284	stone.....	19924
Humbly gritstone.....	10871	Black Brabant marble.....	20742
Craighleith white freestone.....	12846	Very hard freestone.....	20254
Yorkshire paving, with strata.	12856	White Italian veined marble...	20783
Do. against the strata.....	12856	Aberdeen granite, blue kind...	24556
White statuary marble, not		Valencia slate.....	26656
veined.....	18682	Dartmoor granite.....	27680
Brambyfall sandstone, near		Heyton granite.....	31860
Leeds, with strata.....	18682	Herm granite, near Guernsey...	33600

A road made over well dried bogs or naked surface, on account of its elasticity, does not wear as fast as roads made over a hard surface. It has been found that on the road near Bridgewater, England, the part over a rocky bed wears 7, when that over a naked surface wears 5.

The covering of broken stones is, in the words of McAdam, intended to keep the road-bed dry and even.

Some of the material used on the roads near London are brought from the isle of Guernsey and Hudson Bay.

Weight of vehicles, width of tiers, and velocity, have great influence on the wear of roads. In Ireland, two-wheeled wagons or carts are generally used—the weight 6 to 8 cwt., and load 22 to 25 cwt., making a gross load of about 80 cwt. In England, four-wheeled wagons are generally used, and weigh, with their load, from 5 to 6 tons; therefore, the pressure of these vehicles is as 1660 to 3320, on any given point.

It is evident that when the vehicle is made to ascend a large stone, that in falling, it acquires a velocity which is highly injurious to the road, and that there should not be allowed any stone larger than 1½ inches square on the surface.

Table of Uniform Draught.

Description of Surface.	Rate of Inclination.
Ordinary broken stone surface.....	Level.
Close, firm stone paving.....	1 in 48.5
Timber paving.....	1 in 41.5
Timber trackway.....	1 in 31.66
Cut stone trackway.....	1 in 31.66
Iron tramway.....	1 in 29.25
Iron railway.....	1 in 28.5

Explanation. If a power of 90 lbs. will move one ton on a level, broken stone road, it will move the same weight on an iron railway having an inclined plane of 1 in 28½.

FRICTION ON ROADS.

The power required to move a wheel on a well made, level road, depends on the friction of the axles in their boxes, and to the resistance to rolling.

When the axles are well made and oiled, the friction is taken at one-eighteenth of the pressure; but in ordinary cases, it is taken at one-twelfth, $= \frac{W}{12}$, and power $= \frac{W}{12} \times \frac{a}{d} = \frac{W a}{12 d}$. Here power is that force which, if applied at the tier, would just cause the wheel to move. a = diameter of the axis, and d = diameter of the wheel.

The following is Sir John McNeill's formula, given in his evidence before a committee of the House of Lords, for the draught on common roads:

$P = \frac{W + w}{93} + \frac{w}{40} + c V$. Here W = weight of the wagon, w = weight of the load, V = velocity in feet per second, and c = a constant quantity derived from experiments on level roads.

Kind of Road.	Value of c.
For a timber surface.....	2
" paved road.....	2
" a well made broken-stone road, in a dry state.....	5
" " " " covered with dust.....	8
" " " " wet, and covered with mud.....	10
" gravel or flint road, when wet.....	18
" " " " very wet, and covered with mud.....	32

Let $W = 720$, $w = 3000$, paved road; let $V = 4$ feet. Here $c = 2$, and we have—

$$P = \frac{720 + 3000}{93} + \frac{3000}{40} + 2 \times 4.$$

$P = 40 + 75 + 8 = 123$ = draught, or the force necessary to overcome the combined friction of the axle in the box and the wheel in rolling on the surface. This force is one-thirtieth of the total load of weight and wagon.

By *McNeill's Improved Dynamometer*, the following results have been obtained. Weight of wagon and load = 21 cwt.

Kind of Road.	Force in Lbs.	Ratio of Draught to the Load.
Gravel road laid on earth.....	147	= 1-16th of the load.
Broken stones.....	65	= 1-86th "
" on a paved foundation.....	46	= 1-51st "
Well made pavement.....	38	= 1-71st "
Best stone track ways.....	12½	= 1-179th "
Best form of railroad.....	8	= 1-280th "

M. Poncelet gives the following value of draught or force to overcome friction:

On a road of sand and gravel.....	1-16th of the total load.
On a broken stone road, ordinary condition	1-25th "
" " in good condition.....	1-67th "
On a good pavement, at a walk.....	1-54th "
" " at a trot.....	1-42d "
On a road made of oak planks.....	1-98th "

Table showing the Lengths of Horizontal Lines Equivalent to several Ascending and Descending Planes, the Length of the Plane being Unity.

In calculating this table, Mr. Leahy has assumed that an ordinary horse works 8 hours per day, and draws a load of 8000 pounds, including the weight of the wagon, making the net load 1 ton.

One in	One-horse Cart.		Stage Coach.		Stage Wagon.		Angle of Elevation
	Ascend'g.	Desc'nd'g.	Ascend'g.	Desc'nd'g.	Ascend'g.	Desc'nd'g.	
5	8.82	8.27					0 1 11
10	4.16	1.65	2.85		6.07		5 42 58
15	2.90	1.06	2.23		4.89		8 48 51
20	2.08	0.83	1.93	0.07	3.54		2 51 21
25	1.66	0.70	1.74	0.26	3.04		2 17 26
30	1.55	0.74	1.62	0.39	2.70		1 54 37
35	1.45	0.77	1.53	0.47	2.46		1 38 14
40	1.40	0.79	1.46	0.54	2.27		1 25 57
45	1.35	0.81	1.41	0.59	2.13		1 16 24
50	1.31	0.83	1.37	0.63	2.02		1 8 6
55	1.29	0.84	1.34	0.66	1.93	0.07	1 2 30
60	1.26	0.85	1.31	0.69	1.85	0.15	0 57 18
65	1.24	0.86	1.29	0.71	1.78	0.22	0 52 54
70	1.22	0.87	1.27	0.72	1.72	0.78	0 49 7
75			1.68	0.82	1.25	0.75	0 45 51
80	1.19	0.88	1.64	0.86	1.23	0.77	0 42 58
85			1.60	0.40	1.22	0.78	0 40 27
90	1.17	0.89	1.57	0.43	1.21	0.79	0 38 12
95			1.54	0.46	1.20	0.80	0 36 11
100	1.15	0.90	1.51	0.49	1.19	0.81	0 34 23
110			1.45	0.55	1.17	0.83	0 31 15
120			1.43	0.58	1.15	0.85	0 28 39
130			1.39	0.61	1.14	0.86	0 26 27
140			1.36	0.64	1.13	0.87	0 24 33
150	1.10	0.92	1.34	0.66	1.12	0.88	0 22 55
160			1.32	0.68	1.12	0.88	0 21 29
170			1.30	0.70	1.11	0.89	0 20 18
180			1.28	0.72	1.10	0.90	0 19 6
190			1.27	0.73	1.10	0.90	0 18 6
200	1.07	0.93	1.26	0.75	1.09	0.91	0 17 11
210			1.24	0.76	1.09	0.91	0 16 22
220			1.23	0.77	1.08	0.92	0 15 37
230			1.22	0.78	1.08	0.92	0 14 57
240			1.21	0.79	1.08	0.92	0 14 19
250			1.20	0.80	1.07	0.93	0 13 45
260			1.20	0.80	1.07	0.93	0 13 13
270			1.19	0.81	1.07	0.93	0 12 44
280			1.18	0.82	1.07	0.94	0 12 17
290			1.18	0.82	1.06	0.94	0 11 51
300			1.17	0.83	1.06	0.94	0 11 28
350			1.15	0.85	1.05	0.95	0 9 49
400			1.13	0.87	1.05	0.95	0 8 36
450			1.11	0.89	1.04	0.96	0 7 38
500			1.10	0.90	1.04	0.96	0 6 53
550			1.09	0.91	1.03	0.97	0 6 15
600			1.09	0.92	1.03	0.97	0 5 44

Pressure of a load on an inclined plane is found by multiplying the weight of the load by the horizontal distance, and dividing the product by the length of the inclined plane.

Corollary. Hence appears that on an inclined plane, the pressure is less than the weight of the load.

M. Morin's Experiments.

Vehicle used.	Routes passed over.	Pressure	Draught in pounds.	Ratio of draught to load.
Artillery ammunition wagon,	Broken stone,	18215	898.4	$\frac{1}{88.1}$
	in good order,	13541	352.6	$\frac{1}{88.4}$
	and dusty,.....	10101	250.7	$\frac{1}{40.2}$
Wagon without springs,.....	Solid gravel, very dry,.....	15716	306.8	$\frac{1}{51.8}$
" " "		12087	245.9	$\frac{1}{48.9}$
" " "		9814	205.5	$\frac{1}{47.7}$
" " "		7565	150.8	$\frac{1}{50.1}$
Wagon with springs.....	Paved, in good	8528	86.6	$\frac{1}{40.8}$
" " "	order, with wet	7260	196.7	$\frac{1}{36.9}$
" " "	mud,	11018	299.9	$\frac{1}{36.8}$

The greatest inclination ought not to exceed 1 in 80, and need not be less than one in 100, for a horse will draw as well on a road with a rise of 1 in 100 as on a level road. Where the road curves or bends, it should be wider, as follows: When the two lines make an angle of deflection of 90° to 120°, increase the road-bed one-fourth.

Example. Let us suppose that we ascend a hill 1 mile long at the rate of 1 foot in 80, and that we descend 1 mile with an inclination of 1 in 40. Here we have for a one-horse cart or vehicle ascending = 1.66, descending = 0.70, sum = 2.36, mean = 1.18. That is, passing over the hill of 2 miles with the above rise and fall, is equivalent to hauling over 2.36 miles of a horizontal road.

The inclined road is easily drained, and requires less material in construction and annual repair, and avoids curves.

The engineer will be able to judge which is the most economical line from the above table.

M. Morin's experiments show that—

1st. The traction is directly proportional to the load. The traction is inversely proportional to the diameter of the wheel.

2d. Upon hard roads, the resistance is independent of the width of the tire when it exceeds 8 to 4 inches.

3d. At a walking pace, the traction is the same, under the same circumstances, for carriages with and without springs.

4th. Upon hard macadamised and paved roads, the traction increases with the velocity, when above 2½ miles per hour.

5th. Upon soft roads, the traction is independent of the velocity.

6th. Upon a pavement of hewn stones, the traction is three-fourths of that upon the best macadamised roads, at a pace but equal to it at a trot.

7th. The destruction of the road is greater as the diameter of the wheels is less, and is greater with carriages without than with springs.

TABLE C.—For Laying Out Curves. Chord A B = 200 feet or links, or any multiple of either. (See Fig. A, Sec. 819x.)

Rad. of curve.	1/2 angl. of deflect'n ° ' "	D C	F E	H G	W S	Rad. of curve.	1/2 angl. of deflect'n ° ' "	D C	F E	H G	W S
700	8 12 48	7.18	1.79	0.448	0.112	1900	8 01 01	2.63	0.66	0.17	.04
20	7 59 01	6.98	.747	.487	.109	20	2 59 08	.606	.652	.163	.04
40	45 59	.788	.699	.425	.106	40	57 17	.579	.642	.161	
60	38 84	.604	.658	.418	.108	60	55 28	.553	.688	.160	
80	21 57	.488	.614	.408	.101	80	53 48	.530	.633	.158	
800	10 50	.274	.570	.398	.098	2000	51 57	.503	.626	.156	.08
20	01 16	.148	.538	.385	.096	20	50 15	.477	.619	.155	
40	50 14	5.97	.495	.374	.098	40	48 38	.452	.613	.153	.08
60	6 40 39	.844	.460	.365	.091	60	46 57	.429	.607	.152	
80	31 80	.701	.426	.357	.089	80	45 20	.405	.601	.150	.08
900	22 46	.570	.394	.348	.087	2100	43 46	.382	.596	.149	
20	14 25	.486	.364	.341	.085	20	42 18	.357	.589	.147	
40	06 25	.310	.334	.334	.088	40	40 42	.339	.585	.146	.08
60	5 58 45	.222	.307	.327	.082	60	39 12	.316	.579	.145	
80	51 24	.142	.279	.320	.080	80	37 45	.296	.574	.143	
1000	44 20	.012	.254	.318	.078	2200	36 19	.275	.563	.142	.08
20	37 84	4.91	.229	.307	.077	20	34 54	.253	.558	.141	
40	31 04	.817	.205	.301	.075	40	33 31	.232	.553	.139	
60	24 48	.727	.183	.296	.074	60	32 10	.213	.549	.138	.08
80	18 46	.640	.160	.292	.073	80	30 50	.194	.544	.137	
1100	12 57	.556	.140	.285	.071	2300	29 30	.174	.542	.136	
20	07 21	.478	.117	.279	.070	20	28 14	.157	.534	.135	
40	01 57	.396	.099	.275	.069	40	26 57	.138	.530	.134	.08
60	4 56 44	.319	.080	.270	.068	60	25 42	.119	.526	.132	
80	51 41	.247	.062	.265	.066	80	24 29	.102	.521	.131	
1200	46 49	.174	.044	.261	.065	2400	23 17	.084	.517	.130	.08
20	42 06	.105	.027	.257	.064	20	22 06	.067	.518	.129	
40	37 32	.029	.010	.252	.063	40	20 56	.051	.508	.128	
60	33 07	8.98	.0994	.248	.062	60	19 44	.038	.505	.127	
80	28 51	.914	.078	.245	.061	80	18 39	.018	.500	.126	.08
1300	24 42	.853	.963	.241	.060	2500	17 33	.001	.496	.125	
20	20 41	.798	.949	.237	.059	20	16 27	1.99	.492	.124	
40	16 47	.737	.935	.234	.058	40	15 23	.969	.489	.123	.08
60	13 00	.681	.920	.230	.057	60	13 19	.954	.485	.122	
80	09 20	.628	.907	.227	.056	80	13 17	.939	.481	.121	
1400	05 46	.574	.894	.224	.055	2600	12 15	.924	.477	.120	
20	02 18	.526	.882	.221	.055	20	11 14	.909	.474	.119	
40	8 59 05	.481	.870	.218	.054	40	10 15	.895	.470	.118	.02
60	55 39	.429	.857	.214	.053	60	9 16	.880	.466	.117	
80	52 27	.382	.846	.212	.052	80	8 16	.865	.463	.117	
1500	49 20	.337	.834	.209		2700	7 22	.851	.460	.116	
20	46 20	.293	.823	.206	.051	20	6 25	.839	.456	.115	
40	43 23	.250	.818	.203	.050	40	5 29	.825	.453	.114	
60	40 31	.208	.802	.201	.049	60	4 35	.812	.450	.113	.02
80	37 43	.169	.792	.198		80	3 42	.799	.447	.113	
1600	35 00	.128	.782	.196	.048	2800	2 48	.786	.443	.112	
20	32 19	.089	.772	.193		20	1 56	.773	.440	.111	
40	29 45	.052	.763	.191	.047	40	1 04	.760	.437	.110	
60	27 13	.011	.753	.188		60	0 18	.747	.434	.109	.02
80	24 45	2.98	.745	.186	.046	80	59 23	.735	.431	.109	
1700	22 20	.948	.736	.184		2900	58 34	.725	.429	.108	
20	19 59	.910	.728	.182	.045	20	57 45	.714	.425	.107	
40	17 41	.876	.719	.180		40	56 57	.703	.423	.106	
60	15 26	.848	.711	.178	.044	60	56 10	.692	.420	.106	
80	13 14	.812	.703	.176		80	55 23	.681	.417	.105	.02
1800	11 05	.777	.694	.174	.043	3000	54 37	.669	.415	.104	
20	8 59	.749	.687	.172		20	53 51	.658	.412	.104	
40	6 55	.719	.680	.170	.042	40	53 07	.647	.409	.103	
60	4 55	.685	.671	.168		60	52 22	.636	.406	.102	
80	2 57	.662	.666	.167	.041	80	51 38	.625	.404	.102	

TABLE C.—For Laying Out Curves. Chord $AB = 200$ feet or links, or any multiple of either. (See Fig. A, Sec. 819z.)

Rad. of curve.	Angle of defect in ° ' "	DC	FE	HG	WS	Rad. of curve.	Angle of defect in ° ' "	DC	FE	HG	WS
8100	1 50 55	1.61	.404	.101	.025	4800	1 19 57	1.16	.291	.078	.018
20	50 18	.608	.401	.100		20	19 35	.157	.289	.072	
40	49 30	.593	.398	.099		40	19 18	.152	.288	.072	
60	48 48	.583	.396	.099		60	18 51	.146	.287	.072	
80	48 07	.578	.393	.098		80	18 30	.141	.285	.071	
8200	47 27	.568	.391	.098		4400	18 08	.136	.284	.071	
20	46 47	.558	.388	.097	.024	20	17 47	.131	.283	.071	
40	46 07	.548	.386	.097		40	17 26	.126	.282	.071	
60	45 28	.534	.384	.096		60	17 05	.121	.280	.070	
80	44 50	.525	.381	.095		80	16 45	.116	.279	.070	
8300	44 11	.516	.379	.095		4500	16 24	.111	.278	.070	
20	43 34	.506	.377	.094		20	16 04	.106	.277	.069	.017
40	42 57	.497	.374	.094	.028	40	15 44	.102	.276	.069	
60	42 20	.489	.372	.093		60	15 24	.097	.274	.069	
80	41 43	.480	.370	.093		80	15 04	.092	.273	.068	
8400	41 08	.471	.368	.092		4600	14 44	.087	.272	.068	
20	40 32	.462	.366	.092		20	14 25	.082	.271	.068	
40	39 54	.453	.363	.091		40	14 06	.077	.269	.067	
60	39 22	.445	.361	.090		60	13 47	.073	.268	.067	
80	38 48	.437	.359	.090		80	13 28	.069	.267	.067	
8500	38 14	.429	.357	.089	.022	4700	13 09	.064	.266	.067	
20	37 41	.421	.355	.089		20	12 51	.059	.265	.066	
40	37 08	.413	.353	.088		40	12 32	.054	.264	.066	
60	36 35	.405	.351	.088		60	12 14	.050	.263	.066	
80	36 03	.397	.349	.087		80	11 55	.046	.262	.066	
8600	35 30	.389	.347	.087		4800	11 38	.042	.261	.065	.016
20	34 59	.381	.345	.086		20	11 20	.038	.260	.065	
40	34 27	.374	.344	.086	.021	40	11 02	.034	.259	.065	
60	33 57	.366	.342	.086		60	10 44	.030	.258	.065	
80	33 28	.358	.339	.085		80	10 27	.026	.257	.064	
8700	32 55	.351	.338	.085		4900	10 10	.022	.256	.064	
20	32 25	.344	.336	.084		20	9 53	.018	.255	.064	
40	31 56	.337	.334	.084		40	9 36	.018	.253	.063	
60	31 27	.330	.333	.083		60	9 19	.008	.252	.063	
80	30 57	.323	.331	.083		80	9 02	.004	.251	.063	
8800	30 29	.316	.329	.082		5000	8 46	1.00	.250	.063	
20	30 00	.309	.327	.082		20	8 29	.996	.249	.062	
40	29 32	.302	.326	.082		40	8 13	.992	.248	.062	
60	29 04	.295	.324	.081	.020	60	7 55	.988	.247	.062	
80	28 37	.288	.322	.081		80	7 41	.984	.246	.062	
8900	28 09	.282	.321	.080		5100	7 25	.981	.245	.061	.015
20	27 43	.276	.319	.080		20	7 09	.977	.244	.061	
40	27 16	.269	.317	.079		40	6 53	.973	.243	.061	
60	26 49	.262	.316	.079		60	6 38	.969	.242	.061	
80	26 23	.256	.314	.079		80	6 22	.965	.241	.060	
4000	25 57	.250	.312	.078	.019	5200	6 07	.962	.241	.060	
20	25 21	.243	.311	.078		20	5 52	.958	.240	.060	
40	25 06	.237	.309	.077		40	5 37	.954	.239	.060	
60	24 41	.231	.308	.077		60	5 22	.950	.238	.059	
80	24 16	.225	.306	.077		80	5 07	.947	.237	.059	
4100	23 52	.220	.305	.076		5300	4 52	.944	.236	.059	
20	23 27	.214	.304	.076		20	4 37	.940	.235	.059	
40	23 03	.208	.302	.076		40	4 23	.936	.234	.059	
60	22 39	.202	.301	.075		60	4 09	.933	.233	.058	
80	22 14	.196	.299	.075		80	3 54	.929	.232	.058	
4200	21 52	.191	.298	.075		5400	3 40	.926	.232	.058	
20	21 28	.185	.296	.074		20	3 26	.923	.231	.058	
40	21 05	.179	.295	.074		40	3 12	.919	.230	.058	
60	20 42	.173	.293	.073		60	2 58	.916	.229	.057	
80	20 20	.168	.291	.073	.018	80	2 44	.912	.228	.057	.014

TABLE C.—For Laying Out Curves. Chord A B=200 feet or links, or any multiple of either. (See Fig. A, Sec. 819z.)

Rad. of curve.	1/2 angl. of deflect'n ° / ' / "	D C	F E	H G	W S	Rad. of curve.	1/2 angl. of deflect'n ° / ' / "	D C	F E	H G	W S
5500	1 2 31	.909	.227	.057	.014	6700	0 51 19	.746	.187	.047	.012
20	2 17	.905	.226	.057		20	51 10	.744	.186	.047	
40	2 08	.902	.226	.057		40	51 00	.742	.186	.047	
60	1 50	.899	.225	.056		60	50 52	.740	.185	.046	
80	1 37	.896	.224	.056		80	50 42	.738	.185	.046	
5600	1 24	.93	.223	.056		6800	50 33	.736	.184	.046	
20	1 10	.89	.222	.056		20	50 25	.733	.183	.046	
40	0 57	.86	.222	.056		40	50 16	.731	.183	.046	
60	0 44	.83	.221	.055		60	50 07	.728	.182	.046	
80	0 32	.80	.220	.055		80	49 58	.726	.182	.046	
5700	0 19	.77	.219	.055		6900	49 50	.724	.181	.045	.011
20	1 06	.74	.219	.055		20	49 41	.722	.181	.045	
40	0 59 54	.71	.218	.055		40	49 32	.720	.180	.045	
60	59 41	.68	.217	.054		60	49 24	.718	.179	.045	
80	59 29	.65	.216	.054		80	49 15	.716	.179	.045	
5800	59 16	.62	.216	.054		7000	49 07	.714	.179	.045	
20	59 04	.59	.215	.054		20	48 58	.712	.178	.045	
40	58 52	.56	.214	.054		40	48 50	.710	.178	.045	
60	58 40	.53	.213	.053	.018	60	48 42	.708	.177	.044	
80	58 28	.50	.213	.053		80	48 33	.706	.177	.044	
5900	58 16	.47	.212	.053		7100	48 25	.704	.176	.044	
20	58 04	.44	.211	.053		20	48 17	.702	.175	.044	
40	57 53	.42	.211	.053		40	48 09	.700	.175	.044	
60	57 41	.40	.210	.053		60	48 01	.698	.175	.044	
80	57 29	.38	.209	.052		80	47 52	.694	.174	.044	
6000	57 18	.34	.209	.052		7200	47 45	.692	.174	.044	
20	56 07	.31	.208	.052		20	47 37	.690	.173	.043	
40	56 55	.29	.207	.052		40	47 29	.688	.173	.043	
60	56 44	.26	.207	.052		60	47 21	.686	.172	.043	
80	56 33	.23	.206	.052		80	47 13	.684	.172	.043	
6100	56 22	.20	.205	.051		7300	47 06	.682	.171	.043	
20	56 11	.18	.205	.051		50	47 47	.679	.169	.042	
40	55 00	.15	.204	.051		7400	46 28	.676	.169	.042	
60	55 49	.13	.203	.051		50	46 09	.672	.168	.042	
80	55 38	.10	.203	.051		7500	45 51	.668	.167	.042	
6200	55 27	.07	.202	.051		50	45 32	.668	.166	.042	
20	55 16	.04	.201	.050		7600	45 14	.668	.165	.041	.010
40	55 06	.01	.200	.050	.012	50	44 57	.664	.164	.041	
60	54 55	.79	.200	.050		7700	44 39	.660	.163	.041	
80	54 45	.76	.199	.050		50	44 22	.646	.162	.041	
6300	54 34	.74	.199	.050		7800	44 05	.642	.160	.040	
20	54 24	.71	.198	.050		50	43 48	.638	.160	.040	
40	54 14	.78	.197	.049		7900	43 31	.634	.158	.040	
60	54 08	.78	.197	.049		50	43 15	.629	.157	.039	
80	53 58	.78	.196	.049		8000	0 42 58	.624	.157	.039	
6400	53 48	.78	.195	.049		50	42 42	.621	.155	.039	
20	53 38	.77	.195	.049		8100	42 27	.617	.154	.039	
40	53 28	.77	.194	.049		50	42 11	.614	.153	.038	
60	53 18	.77	.194	.049		8200	41 55	.611	.153	.038	
80	53 08	.77	.193	.048		50	41 40	.608	.152	.038	
6500	52 58	.76	.192	.048		8300	41 25	.605	.151	.038	.009
20	52 44	.76	.192	.048		50	41 10	.602	.150	.037	
40	52 34	.76	.191	.048		8400	40 56	.599	.150	.037	
60	52 24	.76	.191	.048		50	40 41	.596	.149	.037	
80	52 15	.76	.190	.048		8500	40 27	.593	.148	.037	
6600	52 08	.75	.189	.047		50	40 13	.589	.147	.037	
20	51 56	.75	.189	.047		8600	39 58	.585	.146	.037	
40	51 47	.75	.188	.047		50	39 45	.581	.145	.036	
60	51 37	.75	.188	.047		8700	39 31	.577	.144	.036	.009
80	51 28	.74	.187	.047							

TABLE C.—For Laying Out Curves. Chord A B = 200 feet or links, or any multiple of either. (See Fig. A, Sec. 819x.)

Rad. of curve.	† angl of deflect'n ° / ' / "	D C	F E	H G	W S	Rad. of curve.	† angl of deflect'n ° / ' / "	D C	F E	H G	W S
8750	0 39 17	.578	.143	.086	.009	14600	0 23 33	.842	.086	.022	.006
8800	39 04	.578	.143	.086		14700	23 28	.840	.085	.021	
8850	38 51	.566	.141	.085		800	23 14	.838	.085	.021	
8900	38 37	.563	.141	.085		900	23 04	.836	.083	.021	
9000	38 12	.557	.139	.085		15000	22 55	.834	.083	.021	
9100	37 47	.549	.137	.084		100	22 46	.832	.082	.021	
9200	37 22	.543	.136	.084		200	22 37	.830	.082	.021	
9300	36 58	.537	.134	.084		300	22 28	.828	.081	.020	
9400	36 35	.531	.133	.083	.008	400	22 19	.826	.081	.020	
9500	36 11	.525	.131	.083		500	22 12	.824	.080	.020	
9600	35 49	.519	.130	.083		600	22 02	.822	.080	.020	
9700	35 26	.513	.128	.082		700	21 54	.820	.079	.020	
9800	35 05	.508	.127	.082		800	21 46	.818	.079	.019	
9900	34 44	.504	.126	.082		900	21 37	.816	.078	.019	
10000	34 23	.500	.125	.081		16000	21 30	.814	.078	.019	
100	34 02	.495	.124	.081		100	21 21	.812	.078	.019	
200	33 42	.491	.123	.081		200	21 13	.810	.077	.019	
300	33 23	.486	.122	.081		300	21 05	.808	.077	.019	
400	33 03	.481	.120	.080		400	20 58	.806	.076	.019	
500	32 44	.476	.119	.080		500	20 50	.804	.076	.019	
600	32 26	.471	.118	.080		600	20 43	.802	.075	.018	
700	32 08	.467	.117	.029	.007	700	20 35	.800	.075	.018	
800	31 50	.463	.116	.029		800	20 28	.798	.074	.018	
900	31 33	.459	.115	.029		900	20 21	.796	.074	.018	
11000	31 15	.455	.114	.028		17000	20 13	.794	.073	.018	
100	30 58	.451	.113	.028		100	20 07	.792	.073	.018	
200	30 42	.447	.112	.028		200	19 59	.790	.072	.018	
300	30 25	.443	.111	.028		300	19 52	.788	.072	.018	
400	30 09	.439	.110	.028		400	19 45	.786	.072	.018	
500	29 54	.435	.109	.027		500	19 39	.784	.071	.018	
600	29 38	.431	.108	.027	.007	600	19 32	.782	.071	.017	
700	29 23	.427	.107	.027		700	19 26	.781	.071	.017	
800	29 08	.424	.106	.027		800	19 19	.780	.070	.017	
900	28 53	.421	.105	.026		900	19 12	.779	.070	.017	
12000	28 40	.418	.104	.026		18000	19 06	.778	.069	.017	.004
100	28 25	.414	.104	.026		100	0 19 00	.776	.069	.017	
200	28 11	.411	.103	.026		200	18 53	.775	.069	.016	
300	27 57	.407	.102	.026		300	18 47	.773	.068	.016	
400	27 43	.403	.101	.025		400	18 41	.772	.068	.016	
500	27 30	.399	.100	.025		500	18 35	.770	.067	.016	
600	27 17	.396	.099	.025		600	18 29	.769	.067	.016	
700	27 04	.393	.098	.025		790	18 23	.768	.067	.016	
800	26 51	.390	.098	.025		800	18 17	.767	.067	.016	
900	26 39	.387	.097	.024		900	18 11	.765	.066	.016	
13000	26 27	.385	.096	.024		19000	18 06	.764	.066	.016	
100	26 14	.382	.096	.024		100	18 00	.762	.066	.016	
200	26 03	.379	.095	.024		200	17 54	.761	.065	.016	
300	25 51	.376	.094	.024		300	17 49	.759	.065	.015	
400	25 39	.373	.093	.023		400	17 43	.758	.065	.015	
500	25 28	.370	.092	.023		500	17 38	.756	.064	.015	
600	25 17	.367	.091	.023		600	17 32	.755	.064	.015	
700	25 06	.364	.090	.023		700	17 27	.753	.063	.015	
800	24 55	.361	.090	.023		800	17 22	.752	.063	.015	
900	24 44	.358	.089	.022		900	17 17	.751	.063	.015	
14000	24 33	.356	.089	.022	.006	20000	17 11	.749	.062	.015	
100	24 23	.353	.088	.022		21000	16 21	.738	.059	.015	
200	24 13	.350	.088	.022		21120	16 16	.737	.059	.020	.004
300	24 02	.348	.087	.022		15840	21 42	.816	.079	.029	.006
400	23 52	.346	.087	.022		10560	32 33	.478	.118	.069	.007
500	23 43	.344	.086	.022	.005	5280	1 5 07	.947	.237	.119	.080

CANALS.

320. In *locating* a canal, reference must be had to the kind of vessels to be used thereon, and the depth of water required; the traffic and resources of the surrounding country; the effect it may have in draining or overflowing certain lands; the feeders and reservoirs necessary to keep the summit level always supplied, allowing for evaporation and leakage through porous banks, etc. The canal to have as little inclination as possible, so as not to offer any resistance to the passage of boats. To be so located that its distance will be as short as possible between the cities and towns through or near which it is to pass. To have its cutting and filling as nearly equal as the nature of the case will allow. To have sufficient slopes and berms as will prevent the banks from sliding. The bottom width ought to be twice the breadth of the largest boat which is to pass through it. The depth of water 18 inches greater than the draft or depth of water drawn by a boat.

Tow-path. About 12 feet wide, being between 2 and 4 feet above the level of the water, and having its surface inclined towards the canal sufficiently to keep it dry. Vegetable soil, and all such as are likely to be washed in, are to be removed. Where there is no tow-path, a berm or bench, 2 feet wide, is left in each side, about 18 inches above the water.

Feeders may have an inclination not more than 2 feet in a mile, to be capable of supplying four or five times the necessary quantity of water to feed the *summit level*.

Reservoirs, or basins, may be made by excavation, or, in a hilly country, by damming the ravines. There are many instances of this on the Rideau Canal in Canada; also, on that built by the author, connecting the Chats and Chaudiere lakes, on the river Ottawa, in the same country.

This necessarily requires that an Act of the Legislature should empower them to enter on any land, and overflow it if necessary, and have commissioners to assess the benefit and damages.

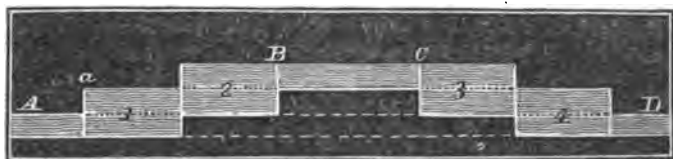
Draft is the depth of water required to float the boat.

Lift is the additional quantity required to pass the boat from one lock into another.

A boat ascending to the summit has as many lifts as there are drafts.

A boat descending from a summit to a lower level has one more lift than drafts.

Let the annexed figure represent a canal, where there are two locks ascending and two descending; there are four lifts and three drafts.



To Ascend from A to B of Lock 1. (See annexed figure.) Boat arrives at gate *a*; finds in it one prism of draft, and the other lock empty. Now, all these locks must be filled to enable the boat to arrive at the summit level B C. Let *L* = prism of lift, and *D* = prism of draft; then it is plain that to ascend from A to B requires two prisms of lift and one of draft, and putting $n = 2$, or the number of locks, the quantity required to pass the boat = $n L + (n - 1) D$.

To Descend from C to D = 2 locks. In lock 3, one prism of lift will be taken, and one of draft. The prism of lift passes into lock 4, together with one of draft, thus using two prisms of draft and one of lift, which is sufficient to pass the boat from C to D = $L + 2 D$. Or,

To ascend = $n L + (n - 1) D$.

To descend = $L + 2 D$. Add these two equations. The whole quantity from A to D = $(n + 1) L + (n + 1) D = (n + 1) \cdot (L + D)$.

Each additional boat passing in the same order requires two prisms of lift and two of draft; that is, the additional discharge = $2 (N - 1) (L + D)$. Here N = number of boats; therefore the whole discharge = $(n + 1) (L + D) + (2 N - 2) (L + D) = (2 N + n - 1) \cdot (L + D)$. To this must be added the loss by evaporation and leakage. Evaporation may be taken at half an inch per day. From one-third to two-thirds of the rain-fall may be collected.

The engineer will, when the channel is in slaty or porous soil, cover it with a layer of flat stones laid in hydraulic mortar, having previously covered it with fine sand.

Locks to be one foot wider than the width of beam, 18 inches deeper than draft of boat, and to be of a sufficient length to allow the rudder to be shifted from side to side.

Bottom to be an inverted arch where it is not rock. Where the bottom is not solid, drive piles, on which lay a sheeting of oak plank to receive the masonry.

The channel to have recesses to receive the lock gates.

The lock gates to make an angle of $54^{\circ} 44'$ with one another, being that which gives them the greatest power of resisting the pressure of the prism of water.

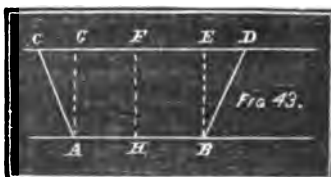
Reservoirs are made in natural ravines which may be found above the summit level, or they are excavated at the necessary heights above the summit.

Dams are made of solid earth or masonry. When of earth, remove the surface to the depth where a firm foundation can be had; then lay the earth in layers of eight or twelve inches; have it puddled and rammed, layer after layer, to the top. *Slope* next the water to be three or four base to one perpendicular (see sec. 147). *Outside slope* about two or two and a half base to one perpendicular. The face next the dam is faced with stone. For *thickness of the top of the dam*, see *Embankments* (sec. 319).

To Set Out the Section of a Canal when the Surface is Level.

321. Let the bottom width $AB = 30$ feet, height of cutting on the centre stake $HF = 20$ feet = h , ratio of slopes 2 to 1 = r —that is, for 1 foot perpendicular there is to be 2 feet base, $20 \times 2 = 40 =$ base for each slope = $CG = ED$, and $20 \times 2 \times 2 = 80 =$ total base for both slopes. Bottom width = 30; therefore, $80 + 30 = 110 =$ width of cutting at top = CD ; and $110 + 30 \div 2 \times 20 =$ sectional area = 1400. In general,

$S = (b + h r) h =$ sec'l area in ft.
 $C = (b + h r) h L =$ cubic content.
 Here S = transverse sectional area.
 C = content of the section, b = bottom width, h = height, r = ratio of slope, and L = length of section.



To Set Out a Section when the Surface is an Inclined Plane, as in fig. 44.

821a. This case requires a cutting and an embankment. We will suppose the slopes to be the same in both.

Let the surface of the land be R Q, the canal A B = bottom = b = 30 feet. Height H G = 20, ratio of slopes of excavation and embankment = $1\frac{1}{2}$ base to 1 height—that is, ratio of slopes = r = $1\frac{1}{2}$ to 1.

At the centre G set up the level; set the leveling staff at N; found the height S N = 5 feet; measured G S = 20.61, and G N = 20; because the slopes being $1\frac{1}{2}$ to 1, the slope to 5 feet = $7\frac{1}{2}$; $\therefore$ G F = $12\frac{1}{2}$, and G M = $27\frac{1}{2}$ feet; and the slope corresponding to H G = 20×1.5 = 35, which added to half the bottom, gives G C = 45.

To Find G E and G Q.

G M : G S :: G C : G E; that is,
27.5 : 20.61 :: 45 : G E = 83.72 feet.

Let the top of embankment P C = 20 feet; then G P = 65.
G F : G S :: G P : G Q; that is,
 $12\frac{1}{2}$: 20.61 :: 65 : G Q = 107.17 feet.

Having G E, G Q, G S and S N, we can find the perpendicular Q V.
G S : S N :: G Q : Q V.
20.61 : 5 :: 107.17 : Q V = 26, which is perpendicular to the surface G V.
20.61 : 5 :: G E = 83.72 : E F = 8.18 feet.
 $G V^2 = G Q^2 - Q V^2$; $\therefore$ we can find G V = 103.96; and by taking 65 from the value of G V, we find $103.96 - 65 = 38.96 = P V$.

To Find the Point R.

We find, when the slope G Q continues to R, that by taking G s = 20.61, n s = 5, n t = $7\frac{1}{2}$, G t = $12\frac{1}{2}$, and s t is parallel to B R; $\therefore$ G t : G s :: G D : G R; but G D = $15 + 20 \times 1\frac{1}{2} = 45$, $\therefore$
12.5 : 20.61 :: 45 : G R = 74.19.

To Find G d = H a, and Area of Cutting.

We have G s : G n :: G R : G d; that is,
20.61 : 20 :: 74.19 : G d = H a = 71.99.
G n : n s :: G d : R d; that is,
20 : 5 :: 71.99 : R d = 17.9975.
But H G = a d = 20; therefore R a = 37.998;
and H a — H B = 71.99 — 15 = B a = 56.99. Let us put 18 = 17.9975.

$$\text{Area of sec. H G R a} = \frac{G H + R a}{2} \times H a = \frac{20 + 38}{2} \times 71.99 = 2087.71$$

$$\text{Deduct the } \triangle B R a = 56.99 \times 19 = \underline{1082.81}$$

$$\text{Area of the figure G H B R} = \underline{1004.90}$$

$$\text{Area G H A C} = (G C + A H) \times \frac{H G}{2} = (45 + 15) \times 10, \quad 600$$

$$\text{Area of the figure C G R B A} = \underline{1604.90}$$

$$\text{Deduct triangle G E C} = 45 \times \text{half of E f} = 45 \times 4.09, \quad 184.05$$

$$\text{Area of B A E G R} = \underline{1420.85}$$

Or thus :

We have $R a$ by calculation or from the level book, 38 nearly. Also, $E g = g f - E f = 20 - 8.18 = 11.82$, which multiplied by ratio of slope, gives $A g = 17.78$, and $H g = 33.72$. But from above we have $H a = 71.99$; $\therefore 71.99 + 32.73 = a g = 104.72$.

$$\frac{104.72}{2} \times (E g + R a) = 52.36 \times (11.82 + 38) = E g a R = 2608.58$$

$$\text{Deduct } \triangle E g A + \triangle B R a; \text{ i.e., } \frac{11.82 \times 17.78}{2} + 56.99 \times 19 = 1187.59$$

$$\text{Area of the section } R E A B = 1420.99$$

Nearly the same area as above. The difference is due to calling 17.9975 = 18.

To Find the Embankment.

We have $Q V = 26$, $P V = 38.96$, $E f = 8.18$, $P C = 20$, $G F = 32.72$, and $C F = G C - G F = 45 - 32.72 = 12.28$

$$G V = 45 + 20 + 38.96 = G C + C P + P V = 103.96$$

$G S : G N :: G E : G f$; that is,

$20.61 : 20 :: 33.72 : G f = 33.72$. This taken from $G C$ or 45 will give

$$C F = 12.28; \therefore f V = 12.28 + 20 + 38.96 = 71.24$$

$$\frac{1}{2} (Q V + E f) = \frac{1}{2} (26 + 8.18) = 17.09$$

$$\text{The product} = \text{area of } Q V F E = 1217.4916$$

$$\text{Deduct } \triangle C f E = 4.09 \times 12.28 = \frac{1}{2} E f \times C f = 50.22$$

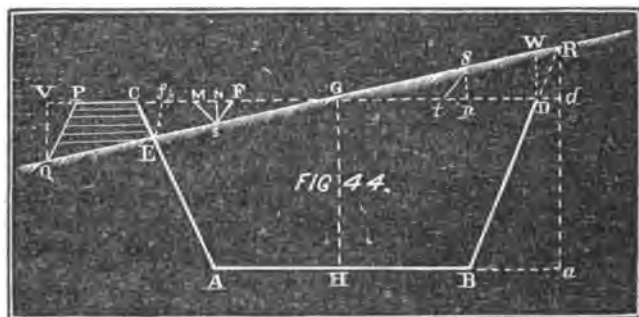
$$\text{Also deduct } \triangle Q V P = 38.96 \times 13 = 506.48$$

$$\text{Sum to be subtracted, } 556.70$$

$$\text{Area of section } Q P C E = 660.79$$

To Set Off the Boundary of a Canal or Railway.

321b. Let the width from the centre stump or stake G to boundary



line = 100 feet, if the ground is an inclined plane, as fig. 44. We can say, as $G N : G S :: G f : G E$; i.e., $20 : 20.61 :: 100 : G E = 103.05$.

Otherwise, take a length of 20 or 30 feet, and, with the assistant, measure carefully, dropping a plumb-line and bob at the lower end, and thus continue to the end. This will be sufficiently accurate.

To Find the Area of a Section of Excavation or Embankment such as A B D C.
(See Fig. 46.)

322. Let r = ratio of slopes, D = greater and d = lesser depth, and b = bottom width.

We have $d r = A E$, and $D r = B F$; $\therefore (D + d) r + b = E F$. But $E F \times (D + d) =$ twice the area of C E F D; i. e., $\{(D + d) r + b\} \cdot (D + d) =$ double area of C E F D.

$(D^2 + 2 D d + d^2) r + (D + d) b =$ double area of C E F D.

$d^2 r = 2 \triangle A C E$, and $D^2 r = 2 \triangle B D F$; these taken from the value of twice the area of C D F D, gives the required area of A C D B = $2 D d r$. This divided by 2 will give the area of

$$A B C D = D d r + \left(\frac{D + d}{2}\right) b.$$

Rule. Multiply the heights and ratio together; to the product add the product of half the heights multiplied by the base. The sum will be the area of A B C D, when the slopes on both sides are equal.

Example. Let bottom $b = 80$, $d = 10$, $D = 20$, ratio of base to perpendicular = $r = 2$, to find the area of the section.

$$D d r = 10 \times 20 \times 2 = 400$$

$$\left(\frac{D + d}{2}\right) \times b = 15 \times 80 = 450$$

$$\text{Area of section A B D C} = 850$$

322a Let the slopes of A C and B D be unequal; let the ratio of slope for A C = r , and that for B D = R . Required area of A B D C = $\frac{b}{2} \cdot (D + d) + \frac{R + r}{2} \cdot (D d)$.

Rule. Multiply the sum of the two heights by half the base, and note the product.

Multiply the product of the heights by half the sum of the ratios, and add the product to the product above noticed. The sum of the two products will be the required area.

Example. Let the heights and base be as in the last example; ratio of slope A C = 2, and that of slope B D = 3.

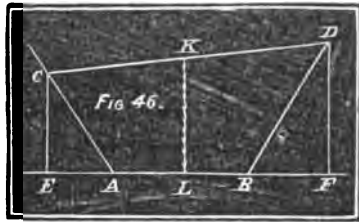
$$\frac{b}{2} (D + d) = 15 \times 80 = 450$$

$$\frac{R + r}{2} \cdot D d = 2.5 \times 200 = 500$$

$$\text{Area of A F D C} = 950$$

Let the Surface of the Side of a Hill Cut the Bottom of the Canal or Road Bed, as in Fig. 47.

322b. Here A B is the bottom of the canal or road, A C and B D its sides, having slopes of r . D E = the surface of the ground, G F = d = lesser height below the bottom, and to the point where the slope A C produced will meet the surface of the ground. D H = D = greater height above the bottom.



Through F, draw F K parallel to A H; then D K = D + d, and A H = b + r D, and A G = r d; therefore F K = G H = b + r D - r d = b + (D - d) r, and by similar triangles.

D K : K F :: D H : M H; that is,

$$D + d : b + r D - r d :: D : M H = \frac{B D + r D^2 - r d D}{D + d}$$

But M H × D H = twice the area of $\triangle M D H$, and twice the area of $\triangle B D H = B H \times D H = R d \times D = r D^2$;

$$\therefore \text{twice area of } \triangle M D B = \frac{b D^2 + r D^3 - r d D^2}{D + d} - r D^2$$

$$= \frac{b D^2 + r D^3 - r d D^2 - r D^3 - r d D^2}{D + d}$$

$$= \frac{b D^2 - 2 r d D^2}{D + d}$$

$$= \frac{(b - 2 r d) D^2}{D + d} \text{ that is,}$$

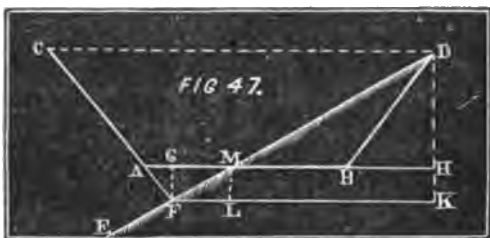
$$\text{Area of } \triangle M D B = \left(\frac{(b - 2 r d) D^2}{2 (D + d)} \right)$$

$$\text{Or} = \frac{\frac{1}{2} (b - r d) D^2}{D + d} \text{ which is that given by Sir}$$

John McNeil in his valuable tables of earthwork.

Rule. From half the base take the product of the ratio of slopes and height below the bed; multiply the difference by the square of the height above the bed of road or canal; divide this product by the sum of the two heights; the quotient will be the area of the section M D H.

Example. Let base = 40, ratio of slopes $1\frac{1}{2}$ to 1, height G F below the bed = $5\frac{1}{2}$, height D H above the bed = 20 feet, to find the area of the section M D B. (See figure 47.)



Half the base =	20
$r d = 5.5 \times 1.5 =$	8.25
	11.75
$D^2 = 20 \times 20 =$	400
	4700

Divide 4700 by $D + d = 20 + 5.5 = 25.5$

The quotient = area of M D B = 184.318 feet.

To Find the Mean Height of a Given Section whose Area = A, Base = b, Ratio of Slopes = r.

828. Let x = required mean height; then mean width = $b + r x$; this multiplied by the mean height, gives $b x + r x^2 = A$ = given area.

$x^2 + \frac{b}{r}x = \frac{A}{r}$ Complete the square:

$$x^2 + \frac{b}{r}x + \frac{b^2}{4r^2} = \frac{A}{r} + \frac{b^2}{4r^2} =$$

$$\frac{4Ar^2 + r b^2}{4r^2} = \frac{4Ar + b^2}{4r^2}$$

$$x + \frac{b}{2r} = \frac{\sqrt{4Ar + b^2}}{2r}$$

Mean height = $x = \frac{(4Ar + b^2)^{\frac{1}{2}}}{2r} - \frac{b}{2r}$, and by substituting the value of A in sec. 322,

$$x = \left(\frac{\{(4Ddr^2 + (D+d)2br + b^2)\}^{\frac{1}{2}} - b}{2r} \right)$$

Rule. To the square of the base, add four times the area multiplied by the ratio of the slopes; take the square root of the product; divide this root by twice the ratio, and from the quotient take the base divided by twice the ratio. The difference will be the required mean height.

Example. Let us take the last example, where the base $b = 40$, ratio $r = 1\frac{1}{2}$, area = 184.818 square feet.

$$4Ar = 184.818 \times 4 \times 1.5 = 1105.878$$

$$b^2 = 40 \times 40 = 1600$$

$$2705.878$$

$$\text{Square root of } 2705.878 = 52.018$$

$$\text{This root divided by } 2r = 3 \text{ gives } = 17.339$$

$$\text{From this take } \frac{b}{2r} = \frac{40}{3} = 13.333$$

$$\text{Gives the mean height} = 4.006, \text{ or } = 4 \text{ feet nearly.}$$

$$4r = 6, \text{ to which add base } 40, \text{ sum } = 46$$

$$\text{Approximate mean height, } = 4$$

$$184$$

Area nearly as above.

It need not be observed that if we took the mean height ≈ 4.009 , we would find 184.818 nearly. Our object here is to show the method of applying the formula to those who have no knowledge of algebraic equations.

Or by plotting the section on a large scale on cartridge paper, the area and mean depth can be computed by measurement. The mean heights are those used in using McNeil's tables of earthwork, and also in finding the middle area, necessary for applying the prismoidal formula.

Rule 2. To four times the product of the heights and ratio add the continual product of the sum of the two heights by twice the base multiplied by the ratio; to this sum add the square of the base; from the square root of this last sum subtract the base, and divide the difference by twice the ratio. The quotient will be the mean height.

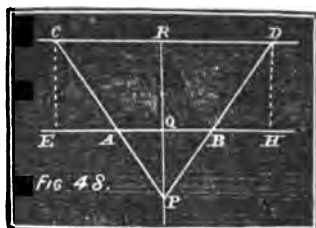
Example. $D = 70, d = 30, b = 40, r = 1.$

$$70 \times 30 \times 4 \times (70 + 30) \times 80 = 16400$$

$$\text{Square of base } = 1600$$

$$18000$$

The square root = 184.164, which, divided by 2, gives 47.082, the mean height.



Another Practical Method.

824. Let $AB = \text{base} = b$, $CDBA = \text{required section}$, whose area $= A$, and mean height QR is required; ratio of slopes perpendicular to base is as 1 to r . (See fig. 48.)

We have $PQ \times 2r = AB = b$; that is,

$$PQ = \frac{b}{2r}; \text{ this } \times \text{ by the base gives twice area of } \triangle ABP = \frac{b^2}{2r};$$

therefore, area $\triangle ABP = \frac{b^2}{4r}$; consequently,

$$\text{area of } \triangle CPD = \frac{b^2}{4r} + A, \text{ or putting area of } \triangle ABP = a,$$

we have area $\triangle CPD = A + a$, and by Euclid VI, prop. 19,

$$\triangle ABP : \triangle PCD :: PQ^2 : PR^2;$$

$$\text{that is, } a : A + a :: \frac{b^2}{4r^2} : PR^2.$$

$$PR^2 = \frac{(A + a)b^2}{4ar^2} \text{ take the square root,}$$

$$PR = \frac{(A + a)^{\frac{1}{2}}}{\sqrt{a}} \cdot \frac{b}{2r}$$

$$PB = \left(\left(\frac{A + a}{a} \right)^{\frac{1}{2}} \cdot \frac{b}{2r} \right)$$

$$QR = \left(\left(\frac{A + a}{a} \right)^{\frac{1}{2}} \cdot \frac{b}{2r} - \frac{b}{2r} \right) = \text{mean height.}$$

Example. Let $AB = b = 20$, ratio $= 2$. Given area of the section 1200, which is to be equal to the section $ABCD$, whose mean height is required.

The constant area of $\triangle ABP$ is always $= \frac{b^2}{4r} = 50$.

$$\frac{(A + a)^{\frac{1}{2}}}{a} = \frac{(1200 + 50)^{\frac{1}{2}}}{50} = \frac{(1250)^{\frac{1}{2}}}{50} = \sqrt{25} = 5.$$

$$\text{Multiply by } \frac{b}{2r} = \frac{20}{4}$$

$$- \frac{b}{2r}$$

$$QR = \text{mean height} =$$

$$\begin{array}{r} 5. \\ 25, \text{ product.} \end{array}$$

$$\begin{array}{r} 5. \\ 20. \end{array}$$

In this example and formula the slopes are the same on both sides.

Let $R = \text{greater}$, and $r = \text{lesser ratio}$;

$$\text{then } QR = \left(\frac{A + a}{a} \right)^{\frac{1}{2}} \cdot \frac{b}{R + r} - \frac{b}{R + r}.$$

When the Slopes are the Same on Both Sides.

825. *Rule.* To the given area above the base add the constant area below the base; divide the sum by the constant area of the $\triangle ABP$; multiply the square root of this quotient by the base divided by twice the ratio of the slope; from this product take the base divided by the ratio of slope. The difference will be the required mean height $= L R$.

When the Slopes are unequal.

Rule. To the given area above the base, add the constant area of the triangle A B P below the base, divide the sum by the constant area of $\triangle$ A B P. Multiply the square root of the quotient, by the base divided by the sum of the ratio of the slopes, from the product subtract the base divided by the sum of the ratios, the difference will be the required mean height = Q R.

Example. Let ratio R = ratio of Q B to Q P = ratio to slope B D = 3, and r = lesser ratio of A Q to P Q = 2.

$$A B = b = 20, \text{ therefore } P Q = \frac{20}{R + r} = 4.$$

Let area of A B D C = 960, and constant area of the triangle under the base = 40 = A = $\triangle$ A B P.

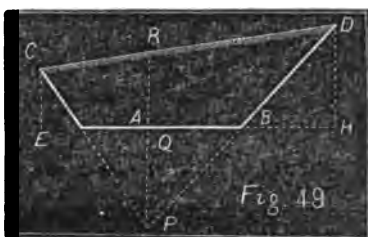
$$\left(\frac{A + a}{a}\right)^{\frac{1}{2}} \cdot \frac{b}{R + r} - \frac{b}{R + r} = \left(\frac{960 + 40}{40}\right)^{\frac{1}{2}} \cdot \frac{20}{5} - \frac{20}{5} = Q R.$$

$$Q R = 5 \times 4 - 4 = 16.$$

826. *Mean height must not be found by adding the heights on each side of the centre stump or stake, and then take half of the sum for a mean height.* This method is commonly used, and is *very erroneous*, as will appear from the following example: Let the greater height D H = 70, (see fig. 49,) the lesser C E = 30, base 40, ratio of slopes 1 to 1.

Correct Method.

70 = greater height	= D
30 = lesser	= d
2) 100, mean height	= 50
30 + 40 + 70 = base E H	= 140
Sectional area of	
C D H E	= 7000
deduct the two triangles	
C E A + D B H	= 2900
Area	4100
	Correct.



Or, by sec. 322, we can find the area

D d r = 70 × 80 × 1	2100
D + d · b = 50 × 40	2000
2	4100, required correct area.

By the Erroneous or Common Method.

$$70 + 30 = 100 = \text{sum of heights.}$$

$$50 = \text{mean height.}$$

$$\text{Half slope} = 50$$

$$100 = \text{mean base.}$$

$$50 = \text{mean height.}$$

$$\text{Area } 6000 \text{ incorrect.}$$

$$\text{Area } 4100 \text{ correct.}$$

$$\text{Difference } 900 \text{ square feet.}$$

From this great difference appears that where the mean height is required, it has to be calculated by the formula in section 323, where

$$x = \text{mean height} = \frac{(4 A r + b^2)^{\frac{1}{2}}}{2 r} - \frac{b}{2 r}$$

n2

Area found by the correct method = 4100

$$\begin{array}{r} 4 \\ 16400 = 4 A \\ 1 = r \\ 16400 = 4 A r \\ 1600 = b^2 \end{array}$$

Square root of 18000 = 184.164,

and 184.164, divided by twice the ratio, gives 67.082, from which take the base, divided by twice the ratio, leaves required mean height = 47.082.

By the common method = 50
Difference, 2.918 feet.

Or thus, by sec. 324: We find the mean height Q R, (fig. 49,) area of triangle A B P, having slopes 1 to 1 = 400, the perpendicular P Q = 20. And from above we have the area of the section A B D C = 4100

$$\begin{array}{r} \therefore \left(\frac{A + a}{a} \right)^{\frac{1}{2}} = \left(\frac{4100 + 400}{400} \right)^{\frac{1}{2}} = \sqrt{\frac{4500}{400}} = \frac{\sqrt{45}}{2} = \frac{6.7082}{2} = 3.3541 \\ \frac{b}{2r} = 20 \\ \text{Less } \frac{b}{2r} = 20 \\ \text{Mean height Q R} = 47.802 \end{array}$$

TO FIND THE CONTENT OF AN EXCAVATION OR EMBANKMENT.

In general, the section to be measured is either a prism, cylinder, cone, pyramid, wedge, or a frustrum of a cone, pyramid, or wedge. The latter is called a prismoid.

A *Prism* is a solid, contained by plane figures, of which two are opposite, equal, similar, and having their sides parallel. The opposite, equal and similar sides are the ends. The other sides are called the lateral sides. Those prisms having regular polygons for bases, are called regular prisms.

Prismoid has its two ends parallel and dissimilar, and may be any figure.

327. *Prism. Rule.* Multiply the area of the base by the height of the section, the product = content, or $S = A l$. Here A = area of the base, and l = the length of the section, and S = sectional area.

328. *Cylinder. Rule.* Square the diameter, multiply it by .7854, then by the height, the product = content = $D^2 \times .7854$. Here D = diameter, solidity = $S = A l$. Here A = area of the base, and l = length.

329. *Cone. Rule.* Multiply the square of the diameter by .7854, and that product by one-third of the height, will give the content = $S = D^2 \times .7854 \times \frac{1}{3}$ Or, solidity = $\frac{A l}{3}$ where A and l are as above.

380. *Frustrum of a Cone. Rule.* To the areas of the two ends, add their mean proportional. Multiply their sum by one-third of the height or length, the product = content.

$$\text{Solidity} = S = \left(A \times a \times \sqrt{A a} \right) \frac{1}{3}$$

$$S = (D^2 + d^2 + D d) 0.2618$$

$$S = \left(\frac{D^2 - d^2}{D - d} \cdot \frac{l}{3} \right) = \left(\frac{D^2 + d^2}{D - d} \right) \times .2618 c. \text{ Here } l = 0.7854, \\ D \text{ and } d = \text{diameters, } l = \text{length, as above.}$$

Example. Let the greater diameter of a frustrum of a cone be $= D = 2$, and the lesser $= d = 1$, and the length $= 15$, to find the content. Dimensions all in feet.

$$A = 4 \times 0.7854 = 3.1416 = 3.1416$$

$$a = 1 \times 0.7854 \quad \underline{0.7854} \quad 0.7854$$

$$\text{Product} = 2.46741264, \text{ square root} = 1.5708$$

$$\underline{5.4978}$$

$$\text{One-third the length, } \underline{5}$$

$$\text{Content or } S = 27.489$$

Or thus :

$$(\text{By sec. 380.}) \quad D^2 + d^2 + Dd = 4 + 1 + 2 = 7$$

$$l = \text{length} \quad = \underline{15}$$

$$\underline{105}$$

$$\frac{0.7859}{8} = \text{tabular number} \quad = \underline{0.2618}$$

$$S = 27.489 = \text{content.}$$

Or thus :

$$\frac{D^2 - d^2}{D - d} = \frac{8 - 1}{1} = 7$$

$$t = \underline{.7854}$$

$$\underline{5.4978}$$

$$\underline{15}$$

$$8)824670$$

$$S = 27.489 = \text{content.}$$

381. *Pyramid. Rule.* Multiply the area of the base by one-third of the length or height, and the product will be the required content. Or, solidity $= S = \frac{Al}{3}$

382. *Frustrum of a Pyramid. Rule.* To the sum of the areas of both ends add their mean proportional, multiply this sum by one-third of their height, the product will be the content, or $S = (A + a + \sqrt{Aa}) \frac{l}{3}$

Let the ends be regular polygons, whose sides are D and d , then,
 $S = \left(\frac{D^2 - d^2}{D - d} \right) \frac{tl}{3}$ Here $D =$ greater and $d =$ lesser side,

$t =$ tabular area, corresponding to the given polygon, and l as above.

Rule. From the cube of the greater side take the cube of the lesser, divide this difference by the difference of the sides, multiply the quotient by the tabular number corresponding to that polygon, and that product by the length or height. One-third of this product will be the required content, the same as for the frustrum of a cone.

Example. Let 3 and 2 respectively be the sides of a square frustrum of a pyramid, and length $= 15$ feet.

$$A + a + \sqrt{Aa} = 9 + 4 + 6 = 19$$

$$\text{One-third the length} = \underline{5}$$

$$\text{Solidity} = S = 95$$

Or thus, by sec. 381 :

$$\frac{D^2 - d^2}{D - d} = \frac{27 - 8}{3 - 2} = \frac{19}{1} = 19$$

$$\text{Tabular number per Table VIII } a = \underline{1}$$

$$\underline{19}$$

$$\text{One-third the length} = \underline{5}$$

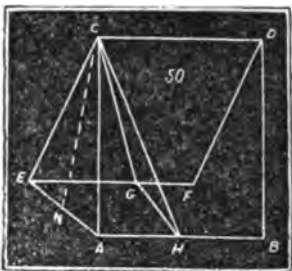
$$S = 95 = \text{content.}$$

383. *Wedge* has a rectangular base and two opposite sides meeting in an edge.

Rule. To twice the length of the base add the length of the edge, multiply this sum by the breadth of the base, and the product by one-sixth of the height, the product will be the solid content, when the base has its sides parallel.

$$S = \frac{1}{6}(2L + l) \cdot b \cdot h. \text{ Here}$$

L = length of the rectangular base
 $A B$, l length of the edge $C D$, b = breadth of base, $B F$ and H = height.



Example. Let $A B = 40$ feet, $B F = b = 10$, $C D = l = 30$, and let the height $N C = 50$ feet = h , to find the content.

$$\begin{array}{r} 2L \times l = 80 + 30 = 110 \\ b \times h = 10 \times 50 = 500 \\ \hline 6)55000 \\ 9166.666 \end{array}$$

9166.666 cubic feet.

Let $C D$, the edge, be parallel to the lengths $A B$ and $E F$, and $A B$ greater than $E F$, $H G$ = perpendicular width.

Rule 2. Add the three parallel edges together, multiply its one-third by half the height, multiplied by the perpendicular breadth, the product will be the required content. Or, $S = \frac{1}{3}(L + L_1 + l) + \frac{l \cdot h \cdot b}{2}$

Here L = greater length of base, L_1 = lesser length, l = length of the edge, h = perpendicular height, and b = perpendicular breadth.

Let us apply this to the last example:

$$\begin{array}{r} \frac{L + L_1 + l}{3} = \frac{40 + 40 + 30}{3} = \frac{110}{3} \\ \frac{h \cdot b}{2} = \frac{50 \times 10}{2} = 250 \end{array}$$

Therefore, content = $\frac{110}{3} \times \frac{250}{1} = \frac{27500}{3} = 9166.666$, as above.

Example 2. Let $A B = 4$, $E F = 2.5$, $C D = 3$, height = 12, and width $H G = 3\frac{1}{2}$, then by Rule 2.

$$4 + 3 + 2.5 \times 12 \times 3.5 = 66\frac{1}{2} \text{ cubic feet.}$$

Note. As Rule 2 answers for any form of a wedge, whose edge is parallel to the base, the opposite sides $A B$ and $E F$ parallel, without any reference to their being equal.

PRISMOID.

334. The prismoid is a frustum of a wedge, its ends being parallel to one another, and therefore similar, or the ends are parallel and dissimilar.

When the section is the frustum of a wedge, it is made up of two wedges, one having the greater end for a base, and the other having the lesser, the content may be found by rule 2 for the wedge.

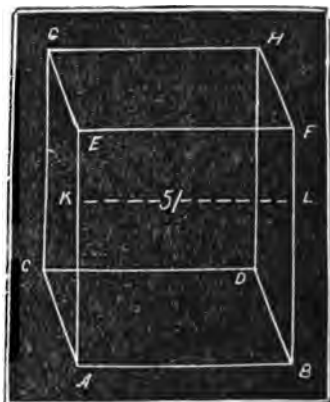
The following rule, known as the prismoidal formula, will answer for a section whose ends are parallel to one another. It is the safest and most expeditious formula now used, and has been first introduced by Sir John MacNeil in calculating his valuable tables on earth work, octavo, pp. 268. T. F. Baker, Esq., C.E., has also given a very concise formula, which, as many perhaps may prefer, I give in the next section. To Mr. Baker, of England, the world is indebted for his practical method of laying out curves.

PRISMOIDAL FORMULA.

$S = (A + a + 4 M) \frac{L}{6}$ Here A = area of greater end, a = area of lesser end, M = area of middle section, and L = length of section, all in feet.

Rule. To the sum of the areas of the two ends, add four times the area of the middle section, multiply this sum by one-sixth of the length, the product will be the required content, or solidity.

Here A = area of $C A B D$,
 a = area of $G E F H$,
 and M = area of section through $K L$.



Example. Let the length L = 400 feet.
 Mean height of section $A B D C$ = 50
 Mean height of section $G E F H$ = 20
 Ratio of slopes = 2 base to 1 perpendicular, and base = 80,

50 = mean height, by sec. 326.	Height	20	50
2		2	20
Half base = 100 for slopes.		40	2)70
80		80	85
Mean br'dth, 180	Mean breadth, 70		2
Height, 50	Height, 20		70
6500	a = 1400		80
	A = 6500		100
	M = 14000		85
	21900	8500 = M .	
	400 = length.		

6)8760000

Content in cubic feet = 9)1460000

3)162222.22

54074.07 cubic yards.

On comparing this with Sir John MacNeil's table, we find 540.72, difference only 2 yards, which is but very little in this large section.

Baker's Method Modified. (See fig. 48.)

$$\text{Solidity} = S = \frac{r}{8} l \left(D^3 + D d + d^3 - \frac{3 B^3}{4 r^3} \right)$$

Here D = greater depth from the vertex, whose slopes meet below the base, d = lesser depth, r = ratio of slopes, B = base, l = length of section, all in feet. The depths D and d are found by adding the perpendicular $P Q$ to the mean height $q R$ of section. (See fig. 48.)

$$\text{Because } \frac{b}{22} = P Q, \therefore \frac{80}{4} = 7.5 = P Q.$$

$$\text{Consequently } D = 50 + 7.5 = 57.5 \\ d = 20 + 7.5 = 27.5$$

$$\begin{aligned}
 D^2 &= 57.5 \times 57.5 &= 3306.25 \\
 d^2 &= 27.5 \times 27.5 &= 756.25 \\
 Dd &= 57.5 \times 27.5 &= 1581.25 \\
 &&5648.75
 \end{aligned}$$

$$\frac{8 B^2}{4 r^2} = \frac{8 \times 80 \times 20}{16} = \frac{2700}{16} = 168.75$$

$$D^2 + Dd + d^2 - \frac{8 B^2}{4 r^2} = 5475$$

$$r l = 2 \times 400 \quad 800$$

$$81)4880000$$

54074.07, the same as that found

above by the Prismoidal formula.

The bases or road beds are, in England, for single track 20, double track 30 feet wide.

And in the United States, in embankments, single track 16, for double track 28 feet. Also in excavation, single track 24, double track 32 feet.

In laying out the line, we endeavor to have the cutting and filling equal to one another, observing to allow 10 per cent for shrinkage; for it has been found that gravel and sand shrink 8 per cent, clay 10, loam 12, and surface soil 15. Where clay is put in water, it shrinks from 30 to 33 per cent.

Rock, broken in large fragments, increases 40 per cent.; if broken into small fragments, increases 60 per cent.

The following, Table a, is calculated from a modified form of Wm. Kelly's formula.

$$\text{Content in cubic yards} = L \left\{ B \cdot \frac{1}{2} \left(\frac{H+h}{27} \right) + \left(\frac{Hh}{27} + \frac{d \cdot d}{3 \cdot 27} \right) s \right\}$$

Here L = length, B = base, H and h = greater and lesser heights, r = ratio of slope, d = difference of heights.

Rule for using Table a. Multiply tabular number of half the height by the base, and call the result = A.

2. Multiply the tabular of either height by the other height, and call the result = B.

3. Multiply the tabular number of the difference of the heights by one-third of the difference, and call the result = C.

Add results B and C together, multiply the sum by the ratio of the slopes, add the product to the result A, and multiply the sum by the length, the product will be the content in cubic yards.

Example as in section 384. Where length = 400, base = 30, heights = 50 and 20, and ratio of slopes = 2.

$$\frac{50 + 20}{2} = 35, \text{ its tabular number, by } 30 = 1.2963 \times 30 = A = 38.889.$$

$$50 \times \text{tabular } 20 = 50 \times 7.7407 = 38.9350 = B.$$

$$10 \times \text{tabular } 80 = 10 \times 1.1111 = 11.1110 = C.$$

$$48.1960 \times 2 =$$

$$96.292$$

$$135.181$$

$$400$$

Length,

$$54072.505 \text{ yds.}$$

By Sir John MacNeil's Table XXIII =

$$54072$$

By his prismoidal formula =

$$54074.072$$

Here we find the difference between table a and the prismoidal formula to be 1 in 38049.

Sir John's tables are calculated only to feet and 2 decimals. William Kelly's (civil engineer, for many years connected with the Ordinance Survey of Ireland) to every three inches, and to three places of decimals. Table a is arranged similar to Mr. Kelly's Table I, but calculated to tenths of a foot, and to four places of decimals. Tables b and c are the same as MacNeil's Tables LVIII and LIX, with our explanation and example.

TABLE a.—For the Computation of Prismoids, for all Bases and Slopes.

Height in Feet.	Tabular No.	Height in Feet.	Tabular No.	Height in Feet.	Tabular No.	Height in Feet.	Tabular No.	Height in Feet.	Tabular No.	Height in Feet.	Tabular No.
1.0	0.0037	6.1	0.2259	12.1	0.4481	18.1	0.6704	24.1	0.8926	30.1	1.1148
2	.0074	2	.2296	2	.4518	2	.6741	2	.8968	2	.1185
3	.0111	3	.2333	3	.4555	3	.6778	3	.9000	3	.1222
4	.0148	4	.2370	4	.4592	4	.6815	4	.9037	4	.1259
5	.0185	5	.2407	5	.4629	5	.6852	5	.9074	5	.1296
6	.0222	6	.2444	6	.4666	6	.6889	6	.9111	6	.1333
7	.0259	7	.2481	7	.4703	7	.6926	7	.9148	7	.1370
8	.0296	8	.2518	8	.4740	8	.6963	8	.9185	8	.1407
9	.0333	9	.2555	9	.4777	9	.7000	9	.9222	9	.1444
1.0	.0370	7.0	.2591	13.0	.4814	19.0	.7037	25.0	.9259	31.0	.1481
1	.0407	1	.2628	1	.4851	1	.7074	1	.9296	1	.1518
2	.0444	2	.2665	2	.4888	2	.7111	2	.9333	2	.1555
3	.0481	3	.2802	3	.4925	3	.7148	3	.9370	3	.1592
4	.0518	4	.2839	4	.4962	4	.7185	4	.9407	4	.1629
5	.0555	5	.2778	5	.5000	5	.7222	5	.9444	5	.1666
6	.0592	6	.2815	6	.5037	6	.7259	6	.9481	6	.1703
7	.0629	7	.2852	7	.5074	7	.7296	7	.9518	7	.1740
8	.0666	8	.2889	8	.5111	8	.7333	8	.9555	8	.1777
9	.0703	9	.2926	9	.5148	9	.7370	9	.9592	9	.1814
2.0	.0741	8.0	.2963	14.0	.5185	20.0	.7407	26.0	.9629	32.0	.1851
1	.0778	1	.3000	1	.5222	1	.7444	1	.9666	1	.1888
2	.0815	2	.3037	2	.5259	2	.7481	2	.9703	2	.1925
3	.0852	3	.3074	3	.5296	3	.7518	3	.9740	3	.1962
4	.0889	4	.3111	4	.5333	4	.7555	4	.9777	4	.1999
5	.0926	5	.3148	5	.5370	5	.7592	5	.9815	5	.2037
6	.0963	6	.3185	6	.5407	6	.7629	6	.9852	6	.2074
7	.1000	7	.3222	7	.5444	7	.7666	7	.9889	7	.2111
8	.1037	8	.3259	8	.5481	8	.7703	8	.9926	8	.2148
9	.1074	9	.3296	9	.5518	9	.7740	9	.9963	9	.2185
3.0	.1111	9.0	.3333	15.0	.5555	21.0	.7778	27.0	1.0000	33.0	.2222
1	.1148	1	.3370	1	.5592	1	.7815	1	.0037	1	.2259
2	.1185	2	.3407	2	.5629	2	.7852	2	.0074	2	.2296
3	.1222	3	.3444	3	.5666	3	.7889	3	.0111	3	.2333
4	.1259	4	.3481	4	.5703	4	.7926	4	.0148	4	.2370
5	.1296	5	.3518	5	.5741	5	.7963	5	.0185	5	.2407
6	.1333	6	.3555	6	.5778	6	.8000	6	.0222	6	.2444
7	.1370	7	.3592	7	.5815	7	.8037	7	.0259	7	.2481
8	.1407	8	.3629	8	.5852	8	.8074	8	.0296	8	.2518
9	.1444	9	.3666	9	.5889	9	.8111	9	.0333	9	.2555
4.0	.1481	10.0	.3704	16.0	.5926	22.0	.8148	28.0	.0370	34.0	.2592
1	.1518	1	.3741	1	.5963	1	.8185	1	.0407	1	.2629
2	.1555	2	.3778	2	.6000	2	.8222	2	.0444	2	.2666
3	.1592	3	.3815	3	.6037	3	.8259	3	.0481	3	.2703
4	.1629	4	.3852	4	.6074	4	.8296	4	.0518	4	.2740
5	.1667	5	.3889	5	.6111	5	.8333	5	.0555	5	.2778
6	.1704	6	.3926	6	.6148	6	.8370	6	.0592	6	.2815
7	.1741	7	.3963	7	.6185	7	.8407	7	.0629	7	.2852
8	.1778	8	.4000	8	.6222	8	.8444	8	.0666	8	.2889
9	.1815	9	.4037	9	.6259	9	.8481	9	.0703	9	.2926
5.0	.1852	11.0	.4074	17.0	.6296	23.0	.8518	29.0	.0741	35.0	.2963
1	.1889	1	.4111	1	.6333	1	.8555	1	.0778	1	.3000
2	.1926	2	.4148	2	.6370	2	.8592	2	.0815	2	.3037
3	.1963	3	.4185	3	.6407	3	.8629	3	.0852	3	.3074
4	.2000	4	.4222	4	.6444	4	.8666	4	.0889	4	.3111
5	.2037	5	.4259	5	.6481	5	.8704	5	.0926	5	.3148
6	.2074	6	.4296	6	.6518	6	.8741	6	.0963	6	.3185
7	.2111	7	.4333	7	.6555	7	.8778	7	.1000	7	.3222
8	.2148	8	.4370	8	.6592	8	.8815	8	.1037	8	.3259
9	.2185	9	.4407	9	.6629	9	.8852	9	.1074	9	.3296
6.0	.2222	12.0	.4444	18.0	.6667	24.0	.8889	30.0	1.1111	36.0	1.3333

TABLE a.—For the Computation of Prismoids, for all Bases and Slopes.

Height in Feet.	Tabular No.	Height in Feet.	Tabular No.	Height in Feet.	Tabular No.	Height in Feet.	Tabular No.	Height in Feet.	Tabular No.	Height in Feet.	Tabular No.
36.1	1.8370	42.1	1.5509	48.1	1.7815	54.1	2.0087	60.1	2.2259	66.1	2.4481
2	.8407	2	.5688	2	.7852	2	.0071	2	.2296	2	.4518
3	.8444	3	.5667	3	.7889	3	.0114	3	.2383	3	.4555
4	.8481	4	.5704	4	.7926	4	.0148	4	.2370	4	.4592
5	.8518	5	.5741	5	.7963	5	.0185	5	.2407	5	.4629
6	.8555	6	.5778	6	.8000	6	.0222	6	.2444	6	.4666
7	.8592	7	.5815	7	.8037	7	.0259	7	.2481	7	.4708
8	.8629	8	.5852	8	.8074	8	.0296	8	.2518	8	.4740
9	.8666	9	.5889	9	.8111	9	.0333	9	.2555	9	.4777
37.0	.8704	43.0	.5926	49.0	.8148	55.0	.0370	61.0	.2592	67.0	.4815
1	.8741	1	.5963	1	.8185	1	.0407	1	.2629	1	.4852
2	.8778	2	.6000	2	.8222	2	.0444	2	.2666	2	.4889
3	.8815	3	.6037	3	.8259	3	.0481	3	.2703	3	.4926
4	.8852	4	.6074	4	.8296	4	.0518	4	.2740	4	.4963
5	.8889	5	.6111	5	.8333	5	.0556	5	.2788	5	.5000
6	.8926	6	.6148	6	.8370	6	.0593	6	.2815	6	.5037
7	.8963	7	.6185	7	.8407	7	.0630	7	.2852	7	.5074
8	.9000	8	.6222	8	.8444	8	.0667	8	.2889	8	.5111
9	.9037	9	.6259	9	.8481	9	.0704	9	.2925	9	.5148
38.0	.9078	44.0	.6295	50.0	.8518	56.0	.0741	62.0	.2968	68.0	.5185
1	.9110	1	.6332	1	.8555	1	.0778	1	.3000	1	.5222
2	.9147	2	.6369	2	.8592	2	.0815	2	.3037	2	.5259
3	.9184	3	.6406	3	.8629	3	.0852	3	.3074	3	.5296
4	.9221	4	.6443	4	.8666	4	.0889	4	.3111	4	.5333
5	.9259	5	.6481	5	.8704	5	.0926	5	.3148	5	.5370
6	.9296	6	.6518	6	.8741	6	.0963	6	.3185	6	.5407
7	.9333	7	.6555	7	.8778	7	.1000	7	.3222	7	.5444
8	.9370	8	.6592	8	.8815	8	.1037	8	.3259	8	.5481
9	.9407	9	.6629	9	.8852	9	.1074	9	.3296	9	.5518
39.0	.9444	45.0	.6667	51.0	.8889	57.0	.1111	63.0	.3333	69.0	.5555
1	.9481	1	.6704	1	.8926	1	.1148	1	.3370	1	.5593
2	.9518	2	.6741	2	.8963	2	.1185	2	.3407	2	.5630
3	.9555	3	.6778	3	.9000	3	.1222	3	.3444	3	.5667
4	.9592	4	.6815	4	.9037	4	.1259	4	.3481	4	.5704
5	.9629	5	.6852	5	.9074	5	.1296	5	.3518	5	.5741
6	.9666	6	.6889	6	.9111	6	.1333	6	.3555	6	.5778
7	.9703	7	.6926	7	.9148	7	.1370	7	.3592	7	.5815
8	.9740	8	.6963	8	.9185	8	.1407	8	.3629	8	.5852
9	.9777	9	.7000	9	.9222	9	.1444	9	.3666	9	.5889
40.0	.9814	46.0	.7037	52.0	.9259	58.0	.1481	64.0	.3704	70.0	.5926
1	.9851	1	.7074	1	.9296	1	.1518	1	.3741	1	.5963
2	.9888	2	.7111	2	.9333	2	.1555	2	.3778	2	.6000
3	.9925	3	.7148	3	.9370	3	.1592	3	.3815	3	.6037
4	.9962	4	.7185	4	.9407	4	.1629	4	.3852	4	.6074
5	.5000	5	.7222	5	.9444	5	.1667	5	.3889	5	.6111
6	.5037	6	.7259	6	.9481	6	.1704	6	.3926	6	.6148
7	.5074	7	.7296	7	.9518	7	.1741	7	.3963	7	.6185
8	.5111	8	.7333	8	.9555	8	.1778	8	.4000	8	.6222
9	.5148	9	.7370	9	.9592	9	.1815	9	.4037	9	.6259
41.0	.5185	47.0	.7407	53.0	.9629	59.0	.1851	65.0	.4074	71.0	.6296
1	.5222	1	.7444	1	.9666	1	.1888	1	.4111	1	.6333
2	.5259	2	.7481	2	.9703	2	.1925	2	.4148	2	.6370
3	.5296	3	.7518	3	.9740	3	.1962	3	.4185	3	.6407
4	.5333	4	.7555	4	.9777	4	.1999	4	.4222	4	.6444
5	.5370	5	.7592	5	.9814	5	.2037	5	.4259	5	.6481
6	.5407	6	.7629	6	.9851	6	.2074	6	.4296	6	.6518
7	.5444	7	.7666	7	.9888	7	.2111	7	.4333	7	.6555
8	.5481	8	.7703	8	.9925	8	.2148	8	.4370	8	.6592
9	.5518	9	.7740	9	.9962	9	.2185	9	.4407	9	.6629
42.0	1.5555	48.0	1.7778	54.0	2.0000	60.0	2.2222	66.0	2.4444	72.0	2.6667

TABLE 6.—For the computation of Prismoids or Earthwork.

Ft	1	2	3	4	5	6	7	8	9	10	11	12	13	Ft
0	2	8	18	32	50	72	98	128	162	200	242	288	338	0
1	6	14	26	42	62	86	114	146	182	222	266	314	366	1
2	14	24	38	56	78	104	134	168	206	248	294	344	398	2
3	26	38	54	74	98	126	158	194	234	278	326	378	434	3
4	42	56	74	96	122	152	186	224	266	312	362	416	474	4
5	62	78	98	122	150	182	218	258	302	350	402	458	518	5
6	86	104	126	152	182	216	254	296	342	392	446	504	566	6
7	114	134	158	186	218	254	294	338	386	438	494	554	618	7
8	146	168	194	224	258	296	338	384	434	488	546	608	674	8
9	182	206	234	266	302	342	386	434	486	542	602	666	734	9
10	222	248	278	312	350	392	438	488	542	600	662	728	798	10
11	266	294	326	362	402	446	494	546	602	662	726	794	866	11
12	314	344	378	416	458	504	554	608	666	728	794	864	938	12
13	366	398	434	474	518	566	618	674	734	798	866	938	1014	13
14	422	456	494	536	582	632	686	744	806	872	942	1016	1094	14
15	482	518	558	602	650	702	758	818	882	950	1022	1098	1178	15
16	546	584	626	672	722	776	834	896	962	1032	1106	1184	1266	16
17	614	654	698	746	798	854	914	978	1046	1118	1194	1274	1358	17
18	686	728	774	824	878	936	998	1064	1134	1208	1286	1368	1454	18
19	762	806	854	906	962	1022	1086	1154	1226	1302	1382	1466	1554	19
20	842	888	938	992	1050	1112	1178	1248	1322	1400	1482	1568	1658	20
21	926	974	1026	1082	1142	1206	1274	1346	1422	1502	1586	1674	1766	21
22	1014	1064	1118	1176	1238	1304	1374	1448	1526	1608	1694	1784	1878	22
23	1106	1158	1214	1274	1338	1406	1478	1554	1634	1718	1806	1898	1994	23
24	1202	1256	1314	1376	1442	1512	1586	1664	1746	18 2	1922	2016	2114	24
25	1302	1358	1418	1482	1550	1622	1698	1774	1862	1950	2042	2138	2238	25
26	1406	1464	1526	1592	1662	1736	1814	1896	1982	2072	2166	2264	2366	26
27	1514	1574	1638	1706	1778	1854	1934	2018	2106	2198	2294	2398	2498	27
28	1626	1688	1754	1824	1898	1976	2058	2144	2234	2328	2426	2528	2634	28
29	1742	1806	1874	1946	2022	2102	2186	2274	2366	2462	2562	2666	2774	29
30	1862	1928	1998	2072	2150	2232	2318	2408	2502	2600	2702	2808	2918	30
31	1986	2054	2126	2202	2282	2366	2454	2546	2642	2742	2846	2954	3066	31
32	2114	2184	2256	2336	2418	2504	2594	2688	2786	2888	2994	3104	3218	32
33	2246	2318	2394	2474	2558	2646	2738	2834	2934	3038	3146	3258	3374	33
34	2382	2456	2534	2616	2702	2792	2886	2984	3086	3192	3202	3416	3534	34
35	2522	2598	2678	2762	2850	2942	3038	3138	3242	3350	3462	3578	3698	35
36	2666	2744	2826	2912	3002	3096	3194	3296	3402	3512	3626	3744	3866	36
37	2814	2894	2978	3066	3158	3254	3354	3458	3566	3678	3794	3914	4038	37
38	2966	3048	3134	3224	3318	3416	3518	3624	3734	3848	3966	4088	4214	38
39	3122	3206	3294	3386	3482	3582	3686	3794	3906	4022	4142	4266	4394	39
40	3282	3368	3458	3552	3650	3752	3858	3968	4082	4200	4322	4448	4578	40
41	3446	3534	3626	3722	3822	3926	4034	4146	4262	4382	4506	4634	4766	41
42	3614	3704	3798	3896	3998	4104	4214	4328	4446	4568	4694	4824	4958	42
43	3786	3878	3974	4074	4178	4286	4398	4514	4634	4758	4886	5018	5154	43
44	3962	4056	4154	4256	4362	4472	4586	4704	4826	4952	5082	5216	5354	44
45	4142	4238	4338	4442	4550	4662	4778	4898	5022	5150	5282	5418	5558	45
46	4326	4424	4526	4632	4742	4856	4974	5096	5222	5352	5486	5624	5766	46
47	4514	4614	4718	4826	4938	5054	5174	5298	5426	5558	5694	5834	5978	47
48	4706	4808	4914	5024	5138	5256	5378	5504	5634	5768	5906	6048	6194	48
49	4902	5006	5114	5226	5342	5462	5586	5714	5846	5982	6122	6266	6414	49
50	5102	5208	5318	5432	5550	5672	5798	5928	6062	6200	6342	6488	6638	50
Ft	1	2	3	4	5	6	7	8	9	10	11	12	13	Ft

TABLE b.—For the computation of Prismoids or Earthwork.

Ft	14	15	16	17	18	19	20	21	22	23	24	25	26	Ft
0	392	450	512	578	648	722	800	882	968	1058	1152	1250	1352	0
1	422	482	546	614	686	762	842	926	1014	1106	1202	1302	1406	1
2	456	518	584	654	728	806	888	974	1064	1158	1256	1358	1464	2
3	494	558	626	698	774	854	938	1026	1118	1214	1314	1418	1526	3
4	536	602	672	746	824	906	992	1082	1176	1274	1376	1482	1592	4
5	582	650	722	798	878	962	1050	1142	1238	1338	1442	1550	1662	5
6	632	702	776	854	936	1022	1112	1206	1304	1406	1512	1622	1736	6
7	686	758	834	914	998	1086	1178	1274	1374	1478	1586	1698	1814	7
8	744	818	896	978	1064	1154	1248	1346	1448	1554	1664	1778	1896	8
9	806	882	962	1046	1134	1226	1322	1422	1526	1634	1746	1862	1982	9
10	872	950	1032	1118	1208	1302	1400	1502	1608	1718	1832	1950	2072	10
11	942	1022	1106	1194	1286	1382	1482	1586	1694	1806	1922	2042	2166	11
12	1016	1098	1184	1274	1368	1466	1568	1674	1784	1898	2016	2138	2264	12
13	1094	1178	1266	1358	1454	1554	1658	1766	1878	1994	2114	2238	2366	13
14	1176	1262	1352	1446	1544	1646	1752	1862	1976	2094	2216	2342	2472	14
15	1262	1350	1442	1538	1638	1742	1850	1962	2078	2198	2322	2450	2582	15
16	1352	1442	1536	1634	1736	1842	1952	2066	2184	2306	2432	2562	2696	16
17	1446	1538	1634	1734	1838	1946	2058	2174	2294	2418	2546	2678	2814	17
18	1544	1638	1736	1838	1944	2054	2168	2286	2408	2534	2664	2798	2936	18
19	1646	1742	1842	1946	2054	2166	2282	2402	2526	2654	2786	2922	3062	19
20	1752	1850	1952	2058	2168	2282	2400	2522	2648	2778	2912	3050	3192	20
21	1862	1962	2066	2174	2286	2402	2522	2646	2774	2906	3042	3182	3326	21
22	1976	2078	2184	2294	2408	2526	2648	2774	2904	3038	3176	3318	3464	22
23	2094	2198	2306	2418	2534	2654	2778	2906	3038	3174	3314	3458	3606	23
24	2216	2322	2432	2546	2664	2786	2912	3042	3176	3314	3456	3602	3752	24
25	2342	2450	2562	2678	2798	2922	3050	3182	3318	3458	3602	3750	3902	25
26	2472	2582	2696	2814	2936	3062	3192	3326	3464	3606	3752	3902	4056	26
27	2606	2718	2834	2954	3078	3206	3338	3474	3614	3758	3906	4058	4214	27
28	2744	2858	2976	3098	3224	3354	3488	3626	3768	3914	4064	4218	4376	28
29	2886	3002	3122	3246	3374	3506	3642	3782	3926	4074	4226	4382	4542	29
30	3032	3150	3272	3398	3528	3662	3800	3942	4088	4238	4392	4550	4712	30
31	3182	3302	3426	3554	3686	3822	3962	4106	4254	4406	4562	4722	4886	31
32	3336	3458	3584	3714	3848	3986	4128	4274	4424	4578	4736	4898	5064	32
33	3494	3618	3746	3878	4014	4157	4298	4446	4598	4754	4914	5078	5246	33
34	3656	3782	3912	4046	4184	4326	4472	4622	4776	4934	5096	5262	5432	34
35	3822	3950	4082	4218	4358	4502	4650	4802	4958	5118	5282	5450	5622	35
36	3992	4122	4256	4394	4536	4682	4832	4986	5144	5306	5472	5642	5816	36
37	4166	4298	4434	4574	4718	4866	5018	5174	5334	5498	5666	5838	6014	37
38	4344	4478	4616	4758	4904	5054	5208	5366	5528	5698	5864	6038	6216	38
39	4526	4662	4802	4946	5094	5246	5402	5562	5726	5894	6066	6242	6422	39
40	4712	4850	4992	5138	5288	5442	5600	5762	5928	6098	6272	6450	6632	40
41	4902	5042	5186	5334	5486	5642	5802	5966	6134	6306	6482	6662	6846	41
42	5096	5238	5384	5534	5688	5846	6008	6174	6344	6518	6696	6878	7064	42
43	5294	5438	5586	5738	5894	6054	6218	6386	6558	6734	6914	7098	7286	43
44	5496	5642	5792	5946	6104	6266	6432	6602	6776	6954	7136	7322	7512	44
45	5702	5850	6002	6158	6318	6482	6650	6822	6998	7178	7362	7550	7742	45
46	5912	6062	6216	6374	6536	6702	6872	7046	7224	7406	7592	7782	7976	46
47	6126	6278	6434	6594	6758	6926	7098	7274	7454	7638	7826	8018	8214	47
48	6344	6498	6656	6818	6984	7154	7328	7506	7688	7874	8064	8258	8456	48
49	6566	6722	6882	7046	7214	7386	7562	7742	7926	8114	8306	8502	8702	49
50	6792	6950	7112	7278	7448	7622	7800	7982	8168	8358	8552	8750	8952	50
Ft	14	15	16	17	18	19	20	21	22	23	24	25	26	Ft

TABLE b.—For the computation of Prismoids or Earthwork.

Ft	27	28	29	30	31	32	33	34	35	36	37	38	Ft
0	1458	1568	1682	1800	1922	2048	2178	2312	2450	2592	2738	2888	0
1	1514	1626	1742	1862	1986	2114	2246	2382	2522	2666	2814	2966	1
2	1574	1688	1806	1928	2054	2184	2318	2456	2598	2744	2894	3048	2
3	1638	1754	1874	1998	2126	2258	2394	2534	2678	2826	2978	3134	3
4	1706	1824	1946	2072	2202	2336	2474	2616	2762	2912	3066	3224	4
5	1778	1898	2022	2150	2282	2418	2558	2702	2850	3002	3158	3318	5
6	1854	1976	2102	2232	2366	2504	2646	2792	2942	3096	3254	3416	6
7	1934	2058	2186	2318	2454	2594	2738	2886	3038	3194	3354	3518	7
8	2018	2144	2274	2408	2546	2688	2834	2984	3138	3296	3458	3624	8
9	2106	2234	2366	2502	2642	2786	2934	3086	3242	3402	3566	3734	9
10	2198	2328	2462	2600	2742	2888	3038	3192	3350	3512	3678	3848	10
11	2294	2426	2562	2702	2846	2994	3146	3302	3462	3626	3794	3966	11
12	2394	2528	2666	2808	2954	3104	3258	3416	3578	3744	3914	4088	12
13	2498	2634	2774	2918	3066	3218	3374	3534	3698	3866	4038	4214	13
14	2606	2744	2886	3032	3182	3336	3494	3656	3822	3992	4166	4344	14
15	2718	2858	3002	3150	3302	3458	3618	3782	3950	4122	4298	4478	15
16	2834	2976	3122	3272	3426	3584	3746	3912	4082	4256	4434	4616	16
17	2954	3098	3246	3398	3554	3714	3878	4046	4218	4392	4574	4758	17
18	3078	3224	3374	3528	3686	3848	4014	4184	4358	4536	4718	4904	18
19	3206	3354	3506	3662	3822	3986	4154	4326	4502	4682	4866	5054	19
20	3338	3488	3642	3800	3962	4128	4298	4472	4650	4832	5018	5208	20
21	3474	3626	3782	3942	4106	4274	4446	4622	4802	4986	5174	5366	21
22	3614	3768	3926	4088	4254	4424	4598	4776	4958	5144	5334	5528	22
23	3758	3914	4074	4238	4406	4578	4754	4934	5118	5306	5498	5694	23
24	3906	4064	4226	4392	4562	4736	4914	5096	5282	5472	5666	5864	24
25	4058	4218	4382	4550	4722	4898	5078	5262	5450	5642	5838	6038	25
26	4214	4376	4542	4712	4886	5064	5246	5432	5622	5816	6014	6216	26
27	4374	4538	4706	4878	5054	5234	5418	5606	5798	5994	6194	6398	27
28	4538	4704	4874	5048	5226	5408	5594	5784	5978	6176	6378	6584	28
29	4706	4874	5046	5222	5402	5586	5774	5966	6162	6362	6566	6774	29
30	4878	5048	5222	5400	5582	5768	5958	6152	6350	6552	6758	6968	30
31	5054	5226	5402	5582	5766	5954	6146	6342	6542	6746	6954	7166	31
32	5234	5408	5586	5768	5954	6144	6338	6536	6738	6944	7154	7368	32
33	5418	5594	5774	5958	6146	6338	6534	6734	6938	7146	7358	7574	33
34	5606	5784	5966	6152	6342	6536	6734	6936	7142	7352	7566	7784	34
35	5798	5978	6162	6350	6542	6738	6938	7142	7350	7562	7778	7998	35
36	5994	6176	6362	6552	6746	6944	7146	7354	7562	7776	7994	8216	36
37	6194	6378	6566	6758	6954	7154	7358	7566	7778	7994	8214	8438	37
38	6398	6584	6774	6968	7166	7368	7574	7784	7998	8216	8438	8664	38
39	6606	6794	6986	7182	7382	7586	7794	8006	8222	8442	8666	8894	39
40	6818	7008	7202	7400	7602	7808	8018	8232	8450	8672	8898	9128	40
41	7034	7226	7422	7622	7826	8034	8246	8462	8682	8906	9134	9366	41
42	7254	7448	7646	7848	8054	8264	8478	8696	8918	9144	9374	9608	42
43	7478	7674	7874	8078	8286	8498	8714	8934	9158	9386	9618	9854	43
44	7706	7904	8106	8312	8522	8736	8954	9176	9402	9632	9866	10104	44
45	7938	8138	8342	8550	8762	8978	9198	9422	9650	9882	10118	10358	45
46	8174	8376	8582	8792	9006	9224	9446	9672	9902	10136	10374	10616	46
47	8414	8618	8826	9038	9254	9474	9698	9926	10158	10394	10634	10878	47
48	8658	8866	9074	9288	9506	9728	9954	10184	10418	10656	10898	11144	48
49	8906	9114	9326	9542	9762	9986	10214	10446	10682	10922	11166	11414	49
50	9158	9368	9582	9800	10022	10248	10478	10712	10950	11192	11438	11688	50
Ft	27	28	29	30	31	32	33	34	35	36	37	38	Ft

TABLE b.—For the computation of Prismoids or Earthwork.

Ft.	39	40	41	42	43	44	45	46	47	48	Ft.
0	8042	8200	8362	8528	8698	8872	4050	4282	4418	4608	0
1	8122	8282	8446	8614	8786	8962	4142	4326	4514	4706	1
2	8206	8368	8534	8704	8878	4058	4238	4424	4614	4808	2
3	8294	8458	8626	8798	8974	4154	4388	4526	4718	4914	3
4	8386	8552	8722	8896	4074	4256	4442	4632	4826	5024	4
5	8482	8650	8822	3998	4178	4362	4550	4742	4938	5138	5
6	8582	8752	8926	4104	4286	4472	4662	4856	4054	5256	6
7	8686	8858	4084	4214	4398	4586	4778	4974	5174	5378	7
8	8794	8968	4146	4328	4514	4704	4898	5096	5298	5504	8
9	8906	4082	4262	4446	4634	4826	5022	5222	5426	5634	9
10	4022	4200	4382	4568	4758	4952	5150	5362	5568	5768	10
11	4142	4322	4506	4694	4886	4082	5282	5486	5694	5906	11
12	4266	4448	4634	4824	5018	5216	5418	5624	5824	6048	12
13	4394	4578	4766	4958	5154	5354	5558	5766	5978	6194	13
14	4526	4712	4902	5096	5294	5496	5702	5912	6126	6344	14
15	4662	4850	5042	5238	5438	5642	5850	6062	6278	6498	15
16	4802	4992	5186	5384	5586	5792	6002	6216	6434	6656	16
17	4946	5138	5334	5534	5738	5946	6158	6374	6594	6818	17
18	5094	5288	5486	5688	5894	6104	6318	6536	6758	6984	18
19	5246	5442	5642	5846	6054	6266	6482	6702	6926	7154	19
20	5402	5600	5802	6008	6218	6432	6650	6872	7098	7328	20
21	5562	5762	5966	6174	6386	6602	6822	7046	7274	7506	21
22	5726	5928	6134	6344	6558	6776	6998	7224	7454	7688	22
23	5894	6098	6306	6518	6734	6954	7178	7406	7638	7874	23
24	6091	6272	6482	6696	6914	7136	7362	7592	7826	8064	24
25	6242	6450	6662	6878	7098	7322	7550	7782	8018	8258	25
26	6422	6632	6846	7064	7286	7512	7742	7976	8214	8456	26
27	6606	6818	7034	7254	7478	7706	7938	8174	8414	8658	27
28	6794	7008	7226	7448	7674	7904	8138	8376	8618	8864	28
29	6986	7202	7422	7646	7874	8106	8342	8582	8826	9074	29
30	7182	7400	7622	7848	8078	8312	8550	8792	9038	9288	30
31	7382	7602	7826	8054	8286	8522	8762	9006	9254	9506	31
32	7586	7808	8034	8264	8498	8736	8978	9224	9474	9728	32
33	7794	8018	8246	8478	8714	8954	9198	9446	9698	9954	33
34	8006	8232	8462	8696	8934	9176	9422	9672	9926	10184	34
35	8222	8450	8682	8918	9158	9402	9650	9902	10158	10418	35
36	8442	8672	8906	9144	9386	9632	9882	10136	10394	10656	36
37	8666	8898	9134	9374	9618	9866	10118	10374	10634	10898	37
38	8894	9128	9366	9608	9854	10104	10358	10616	10878	11144	38
39	9126	9362	9602	9846	10094	10346	10602	10862	11126	11394	39
40	9362	9600	9842	10088	10338	10592	10850	11112	11378	11648	40
41	9602	9842	10086	10334	10586	10842	11102	11366	11634	11906	41
42	9846	10088	10334	10584	10838	11096	11358	11624	11884	12168	42
43	10094	10338	10586	10838	11094	11254	11618	11886	12156	12434	43
44	10246	10592	10842	11096	11354	11616	11882	12152	12426	12704	44
45	10602	10850	11102	11358	11618	11882	12150	12422	12698	12978	45
46	10862	11112	11366	11624	11886	12152	12422	12696	12974	13256	46
47	11126	11378	11634	11894	12158	12426	12698	12974	13254	13538	47
48	11394	11648	11906	12168	12434	12704	12978	13256	13538	13824	48
49	11666	11922	12182	12446	12714	12986	13262	13542	13826	14114	49
50	11942	12200	12462	12728	12998	13272	13556	13832	14118	14408	50
Ft.	39	40	41	42	43	44	45	46	47	48	Ft.

TABLE c.—For calculating Prismoids.

Ft.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	Ft.
0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	0
1	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	1
2	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	2
3	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	3
4	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	4
5	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	5
6	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	6
7	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	7
8	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75	8
9	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75	78	9
10	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75	78	81	10
11	36	39	42	45	48	51	54	57	60	63	66	69	72	75	78	81	84	11
12	39	42	45	48	51	54	57	60	63	66	69	72	75	78	81	84	87	12
13	42	45	48	51	54	57	60	63	66	69	72	75	78	81	84	87	90	13
14	45	48	51	54	57	60	63	66	69	72	75	78	81	84	87	90	93	14
15	48	51	54	57	60	63	66	69	72	75	78	81	84	87	90	93	96	15
16	51	54	57	60	63	66	69	72	75	78	81	84	87	90	93	96	99	16
17	54	57	60	63	66	69	72	75	78	81	84	87	90	93	96	99	102	17
18	57	60	63	66	69	72	75	78	81	84	87	90	93	96	99	102	105	18
19	60	63	66	69	72	75	78	81	84	87	90	93	96	99	102	105	108	19
20	63	66	69	72	75	78	81	84	87	90	93	96	99	102	105	108	111	20
21	66	69	72	75	78	81	84	87	90	93	96	99	102	105	108	111	114	21
22	69	72	75	78	81	84	87	90	93	96	99	102	105	108	111	114	117	22
23	72	75	78	81	84	87	90	93	96	99	102	105	108	111	114	117	120	23
24	75	78	81	84	87	90	93	96	99	102	105	108	111	114	117	120	123	24
25	78	81	84	87	90	93	96	99	102	105	108	111	114	117	120	123	126	25
26	81	84	87	90	93	96	99	102	105	108	111	114	117	120	123	126	129	26
27	84	87	90	93	96	99	102	105	108	111	114	117	120	123	126	129	132	27
28	87	90	93	96	99	102	105	108	111	114	117	120	123	126	129	132	135	28
29	90	93	96	99	102	105	108	111	114	117	120	123	126	129	132	135	138	29
30	93	96	99	102	105	108	111	114	117	120	123	126	129	132	135	138	141	30
31	96	99	102	105	108	111	114	117	120	123	126	129	132	135	138	141	144	31
32	99	102	105	108	111	114	117	120	123	126	129	132	135	138	141	144	147	32
33	102	105	108	111	114	117	120	123	126	129	132	135	138	141	144	147	150	33
34	105	108	111	114	117	120	123	126	129	132	135	138	141	144	147	150	153	34
35	108	111	114	117	120	123	126	129	132	135	138	141	144	147	150	153	156	35
36	111	114	117	120	123	126	129	132	135	138	141	144	147	150	153	156	159	36
37	114	117	120	123	126	129	132	135	138	141	144	147	150	153	156	159	162	37
38	117	120	123	126	129	132	135	138	141	144	147	150	153	156	159	162	165	38
39	120	123	126	129	132	135	138	141	144	147	150	153	156	159	162	165	168	39
40	123	126	129	132	135	138	141	144	147	150	153	156	159	162	165	168	171	40
41	126	129	132	135	138	141	144	147	150	153	156	159	162	165	168	171	174	41
42	129	132	135	138	141	144	147	150	153	156	159	162	165	168	171	174	177	42
43	132	135	138	141	144	147	150	153	156	159	162	165	168	171	174	177	180	43
44	135	138	141	144	147	150	153	156	159	162	165	168	171	174	177	180	183	44
45	138	141	144	147	150	153	156	159	162	165	168	171	174	177	180	183	186	45
46	141	144	147	150	153	156	159	162	165	168	171	174	177	180	183	186	189	46
47	144	147	150	153	156	159	162	165	168	171	174	177	180	183	186	189	192	47
48	147	150	153	156	159	162	165	168	171	174	177	180	183	186	189	192	195	48
49	150	153	156	159	162	165	168	171	174	177	180	183	186	189	192	195	198	49
50	153	156	159	162	165	168	171	174	177	180	183	186	189	192	195	198	201	50
Ft.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	Ft.

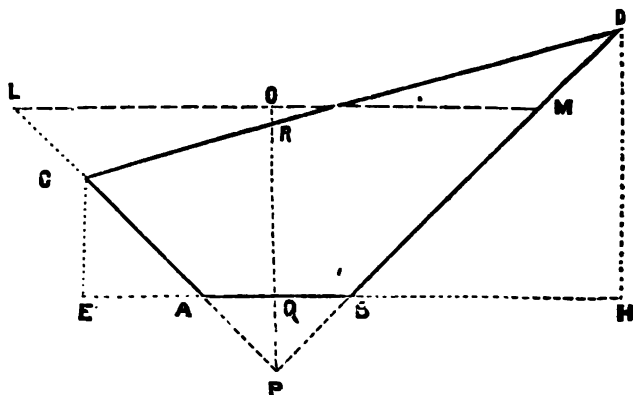
TABLE c.—For calculating Prismoids.

Pt.	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	Pt.
0	54	57	60	63	66	69	72	75	78	81	84	87	90	93	96	99	102	0
1	57	60	63	66	69	72	75	78	81	84	87	90	93	96	99	102	105	1
2	60	63	66	69	72	75	78	81	84	87	90	93	96	99	102	105	108	2
3	63	66	69	72	75	78	81	84	87	90	93	96	99	102	105	108	111	3
4	66	69	72	75	78	81	84	87	90	93	96	99	102	105	108	111	114	4
5	69	72	75	78	81	84	87	90	93	96	99	102	105	108	111	114	117	5
6	72	75	78	81	84	87	90	93	96	99	102	105	108	111	114	117	120	6
7	75	78	81	84	87	90	93	96	99	102	105	108	111	114	117	120	123	7
8	78	81	84	87	90	93	96	99	102	105	108	111	114	117	120	123	126	8
9	81	84	87	90	93	96	99	102	105	108	111	114	117	120	123	126	129	9
10	84	87	90	93	96	99	102	105	108	111	114	117	120	123	126	129	132	10
11	87	90	93	96	99	102	105	108	111	114	117	120	123	126	129	132	135	11
12	90	93	96	99	102	105	108	111	114	117	120	123	126	129	132	135	138	12
13	93	96	99	102	105	108	111	114	117	120	123	126	129	132	135	138	141	13
14	96	99	102	105	108	111	114	117	120	123	126	129	132	135	138	141	144	14
15	99	102	105	108	111	114	117	120	123	126	129	132	135	138	141	144	147	15
16	102	105	108	111	114	117	120	123	126	129	132	135	138	141	144	147	150	16
17	105	108	111	114	117	120	123	126	129	132	135	138	141	144	147	150	153	17
18	108	111	114	117	120	123	126	129	132	135	138	141	144	147	150	153	156	18
19	111	114	117	120	123	126	129	132	135	138	141	144	147	150	153	156	159	19
20	114	117	120	123	126	129	132	135	138	141	144	147	150	153	156	159	162	20
21	117	120	123	126	129	132	135	138	141	144	147	150	153	156	159	162	165	21
22	120	123	126	129	132	135	138	141	144	147	150	153	156	159	162	165	168	22
23	123	126	129	132	135	138	141	144	147	150	153	156	159	162	165	168	171	23
24	126	129	132	135	138	141	144	147	150	153	156	159	162	165	168	171	174	24
25	129	132	135	138	141	144	147	150	153	156	159	162	165	168	171	174	177	25
26	132	135	138	141	144	147	150	153	156	159	162	165	168	171	174	177	180	26
27	135	138	141	144	147	150	153	156	159	162	165	168	171	174	177	180	183	27
28	138	141	144	147	150	153	156	159	162	165	168	171	174	177	180	183	186	28
29	141	144	147	150	153	156	159	162	165	168	171	174	177	180	183	186	189	29
30	144	147	150	153	156	159	162	165	168	171	174	177	180	183	186	189	192	30
31	147	150	153	156	159	162	165	168	171	174	177	180	183	186	189	192	195	31
32	150	153	156	159	162	165	168	171	174	177	180	183	186	189	192	195	198	32
33	153	156	159	162	165	168	171	174	177	180	183	186	189	192	195	198	201	33
34	156	159	162	165	168	171	174	177	180	183	186	189	192	195	198	201	204	34
35	159	162	165	168	171	174	177	180	183	186	189	192	195	198	201	204	207	35
36	162	165	168	171	174	177	180	183	186	189	192	195	198	201	204	207	210	36
37	165	168	171	174	177	180	183	186	189	192	195	198	201	204	207	210	213	37
38	168	171	174	177	180	183	186	189	192	195	198	201	204	207	210	213	216	38
39	171	174	177	180	183	186	189	192	195	198	201	204	207	210	213	216	219	39
40	174	177	180	183	186	189	192	195	198	201	204	207	210	213	216	219	222	40
41	177	180	183	186	189	192	195	198	201	204	207	210	213	216	219	222	225	41
42	180	183	186	189	192	195	198	201	204	207	210	213	216	219	222	225	228	42
43	183	186	189	192	195	198	201	204	207	210	213	216	219	222	225	228	231	43
44	186	189	192	195	198	201	204	207	210	213	216	219	222	225	228	231	234	44
45	189	192	195	198	201	204	207	210	213	216	219	222	225	228	231	234	237	45
46	192	195	198	201	204	207	210	213	216	219	222	225	228	231	234	237	240	46
47	195	198	201	204	207	210	213	216	219	222	225	228	231	234	237	240	243	47
48	198	201	204	207	210	213	216	219	222	225	228	231	234	237	240	243	246	48
49	201	204	207	210	213	216	219	222	225	228	231	234	237	240	243	246	249	49
50	204	207	210	213	216	219	222	225	228	231	234	237	240	243	246	249	252	50
Pt.	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	Pt.

TABLE c.—For calculating Prismoids.

Ft.	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	Ft.
0	105	108	111	114	117	120	123	126	129	132	135	138	141	144	147	150	0
1	108	111	114	117	120	123	126	129	132	135	138	141	144	147	150	153	1
2	111	114	117	120	123	126	129	132	135	138	141	144	147	150	153	156	2
3	114	117	120	123	126	129	132	135	138	141	144	147	150	153	156	159	3
4	117	120	123	126	129	132	135	138	141	144	147	150	153	156	159	162	4
5	120	123	126	129	132	135	138	141	144	147	150	153	156	159	162	165	5
6	123	126	129	132	135	138	141	144	147	150	153	156	159	162	165	168	6
7	126	129	132	135	138	141	144	147	150	153	156	159	162	165	168	171	7
8	129	132	135	138	141	144	147	150	153	156	159	162	165	168	171	174	8
9	132	135	138	141	144	147	150	153	156	159	162	165	168	171	174	177	9
10	135	138	141	144	147	150	153	156	159	162	165	168	171	174	177	180	10
11	138	141	144	147	150	153	156	159	162	165	168	171	174	177	180	183	11
12	141	144	147	150	153	156	159	162	165	168	171	174	177	180	183	186	12
13	144	147	150	153	156	159	162	165	168	171	174	177	180	183	186	189	13
14	147	150	153	156	159	162	165	168	171	174	177	180	183	186	189	192	14
15	150	153	156	159	162	165	168	171	174	177	180	183	186	189	192	195	15
16	153	156	159	162	165	168	171	174	177	180	183	186	189	192	195	198	16
17	156	159	162	165	168	171	174	177	180	183	186	189	192	195	198	201	17
18	159	162	165	168	171	174	177	180	183	186	189	192	195	198	201	204	18
19	162	165	168	171	174	177	180	183	186	189	192	195	198	201	204	207	19
20	165	168	171	174	177	180	183	186	189	192	195	198	201	204	207	210	20
21	168	171	174	177	180	183	186	189	192	195	198	201	204	207	210	213	21
22	171	174	177	180	183	186	189	192	195	198	201	204	207	210	213	216	22
23	174	177	180	183	186	189	192	195	198	201	204	207	210	213	216	219	23
24	177	180	183	186	189	192	195	198	201	204	207	210	213	216	219	222	24
25	180	183	186	189	192	195	198	201	204	207	210	213	216	219	222	225	25
26	183	186	189	192	195	198	201	204	207	210	213	216	219	222	225	228	26
27	186	189	192	195	198	201	204	207	210	213	216	219	222	225	228	231	27
28	189	192	195	198	201	204	207	210	213	216	219	222	225	228	231	234	28
29	192	195	198	201	204	207	210	213	216	219	222	225	228	231	234	237	29
30	195	198	201	204	207	210	213	216	219	222	225	228	231	234	237	240	30
31	198	201	204	207	210	213	216	219	222	225	228	231	234	237	240	243	31
32	201	204	207	210	213	216	219	222	225	228	231	234	237	240	243	246	32
33	204	207	210	213	216	219	222	225	228	231	234	237	240	243	246	249	33
34	207	210	213	216	219	222	225	228	231	234	237	240	243	246	249	252	34
35	210	213	216	219	222	225	228	231	234	237	240	243	246	249	252	255	35
36	213	216	219	222	225	228	231	234	237	240	243	246	249	252	255	258	36
37	217	219	222	225	228	231	234	237	240	243	246	249	252	255	258	261	37
38	219	222	225	228	231	234	237	240	243	246	249	252	255	258	261	264	38
39	222	225	228	231	234	237	240	243	246	249	252	255	258	261	264	267	39
40	225	228	231	234	237	240	243	246	249	252	255	258	261	264	267	270	40
41	228	231	234	237	240	243	246	249	252	255	258	261	264	267	270	273	41
42	231	234	237	240	243	246	249	252	255	258	261	264	267	270	273	276	42
43	234	237	240	243	246	249	252	255	258	261	264	267	270	273	276	279	43
44	237	240	243	246	249	252	255	258	261	264	267	270	273	276	279	282	44
45	240	243	246	249	252	255	258	261	264	267	270	273	276	279	282	285	45
46	243	246	249	252	255	258	261	264	267	270	273	276	279	282	285	288	46
47	246	249	252	255	258	261	264	267	270	273	276	279	282	285	288	291	47
48	249	252	255	258	261	264	267	270	273	276	279	282	285	288	291	294	48
49	252	255	258	261	264	267	270	273	276	279	282	285	288	291	294	297	49
50	255	258	261	264	267	270	273	276	279	282	285	288	291	294	297	300	50
Ft.	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	Ft.

Application. In using either of the foregoing tables, a , b and c , we must use the mean heights of the end sections, as QO in the annexed figure.



Q is the centre of the road bed. R is the centre stump. $CE = d =$ lesser height. $DH = D =$ greater height. P is where the slopes meet on the other side of the road bed.

We find the end area of the section by the formula in sec. 322, where $A = \text{area} = Dd + \frac{D+d}{2} \cdot b$. And the mean height, x , (from formula in sec. 323,) $= \frac{\sqrt{(4Ar + b^2)} - b}{2r}$.

The following tabular form will show how to find the contents of any section or number of sections from Tables b and c .

By Tables b and c . The annexed table shows our method of using Sir John McNeil's tables 58 and 59; which we use as tables b and c . Opposite 47 and under 18 in table b , we find 5978 which we put in column IV.	I	II	III	IV	V	VI
	End Area's	Mean Hgt. in ft.	L'gh in ft.	From Table b .	From Table c .	Sum.
4100	47.08			5978.	180.	
725	18.54	120		17.28	0.24	
				79.92	1.62	
				6075.2	181.86	13,349.6
				$r = 1$	$b = 40$	6,1728
				6075.2	7274.4	82,40451
A	a	L	m	s	t	120
B	b	L	n	a	v	9888.58
C	c	L	o	s	a	content.
			r	b		
			$r s$	$b A$		$rS + bQ$
						L
						$rSL + bQL$
						6,1728

Contents in Cubic Yards.

Find the vertical difference between 47 and 18, and 48 and 18 to be 216, which multiplied by the decimal .08, gives 17.28, which put in col. IV. Find the horizontal difference between 47 and 18, and 47 and 14 to be 148, which multiplied by 0.54 gives 79.92, which is also put in col. IV. In like manner we take from table c , tabular numbers similar to those in col. IV and put them in col. V. Now add the results in col. IV and V, multiply the sum in col. IV by the base b , and that in col. V by the ratio of the slopes, add the two products together, cut off three figures, to the right for decimals, multiply the result by the constant multiplier 6.1728, the product will be the content in cubic yards. When there are several sections having the same length, base, and ratio of slopes, as A , B , C , etc., put their end areas in col. I. Their mean heights in col. II, their lengths in col. III, their tabular numbers from tables b and c , in col. IV and V as above, where S and Q are the sums of columns IV and V. rS is the product of col. IV $\times$ by the ratio of the slopes and $bQ =$ col. V $\times$ by the base. From their sum, cut off 3 places to the right and proceed as in the above example.

SPHERICAL TRIGONOMETRY.

845. A *Spherical Triangle* is formed by the intersection of three great circles on the surface of a sphere, the planes of each circle passing through the centre of the sphere.

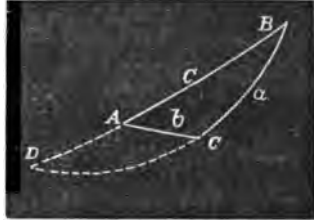
846. A *Spherical Angle* is that formed by the intersection of the planes of the great circles, and is the measure of the angles formed by the great circles.

847. The sides and angles of a spherical triangle have no affinity to those of a plane triangle, for in a spherical triangle, the sides and angles are of the same species, each being measured on the arc of a great circle.

848. As in plane trigonometry, we have isocles equilateral oblique-angled and right-angled triangles.

849. A right-angled triangle is formed by the intersection of three great circles, two of which intersect one another at right angles, that is one great circle must pass through the centre of the sphere and the pole of another of the three circles.

Let the side of the triangle be produced to meet as at D in the annexed figure, the arc B A D and B C D are semi-circles, therefore, the side A D is the supplement of A B, and C D is the supplement of B C and the $\triangle A D C$ is the supplementary or *polar* triangle to ABC.



850. Any two sides of a $\triangle$ is greater than the third. Any side is less than the sum of the other two sides, but greater than their difference.

851. If tangents be drawn from the point B to the arcs B A and B C the angle thus formed will be the measure of the spherical angle A B C.

852. The greater angle is subtended by the greater side.

A right-angled $\triangle$ has one angle of 90° .

A quadrantal $\triangle$ has one side of 90° .

An oblique-angled $\triangle$ has no side or angle $= 90^\circ$.

The three sides of a spherical $\triangle$ are together less than 360°

The three angles are together greater than two, and less than six right-angles.

853. The angles of one triangle if taken from 180° will give the sides of a new supplementary or polar triangle.

If the sides of a $\triangle$ be taken from 180° , it gives the angles of a polar $\triangle$.

854. If the sum of any two sides be either equal, greater or less than 180° , the sum of the opposite angles will be equal, greater or less than 180° .

855. A right-angled spherical $\triangle$ may have either,

One right angle and two acute angles.

One right angle and two obtuse angles.

One obtuse angle and two right angles.

One acute angle and two right angles. Or

Three right angles.

356. If one of the sides of the $\triangle$ be 90° , one of the other sides will be 90° , and then each side will be equal to its opposite $<$. And if any two of its sides are each $=$ to 90° , then the third side is $=$ to 90° .

357. If two of the angles are each 90° , the opposite sides are each equal to 90° .

358. If the two legs of a right-angled $\triangle$ be both acute or both obtuse, the hypotenuse will be less than a quadrant. If one be acute and the other obtuse, that is when they are of different species, the hypotenuse is greater than a quadrant.

359. In any right angled spherical $\triangle$ each of the oblique angles is of the same species as its opposite side, and the sides containing the right angle are of the same species as their opposite angles.

360. If the hypotenuse be less than 90° , the legs are of the same species as their adjacent angles, but if the hypotenuse be greater, then the legs and adjacent angles are of different species.

361. In any spherical $\triangle$ the sines of the angles are to one another as the sines of their opposite sides.

362. SOLUTION OF RIGHT-ANGLED SPHERICAL TRIANGLES.

$$\text{Sin. } a = \text{sin. } c \cdot \text{sin. } A, \text{ Equat. A.}$$

$$\text{tan. } a = \text{tan. } c \cdot \text{cos. } B$$

$$= \text{tan. } A \cdot \text{sin. } B, \text{ Equation B.}$$

$$\text{Sin. } b = \text{sin. } c \cdot \text{sin. } B = \frac{\text{tan. } a}{\text{tan. } A}.$$

$$\text{Equation C.}$$

$$\text{tan. } b = \text{tan. } b \cdot \text{cos. } A$$

$$= \text{tan. } B \cdot \text{sin. } A, \text{ Equation D.}$$

$$\text{Cos. } A = \text{cos. } a \cdot \text{sin. } B,$$

$$\text{Equation E.}$$

$$\text{Cos. } B = \text{cos. } b \cdot \text{sin. } A,$$

$$\text{Equation F.}$$

$$\text{Sin. } B = \frac{\text{cos. } A}{\text{cos. } a}.$$

$$\text{Equation G.}$$

$$\text{Cos. } c = \text{cos. } a \cdot \text{cos. } b,$$

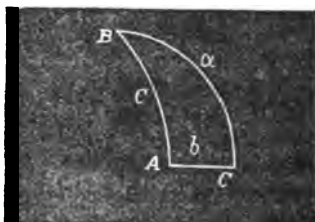
$$\text{Equation H.}$$

$$\text{Cos. } c = \text{cot. } A \cdot \text{cot. } B,$$

$$\text{Equation I.}$$

$$\text{Sin. } c = \frac{\text{sin. } a}{\text{sin. } A}. \text{ Here } c = \text{hypotenuse.}$$

$$\text{Equation K.}$$



363. NAPIER'S RULES FOR THE CIRCULAR PARTS.

Lord Napier has given the following simple rules for solving right-angled spherical triangles.

The sine of the middle part = product of the adjacent parts.

The sine of the middle part = product of the cosines of the opposite parts.

In applying Napier's analogies, we take the complements of the hypotenuse and of the other angles, and reject the right angle. We will arrange Napier's rules as follows, where co. = complement of the angles or hypotenuse.

Sine of the middle part,	Is equal to the product of the tangents of the adjacent parts.	Is equal to the product of the cosines of the opposite parts.
Sine comp. A.	$\text{tan. co. } c \cdot \text{tan. } b.$	$\text{Cos. co. } B \cdot \text{cos. } a.$
Sin. comp. c.	$\text{tan. co. } A \cdot \text{tan. co. } B.$	$\text{Cos. } b \cdot \text{cos. } a.$
Sin. comp. B.	$\text{tan. comp. } c \cdot \text{tan. } a.$	$\text{Cos. } b \cdot \text{cos. } A.$
Sin. a.	$\text{tan. comp. } B \cdot \text{tan. } b.$	$\text{Cos. comp. } A \cdot \text{cos. com. } c$
Sin. b.	$\text{tan. co. } A \cdot \text{tan. } a.$	$\text{Cos. com. } c \cdot \text{cos. com. } B$

It is easy to remember that *adjacent* requires *tangent*, and *opposite* requires *cosine*, from the letter *a* being found in the first syllable of *adjacent* and *tangent*, and *o* being in the first syllable of *opposite* and *cosine*.

Example 1. Given the $\angle A = 23^\circ 28'$ and $c = 145^\circ$ to find the sides *a* and *b*, and the angle *B*.

Comp. $c = \text{comp. } 180 - 145 = 35$ and $55^\circ = \text{comp.}$

Comp. $A = 90^\circ - 23^\circ 28' = 66^\circ 32'$.

Sin. $a = \cos. 55^\circ \times \cos. 66^\circ 32' = 0.57858 \times 0.39822$ and $a = 18^\circ 12' 18'' = \text{natural sine of } 0.22841$.

Having *a* and comp. of *c*, we find $B = 50^\circ 81'$ and $b = 24^\circ 24'$.

Example 2. Given $b = 46^\circ 18' 23''$. $A = 84^\circ 27' 29''$ to find $\angle B$. Answer, $B = 66^\circ 59' 25''$.

Example 3. Given $a = 48^\circ 24' 16''$, and $b = 59^\circ 38' 27''$. We find $c = 79^\circ 23' 42''$.

Example 4. Given $a = 116^\circ 30' 43''$ and $b = 29^\circ 41' 32''$. We find $A = 108^\circ 52' 48''$.

Example 5. Given $b = 29^\circ 12' 50''$, and $\angle B = 37^\circ 26' 21''$. We find $a = 46^\circ 55' 2''$ or $a = 138^\circ 4' 58''$.

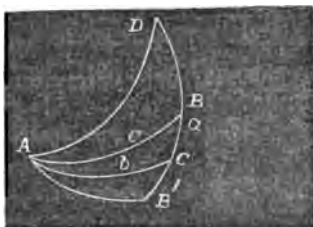
NOTE. We can use either natural or logarithmic numbers.

364.

QUADRANTAL SPHERICAL TRIANGLES.

Let $\angle A D = 90^\circ$, produce *D B* to *C* making $\angle D C = \angle A D = 90^\circ$; therefore the arc *A C* is the measure of the angle $\angle A D B$.

If the $\angle D A B$ is less than 90° , then *D B* is less than 90° . But if the $\angle D A B$ is greater than 90° , then the side *D B* is greater than 90° .



Example. Let the $\angle D = 42^\circ 12' = \text{Arc } A C$ in the triangle $A B C$, and let the $\angle D A B = 54^\circ 48'$, then $90^\circ - 54^\circ 18' = 35^\circ 17' = \angle B A C = \angle A$ in the $\triangle B A C$.

By Napier's analogies, $\sin. \text{comp. } A \times \text{radius} = \tan. b \times \tan. \text{comp. } c$.
i. e., $\text{rad. cos. } A = \tan. b . \cot. c$, and $\cot. c = \frac{\text{Rad. cos. } A}{\tan. b} =$

$\frac{\text{Rad. cos. } 54^\circ 48'}{\tan. 42^\circ 12'} = 48^\circ 0' 9'' = c$. And $\text{Sin. comp. } B = \cos. B = \cos. b . \cos. A = \cos. b . \sin. A$, and having *b* and *A* in the above, we have $\cos. B = \cos. 42^\circ 12' \times \sin. 48^\circ 0' 9'' = 64^\circ 39' 55'' = B$.

Again, $\sin. \text{comp. } B = \tan. a . \tan. \text{comp. } c$ i. e. $\cos. B = \tan. a . \cot. c$.
 $\text{Tan. } a = \frac{\cos. B}{\cot. c} = \frac{\cos. 64^\circ 39' 55''}{\cot. 48^\circ 0' 9''} = 25^\circ 25' 20'' = \text{value of } a$.

$\therefore 90^\circ - 25^\circ 25' 20'' = 64^\circ 34' 40'' = \text{side } D B$.—*Young's Trigonometry.*

365.

OBLIQUE-ANGLED SPHERICAL TRIANGLES.

Oblique-angled triangles are divided into six cases by Thomson and other mathematicians.

- I. When the three sides are given, to find the angles.
- II. When the three angles are given, to find the sides.
- III. When the two sides and their contained angle are given.
- IV. When one side and the adjacent angles are given.
- V. When two angles and a side opposite to one of them.
- VI. When two sides and an angle opposite to one of them.

The following formulas may be solved by logarithms or natural numbers.

866. The following is the fundamental formula, and is applicable to all spherical triangles. Puissant in his *Geodesie*, vol. I, p. 58, says: "Il serait aise de prouver que l'equation est le fondement unique de toute la Trigonometrie spherique."

$$\cos. a = \cos. b \cdot \cos. c + \sin. b \cdot \sin. c \cdot \cos. A.$$

$$\cos. b = \cos. a \cdot \cos. c + \sin. a \cdot \sin. c \cdot \cos. B.$$

$$\cos. c = \cos. a \cdot \cos. b + \sin. a \cdot \sin. b \cdot \cos. C.$$

From these we can find the following equations:

$$\cos. A = \frac{\cos. a - \cos. b \cdot \cos. c}{\sin. b \cdot \sin. c} \quad \text{Equation A.}$$

$$\cos. B = \frac{\cos. b - \cos. a \cdot \cos. c}{\sin. a \cdot \sin. c} \quad \text{Equation B.}$$

$$\cos. C = \frac{\cos. c - \cos. a \cdot \cos. b}{\sin. a \cdot \sin. b} \quad \text{Equation C.}$$

If we have a, b and A given, then side a : sine of $\angle A$: : side b to the sine of $\angle B$.

The following formulas are applicable to natural numbers and logarithms. The symbol $\frac{1}{2}$ = square root.

867. CASE I. Having the three sides given, let s = half the sum of the sides.

$$\text{Sine } \frac{1}{2} A = \left(\frac{\sin. (s-b) \sin. (s-c)}{\sin. b \cdot \sin. c} \right)^{\frac{1}{2}} \quad \text{Equation A.}$$

$$\text{Sin. } \frac{1}{2} B = \left(\frac{\sin. (s-a) \sin. (s-c)}{\sin. a \cdot \sin. c} \right)^{\frac{1}{2}} \quad \text{Equation B.}$$

$$\text{Sine } \frac{1}{2} C = \left(\frac{\sin. (s-a) \sin. (s-b)}{\sin. a \cdot \sin. b} \right)^{\frac{1}{2}} \quad \text{Equation C.}$$

$$\cos. \frac{1}{2} A = \left(\frac{\sin. s \cdot \sin. (s-a)}{\sin. b \cdot \sin. c} \right)^{\frac{1}{2}} \quad \text{Equation D.}$$

$$\cos. \frac{1}{2} B = \left(\frac{\sin. s \cdot \sin. (s-b)}{\sin. a \cdot \sin. c} \right)^{\frac{1}{2}} \quad \text{Equation E.}$$

$$\cos. \frac{1}{2} C = \left(\frac{\sin. s \cdot \sin. (s-c)}{\sin. a \cdot \sin. b} \right)^{\frac{1}{2}} \quad \text{Equation F.}$$

$$\text{Tan. } \frac{1}{2} A = \left(\frac{\sin. (s-b) \cdot \sin. (s-c)}{\sin. s \cdot \sin. (s-a)} \right)^{\frac{1}{2}} \quad \text{Equation G.}$$

$$\text{Tan. } \frac{1}{2} B = \left(\frac{\sin. (s-a) \cdot \sin. (s-c)}{\sin. s \cdot \sin. (s-b)} \right)^{\frac{1}{2}} \quad \text{Equation H.}$$

$$\text{Tan. } \frac{1}{2} C = \left(\frac{\sin. (s-a) \cdot \sin. (s-b)}{\sin. s \cdot \sin. (s-c)} \right)^{\frac{1}{2}} \quad \text{Equation I.}$$

368. CASE II. Having the three angles given, to find the sides.

$$\text{Sine } \frac{1}{2} a = \left(\frac{-\cos. S \cdot \cos. (S - A)}{\sin. B \cdot \sin. C} \right)^{\frac{1}{2}} \quad \text{Equation A.}$$

$$\text{Sine } \frac{1}{2} b = \left(\frac{-\cos. S \cdot \cos. (S - B)}{\sin. A \cdot \sin. C} \right)^{\frac{1}{2}} \quad \text{Equation B.}$$

$$\text{Sine } \frac{1}{2} c = \left(\frac{-\cos. S \cdot \cos. (S - C)}{\sin. A \cdot \sin. B} \right)^{\frac{1}{2}} \quad \text{Equation C.}$$

$$\text{Cos. } \frac{1}{2} a = \left(\frac{\cos. (S - B) \cdot \cos. (S - C)}{\sin. B \cdot \sin. C} \right)^{\frac{1}{2}} \quad \text{Equation D.}$$

$$\text{Cos. } \frac{1}{2} b = \left(\frac{\cos. (S - A) \cdot \cos. (S - C)}{\sin. A \cdot \sin. C} \right)^{\frac{1}{2}} \quad \text{Equation E.}$$

$$\text{Cos. } \frac{1}{2} c = \left(\frac{\cos. (S - A) \cdot \cos. (S - B)}{\sin. A \cdot \sin. B} \right)^{\frac{1}{2}} \quad \text{Equation F.}$$

$$\text{Tan. } \frac{1}{2} a = \left(\frac{-\cos. S \cdot \cos. (S - A)}{\cos. (S - B) \cos. (S - C)} \right)^{\frac{1}{2}} \quad \text{Equation G.}$$

$$\text{Tan. } \frac{1}{2} b = \left(\frac{-\cos. S \cdot \cos. (S - B)}{\cos. (S - A) \cos. (S - C)} \right)^{\frac{1}{2}} \quad \text{Equation H.}$$

$$\text{Tan. } \frac{1}{2} c = \left(\frac{-\cos. S \cdot \cos. (S - C)}{\cos. (S - A) \cos. (S - B)} \right)^{\frac{1}{2}} \quad \text{Equation I.}$$

369. CASE III. When two sides and the angle contained by them are given to find the remaining parts.

Let us suppose the two sides a and b and the contained $\angle = C$.

By Napier's analogies,

$$\text{Cos. } \frac{1}{2} (a + b) : \cos. \frac{1}{2} (a \infty b) :: \cot. \frac{1}{2} C : \tan. \frac{1}{2} (A + B) \quad \text{Equat. J.}$$

$$\text{Sin. } \frac{1}{2} (a + b) : \sin. \frac{1}{2} (a \infty b) :: \cot. \frac{1}{2} C : \tan. \frac{1}{2} (A \infty B) \quad \text{Equat. K.}$$

Tan. of half the sum of the unknown angles =

$$\frac{\cos. \frac{1}{2} (a \infty b) \cdot \cot. \frac{1}{2} C}{\cos. \frac{1}{2} (a + b)} \quad \text{Equation L.}$$

$$\text{Tan. of half the difference of same} = \frac{\sin. \frac{1}{2} (a \infty b) \cdot \cot. \frac{1}{2} C}{\sin. \frac{1}{2} (a + b)}$$

Equation M.

∞ signifies the difference between a and b .

Having determined half the sum and half the difference of the angles, we find the angles A and B .

Then the side c may be found from (Equation F.)

$\sin. B : \sin. b :: \sin. C : \sin. c$, from which c is found.

370. Napier's analogies for finding the side from the angle.

$$\cos. (A + B) : \cos. (A \infty B) : \tan. \frac{1}{2} c : \tan. \frac{1}{2} (a + b) \quad \text{Equation N.}$$

$$\text{or } \sin. (A + B) : \sin. (A \infty B) : \tan. \frac{1}{2} c : \tan. \frac{1}{2} (a - b) \quad \text{Equation O.}$$

$$\text{or } \tan. \frac{1}{2} c = \frac{\cos. (A + B) \cdot \tan. \frac{1}{2} (a + b)}{\cos. (A \infty B)} \quad \text{Equation P.}$$

$$\text{or } \tan. \frac{1}{2} c = \frac{\sin. (A + B) \cdot \tan. \frac{1}{2} (a - b)}{(\sin. A \infty B)} \quad \text{Equation Q.}$$

The value may be found from the general equation.

871. CASE IV. When one side and the adjacent angles are given.

Given A and B and the adjacent side c.

$$\cos. \frac{1}{2} (A + B) : \cos. (A \infty B) : \tan. \frac{1}{2} c : \tan. \frac{1}{2} (a + b)$$

$$\sin. \frac{1}{2} (A + B) : \sin. A (\infty B) : \tan. \frac{1}{2} c : \tan. \frac{1}{2} (a - b)$$

From these we have the sides a and b.

$$\tan. \frac{1}{2} (a + b) = \frac{\cos. (A \infty B) \cdot \tan. \frac{1}{2} c}{\cos. \frac{1}{2} (A + B)} \quad \text{Equation R.}$$

$$\tan. \frac{1}{2} (a - b) = \frac{\sin. \frac{1}{2} (A \infty B) \cdot \tan. \frac{1}{2} c}{\sin. \frac{1}{2} (A + B)} \quad \text{Equation S.}$$

And to find $\angle C$, we have

$$\cot. \frac{1}{2} C = \frac{\cos. \frac{1}{2} (a + b) \cdot \tan. \frac{1}{2} (A + B)}{\cos. \frac{1}{2} (a \infty b)} \quad \text{Equation T.}$$

$$\cot. \frac{1}{2} C = \frac{\sin. \frac{1}{2} (a + b) \cdot \tan. \frac{1}{2} (A - B)}{\sin. \frac{1}{2} (a \infty b)} \quad \text{Equation U.}$$

872. CASE V. When two sides and an angle opposite to one of them are given, as, a, b and the angle A.

$$\sin. a : \sin. b :: \sin. A : \sin. B = \frac{\sin. b \cdot \sin. A}{\sin. a} \therefore \text{we have B.}$$

To find C and c, as we have now a, b and A and B.

$$\text{We have from (Eq. T) } \cot. \frac{1}{2} C = \frac{\cos. \frac{1}{2} (a + b) \cdot \tan. \frac{1}{2} (A + B)}{\cos. \frac{1}{2} (a \infty b)} \quad (V)$$

and from (R) we have the value of c, for

$$\tan. \frac{1}{2} c = \frac{\cos. \frac{1}{2} (A + B) \cdot \tan. \frac{1}{2} (a + b)}{\cos. \frac{1}{2} (A \infty B)} \cdot (W) \quad \text{Having the angles}$$

A, B and C, and the sides a and b, we can find c, because $\sin. B : \sin. C :: \sin. b : \sin. c$.

NOTE. As the value determined by proportion admits sometimes of a double value, because two arcs have the same sine. It is therefore better to use Napier's analogies.

873. CASE VI. When two angles A and B and the side a opposite to one of them are given to find the other parts.

$$\sin. A : \sin. B :: \sin. a : \sin. b \therefore \text{we have side b.}$$

By Eq. (V) we find the $\angle C$.

By Eq. (W) we find c, which may be found by proportion.

NOTE. If cosine A is less than cosine B, B and b will be of the same species, (i. e.,) each must be more or less than 90° in the above proportion. If $\cos. B$ is less than $\cos. A$, then b may have two values.

874. Examples with their answers for each case.

CASE I. Ex. 1. Given $c = 79^\circ 17' 14''$, $b = 58^\circ$ and $a = 110^\circ$ to find A.

Answer. $A = 121^\circ 54' 56''$.

Ex. 2. Given $a = 100^\circ$, $b = 87^\circ 18'$, and $c = 62^\circ 46'$.

Answer. $A = 176^\circ 15' 40''$.

Ex. 3. Given $a = 61^\circ 32' 12''$, $b = 83^\circ 19' 42''$, $c = 23^\circ 27' 46''$ to find A.

Answer. $A = 20^\circ 39' 48''$.

Ex. 4. Given $a = 46^\circ$, $b = 72^\circ$, and $c = 68^\circ$.

Answer. $A = 48^\circ 58'$, $B = 85^\circ 48'$, $C = 76^\circ 28'$.

CASE II. Ex. 1. Given $A = 90^\circ$, $B = 95^\circ 6'$, $C = 71^\circ 36'$, to find the sides.

Answer. $a = 91^\circ 42'$, $b = 95^\circ 22' 30''$, $c = 71^\circ 31' 30''$.

Ex. 2. $A = 89^\circ$, $B = 5^\circ$, $C = 88^\circ$.

Answer. $a = 53^\circ 10'$, $b = 4^\circ$, $c = 53^\circ 8'$.

Ex. 3. $A = 108^\circ 59' 57''$, $B = 46^\circ 18' 7''$, $C = 36^\circ 7' 52''$.

Answer. $a = 42^\circ 8' 48''$.

CASE III. Ex. 1. Given $a = 88^\circ 30'$, $b = 70^\circ$, and $C = 31^\circ 34' 26''$.

Answer. $B = 130^\circ 3' 11''$, $A = 30^\circ 28' 11''$.

Ex. 2. Given $a = 78^\circ 41'$, $b = 158^\circ 30'$, $C = 140^\circ 22'$.

Answer. $A = 133^\circ 15'$, $B = 160^\circ 39'$, $c = 120^\circ 50'$.

Ex. 3. Given $a = 18$, $c = 9^\circ$, $B = 176^\circ$ to find other parts.

Answer. $A = 2^\circ 24'$, $C = 1^\circ 40'$.

CASE IV. Ex. 1. Given $a = 71^\circ 45'$, $B = 104^\circ 5'$, $C = 82^\circ 18'$ to find etc.

Answer. $A = 70^\circ 31'$, $b = 102^\circ 17'$, $c = 86^\circ 41'$.

Ex. 2. $A = 30^\circ 28' 11''$, $B = 130^\circ 3' 11''$, $c = 40^\circ$ to find etc.

Answer. $a = 38^\circ 30'$, $b = 70^\circ$, $C = 31^\circ 34' 26''$.

Ex. 3. Given $B = 125^\circ 37'$, $C = 98^\circ 44'$, $a = 45^\circ 54'$ to find etc.

Answer. $A = 61^\circ 55'$, $b = 138^\circ 34'$, $c = 126^\circ 28'$.

CASE V. Ex. 1. $a = 136^\circ 25'$, $c = 125^\circ 40'$, $C = 100^\circ$ to find etc.

Answer. $A = 123^\circ 19'$, $B = 62^\circ 6'$, $b = 46^\circ 48'$.

Ex. 2. Given $a = 84^\circ 14' 29''$, $b = 44^\circ 18' 45''$, $A = 130^\circ 5' 22''$ to

Answer. $B = 32^\circ 26' 7''$, $C = 36^\circ 45' 28''$, $c = 51^\circ 6' 12''$.

Ex. 3. Given $a = 54^\circ$, $c = 22^\circ$, $C = 12^\circ$ to find etc.

Answer. $b = 73^\circ 16'$, $B = 147^\circ 53'$, $A = 26^\circ 41'$, or

$b = 33^\circ 32'$, $B = 17^\circ 51'$, $A = 153^\circ 19'$.—*Peirce's Trigonometry.*

CASE VI. Ex. 1. Given $A = 103^\circ 16'$, $B = 76^\circ 44'$, $b = 30^\circ 7'$ to find etc.

Answer. $a = 149^\circ 53'$, $c = 164^\circ 50'$, $C = 149^\circ 30'$.—*Thomson.*

Ex. 2. Given $A = 104^\circ$, $C = 95^\circ$, $a = 138^\circ$ to find etc.

Answer. $b = 17^\circ 21'$, $c = 136^\circ 36'$, $B = 25^\circ 37'$, or

$b = 171^\circ 37'$, $c = 43^\circ 24'$, $B = 167^\circ 47'$.—*Peirce.*

Ex. 3. Given $A = 17^\circ 46' 16\frac{1}{2}''$, $B = 151^\circ 43' 52''$, $a = 37^\circ 48'$ to find etc.

Answer. $b = 180^\circ$, $c = 74^\circ 30'$.—*Young's Trigonometry.*

SPHERICAL ASTRONOMY

375. *Meridians*, are great circles passing through the celestial poles and the place of the observer, and are perpendicular to the equinoctial. They are called hour lines, and circles of right ascension.

Altitude of a Celestial Object, is its height above the horizon, measured on the meridian or vertical circle.

Zenith Distance, is the complement of the altitude, or the altitude taken from 90° .

Azimuth or Vertical Circles, pass through the zenith and nadir, and cut the horizon at right angles.

Azimuth or Bearing of a celestial object, is the arc intercepted between the North and South points and a circle of altitude passing through the

place of the body, and is the same as the angle formed at the zenith by the intersection of the celestial meridian and circle of altitude.

Greatest Azimuth or Elongation of a celestial object, is that at which during a short time the azimuth or bearing appears to be stationary, and at which point the object moves rapidly in altitude, but appears stationary in azimuth. When the celestial object is at this point, it is the most favorable situation for determining the true time, and variation of the compass, and consequently the astronomical bearing of any line in surveying. See Table XXII.

Parallax, is the difference of the angles as taken from the surface and centre of the earth. It increases from the horizon to the zenith, and is to be always *added* to the observed altitude. (See Table XVIII.)

Dip, is the correction made for the height of the eye above the horizon when on water, and is always to be *subtracted*. When on land using an artificial horizon, half the observed altitude will be used. (See Table XVI.)

Refraction in altitude, is the difference between the apparent and true altitude, and is always to be *subtracted*. (See Table XVII.)

As the greatest effect of refraction is near the horizon, altitudes less than 26° ought to be avoided as much as possible.

Prime Vertical, is the azimuth circle cutting the East and West points.

Elevation of the Pole, is an arc of the meridian intercepted between the elevated pole and the horizon.

Declination, is that portion of its meridian between the equinoctial and centre of the object, and is either North or South as the celestial object is North or South of the equinoctial.

Polar distance, is the declination taken from 90° .

Right Ascension is the arc of the equinoctial between its meridian and the vernal equinox, and is reckoned eastward.

Latitude of a celestial object is an arc of celestial longitude between the object and the ecliptic, and is North or South latitude according as the object is situated with respect to the ecliptic between the first points of Ares and a circle of longitude passing through that point.

Mean Time, is that shown by a clock or chronometer, The mean day is 24 hours long.

Apparent Solar Days, are sometimes more or less than 24 hours.

Equation of Time, is the correction for changing mean time into apparent time and *visa versa*, and is given in the nautical almanac each year.

Sidereal Time. A sidereal day is the interval between two successive transits of the same star over the meridian, and is always of the same length; for all the fixed stars make their revolutions in equal time. The sidereal is shorter than the mean solar day by $8' 56\frac{1}{2}''$. This difference is owing to the sun's annual motion from *West* to *East*, by which he leaves the star as if it were behind him.

The star culminates $8' 56.5554''$ earlier every day than the time shown by the clock.

Civil Time, begins at midnight and runs to 12 or noon, and then from noon again 12 hours to midnight.

Astronomical or Solar Day, is the time between two successive transits of the sun's centre over the same meridian. It begins at noon and is

reckoned on 24 hours to the next noon, without regarding the civil time. This is always known as apparent time.

Nautical or Sea Day, begins 12 hours earlier than the astronomical.

Example. Civil time, April 8th, 12h. = Ast. 8d. 0h.

Example. Civil time, April 9th, 10h. = Ast. 8d. 22h.

If the civil time be after noon of the given day, it agrees with the astronomical; but when the time is before noon, add 12 hours to the civil time, and put the date one day back for the astronomical. The *nautical* or *sea day* is the same as the *civil time*, the noon of each is the beginning of the astronomical day.

376. To find at what time a heavenly body will culminate, or pass the meridian of a given place. (See 264ⁿ, p. 69.)

From the Nautical Almanac take the star's right ascension, also the R. A. of the mean sun, or sidereal time. From the star's R. A., increased by 24 if necessary, subtract the sidereal time above taken, the difference will be the approximate sidereal time of transit at the station. Apply the correction for the longitude in time to the approximate, by adding for E. longitude, and subtracting for W. longitude, the sum or difference will be the Greenwich date or time of transit. The correction is 0.6571s. for each degree.

Ex. At what time did α Scorpio (Antares) pass the meridian of Copenhagen, in longitude $12^{\circ} 85'$ E. of Greenwich, on the 20th August, 1846?

Star's R. A.	=	16 20 02
Sun's R. A. from sid. col.	=	9 58 45.5
Sidereal interval, at station,	=	6 26 16.5
Cor. for long. = $12^{\circ} 85' \times 0.6571s.$	=	+ 8.27
(Here 8m. 56.55s. divided by 860° = 0.6571s.)		6 26 24.77
This reduced to mean time,	=	6 25 21.46

The correction for long. is added in east and subtracted in west long.

NOTE. The sidereal columns of the Nautical Almanac, are found by adding or subtracting the equation of time, to or from the sun's R. A. at mean noon. What we have given in sec. 264ⁿ, will be sufficiently near for taking a meridian altitude.

377. LATITUDE BY OBSERVATION OF THE SUN.

RULE. Correct the sun's altitude of the limb for index error. Subtract the dip of the horizon. The difference = apparent altitude. From the apparent altitude, take the refraction corresponding to the altitude; the difference = true altitude of the observed limb. To this altitude, add or subtract the sun's semi-diameter, taken from p. 2 of the Nautical Almanac, the sum or difference = true altitude of the sun's centre. Add the sun's semi-diameter when the lower limb is observed, and subtract for the upper.

From 90, subtract the true altitude, the difference will be the zenith distance, which is north, if the zenith of the observer is north of the sun, and south, if his zenith is south of the sun.

From the Nautical Almanac, take the sun's declination, which correct for the longitude of the observer; then if the corrected declination and the zenith distance be of the same name, that is, both north or south, their sum will be the latitude; but if one is north and the other south, their difference will be the latitude.

Example. From Norie's Epitome of Navigation, August 30, 1851, in long. 129° W., the meridian altitude of the sun's lower limb was $57^{\circ} 18' 30''$, the observer's zenith north of the sun. Height of the eye above the horizon, 18 feet. Require the latitude.

Observed altitude,	$57^{\circ} 18' 30''$	
Dip of the horizon, correction from Table XVI,	—	$4' 08''$
Apparent altitude of sun's lower limb =	$57^{\circ} 14' 22''$	
Correction from Tables XVII and XVIII for refraction and parallax,	—	$82''$
True altitude of the sun's lower limb =	$57^{\circ} 13' 40''$	
Sun's semi-diameter from N. A. for the given day +	$15' 52''$	
True altitude of sun's centre =	$57^{\circ} 29' 42''$	
Zenith distance = 90° — alt.	=	$82^{\circ} 30' 18''$ N.
Declination on 30th August, is	N.	$9^{\circ} 08' 30''$
Declination on 31st August, is	N.	$8^{\circ} 46' 58''$
Decrease in 24 hours,		$0^{\circ} 21' 32''$
$360^{\circ} : 21' 32'' :: 129^{\circ} : 7' 48''$.		
Declination, 30th August, 1851,	=	N. $9^{\circ} 08' 30''$
Correction for W. longitude 129° =	—	$7' 48''$
Correct declination at station	=	$9^{\circ} 00' 42''$ N.
From above, the zenith distance	=	$82^{\circ} 30' 18''$ N.
North latitude	=	$41^{\circ} 31' 05''$

Norie gives $41^{\circ} 30' 58''$, because he does not use the table of declination in the N. A., but one which he considers approximately near.

As the Nautical Almanacs are within the reach of every one, and the expense is not more than one dollar, it is presumed that each of our readers will have one for every year.

Example 2. On the 17th November, 1848, in longitude 80° E., meridian altitude of sun's lower limb was $50^{\circ} 6'$ south of the observer, (that is, south of his zenith) the eye being 17 feet above the level of the horizon. Required the latitude. Answer, $20^{\circ} 32' 58''$.

NOTE. On land we have no correction for dip.

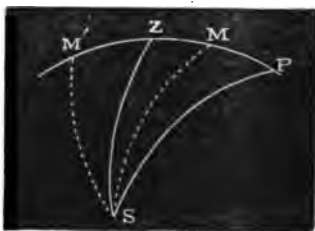
378. To find the latitude when the celestial object is off the meridian, by having the hour angle between the place of the object and meridian, the altitude and declination or polar distance.

Let S = place of the star. P the elevated pole. Z = the zenith.

Here $P S = p$ = codeclination = polar distance.

$Z S = z$ = zenith distance and $P Z$ is the colatitude = P , and the hour angle, $\angle P S Z = h$.

By case VI, we have p , z , and the hour angle $\angle P S Z = h$, to find $P Z$. Let fall the perpendicular $S M$. Let it fall within the $\triangle S P Z$, then we have



$\tan. PM = \cos. h \times \cotan. \text{declination} = \cos. h \cdot \tan. \text{pol. dist.}$

$\cos. ZM = \cos. PM \times \sin. \text{alt.} \times \text{cosecant of declination.}$

Colatitude = $PM + ZM$ when the perp. falls within $\triangle PSZ$.

Colatitude = $PM - ZM$ when the perp. falls without the same.

It is to be observed that there may be an ambiguity whether the point M would fall inside or out of the $\triangle PSZ$. This can only happen when the object is near the prime vertical, that is due E. or W. As the observation should be made near the meridian, the approximate latitude will show whether M is between the pole, P and zenith, Z or not.

Having the two sides p and z , and the $\angle h = \angle SPZ$, we find PZ the colat. by sec. 372.

379. Latitude from a double altitude of the sun, and the elapsed time.

The altitudes ought to be as near the meridian as possible, and the elapsed time not more than two hours. When not more than this time, we may safely take the mean of the sun's polar distance at the two altitudes.

Let S and S' be the position of the object at the time of observations.

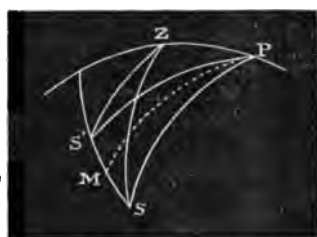
ZS and $ZS' =$ zenith distances.

PS and $PS' =$ the polar distances.

Angle $SPS' =$ elapsed time.

To find the colatitude = PZ .

Various rules are published for the solution of this problem, but we will follow the immortal *Delambre*.



Delambre, who has calculated more spherical triangles than any other man, found, after investigating the many formulas, that the direct method of resolving the triangle was the best and most accurate method. We now have the following:

PS and $PS' =$ polar distances.

ZS and $ZS' =$ colatitudes.

Hour angle = SPS' .

Half of $PS + PS' =$ mean polar distance = p .

One-half the elapsed time in space = h .

Draw the perpendicular PM , then we have

$\log. \sin. SM = \log. \sin. \text{mean polar distance} + \log. \sin. \text{one-half hour angle in space, and having } SM = S'M, \text{ we have the base, } SM S'.$

Consequently, in the $\triangle SZS'$, we have the three sides given to find the angles, and also the three sides of the triangle $PS S'$. By sec. 367, we find the angles $PS S'$ and $ZS S'$ $\therefore$ the $\angle PSZ$ is found, and the sides PS and ZS is found by observation, then we have in the triangle PSZ the two sides PS , SZ and the angle PSZ , to find the colat. PZ , which can be found by sec. 369.

380. To find the latitude by a meridian altitude of *Polaris*, or any other circumpolar star.

Take the altitude of the object above and below the pole, where great accuracy is required. Let their apparent zenith distances be z and z' respectively, and also, r and r' , the refractions due to the altitudes, then

Colatitude = correct zenith distance = $\frac{1}{2}(z + z' + r + r')$

Let A and A' be the correct altitudes, then we have

Colatitude = $\frac{1}{2}(180 - (A + A') + (r + r'))$

NOTE. Here we do not require to know the declination of the object.

By this method, we observe several stars, from a mean of which the latitude may be found with great accuracy. The instrument is to be placed in the plane of the meridian as near as possible. The altitude will be the least below the pole, and greatest above it, at the time of its meridian transit or passage.

881. *To find the latitude by a meridian altitude of a star above the pole.*

Correct the altitude as above for the sun. From this, take the polar distance, the difference = the required latitude.

Let A and A' = corrected altitudes above and below the pole.

p = polar distance of the object. Then

Latitude = $A - p$ when $*$ is above the pole.

Latitude = $A + p$ when $*$ is below the pole.

882. *To find the latitude by the pole star, at any time of the day.*

The following formula is given in the British Nautical Almanac since 1840, and is the same in Schumacher's Ephemeris :

$$L = a - p \cdot \cos. h + \frac{1}{2} \sin. 1'' (p \sin. h)^2 \tan. a.$$

$$- \frac{1}{2} \sin. 2 1'' (p \cos. h) (p \sin. h)^2.$$

If we reject the fourth term, it will never cause an error more than half a second. Then we have

$$L = a - p \cdot \cos. h + \frac{1}{2} \sin. 1'' (p \sin. h)^2 \cdot \tan. a.$$

Here L = latitude, a = true altitude of the star.

p = apparent polar distance, expressed in seconds.

h = star's hour angle = $S - r$.

S = sidereal time of observation.

r = right ascension of the star.

p is *plus* when the $*$ is W. of the meridian, and negative when E.

Example. In 1858, Jan. 21, in longitude 80° W., about 2 hours after the upper transit of Polaris, its altitude, cleared of index error, refraction and parallax, was observed = $40^\circ 10'$. Star's declination = $88^\circ 31' 47''$. Mean time of observation by chronometer = 7h. 0m. 32.40s. To find the latitude.

	h m s
1858, Jan. 21, Polaris' R. A.,	1 5 36.79
Sidereal time, mean noon, Greenwich,	20 8 2.78
Sid. interval from mean noon at Greenwich =	5 2 34.06
Cor. $80^\circ \times 0.6571$, to be subtracted in W. long.	52.57
Sidereal interval of meridian passage at station,	5 1 41.49
Mean time of observation, 7h. 0m. 32.40s. which,	
reduced to sidereal time by Table XXXI, =	7 1 41.49
Hour angle h in arc = 30° =	in time, 2 0 00
$p = 5292.6''$, its log. = 3.7236691	
$h = 30^\circ$ its log. cosine, 9.9375806	

$$\text{Log. of } p \cos. h = 8.6611997 = 4583.5 = \text{first correction.}$$

$$4583.5'' = 1^\circ 16' 23.5'' = \text{negative} = -1^\circ 16' 23.5'' = \text{first cor.}$$

To find the second correction.

$$\begin{aligned} \text{Log. sin. } h = 30^\circ &= 9.6989700 \\ \text{Polar dis. } p = 5292.6, \text{ log} &= 3.7236691 \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{Log. sin. } h = 30^\circ &= 9.6989700 \\ \text{Polar dis. } p = 5292.6, \text{ log} &= 3.7236691 \end{aligned}} \right\} = 3.4226291$$

$$\begin{aligned}
 (p \sin. h)^2 &= 3.4226291 \times 2 = 6.8452782 \\
 \frac{1}{2} \sin. 1'' &= 4.8845449 \\
 \tan. \text{ of alt. } 40^\circ 10' &= 9.9268778 \\
 \frac{1}{2} \sin. 1'' (p \sin. h)^2 \cdot \tan. a &= 1.1562009 \\
 &= + 14.81'' = \text{second cor.} \\
 \text{Altitude,} &40 \ 10 \ 00 \\
 \text{First correction} &- \ 1 \ 16 \ 23.50 \\
 \hline
 &88 \ 53 \ 86.50 \\
 \text{Second correction} &+ \ 0 \ 0 \ 14.81 \\
 \hline
 &88 \ 53 \ 50.81 = \text{required latitude.}
 \end{aligned}$$

NOTE. Here we rejected the fourth term as of no consequence.

The longitude may be assumed approximately near; for an error of one degree in longitude, makes but an error of 0.68a. in the hour angle.

388. To find the variation of the compass by an azimuth of a star.

At sec. 264c and 264H, we have shown how to find the azimuth, when the star was at its greatest elongation. To find the azimuth at any other time, we take the altitude, and know the polar distance of the star and the colatitude of the place; that is, we have the

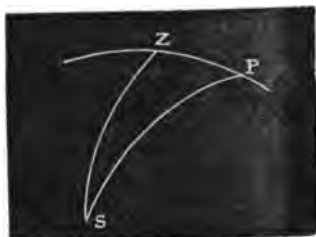
Polar distance, P S

Colatitude, P Z

Zenith distance, Z S

To find the

Azimuth angle P Z S.



We find the required angle P Z S by sec. 367.

By Table XXIII, we can find the azimuth from the greatest elongation of certain circumpolar stars.

384. To find at what time Polaris or any other star will be at its greatest eastern or western elongation or azimuth. Its true altitude and greatest azimuth at that time. Also to determine the error of the chronometer or watch.

In the following example, let P = polar distance, L = latitude, R. A. = right ascension, and G. A. = greatest azimuth.

Given the latitude of observatory house in Chicago = $41^\circ 50' 30''$ N. longitude, $87^\circ 34' 7''$ W. on the 1st December, 1866, to find the above.

Polaris, polar distance = $1^\circ 24' 4''$.

NOTE. In determining the greatest azimuth, we select a star whose polar distance does not exceed 16° , and for determining the true mean time, we take a star whose polar distance will be greater than 16° or about 20 to 30° , and which can be used early in the night. Calculating the altitude and time of the star's greatest azimuth, is claimed by us as new, simple and infallibly true, and can be found by any ordinary surveying instrument whose vertical arc reads to minutes.

It is generally believed by surveyors, that when Polaris, Alhion in Ursa Majoris, or Gamma in Cassiopeae, are in the same plane or vertical line, Polaris is then on the meridian.

It is to be much regretted that the above two last named stars so much used by surveyors, have not found place in the British or American Ephemeris. However, we have calculated the R. A. and declination of them till 1940. See Table XXV.

NOTE. We will send a copy of this part of our work to the respective Nautical Almanac offices above named, urging the necessity of giving the right ascension and declination of these two stars. With what success, our readers will hereafter see.

Time from Merid. Passage.		Altitude at G. A.	Greatest Azimuth.
Tan. p	8.388487	Radius,	10.000000
Tan. L +	9.951028	Sine L +	9.824174
	18.389460		19.824174
Less	10.	Cos. p —	9.999870
Cosine =	8.839460	Sine =	9.824804
88° 44' 58''		True alt.	41° 51' 25''
Sid. 5h. 54m. 59.58s.		Cor. tab. XII +. 1 8	1° 52' 51''
		Appt. alt.	41° 52' 88''
			Greatest azimuth.

Polaris R. A. =	1h. 10m. 54.80s.
Sun's R. A. = sid. column,	16 41 25.04
	8 29 29.26
Cor. for 87° 34' 7'' at 0.6571s. for each deg. —	57.54
Upper transit in sidereal time =	8 28 31.72
Time from meridian passage to G. E. A. =	5 54 59.58
This would be in day time, for G. E. A.,	2 38 32.19
This is after midnight, for G. W. A.,	14 28 21.25
Or, December 2d,	2 28 21.25
Which, if reduced to mean time, gives	2 22 57.70

885. To find the azimuth or bearing of Polaris from the meridian, when Polaris and Alioth (*Epsilon in Ursa Majoris*) are on the same vertical line.

Example. The latitude of observatory house in Chicago, (corner of 26th and Halsted streets,) is 41° 50' 30''. Required the azimuth of Polaris when vertical with Alioth, on the first day of January, 1867.

Right Ascension.	Ann. variation.	N. P. D.	Ann. variation.
Polaris, 1h. 10m. 17s.	+ 19.664s.	1° 28' 59''	— 19.12''
Alioth, 12h. 48m. 10s.	+ 2.661s.	83° 19' 05''	— 19.67''
Gamma, 0h. 48m. 42s.	+ 3.561s.	80° 0' 15''	— 19.613''

Latitude, 41° 50' 30'' ∴ colatitude = 48° 9' 30''.

Polaris N. P. D. 1° 24' and colat. less polar distance = Z.

Altitude above the pole = 43° 14' 29''

48° 9' 30'' — 1° 24' = 46° 45' 30'', zenith dist. of Polaris.

To find Alioth's zenith distance.

Latitude,	41° 50' 30''	
Alioth below the pole,	83° 19' 05''	= polar distance.
Alioth's altitude,	8° 31' 25''	under transit.
Alioth's zenith distance,	81° 28' 35''	
Polaris' upper transit, 1st January, 1867,	1h. 10m. 17s.	
Alioth's upper transit, 12h. 48m. 10s.	Under at 0h. 48m. 10s.	
Hour angle in space = 5° 31' 45'', in time =	22m, 07s.	

Here we find that Alioth passes the meridian below the pole 22m, 7s, earlier than Polaris will pass above it, consequently, they will be vertical E. of the meridian.

As Polaris moves about half a minute of a degree in one minute of time, it is evident that we may take the zenith distances of both stars the same as if taken on the meridian without any sensible error.

We have in the $\triangle P Z S$, fig. in sec. 383, the sides

$P S$ = polar distance. $Z S$ = zenith distance. And the hour angle $S P Z$, in space, to find the azimuth angle $S Z P$. By sec. 372,

$$\text{we have } \sin. < S Z P = \frac{\sin. < S P Z \cdot \sin. P S}{\sin. Z S} = \frac{\sin. h \times \sin. p}{\sin. z}$$

$$\sin. < S Z P = \frac{\sin. 5^{\circ} 31' 45'' \times \sin. 1^{\circ} 24'}{\sin. 46^{\circ} 45' 30''} = 0^{\circ} 11'.$$

That is, the azimuth of Polaris is 11' E. of the meridian, when Alioth is on it below the pole. Alioth is going E. and Polaris going W., therefore, they meet E. of the meridian. Their motions are

$$\begin{array}{l} \text{As } \frac{\text{sine polar distance of Polaris}}{\text{sine of its zenith distance}} \cdot \frac{\text{sine polar distance of Alioth.}}{\text{sine of its zenith distance.}} \text{ Or} \\ \text{As } \frac{\text{sine } 1^{\circ} 24'}{\text{sine } 46^{\circ} 45' 30''} \cdot \frac{\text{sine } 38^{\circ} 19' 05''}{\text{sine } 81^{\circ} 28' 35''} \cdot \cdot \cdot .0244 \cdot .5468 \\ \cdot \cdot \cdot .7285 \cdot .9889 \end{array}$$

Or as $0.0244 \times 0.9889 : 0.5468 \times 0.7285$. Or 1 : 16.

And 17 : 11' : : 1 : Polaris' space moved west = 39'' nearly.

Therefore, $11' - 39'' = N. 10' 21'' E.$ = required azimuth.

386. To find the azimuth of Polaris when on the same vertical plane with γ in Ursa Majoris, in Chicago, on the 1st Jan., 1867: Lat. $41^{\circ} 50' 30''$.

R. A. of Polaris at upper transit, 1h, 10m, 17s.

R. A. of γ Ura. Maj. at upper transit, 11h, 46m, 49s.

“ “ “ “ under transit, 23h, 46m, 49s.

Hour angle in space, $20^{\circ} 52'$ = in sidereal time to, 1h, 23m, 28s.

Polaris' polar dist. above the pole = $1^{\circ} 24'$ $\therefore$ its alt. = $48^{\circ} 14' 30''$, and the altitude taken from 90° , gives the zenith dist. = $46^{\circ} 45' 30''$. Gamma's polar distance, from Nautical Almanac, $85^{\circ} 34'$ below the pole $\therefore$ its altitude = $41^{\circ} 50' 30'' - 85^{\circ} 34' = 6^{\circ} 16' 30''$, and its zenith distance, $83^{\circ} 43' 30''$.

In the $\triangle S P Z$, we have the hour $< S P Z = h$, equal to $20^{\circ} 52'$, $P S = 1^{\circ} 24'$, and $Z P = 48^{\circ} 14' 30''$. By sec. 372,

$$\sin. < S Z P = \frac{\sin. 20^{\circ} 52' \times \sin. 1^{\circ} 24'}{\sin. 46^{\circ} 45' 30''} \quad \text{By using Table A,}$$

$$\text{we have } \sin. S Z P = \frac{.35619 \times .02448}{.72887} = .01195 = 41'$$

Angular motion of Polaris is to the angular motion of γ nearly

$$\text{as } \frac{\text{sine polar dist. of Polaris}}{\text{sine of its zenith dist.}} \cdot \frac{\text{sine polar dist. of } \gamma}{\text{sine of its zenith dist.}}$$

$$\text{that is, } \frac{\sin. p}{\sin. z} \cdot \frac{\sin. P}{\sin. Z} \cdot \cdot \cdot \frac{\sin. P \times \sin. z}{\sin. p \times \sin. Z} \cdot 1. \quad \text{By Table A,}$$

$$\sin. P = \sin. 85^{\circ} 34' = .5817$$

$$\sin. z = \sin. 46^{\circ} 45' 30'' = .7284. \quad \text{Their product} = .42871028 = B.$$

$\sin. p \times \sin. Z = \sin. 1^{\circ} 24' \times \sin. 85^{\circ} 43' 30'' = .0244 \times .294 = .02428342 = C$, divided into B, gives the value of the 4th number = 27. As γ moves E. $27'$ and Polaris moves W. $1'$ in the same time, making a total distance of $28' \therefore 28 : 41' :: 1 : 1' 28''$, which, taken from the above $41'$, leaves the azimuth of Polaris N. $89^{\circ} 32''$ E. of the meridian.

Table XXIII gives the greatest azimuths of certain stars near the North and South Poles; by which the true bearing of a line and variation of the compass can be found several times during the night. There are several bright stars near the North Pole. The nearest one to the South Pole is β Hydri, which is now about 12° from it. This circumstance led us to ask frequently why there should not be the same means given those south of the Equator as to those north of it. It was on the night of the 18th January, 1867, as we revelled in a pleasant starry dream, that we heard the words—*God has given the Cross to man the emblem of and guide to salvation. He has also made the Southern Cross a guide in Surveying and Navigation.* Not a moment was lost in seeing if this was so. We found from our British Association's Catalogue of Stars, that when α' (a star of the first magnitude) in the foot of the Southern Cross was vertical with β (a bright star) in the tail of the Serpent, that then, in lat. 12° , they were within $1' 12''$ of the true meridian, and that their annual variations are so small as to require about 50 years to make a change of half a minute in the azimuth or bearing of any line.

We rejoice at the valuable discovery, but struck with awe at the forethought of the Great Creator in ordaining such an infallible guide, and brought once more to mind the expression of Capt. King, of the Royal Engineers, who, after taking the time according to our new method, in 1846, near Ottawa, Canada, and seeing the perfect work of the heavens, said—*"Who dares say there is no God?"*

Our readers will perceive that Tables XXIII, XXVI, XXVII and XXVIII are original, and the result of much time and labor.

Table XXVI gives the azimuth of α' Crucis when vertical with β Hydra in the southern hemisphere until the year 2150.

Table XXVII gives the azimuth of Polaris when vertical with Alioth in Ursa Majoris until the year 1940.

Table XXVIII, when Polaris is vertical with γ in Cassiopea till 1940.

387.

TO DETERMINE THE TRUE TIME.

The true time may be obtained by a meridian passage of the sun or star. When the telescope is in the plane of the meridian, as in observatories, we find the meridian transit of both limbs of the sun, the mean of which will be the apparent noon, which reduce to mean time by adding or subtracting the equation of time. If we observe the meridian passage of a star, we compare it with the calculated time of transit, and thereby find the error of the chronometer or watch.

388. *By equal altitudes* of a star, the mean of both will be the apparent time of transit, which, compared with the calculated time of transit, will give the error of the watch, if any.

389. *By equal altitudes of the sun*, taken between 9 A. M. and 3 P. M. In this method we will use Baily's Formula, and that part of his Table XVI, from 2 to 8 hours elapsed time between the observations.

$X = \pm A \delta \tan. L + B \delta \tan. D$. Here

T = time in hours, L = latitude of place, *minus when south*.

D = dec. at noon, also *minus when south*.

δ = double variation of dec. in seconds, deduced from the noon of the preceding day to that of the following. *Minus when the sun is going S.*

X = correction in seconds. A is *minus* if the time for noon is required, and *plus* when midnight is required. The values of A and B for time T , may be found from Table XXVIII, which is part of Baily's Table XVI, and agrees with Col. Frome's Table XIV, in his Trigonometrical Surveying, and also with Capt. Lee's Table of Equal Altitudes. We give the values of A and B but for 6 hours of elapsed time or interval, for before or after this time, (that is, before 9 A. M. or after 3 P. M.) it will be better to take an altitude when the sun is on or near the prime vertical, which time and altitude may be found from Tables XXI and XXII of this work.

890. To determine the time at Tasche in lat. $45^{\circ} 48'$ north, on the 9th of August, 1844, by equal altitudes of the sun.

Alt. U. L.	° /	Chronometer Time.		Elap. time T.	Value of X.
		A. M.	P. M.		
		h m s	h m s	h m	s
78 50		1 28 23	8 08 16.5	6 33	10.63
"	79 19.30	1 29 52.8	8 01 46.5		
"	85 36.00	1 49 33	7 42 18	5 48	10.1
"	87 02.10	1 53 58.5	7 37 46.2		

Here the sun is going south, therefore D is *minus*. The lat. is north, $\therefore L$ is *plus*. Also δ is *minus*. We want the time of noon, $\therefore$ the value of A is *minus*, and $-A \times -\delta \times +L$, will be positive or *plus*, and also, $B \times -\delta \times -D$, will be *plus* in the following calculation, where we find $\delta = 2094''$ —from the Nautical Almanac:

$T = 6\text{h. } 3\text{m.}$ its log. $A = -7.7793$, and log. $B = -7.5951$.

$\delta = 2094''$, its log. $= -3.3210$, log. $\delta = -3.310$.

$L = 45^{\circ} 48'$ log. tan. $= +0.0121$, log. tan. $D = -9.4433$.

First correction $+12.95\text{s.} = 1.1124$. $2.32\text{s.} = -0.3654$.

Second correction 2.32

$X = 10.08$

Time A. M. $= t = 1\text{h. } 28\text{m. } 23.0\text{s.}$

Time P. M. $= t' = 8 \quad 08 \quad 16.5$

$t + t' = 9 \quad 31 \quad 39.5$

$t + t' = 4 \quad 45 \quad 49.75$

2
 $X = + 10.63$

4 46 00.38 chronometer time of app't noon.

05 09.09 equat. time from Naut. Almanac.

4h. 40m. 51.29s. chronom. fast of mean time, at
app't noon, August 9, 1844.

Correct this for the daily rate of loss or gain by the chronometer, result will be the true mean time of chronometer at apparent noon. This time converted into space, will give the long. W. of the meridian, whose mean time the chronometer is supposed to keep. The above is one of Col. J. D. Graham's observations, as given by Captain Lee, U. S. T. E. in his Tables and Formulas.

Time by Equal Altitudes. (See sec. 388.)

We set the instrument to a given altitude to the nearest minute in advance of the star, and wait till it comes to that altitude.

Example from Young's Nautical Astronomy.

Observations made on the star Arcturus, Nov. 29, 1858, in longitude $98^{\circ} 30'$ E. to find the time :

Altitudes E. and W. of the Meridian.	Times shown by Chronometer.	Sum of Times.
	h. m. s.	h. m. s.
48 10	{ 11 55 47 } { 18 11 55 }	30 7 42
48 30	{ 11 57 57 } { 18 9 45 }	30 7 42
43 50	{ 12 0 7 } { 18 7 35 }	30 7 42
From the sum of the times, we get the chronometer time of the star's meridian passage, or transit, equal to		15h. 8m. 51s.
Arcturus, R. A. Nov. 29,	h. m. s. 14 9 13	
R. A. of mean sun, sid. col.,	16 20 48	Diff. for 1h. = + 10.76s.
Mean time of transit at station,	21 48 25 nearly.	15½ hours.
Long. $98^{\circ} 30'$ E in time,	6 24 00 subtract.	164.09
Mean time at Greenwich,	15 14 25 nearly.	or 2m. 44s.
Cor. for 15½ hours,	— 2 44 subtract, because R. A. is increasing.	
Mean time at Greenwich,	15 11 41	
Mean time by chronometer,	15 8 51	
Error on mean time,	— 7 50 at station.	
Mean time of transit at place,	h. m. s. 21 48 25 nearly.	
Cor. for increase in R. A.,	— 2 44	
	21 45 41	
Mean time as shown by chrcnometer,	15 8 51	
Error of chronometer on mean time,	• 6 41 50 at station.	

By sec. 388. Set the altitude to a given minute in advance, and wait till the star comes to this, and note the mean time.

Time before Midnight.	Altitudes of Star.	Time after Midnight.
h. m. s.	° ' "	h. m. s.
9 50 10	50 0	2 7 40
9 50 20	50 10	2 7 30
9 50 21	50 20	7 19
9 50 20.8		2 7 29.7 Mean.
14 7 29.7		+ 12
2) 23 57 50.0		14 7 29.7

11 58 55 . Mean time by clock at station.

390.*

True time by a Horizontal Dial.

This dial is made on slate or brass, well fastened on the top of a post or column, and the face engraved like a clock. (See fig. 49.) It may be set by finding the true mean time and reducing it to the apparent, by means of the equation of time, found in all almanacs. Having the correct apparent noon by clock, set the dial.

Otherwise. Near the dial make a board fast to some horizontal surface, on which paste some paper, and draw thereon several cocentric circles. Perpendicular to this, at the common centre, erect a piece of fine steel wire, and watch where the end of its shadow falls on the circles between the hours of 9 and 3. Find the termini on two points of the same or more circles; bisect the spaces between them, through which, and the centre of the circles, draw a line, which will be the 12 o'clock hour line, from which, at any future time, we may find the apparent, and hence the true mean time.

A brass plate may be fastened to an upper window sill, in which set a perpendicular wire as gnomon, and draw the meridian.

Calculation. We have the latitude, hour angle and radius to find the hour arc from the meridian.

Rule. Rad. : sin. lat. :: tan. hour angle : tan. of the hour arc from the meridian.

Example. Lat. 41° . Hour angle between 10 and 12 = 2 hours = 30° . As 1 : .65606 :: .57735 : tan. hour arc = .37878, whose arc is = $20^\circ 44' 55''$.

In like manner we calculate the arc from 12 to each of the hours, 1, 3 and 5, which are the same on both sides. The morning and evening hours are found by drawing lines (see fig. 49) from 3, 4 and 5 through the centre or angle of the style at *c*. These will give the morning hours. For the evening hours, draw the lines through 7, 8, 9, and centre *d*, at the angle of the style. The half and quarter hours are calculated in like manner. The slant of the gnomon, *d f*, must point to the elevated pole, and the plate or dial be set horizontal for the lat. for which it is made. The $\angle$ of the gnomon is equal the latitude. A horizontal dial made for one latitude may be made to answer for any other, by having the line *d f* point to the elevated pole. *Example.* One made for lat. 41° may be used in lat. 50° , by elevating the north end of the dial plate 9° , and *vice versa*.

The following table shows the hour arcs at four places:

LAT. 41° .	LAT. 49° .	LAT. $54^\circ 36'$. BELFAST, IRELAND	LAT. $55^\circ 52'$. GLASGOW, SCOTL'D.
1h. = $9^\circ 58'$	$11^\circ 26'$	$12^\circ 19'$	$12^\circ 30'$
2 20 45	23 33	25 12	25 32 $\frac{1}{2}$
3 33 16	37 03	39 11	39 37 $\frac{1}{2}$
4 48 39	52 35	54 41	55 06 $\frac{1}{2}$
5 67 47	70 27	71 48	72 04
6 90 00	90 00	90 00	90 00

To set off these hour arcs, we may, from *c*, set off on line *c n* the chord of 60° and describe a quadrant, in which set off from the line *c n* the hour arcs above calculated.

In our early days we made many dials by the following simple method:

We draw the lines, *c n* and *g h*, so that *c g* will be 5 inches, and described the quadrants, *c, g, k*,

We have, by using a scale of 20 parts to the inch, a radius *c k* = 100.

As the chord of an arc is twice the sine of that arc, we find the sines of half the above hour arcs in Table A; double it; set the decimal mark two places ahead; those to the left will be divisions on the scale to be set off from *k* in the arc *k g*. *Example*—

Let half of the hour arc = $4^\circ 59'$, twice its sine = .17374, which give 17.4 parts for the chord to be set off.

391. *By our new method*, we select one of the bright circumpolar stars given in the N. A., whose polar distance is between 15 and 80 degrees. (See our Time Stars in Table XXIV.)

By sec. 264c, we find the sidereal time of its meridian passage = T .

By sec. 264b, we find its hour angle from ditto = t .

By sec. 264f, we have its true altitude A , when at its greatest azimuth or elongation from the meridian.

Example. Star, 8, on a given day, in latitude, L , passed the meridian at time, T , and took time, t , to come to its greatest azimuth, east or west.

We now reduce the sidereal time to mean time.

Greatest eastern azimuth was at time $T - t$. Mean time.

Greatest western ditto, $T + t$. Ditto.

True altitude of its greatest azimuth = A .

Let r = refraction and i index error, then App. alt. = $A + r \pm i$.

We now set the instrument a few minutes before the calculated sidereal time reduced to mean time, and elevate the telescope to the alt. = $A + r \pm i$, and observe when the star comes to the cross hairs at time T' .

The difference between mean time, $T \pm t$ and T' gives the error of time as shown by the watch or chronometer.

This method is extremely accurate, because the star changes its altitude rapidly when near its greatest elongation. As we may take several stars on the same night, we can have one observation to check another.

Now having the true time at station and an approximate longitude, we can find a new longitude, and with it as a basis, find a second, and so on to any desired degree of accuracy.

392. *To find the difference of Longitude.*

1. By rockets sent up at both stations, the observers having previously compared their chronometers and noted the time of breaking.

2. As the last, but instead of rockets, flashes of gunpowder on a metal plate is used. This signal can be seen under favorable circumstances, a distance of forty miles.

3. By the electric telegraph.

4. By the Heliostat.

5. By the Drummond light.

6. By moon culminating stars.

7. By lunar observations.

In 7, we require the altitudes of the moon and star, and the angular distance between the moon's bright limb and the star at the same time, thus requiring three observers. If one has to do it alone, he takes the altitudes first, then the lunar distance, note the times, and repeat the observations in reverse order, and find the mean reduced altitude, also the mean lunar distance.

8. By occultation or eclipse of certain stars by the moon.

393. *By the Electric Telegraph.*

The following example and method used by the late Col. Graham is so very plain, that we can add nothing to it. No man was more devoted to the application of astronomy to Geodesy than he:

LONGITUDE OF CHICAGO AND QUEBEC.

The following interesting letter of Col. Graham, Superintendent of U. S. Works on the Northern Lakes, is in reference to the observations made by him, in conjunction with Lieut. Ashe, R. N., in charge of the observatory at Quebec, to ascertain the difference of longitude between this city and Quebec :

CHICAGO, June 5, 1857.

To the Editor of the Chicago Times: A desire having been expressed by some of the citizens of Chicago for the publication of the results of the observations made conjointly by Lieut. E. D. Ashe, Royal Navy, and myself, on the night of the 15th of May, ult., for ascertaining by telegraphic signals the difference of longitude between Chicago and Quebec, I herewith offer them for your columns, in case you should think them of sufficient interest to be announced. All the observations at Quebec were made under the direction of Lieut. Ashe, who has charge of the British observatory there, while those at this place were made under my direction.

The electric current was transmitted *via* Toledo, Cleveland, Buffalo, Toronto and Montreal, a distance, measured along the wires, of 1,210 miles, by one entire connection between the two extreme stations, and without any intermediate repetition; and yet all the signals made at the end of this long line were distinctly heard at the other, thus making the telegraphic comparisons of the *local time* at the two stations perfectly satisfactory.

This "local time" was determined (also on the night of the 15th ultimo) by observations of the meridian transits of stars, by the use of transit instruments and good clocks or chronometers at the two stations. The point of observation for the "time" at Quebec was the citadel, and at Chicago the Catholic church on Wolcott street, near the corner of Huron.

The following is the result :

1. CHICAGO SIGNALS RECORDED AT BOTH STATIONS. ELECTRIC FLUID TRANSMITTED FROM WEST TO EAST.

Correct Chicago sidereal time of signals.	Correct Quebec sidereal time of signals.	Difference of longitude. Electric fluid transmitted from west to east.
h. m. s.	h. m. s.	h. m. s.
16 11 13.19	16 16 54.83	1 05 41.64
15 42 18.28	16 47 59.83	1 05 41.55
Mean; electric fluid transmitted from west to east,		1 05 41.596

2. QUEBEC SIGNALS RECORDED AT BOTH STATIONS—ELECTRIC FLUID TRANSMITTED FROM EAST TO WEST.

Correct Quebec sidereal time of signals.	Correct Chicago sidereal time of signals.	Difference of longitude. Electric fluid transmitted from east to west.
h. m. s.	h. m. s.	h. m. s.
16 24 15.83	15 18 34.40	1 05 41.43
16 54 45.83	15 49 04.89	1 05 41.41
Mean; electric fluid transmitted from east to west.		1 05 41.435
Mean; electric fluid transmitted from west to east, as above,		1 05 41.596
RESULT—Chicago west, in longitude from Quebec,		1 05 41.515
Difference between results of electric fluid transmitted east and west = 0.16 and half diff. = 0.08.		

From which it would appear that the electric fluid was transmitted along the wires between Chicago and Quebec in 8-100ths of a second of time. At this rate it would be only $1\frac{1}{2}$ seconds of time in being transmitted around the circumference of the earth.

I will now proceed to a deduction of the longitude of Chicago, west of the meridian of Greenwich, by combining the above result with a determination of the longitude of Quebec made by myself in the year 1842, while serving as commissioner and chief astronomer on the part of the United States for determining our northwestern boundary, which will be found published at pages 368-369 of the American Almanac for the year 1848. That determination gave for the longitude of the centre of the citadel of Quebec west of Greenwich :

	h.	m.	s.
Difference of longitude between the same point and the Catholic Church on Wolcott street, near the intersection of Huron street, Chicago, by the above described operations,	4	44	49.66
Longitude west of Greenwich, of the Catholic Church on Wolcott street, street, near Huron street, Chicago, Illinois,	1	06	41.51
	5	50	31.16

That is to say, five hours, fifty minutes, thirty-one and sixteen-hundredth seconds of time, or in arc, 87deg. 37min. 47 4-10sec.

J. D. GRAHAM,
Major Topographical Engineers, Brevet Lieut. Col. U. S. Army.

By the Heliostat.

This instrument consists of a mirror, pole, Jacob staff or rod, and a brass ring with cross wires. The brass ring used in our Heliostat, is $\frac{3}{4}$ of an inch thick and $3\frac{1}{2}$ inches diameter. In this is fixed a steel point 2 inches long. There are 4 holes in the ring for to receive cross wires or silk threads made fast by wax. The flag-staff is bored at every 6 inches on both sides to receive the ring, which ought to be at a sufficient distance from the side of the pole so as not to obstruct the direction of the reflected rays of the sun. The pole and ring are set in direction of station B, about 30 to 40 feet in advance of the mirror placed over station A, and the centre of the ring in direction of B, as near as possible. The ring can be raised or lowered to get an approximate direction to B. It will be well to remove the rings from side to side, till the observer at B sees the flash given at A, when B sends a return flash to A.

The mirror is of the best looking-glass material, $3\frac{1}{2}$ inches in diameter, set in bronzed brass frame or ring, $4\frac{1}{2}$ inches outer diameter, $3\frac{1}{2}$ inches inner diameter, and three-tenths of an inch thick. This is set into a semicircular ring, four-tenths of an inch thick, leaving a space between it and the mirror of two-tenths of an inch; both are connected by two screws, one of which is a clamping screw. Both rings are attached to a circular piece of the same dimensions as the outer piece, $1\frac{1}{2}$ inches long; and to this is permanently fixed a cylindrical piece, $\frac{1}{2}$ inch in diameter and $1\frac{1}{2}$ inches long, into which there is a groove to receive the clamping screw from the tube or socket.

The socket or tube, is 8 inches long, and $\frac{1}{2}$ inch inner diameter, having two clamping screws, one to clamp the whole to the rod or Jacob staff, and the other to allow of the mirror being turned in any direction.

By these three clamping screws, the mirror is raised to any required height, and turned in any direction. The back of the mirror is lined with brass, in the centre of which there is a small hole, opposite to which the silvering is removed. The observer at A sets the centre of the mirror over station A, looks through the hole and through the centre of the cross, and elevates one or both, till he gets an approximate direction of the line, A, B. Our Heliostat, with pouch, weighs but $3\frac{1}{2}$ pounds.

A mirror of 4 inches will be seen at a distance of 40 miles. One of 8 to 10 inches will be seen at a distance of 100 miles.

We use a mirror of 4 inches diameter, fitted up in a superior style by Mr. B. Kratzenstein, mathematical instrument maker, Chicago. Like all his work, it reflects credit on him. We have found it of great use in large surveys, such as running long lines on the prairies, where it is often required to run a line to a given point, call back our flagman,

or make him move right or left. We are indebted to Mr. James Reddy, now of Chicago, formerly civilian on the Ordnance Surveys of Ireland, England and Scotland, for many hints respecting the construction and application of the Heliostat.

Example. Let A be the east and B the west station. Observer A shuts off the reflection at 2h. p. m.—2h. 1m.—2h. 2m., etc., which B observes to agree with his local time 1h.—1h. 1m.—1h. 2m., etc., showing a difference in time of 1h. or 15 degrees of longitude.

The Drummond Light.

This light was invented by Captain Drummond, of the Royal Engineers, when employed on the Irish Ordnance Survey. It is made by placing a ball of lime, about a quarter of an inch in diameter, in the focus of a parabolic reflector. On this ball a stream of oxy-hydrogen gas is made to burn, raising the lime to an intense heat, and giving out a brilliant light. This has been used in Ireland, where a station in the barony of Ennishowen was made visible in hazy weather, at the distance of 67 miles. Also, on the 31st December, 1848, at half-past 3 p. m., a light was exhibited on the top of Slieve Donard, in the County Down, which was seen from the top of Snowdown, in Wales, a distance of 108 miles. On other mountains, it has been seen at distances up to 112 miles. As the apparatus is both burdensome and expensive, and the manipulation dangerous, unless in the hands of an experienced chemist, we must refer our readers to some laboratory in one of the medical colleges. The Heliostat is so simple and so easily managed, that it supersedes the Drummond light in sunny weather. (See *Trigonometrical Surveying*.)

To find the Longitude by Moon Culminating Stars.

394. We set the instrument in the plane of the meridian by Polaris at its upper or lower transit, or its greatest eastern or western elongation, or azimuth. If we cannot use Polaris, take one of the stars in Ursa Minoris at its greatest azimuth, as calculated in Table XXIII. When the instrument is thus set, let there be a permanent mark made at a distance from the station, so as to check the instrument during the time of making the observations. If the instrument be within a few minutes of the meridian, it will be sufficiently correct for our purpose; but by the above, it can be exactly placed in the meridian.

Moon culminating stars are those which differ but little in declination from the moon, and appear generally in the field of view of the telescope along with the moon. We observe the time of meridian passage of the moon's bright limb and one of the moon culminating stars, selected from the Nautical Almanac for the given time.

Let L = longitude of Greenwich or any other principal meridian.

l, longitude of the station.

A, the observed difference of R. A. between the moon's bright limb, and star at L, from Nautical Almanac.

a, observed difference R. A. between the same at the station.

d, difference of longitude.

h, mean hourly difference in the moon's R. A. in passing from L to l.

$$\text{Then we have} \quad d = \frac{A - a}{h}$$

The following example and solution is from Colonel Frome's Trigonometrical Surveying, p. 238. London, 1862.

At Chatham, March 9, 1838, the transit of α Leonis was observed by chronometer at 10h. 20m. 7s.; the daily gaining rate of chronometer being 1.5s. to find the longitude.

Eastern Meridian, Chatham. Observed transits.

	h. m. s.
α Leonis,	10 52.46
Moon's bright limb,	11 20 7.5
	0 27 21.5
On account of rate of chronometer,	— 0 0 0.03
As 24h : 1.5s. : $\frac{1}{2}$ h. : 0.03s.	0 27 21.47
Equivalent in sidereal time,	— a , 0 27 25.96

Western Meridian, Greenwich. Apparent right ascension.

	h. m. s.
α Leonis,	9 59 46.18
Moon's bright limb,	10 27 16.76
	A, 0 27 30.58
Observed transits,	a, 0 27 25.96

Difference of sidereal time between the intervals = $A - a = 0 \ 0 \ 4.62$

Due to change in time of moon's semidiameter passing the

meridian, (N. A., Table of Moon's Culminating Stars,) + $0 \ 0 \ 0.01$

Difference in moon's right ascension, $0 \ 0 \ 4.63$

Variation of moon's right ascension in 1 hour of terrestrial longitude is, by the Nautical Almanac, 112.77 seconds.

Therefore, As 112.77 : 1h. : : 4.63s. : : 147.80 = 2m. 27.8s., the difference of longitude.

When the difference of longitude is considerable, instead of using the figures given in the list of moon culminating stars for the variation of the moon's right ascension in one hour of longitude, the right ascension of her centre at the time of observation should be found by adding to or subtracting from the right ascension of her bright limb at the time of Greenwich transit, the observed change of interval, and the sidereal time in which her semidiameter passes the meridian. The Greenwich mean time corresponding to such R. A., being then taken from the N. A. and converted into sidereal time, will give, by its difference from the observed R. A., the difference of longitude required. From above:

	h. m. s.
Moon's R. A. at Greenwich transit,	10 27 16.76
Sidereal time of semidiameter passing the meridian	+ 0 1 2.26
	Moon's R. A. at Greenwich transit,
	10 28 19.02
Observed difference,	0 0 4.62
	Moon's R. A. at the time, and sid. time at station,
	10 28 14.40
Greenwich mean time, corresponding to the above R. A.,	
taken from Nautical Almanac, (Table, Moon's R. A. and	
Dec.), 11h. 17m. 0.5s., or sidereal time,	10 25 46.5
	Difference of longitude, .
	0 2 27.9

Longitude by Lunar Distances.—Young's Method.

395. In this method we take the altitudes of the moon and sun, or one of the following bright stars, and the distance between their centres. In the Northern Hemisphere we have

α Arietes, α Tauri (*Aldebaran*,) β Geminorum (*Pollux*,) α Leonis (*Regulus*,) α Virginis (*Spica*,) α Scorpii (*Antares*,) α Aquilae (*Allair*,) α Piscis Australis (*Fomalhaut*,) and α Pegasi (*Markab*,)

We observe the moon's bright limb, and add the semidiameter of the moon, sun, or planet, and thereby find the apparent distance between their centres. This has to be corrected so as to find the true altitude and distance of the centres.

The following formula by Professor Young, formerly of Belfast, Ireland, appears to us to be easily applied, by either using the tables of logarithms, or natural sines and cosines, given in Table A.

Let a , a' , and d represent the apparent altitudes and distance of the moon and star. A , A' , and D the true altitudes and distance.

D is the required lunar distance and ∞ = symbol for difference,

$$D = \left\{ \cos. d + \cos. (a + a') \right\} \left\{ \frac{\cos. (A + A') + \cos. A \infty A'}{\cos. (a + a') + \cos. a \infty a'} \right\} - \cos. (A + A').$$

Example from Young's Nautical Astronomy:—

Let the apparent altitude of the moon's centre,	$24^{\circ} 29' 44'' = a'$
The true altitude,	$25^{\circ} 17' 45'' = A$
The apparent altitude of the star = a' ,	$45^{\circ} 9' 12'' = a'$
Its true altitude,	$45^{\circ} 8' 15'' = A'$
The apparent distance of the star and centre of the moon,	$63^{\circ} 35' 14'' = d$

Here we have,

$\cos. d = \cos. 63^{\circ} 35' 14''$,	nat. cos. 444835
$\cos. (a + a') = \cos. 69^{\circ} 38' 56''$	“ “ 347772
$\cos. d + \cos. (a + a')$	= sum, .792607 = S.
$\cos. (A \infty A') = \cos. 19^{\circ} 50' 30'' = \text{nat. cos. } 940634$	
$\cos. (A + A') = \cos. 70^{\circ} 26' 0'' = \text{nat. cos. } 334903$	
$\cos. (A + A') + \cos. (A \infty A')$	sum, 1275537 = S'
and S multiplied by S' = 127537 $\times$ 792607 =	P
$\cos. (a + a') = \text{from above, } 347772$	
$\cos. (a \infty a') = \cos. 20^{\circ} 29' 28'' = 935704$	

$\cos. (a + a') + \cos. (a \infty a') = 1283476 = S''$. Divide P by S'', and it will give .45280, which is the nat. cos. of $63^{\circ} 4' 45'' = D$

396. *Example.* September 2, 1858, at 4h. 50m. 11s., as shown by the chronometer, in Lat. $21^{\circ} 30' N.$, the following lunar observations were taken:—

Height of the eye above the horizon, 24 feet.

Alt. Sun's L.L.	Obs. Alt. Moon's L.L.	Dist. of Near Limbs.
$58^{\circ} 40' 30''$	$32^{\circ} 52' 20''$	$65^{\circ} 32' 10''$
Index cor. + 2 10	+ 3 40	- 1 10
Sun's noon, Dec., at Greenwich, $7^{\circ} 56' 46'' 5 N.$	Diff. for 1 hour, = $-54^{\circ} 96$	
Cor. for 4h. 50m.,	- 4 26	5
Dec.	7 52 21	For 5 hours = 27480
	90	For 10 m. = 916
Polar dist.	82 7 39	60) 26 5 64
		- 4' 26''

Sun's semidiam.	15' 53", 8	Moon's semidiam.	16' 17"
Equa. of time,	25s. 35	Diff. for 1h.,	+ 0' 796
Cor. for 4h. 50m.,	3 85		5
Corrected eq. of time,	29 2	Sub.	
		For 5 hours,	3980
		For 10 m.,	133
			+ 3 847

Moon's Hor. Parallax,	59' 35" 1	Diff. for 12h.,	= 5" 7
Cor. for 5 hours,	2"	Diff. for 5h.,	= 2"

Hor. Parallax corrected, 59 37

Minutes and seconds may be easily obtained, but there is a table for furnishing this difference in the Nautical Almanac, p. 520.

The difference between the moon's R. A. at 23h., and at the following noon is by (Naut. Alm.) + 2m. 5s., the proportional part of which, for 7m. 42s., is + 16s.

Also, the difference between the two declinations is - 8' 1", the proportional part of which is 7m. 42s., is 1' 2".

1. For the Apparent and True Altitudes.

SUN.		MOON.	
Obs. Alt. L. L.	58° 42' 40"	Obs. Alt. L. L.,	32° 56' 0"
Dip - 4' 49"	- 4' 49" } + 11 5	Dip, - 4 49	
Semidiam. + 15 54		Semidiam., + 16 17	+ 11 37
Apparent Alt.,	58 53 45	Augment. n + 9	
Refrac.—less parallax,	- 30	Apparent Alt.,	33 7 37
True Alt.,	58 53 15	Cor. for Alt.,	+ 48 26
		True Alt.,	35 56 3

2. For the Mean Time at Ship.

Sun's Alt.,	58° 53' 15"	Tab.	Parts
Lat.,	21 30 0	Compliment of cosine, 0.0312 32	diff. for secants
Pol. dist.,	82 7 39	" " 0. 041 24 29 -	1131

2) 162 30 54

½ sum, =	81 15 27	cosine,	9.182196	1369 -	36962
½ sum - alt.	22 22 12	sine,	9.580392	511 +	6132
			18.798034		31962
			320		

2) 18.797714

½ hour angle 14° 30' 31½"	sine, 9.398857
Hour angle, 29 1 3	= 1h. 56m. 4s., apparent time at ship.
Equa. of time,	29

Mean time at ship, 1h. 55m. 35s.

3. For the True Distance, the G. Time, and the Longitude.

Obs. dist.	65° 01' 0"	Appt. dist.	66° 3' 20"	nat. cos.	403850 = y
Sun's semi,	+ 15 54	Appt. alt.	58 53 45		
Moon's + Augm. +	16 26		33 7 37		

Sum, 92 1 22 nat. cos. - 035297 = x

Multiplier = y - x = 370553

True Alt., $\left\{ \begin{array}{l} 58^{\circ} 53' 15'' \\ 33 \ 56 \ 3 \end{array} \right.$ Diff. $25^{\circ} 46' 18''$ nat. cos. $900556 + = W$
 $w - x = 865259 = \text{Divisor.}$
 Sum, $92 \ 49 \ 18$ nat. cos. $- 049228 = v$
 Diff. $24 \ 57 \ 12$ nat. cos. 906652

Multiplier, 370553, inverted = $\begin{array}{r} 857424 \\ 255073 \end{array}$ Multiplicand.
 Multiplier.

NOTE.—This rapid method is }
 done by throwing off a figure }
 in each line as we proceed. }

$\begin{array}{r} 2672272 \\ 600197 \\ 4287 \\ 429 \\ 26 \end{array}$

Divisor, 865259 $\begin{array}{r} 3177211 \\ 2595777 \end{array}$ $\left\{ \begin{array}{l} 367198 = \text{Quotient.} \\ + 049228 = v \end{array} \right.$

NOTE.—The division is abridged }
 by rejecting a figure each time, }
 in the divisor. }

$\begin{array}{r} 581434 \\ 519155 \\ 62279 \\ 60568 \\ 1711 \\ 865 \\ 846 \\ 779 \\ 67 \\ 69 \end{array}$ nat cos. $65^{\circ} 23' 27''$.

True distance, $65^{\circ} 23' 27''$
 Dist. at 3h. (Naut. A.) $66 \ 24 \ 23$ Proportional Log. of diff. 2537
 $\begin{array}{r} 1 \ 0 \ 56 \\ 4704 \end{array}$
 Interval of time, 1h. 49m. 18s. P L = 2167
 $+ 1$

Mean time at Green., 3h. + 1 49 19
 $\begin{array}{r} 1 \ 55 \ 35 \end{array}$

Long. W. in time, $2 \ 53 \ 44$ Long. = $43^{\circ} 28' W$.

And the error of the chronometer is 52s. *fast* on Greenwich mean time.

A base line is selected as level as can be found, and as long as possible, this is lined, leveled, and measured with rods of Norway pine, with platt inum plates and points to serve as indices to connect the rods. They are daily examined by a standard measure, reference being had to the change of temperature. (See p. 165.) At each extremity stones are buried, and at the trig. points are put discs of copper or brass, with a centre point in them. From these extreme points angles are taken to points selected on high places, thus dividing the country into large triangles, and their sides calculated.

These are again subdivided into smaller triangles, whose sides may range from one mile to two miles. These lines are chained, horizontally, by the chain and plumb-line; or, as on the ordnance survey of Ireland, the lines of slopes are measured, and the angles of elevation and depression taken. Spires of churches, angles of towers and of public buildings are observed.

On the main lines of the triangles, the heights of places are calculated from the field book, and marked on the lines. When inaccessible points are observed from other points, we must take a station near the inaccessible one, and reduce it to the centre by (sec. 244.) On the second or third pages of the field book, we sketch a diagram of the main triangle, and all chain lines, with their numbers written on the respective lines, in the direction in which the lines were run. The main triangle may be subdivided in any manner that the locality will allow. See Fig. 64 is the best.

Here we have three check-lines, D F, D E, and F E, on the main triangle, and having the angles at A, B, and C, with the distances, A D, D C, C E, B E, B F, and F D, we can calculate F D, D E, and F E, insuring perfect accuracy. We chain as stated in Section 211.

In keeping our field book we prefer the ordnance system of beginning at the bottom, and enter toward the top the offsets and inlets, stating at what line and distance we began, and on what; we note every fence and object that we pass over or near; leave a mark at every 10 chains, or 500 feet, and a small peg, numbered as in the field book.

398. See the diagram (figure 65).

Here we began 114 feet farther on line 1 than where we met our picket and peg at 3500 feet, and closed on line 3 at 870, where we had a peg and a long Isocetes' triangle dug out of the ground.

We write the bearings of lines as on line 3, and also take the angles, and mark them as above.

When there are Woods. Poles are fastened to trees, and made to project over the tops of all the surrounding ones. The position of these are observed or *Trigged*. The roads, walks, lakes, etc., in these woods can be surveyed by traversing, closing, from time to time, on the principal stations or Trig. points, but we require one line running to one of the forest poles, on which to begin our traverse, and continue, closing occasionally on the main lines and Trig. points.

399. *Traverse Surveying.* See Secs. 216, 217, 255.

The bearing of the most westerly station is taken. At Sec. 216 is given a good example where we begin at the W. line of the estate, making its bearing 0, and the land is kept on the right. There we began with zero and closed with 180, showing the work to close on the assumed bearing.

400. *To Protract these Angles at Sec. 216.* Draw the line A B through the sheet; let A be S, and B, N. On this lay off other lines parallel to A B, according to the number of bearings, size of protractor and scale. We lay down A B, then from B set off four, five, or more angles, L, K, I, and H. Lay the parallel ruler from A to L, draw a line and mark the distance A L of the second line on it. Lay the ruler from A to K, move one edge to pass through L, draw a line, mark the third line L K on it. Lay the ruler on A I, move the other edge to pass through K, draw the line K I, equal to the fourth line. Lay the ruler on A to H, make the other edge pass through I, and mark the fifth line, I H. Now, we suppose that we are getting too far from our first meridian, A B. We now remove the protractor to the next meridian, and select a point opposite H, and then lay off the bearings, G, F, E, D, etc.

Now, from this new station, which we will call X, we lay the parallel ruler to F and make the other edge pass through H, and set off the sixth line H G. Lay the parallel ruler from X to F, and move the other edge through G, and mark the seventh line, G F, and so proceed.

We have used a heavy circular protractor made by Troughton & Simms, of London, it is 12 inches diameter, with an arm of 10 inches, this, with a parallel ruler 4 feet long, enabled us to lay down lines and angles with facility and extreme accuracy.

401. By a table of tangents we lay off on one of the lines, A B, the distance, 20 inches, on a scale of 20 parts to the inch. Then find the nat. tangent to the required angle, and multiply it by 400 divisions of the scale, it will give the perp., B C, at the end of the base. Join A and C, and on A C lay off the given distance, and so proceed.

By this means we can, without a protractor, lay off any required angle.

REGISTERED SHEET FOR COMPUTATION.

Plans and Plats.	Triangles and Trapeziums.	1st side.	2d side.	3d side.	Contents in Chains.
Plat 1	Triangle A C B,	4454 lks	3398	4250	679.5032
	" A F D,	2234	1766	1684	143.0516
Division K	On line D F,	2234	10	98	0.0490
of		90	70	400	3.2000
Thos. Linskey's	Additives,	70	50	900	5.4000
Farm.			50	600	1.5000
			Total Additives,		158.2006
Div. K,	D F,		20	140	1400
"	Negatives, D F,	20	100	260	9600
		100	80	500	4.5000
			80	500	2.0000
					7.6000
					150.6006

Area, 15.06006 Acres.

There is always a content plat or plan made, which is lettered and numbered, and the Register Sheet made to correspond with it.

403. *Computation by Scale.* Where the plats or maps for content are drawn on a large scale, of 2 or 3 chains to the inch, we double up the sheet by bringing the edges together. Draw a line about an inch from the margin; on this line mark off every inch, and dot through; now open the sheet and draw corresponding lines through these dots; make a small circle around every fifth one, and number them in pencil mark.

Lines are now drawn through the part to be computed. Where every pair of lines meet the boundaries, the outlines are then equated with a piece of thin glass having a perpendicular line cut on it, or, better, with a piece of transparent horn. When all the outlines of the figure are thus equated, we measure the length in chains, which, multiplied by the chains to one inch, will give the content in square chains. This gives an excellent check on the contents found by triangulation or traversing. It will be very convenient to have a strip of long drawing paper, on the edge of which a scale of inches is made. We apply zero to the left-hand side of the first parallel, and make a mark, a, at the other end; then bring mark a to the left side of the second parallelogram, and make a mark, b, at the other end, and so continue to the end. Then apply the required scale to the fractional part, to find the total distance.

The English surveyors compute by triangulation on paper, and sometimes by parallels having a long scale, with a movable vernier and cross-hairs, to

equate the boundaries. We do not wish to be understood as favoring computation from paper.

The Irish surveyors always draw the parallel lines on the content plat or map, and mark the scale at three or four places, to test the expansion or contraction of the sheet during the construction or calculation. We prefer, when possible, 3 chains, or 200 feet, to an inch for estates in the country, and 40 feet for city property.

403a. *Division of Land.*

When the area A is to be cut off from a rectangular tract, the sides of which are a and b . Then corresponding sides of the tract,

$$S = \left\{ \frac{A}{a} \text{ and } \frac{A}{b} \right\} \text{ respectively, the required side, } S.$$

404. When the area A , = triangle ADE , is to be cut off from the triangle ABC , by a line parallel to one of its sides. (Fig. 66.)

Then triangle ABC : triangle ADE :: AB^2 : AD^2 .

405. From a given point, D , in the triangle, ABC , to draw a line, dividing it into two parts, as A and B . (See Fig. 66.) We find the angle ABC . By (Sec. 29,) $AD \times AE \times \frac{1}{2} \sin. A$ = area B

(i. e.) $AD \times AE \sin. A = 2B$

$$AE = \left\{ \frac{2B}{AD \sin. A} \right\}$$

NOTE.—We prefer this to any other complicated formula, in cutting off a given area from a quadrilateral or triangular field.

406. When the area B or A is to be cut off by the line DE , (Fig. 66,) making a given angle, C , with the line AB , let area = S .

Let the angle at $A = b$, that at $D = c$, and that at $E = d$, and AD , the required side.

$$AD = x, \text{ and } AE = \frac{\sin. c \cdot x}{\sin. d}$$

$$DE = \frac{\sin. b \cdot x}{\sin. d} \text{ but } AD \times DE \times \frac{1}{2} \sin. c = \text{Area} = B$$

$$\frac{\sin. b \cdot x}{\sin. d} \cdot \sin. c \cdot x = 2B$$

$$x^2 \sin. c \sin. b = 2B \sin. d$$

$$AD = x = \left\{ \frac{2B \sin. d}{\sin. c \sin. b} \right\}^{\frac{1}{2}}$$

From the value of x we find AE and DE from above.

Having AD and AE from these formulas, let us assume $AD = 10$ chains, and having found the value of AE by substituting 10 chains for x .

Multiply the numerical value of AE by 10 chains, and again by $\frac{1}{2}$ the natural sine of the angle DAB , let its area = s , L ,

Then s : S :: AD^2 : the required AE^2 ,

$$s : S :: 100 : AD^2$$

As s , S , and 100 are given, we have

$$AD = \left\{ \frac{100 S}{s} \right\}^{\frac{1}{2}}$$

This useful problem was proposed to us in Dublin, at our examination for Certified Land Surveyor, September, 1835, by W. Longfield, Esq., Civil Engineer and Surveyor.

NOTE.—When the given area is to be cut off by the shortest line, D E, in the triangle A D E, (Fig. 66.) then A D = D E.

407. When the area B is to be cut off by the line D E, starting from the point D. (Fig. 66.)

$$A D = \frac{2 B}{A E \sin. A} \quad A E = \frac{2 B}{A D \sin. A}$$

408. From the quadrilateral, (Fig. 67,) A B C D, to cut off the area A by the line F E, parallel to the side B C.

Produce the lines B A and C D to meet at G. Take the angles at B, C, D and A, and, as a check, take the angle G. Measure G D and G A. We have the area of the quadrilateral = A + B, and of the triangle G D A = C, and the line G B is given. By Sec. 404 we find the line A F or G E. For triangle G C B : triangle G F E :: G B² : G F² or :: G C² : G E².

By taking the square roots we find G F and G E.

409. To divide any quadrilateral figure into any number of equal parts, by lines dividing one of the sides into equal parts.

Let A B C D be the required figure, (see Fig. 70,) whose angles, sides, and areas are given, produce the the sides C D and B A to meet in E. As the angles at A and D are given, we find the angle E, and consequently the sides A E and D E, and area B of the triangle A E D. We have the distances E A, E F, and E G, and areas B + A = triangle E F K, and B + 2 A = triangle E G H : and by Sec. 29.

$$E K = \frac{F E \cdot \frac{1}{2} \times s}{B + A} \quad \text{and} \quad E H = \frac{B + 2 A}{G E \cdot \frac{1}{2} \sin. E}$$

410. If, in the last problem, it were required to have the sides B A and C D proportionally divided so as to give equal areas,

Let B A = a, C D = n a, A E = b, D E = c, and $\frac{1}{2} \sin. E = S$, and x = A F, then we have, by Sec.

$$(b + x)(c + n x) = \frac{A}{s} \quad \text{from which we have}$$

$$b c + (b n + c) x + n x^2 = \frac{A}{s}$$

$$x^2 + \left\{ \frac{b n + c}{n} \right\} x = \frac{A - b c s}{s} \quad \text{put} \quad \frac{b n + c}{n} = 2 m, \text{ and complete}$$

the square, and find the square root.

$$x \div 2 m x + m^2 = \frac{A - b c s + m^2}{s}$$

$$x = -m \pm \sqrt{\frac{A - b c s + m^2}{s}} = A F \text{ and } n \times A F = K D.$$

In like manner we find the points G and H.

411. *Contouring.* (Fig. 70a.)

Three points forming the vertexes of a triangle, A B C, whose altitudes above the sea, or *datum* line, are given. Lines are chained from A to B, B to C, and C to A, and stations marked at given distances, and contour points made at every change of altitude equal to 10, 20, or 30 feet. Lines are chained down the side of the hill, and connected with check-lines. The level of station *a* is carried around the hill, showing where the contour line intersects each chain line, to the place of beginning. Begin again at the next station, *b*, below, and proceed as in the above, and so to the lowest station. The contour lines will be the same as if water raised to different heights around the hill, leaving flood-line marks on the hill. The plotting is similar to triangular surveying. The shading of the hill requires practice.

Final Examination. When a plan is ready for final examination, tracings are taken, of such size as to cover a sheet of letter paper, or white card-board of that size, made to fit an ordinary portfolio. In the field, the examiner puts himself in the direction of two objects, such as fences or houses, and paces the distance to the nearest fixed corner, and, by applying his scale, he can find if it is correct; by these means he will detect all omissions and errors. He will be able to put on the topography of the survey. He generally finds pacing near enough to discover errors, but where errors occur, he chains the required distances.

412. In plotting in detail we use two scales, one flat, 12 inches long, but having the same scale on both sides, such as one chain to an inch, or three chains to an inch. The other scale is 2 inches long, for plotting the offsets graduated on both sides of the index in the middle, ends not beveled. If the index is one inch from each end, we draw a line parallel to the chain line, one inch distant. If the index is two inches, we draw it two inches from the line. On each end of the small scale we have, at two chains' distance, lines marked on it to check the reading on the large scale. At each end of the chain line, perpendiculars are drawn to find the point of beginning. The large scale in position, the small one slides along its edge to the respective distances where the offset can be set off on either side of the chain line.

413. *Finishing the Plans or Map.*

Indian ink, made fresh, to which add a little Prussian blue, expose to the sun or heat for a short time, to increase its blackness.

1 and 2. **FORESTS AND WOODS.**—*Jaune jonquille*, composed of gum gamboge, 8 parts; Prussian blue, 3 parts; water, 8 parts. The woods have not the trees sketched as heavily as forests.

3. **BRAMBLES, BRIARS, BRUSHWOOD.**—Same as No. 1, but lighter, by adding 4 parts of water.

4. **TURF-PIT.**—The water pits by Prussian blue, and the bog by sepia and blue.

5. **MEADOWS OR PRAIRIES.**—Prussian blue, 6 parts; gamboge, 2 parts; and water, 8 parts.

6. **SWAMP.**—In addition to dashes of water, we pass a light tint of Prussian blue.

7. **CULTIVATED LAND.**—Sepia, 6 parts; carmine, 1 part; gamboge, $\frac{1}{2}$ part.

8. **CULTIVATED LAND, BUT WET.**—Same as above, except that dashes of water are marked with blue.

9. TREES.—Same as 1 and 2; sketched on, and shaded with sepia.
 10. HEATH, FURZE.—Une teinte panachée, nearly green, and light carmine.

Teinte panachée is where two colors are taken in two brushes, and laid on carefully, coupled together.

11. MARSH.—The blue of water, with horizontal spots of grass green, or to No. 5 add 2 parts of water.

12. PASTURES.—To No. 5 add 4 parts of water.

13. VINEYARDS.—Carmine and Prussian blue in equal parts.

14. ORCHARDS.—Prussian blue and gamboge in equal parts.

15. UNCULTIVATED LAND, FILLED WITH WEEDS.—Same as No. 3.

16. FIELDS OR ENCLOSURES.—Walled in are traced in carmine, and if boarded, in sepia. Hedges, same as for forests, to which is added 2 parts of green meadow.

17. HABITATIONS.—A fine, pale tint of carmine, light, for massive buildings, and heavier for house of less importance.

18. VEGETABLE GARDENS.—Each ridge or square receives a different color of carmine, sepia, gamboge—the color for woods and meadows.

19. PLEASURE GARDENS, FLOWER GARDENS.—Are colored with meadow color, and wood color for massive trees; the alley, or walks, are white, or gamboge with a small point of carmine.

20. The colors used are, generally, Indian Ink, Carmine, Gamboge, Prussian Blue, Sepia, Minum, Vermillion, Emerald Green, Cobalt Blue, Indian Yellow.

414. *Levelling.*

THE ENGLISH AND IRISH BOARDS OF WORKS METHODS.

DISTANCES.					Height of Collimation.	Reduced Level.	Required Grade.	Cut.	Fill	REMARKS.
Left.	Centre.	Right.	Staff.	Soundings.						
60	10.00		2.44		97.03	94.59	90.60	3.99		Bench Mark. 94.59, at Station, 900 ft..
	10.50		8.84		97.03	88.19				
	11.00		2.83		97.03	94.20	90.50	3.70		
174	11.50		0.74		94.94	94.94				Bank of Creek. Middle of Creek.
	12.00		2.18	8.30	94.94	92.76		2.36		
	13.00		5.35		94.94	89.59				
	14.00		6.77		96.36	96.36	90.10		2.69	B.M., Peg and Stake in Meadow.
	15.00		3.57		96.36	92.79				
	15.00	120	6.32		96.36	90.04				
	15.00	136	8.27		96.36	88.09				
	15.70	130	2.63		96.36	93.73				

This method of keeping a field-book was used by the English and Irish Board of Works. Size of books 8 by 6¼ inches.

Many Engineers there kept their books thus: ruled from left to right, Back Sights, Fore Sights, Rise, Fall, Reduced Level, Distance, Permanent Reduced Levels, and Remarks. Book, 7½ by 5 inches.

414a. Colonel Frome, Royal English Engineer, in his Treatise on Surveying, gives, from left to right, Distances, B. S., F. S., +, -, Rise, Fall, Remarks. The columns Rise and Fall show the elevation at any station above *datum*, that assumed at the beginning.

Sir John McNeill's plan of showing the route for the road, and a profile of the cutting and filling on the same: the line is not less than a scale of 4 inches to 1 mile, and the vertical sections not less than 100 feet to an inch.

415. *Our Method.*

Left Centre	Dist. Centre	Right	Bench.	B. S.	Height of Collimation.	F. S.	Elevation.	Grade.	Cut.	Fill.	REMARKS.
	00		50.0	10.0	60.00	4.00	56.00	60.00		4.00	Bench Mark, nail in root of ash tree at the S.-W. corner of Thos. King's garden.
	100					4.50	55.50	60.30		4.80	
	200					9.00	51.00	60.60		9.60	
	300					11.50	48.50	60.90		11.40	NOTE.—At 697 came to bank of Flaskagh river, 2 feet above level of water; depth, 8 feet, width, 67 feet. (Here make a sketch showing depth of water for every 10 feet.)
	400		48.50	8.00	56.50	0.10	56.40	61.20		4.80	
	500					0.20	56.30	61.50		5.20	
	600					0.30	56.20	61.80		5.60	
	600	30				1.10	55.40	61.80		6.40	
	600	90				2.40	54.10	61.80		7.70	B. M. at elevation 38.50, opposite distance, 850 feet, on the S.-W. corner of John McCabe's residence.
40	600					0.40	56.10	61.80		5.70	
80	600					0.70	55.80	61.80		6.00	
	697					30.90	25.60	62.10		36.50	
	764					22.40	34.10	62.20		28.10	
	800					20.10	36.40	62.40		26.00	B. M., 38.00, door-step of Mr. Jas. Roger's school-house.
	850					18.00	38.50				
	900		38.50	6.50	45.00	5.40	39.60	62.70		23.10	
	1175		38.50			7.10	37.90	63.22		25.32	
	1175	170	38.50			7.00	38.00				

B. M. + Back sights = 50.0 + 10. + 8. + 6.50 = 74.50

F. sights at turning points 11.50 + 18.00 + 7.00 = 36.50

Proof, 38.00

416. *Levelling by Barometrical Observations.*

BAROMETRICAL MEASUREMENT OF HEIGHTS. — BAILY.

TABLE A.
THERMOMETERS IN OPEN AIR.

$t + t'$	A	$t + t'$	A	$t + t'$	A	$t + t'$	A	$t + t'$	A
1	4.74914	37	4.76742	73	4.78497	109	4.80183	145	4.81807
2	966	38	792	74	544	110	229	6	851
3	4.75017	39	842	75	592	1	275	7	895
4	069	40	891	76	640	2	321	8	939
5	120	41	941	77	688	3	367	9	983
6	172	42	990	78	735	4	412	150	4.82027
7	223	43	4.77039	79	783	5	458	1	071
8	274	44	089	80	830	6	504	2	115
9	326	45	138	81	878	7	550	3	159
10	377	46	187	82	925	8	595	4	203
11	428	47	236	83	972	9	641	5	247
12	479	48	285	84	4.79019	120	687	6	291
13	531	49	334	85	066	1	732	7	335
14	582	50	383	86	113	2	777	8	379
15	633	51	432	87	160	3	822	9	423
16	684	52	481	88	207	4	867	160	466
17	735	53	530	89	254	5	912	1	510
18	786	54	579	90	301	6	957	2	553
19	837	55	628	91	348	7	4.81002	3	596
20	888	56	677	92	395	8	047	4	640
21	938	57	726	93	442	9	092	5	683
22	989	58	774	94	488	130	137	6	727
23	4.76039	59	823	95	535	1	182	7	770
24	090	60	871	96	582	2	227	8	813
25	140	61	919	97	629	3	272	9	857
26	190	62	968	98	675	4	317	170	900
27	241	63	4.78016	99	722	5	362	1	943
28	291	64	065	100	768	6	407	2	986
29	342	65	113	101	814	7	452	3	4.83030
30	392	66	161	102	860	8	496	4	073
31	442	67	209	103	907	9	541	5	116
32	492	68	257	104	953	140	585	6	159
33	542	69	305	105	999	1	630	7	201
34	592	70	352	106	4.80045	2	675	8	244
35	642	71	400	107	091	3	719	9	287
36	4.76692	72	4.78449	108	4.80137	144	763	180	329

NOTE. t = temperature of the air at the lower station; t' = that at the upper station; A = correction for temperature, dependent on $t + t'$.

And for Table B.: r = temperature of mercury at the lower station; r' = that at the upper station; B = correction due to the mercury dependent on $r - r'$; C = correction for the latitude of the place; D = latitude; R = height of barometer at lower station; R' = height of barometer at upper station. For Table B. see next page.

BAROMETRICAL MEASUREMENT OF HEIGHTS.

TABLE B.

417. ATTACHED THERMOMETERS.

$r - r'$	B	$r - r'$	B	$r - r'$	B	Lat.	C
0	0.00000	20	0.00087	40	0.00174	0	0.00117
1	04	21	91	41	78	5	115
2	09	22	96	42	82	10	110
3	13	23	100	43	87	15	100
4	17	24	104	44	91	20	090
5	22	25	0.00109	45	95	25	075
6	26	26	13	46	0.00200	30	058
7	30	27	17	47	04	35	040
8	35	28	22	48	08	40	020
9	39	29	26	49	13	45	0.00000
10	43	30	30	50	17	50	9.99980
11	48	31	35	51	21	55	62
12	52	32	39	52	26	60	42
13	56	33	43	53	30	65	25
14	0.00061	34	48	54	34	70	10
15	65	35	52	55	39	75	9.99900
16	69	36	56	56	43	80	890
17	74	37	61	57	47	85	85
18	78	38	65	58	52	90	9.99883
19	0.00083	39	0.00169	59	0.00256		

418. Example from Colonel Frome's TRIGONOMETRICAL SURVEYING.

SURVEYING p. 110.				Correction for Capacity.	Corrected Barometer.	Diff. of Level.	Remarks.
Stations.	A F	D F	Bar.				
High Water Mark	61°	58'	30.405	.004	30.409		
Parade, Brompton Barracks, . . .	60°	57°	30.276	.002	30.278	116.6	

$$t + t' = 58 + 57 = 115. \quad \text{From Table A} = 4.80458$$

$$r - r' = 61 - 60 = 1.$$

$$B = 0.00004 = m$$

$$\text{Lat. } 51^\circ 24'$$

$$9.99974 = n$$

$$\text{Log. of } R = \text{Log. } 30.409$$

$$1.48300 = p$$

$$\text{Log. of } R' = \text{Log. } 30.278 \left\{ \begin{array}{l} + B \\ 00004 \end{array} \right.$$

$$1.48117 = q$$

$$D = 0.00183 = p - q$$

$$\text{Log. } D = 3.26245$$

$$A = 4.80458$$

$$C = 9.99974$$

$2.06677 = 116.6 = \text{altitude in feet, which was found by the spirit level to be 115 feet.}$

These Tables are from the Smithsonian Meteorological and Physical Tables, published in Washington, 1858.

In 1844, in Ottawa, Canada, Mrs. McDermott, in my absence, kept a record of numerous observations of the state of thermometer and mountain barometer, for Sir William Logan, Provincial Geologist, then making a tour of the valley of the River Ottawa and its tributaries. (See his Geological Reports.) The observations were made at the hours of 7, 9, noon, 3, and 6, to be used for the lower Station, at Montreal.

419. *To find the Altitude of one Station above another, from the Temperature of the Boiling of Water.*

This method is not so reliable as that by barometrical observations, although Colonel Sykes, in Australia, has found altitudes above the sea agree with those found by triangulation closer than he had anticipated.

There are very valuable tables in the Smithsonian Institute's Meteorological and Physical Tables—Tables XXIV, XXV, and XXVI—for finding the altitudes by this method.

Take any tin pot and lay a piece of board across the top, having groove to receive the thermometer, and a button or slide to keep it steady, at about two inches from the bottom. Take several observations, carefully noting them, and at the same time the temperature of the surrounding air. Use Fahrenheit's thermometer.

TABLE A.

TABLE B.

Boiling point of Water.	Pressure of Barometer in inches.	Altitude above the Sea, (from 30 in.)	Temperature of the air.	Multiplier.	Temperature of the air.	Multiplier.
185°	17.048	14.548	32°	1.000	62°	1.062
86	.423	13.977	33	2	3	64
87	.809	.408	34	4	4	66
88	18.195	12.843	5	6	5	69
89	.592	.280	6	8	6	71
190	.996	11.719	7	10	7	73
91	19.407	.161	8	12	8	75
92	.825	10.606	9	15	9	77
93	20.251	10.053	40	17	70	79
94	.685	9.502	1	1.019	71	81
95	21.126	8.953	2	21	2	1.083
96	.576	8.407	3	23	3	85
97	22.033	7.864	4	25	4	87
98	.498	7.324	45	27	75	89
99	.971	6.786	6	29	6	91
200	23.453	6.250	7	31	7	94
1	.943	5.716	8	33	8	96
2	24.442	5.185	9	35	9	98
3	.949	4.657	50	37	80	1.100
4	25.465	4.131	1	1.039	81	102
5	.990	3.607	2	42	2	104
6	26.523	3.085	3	44	3	106
7	.966	2.566	4	46	4	108
8	27.618	2.049	55	48	85	110
9	.980	1.534	6	50	6	112
10	28.751	1.021	7	52	7	114
11	.931	.509	8	54	8	116
12	29.922	0	9	56	9	117
13	.952	.507	60	58	90	121
214	31.194	1.013	61	1.060	91	123

NOTE.—The first two columns of Table B are from the Smithsonian Tables referred to above. The third column and Table B is from Col. Frome's Treatise on Trigonometrical Surveying, 3d edition, 1862.

Example.—Boiling point, upper station, 209°, lower, 202°; temperature of the air at upper station, 72°, lower, 84°, mean temperature, 78°.

From Table A, 209°. Alt., 1534 ft.
202. " 5185

Approximate height, 3651
Mean temperature, 78°. Multiplier from Table B, 1096

Product, 4001 ft.

Where the degrees are taken to tenths, then we interpolate.

419a. — *Continued from Sec. 410. Having one side, A B, and the adjacent angles, — to find the area*—Let the triangle A B C (Fig. 68,) be the triangle; the side A B = s , and the angles A and B are given, also the angle C.

$$\text{Sin. C} : S :: \text{sin. A} : B C = \frac{S \cdot \text{Sin. A}}{\text{Sin. C}}, \text{ and } A C = \frac{S \cdot \text{Sin. B}}{\text{Sin. C}}$$

$$\text{By Sec. 29. } S. = \frac{S \cdot \text{Sin. A}}{2 \text{ Sin. C}} \cdot \text{Sin. B} = \frac{S^2 \cdot \text{Sin. A} \cdot \text{Sin. B}}{2 \text{ Sin. C}} = \text{area.}$$

420. *From a point, P, within a given figure, to draw a line cutting off any part of it by the line F G.*—Let the figure I G B A E = the required area. (See Fig. 69.)

Let the A B C D E F the tract be plotted on a scale of ten feet to an inch from which we can find the position of the required line very nearly, with reference to the points B and E. Run the assumed line, A S, through P, finding the distances A P = m and P S = n , also the angles P T A, P S G, and that the tract A S B A T is too great, by the area d . Hence the true line, T P G, must be such that the triangle P S G - P A T = d .

Assume the angle S P G = P, then we find the angles T and G, and by Sec. 409 we find the areas of the triangles P S G and P A T. If the difference is not = d , again, calculate the sides P G and P T.

420a. *From the triangle A B C to cut off a given area (say one-third,) by a line drawn through the given point, D.* (Fig. 69a.)

Through D draw the line D G parallel to A C.

Now all the angles at A, B, and C are given, and the line D G is given to find the point I or H, through which, and the given point D, the line I D H will cut off the triangle A H I = to one-third the area of the triangle A B C. (Fig. 69a.) Make A F one-third of A C, then the triangle A B F = one-third of the triangle A B C, which is to be = to triangle A I H.

The triangle A H I = A H × A I × $\frac{1}{2}$ Nat. Sin. angle A.

The triangle A B F = A B × A F × $\frac{1}{2}$ Nat. Sin. A.

$$A H \times A I = A B \times A F, \text{ and } A I = \frac{A B \times A F}{A H} \text{ and as the triangles H G D and H A I are similar.}$$

$$H G : G D :: H A : A I : H A : \frac{A B \times A F}{H A}$$

H G : G D :: H A² : A B × A F, and by Euclid, 6-16,

$$G D \times H A^2 = H G \times A B \times A F = (H A - A G) \cdot A F \cdot A B$$

$$= H A \cdot A F \cdot A B - A G \cdot A F \cdot A B$$

$$\text{and } H A^2 = \frac{A B \cdot A F}{G D} \cdot H A - \frac{A B \cdot A F}{G D} \cdot A G. \text{ Let } P = \frac{A B \cdot A F}{G D}$$

Now we have P and A G given, to find A H or A I,

$$A H^2 = P \times H A - P \times A G.$$

$$H A^2 = P \times H A = - P \times A G. \text{ Complete the square}$$

$$H A^2 - P \times H A + \frac{P^2}{4} = \frac{P^2}{4} - P \times A G.$$

$$H A - \frac{P}{2} = \left\{ \frac{P^2}{4} - A G \times P \right\}^{\frac{1}{2}} \text{ Whence}$$

$$A H = \frac{1}{2} P + \left(\frac{1}{4} P^2 - A G \times P \right)^{\frac{1}{2}}, \text{ when D is inside the triangle.}$$

$$A H = \frac{1}{2} P + \left(\frac{1}{4} P^2 + A G \times P \right)^{\frac{1}{2}}, \text{ when } d \text{ is outside.}$$

421. Through the point D to draw the line G D E so that the triangle B G E will be the least possible. Through D draw H D I parallel to B C, make B H = H G, and draw G D E, which is the required line. Fig. 69a.

Geodetical Jurisprudence, p. 72, B.

Chief Justice Caton's opinion adds the following in support of established lines and monuments:—

Dreer v. Carskaddan, 48 Penn. State, 28.

Bartlett v. Hubert, 21 Texas, 8.

Thomas v. Patten, 13 Maine, 329.

To Divide pro rata.

After Bailey v. Chamblin, 20 Indiana, 33, add

Jones v. Kemble, 19 Wisconsin, 429.

Francoise v. Maloney, Illinois, April Term, 1871.

Withham v. Cutts, 4 Greanleaf R., Maine, 9.

309Me. After English Reports, 42, p. 307, add

Knowlton v. Smith, 36 Missouri, 520.

Jordan v. Deaton, 23 Arkansas, 704.

United States Digest, Vol. 27—where an owner points out a boundary, and allows improvements to be made according to it, cannot be altered when found incorrect by a survey.

For Laying Out Curves.

Example after p. 72. Let radius = 2000 feet; chord, 200; then tangential angle = $2^{\circ} 51' 57''$; versed sine at the middle, 2,503 feet. If the ground does not admit of laying off long chord of 200 feet, make $200 = 200$ half feet = 100, then for radius 4000 find the versed sine = 1,251 and the tang. angle = $1^{\circ} 25' 57''$. If we use the chord of 200 feet, half feet, or links, then we are to take the ordinates in Table C as feet, half feet, or links.

Canals.

The Illinois and Michigan locks are 128 feet long, 18 feet wide, and 6 feet deep, bottom 36, surface 60, tow-path 15, berm 7, tow-path above water, 3 feet.

The New York Canals.—Erie Canal, 363 miles long, when first built, 40 feet at top, 28 at bottom, 4 feet deep, 84 locks, each 90x15, lockage 688, 8 large feeders, 18 aqueducts. The aqueduct across the Mohawk is 1188 feet in length.

The Pennsylvania Canal—top 40, bottom 28, depth 4, locks 90x15, and some, 90x17.

The Ohio and Erie Canal—40 feet at top, 4 feet deep.

Rideau Canal, in Canada—129½ miles long, 53 locks, each 134x33.

Welland Canal, in Canada—locks, large enough to admit large vessels. It is now in progress of widening and deepening, so as to admit of the largest vessels that may sail on the lakes, and to correspond with the canals and lakes at Lachine, and on the River St. Lawrence.

CORRECTIONS.

Page 43, example 2, read the polygon a b c d e f g h. Fig. 38.

Page 72B53, *soda* No O read *soda* Na O.

72B55, 4th line, read felspathic.

72B111, after the 8th line insert Sir William Bland makes it as 17 to 13, egg-shaped.

72s, begin at 8th line from bottom and put mean base = $50 + 40 = 90$
50

4500

4100

Difference, square feet, 400

72r, in 4th equation from bottom read solidity $s = (A \times a + \sqrt{Aa}) \frac{1}{3}$

$$s = (D^2 + d^2 + Dd) \cdot 0.2618b.$$

$$s = \frac{D^3 - d^3}{D - d} \times \frac{1}{3} = \left\{ \frac{D^3 - d^3}{D - d} \right\} \times .2618b.$$

72w, in 3d equation from the bottom read Because $\frac{b}{2r}$

72H*, at 16th line from bottom, for $rS - bA$, read $rS + bQ$.

72H*10, at 14th from bottom, for product of the adjacent parts, read product of tan of adjacent parts.

72H*24, Sec. 388, for apparent, read mean.

72H*30, by the Helio-stat, insert after Helio-stat Fig. H.

72R*, under 82° , opposite 48, for 2921 put 9921.

104, under 2, opposite 12, make it 1.9300, and opposite 13 make it 1.9199.

110, under 2, opposite 12, put 1.8999.

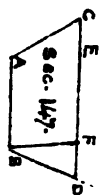
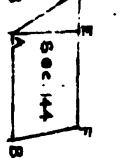
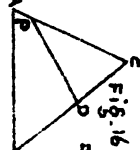
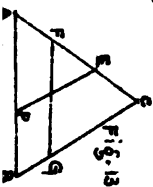
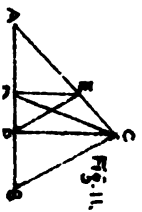
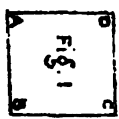
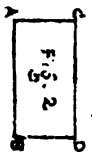
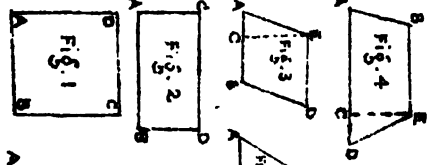
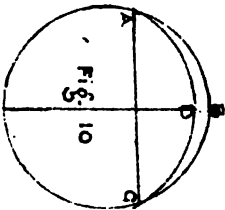
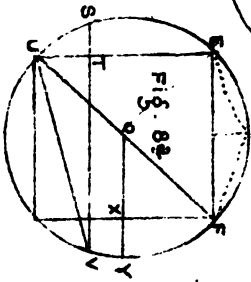
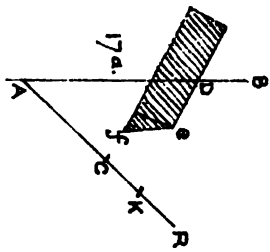
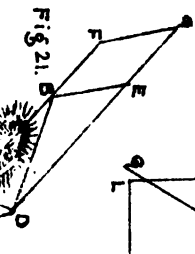
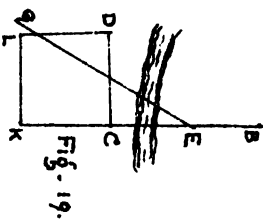
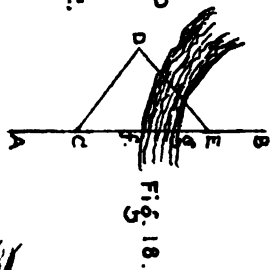
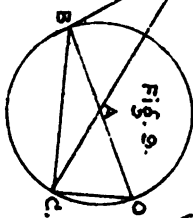
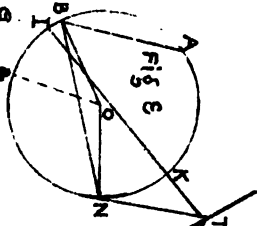
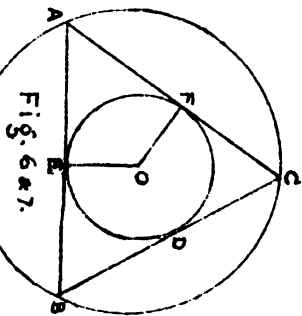
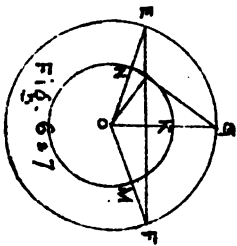
113, under 9 put 2.9722.

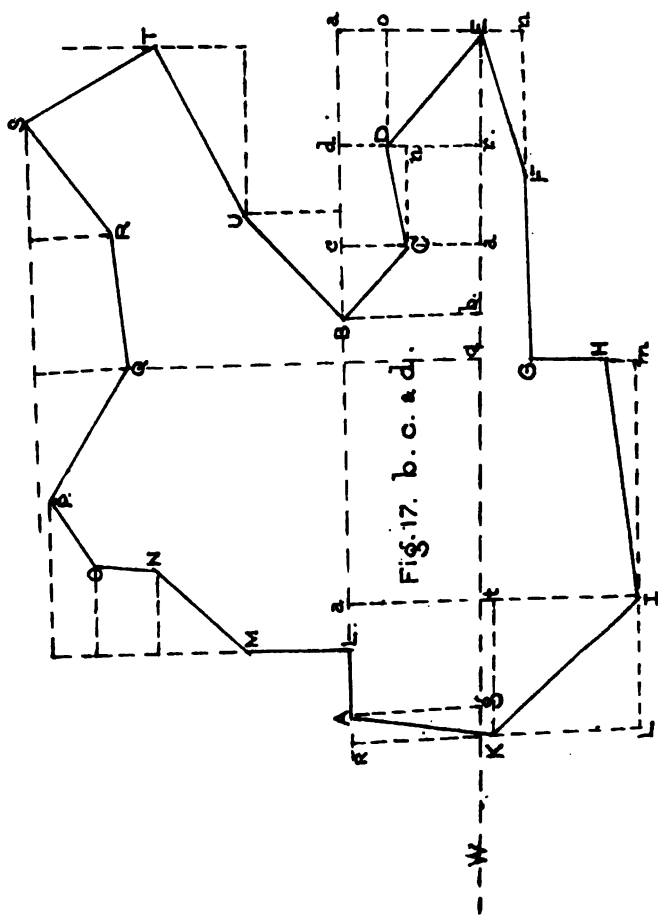
264, after the words *From the above we have* insert

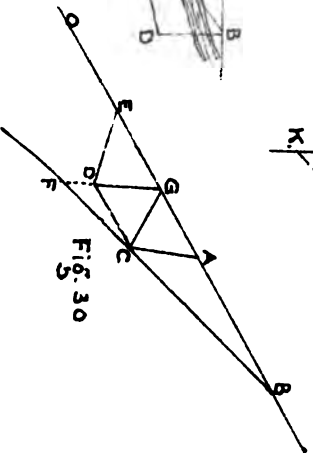
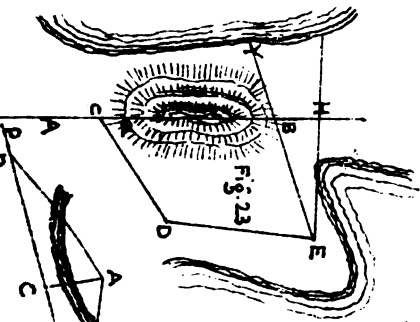
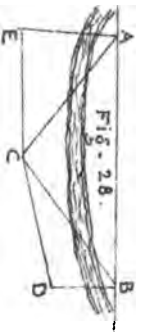
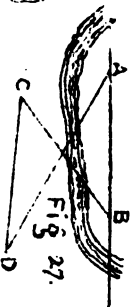
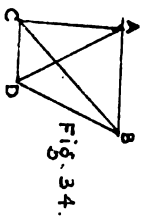
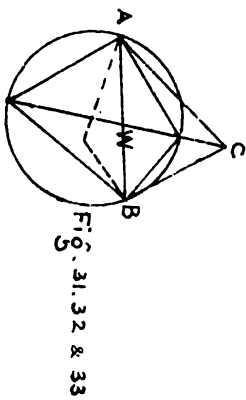
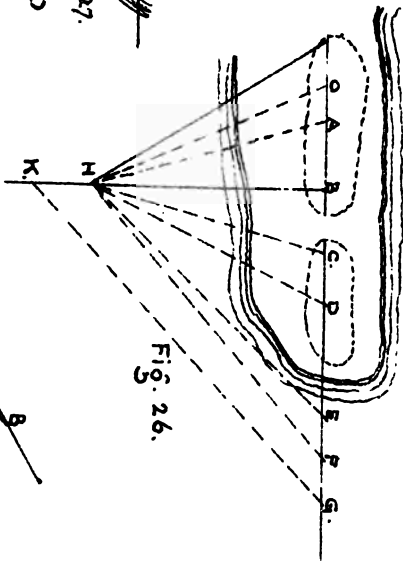
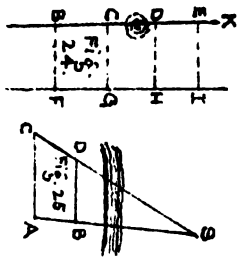
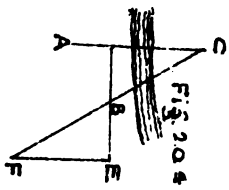
$$V \left\{ \frac{Ri}{0.000507 + \frac{0.00000647}{R}} \right\}^{\frac{1}{2}} \text{ in metres in terms of its radius.}$$

$$V \left\{ \frac{Di}{0.000309 + \frac{0.00000648}{D}} \right\}^{\frac{1}{2}} \text{ in terms of English feet.}$$

$$V \left\{ \frac{ri}{0.00007726 + \frac{0.00000162}{r}} \right\}^{\frac{1}{2}} \text{ in terms of its hydraulic mean depth.}$$







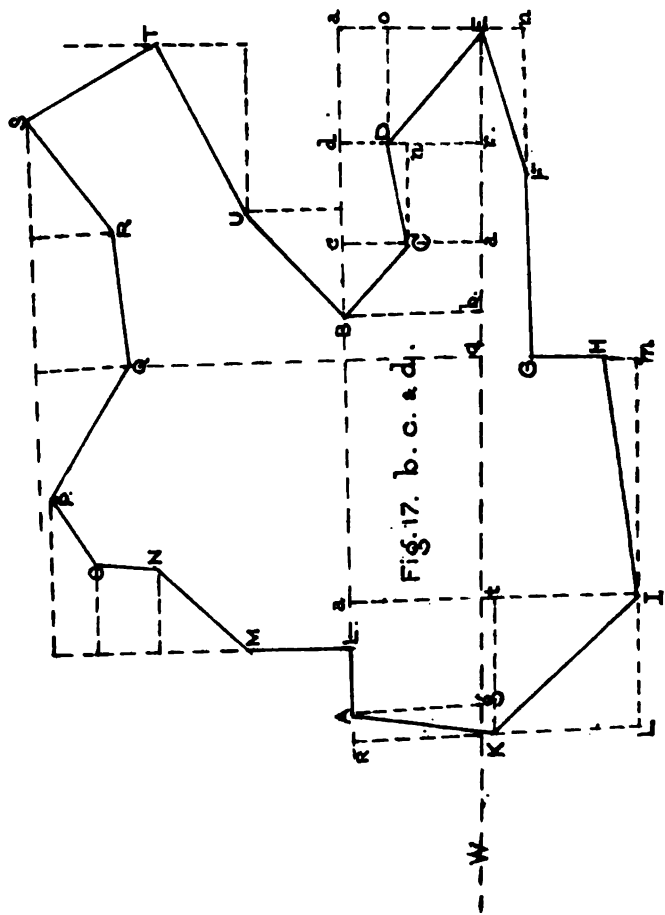


Fig. 17. b. o. a d.

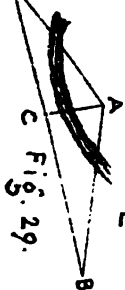
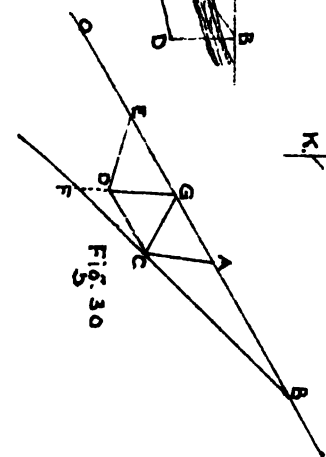
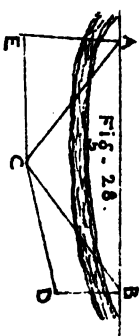
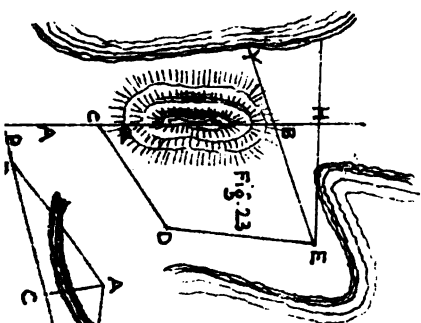
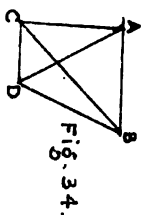
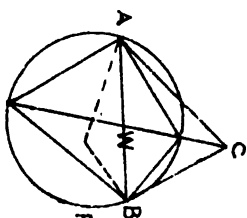
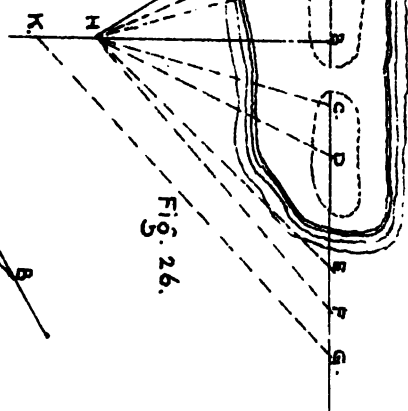
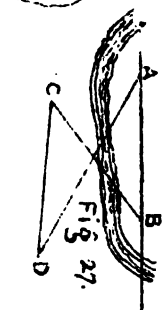
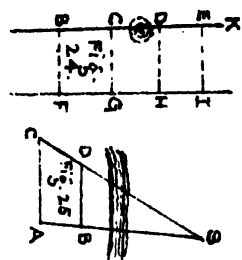


Fig. 36.37.

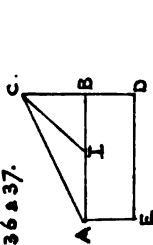


Fig. 38.

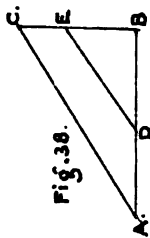
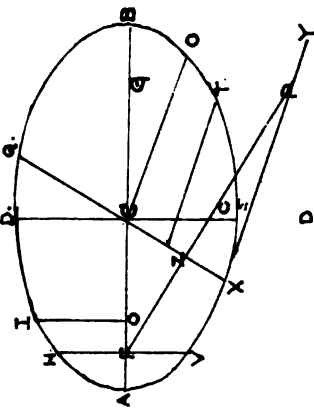
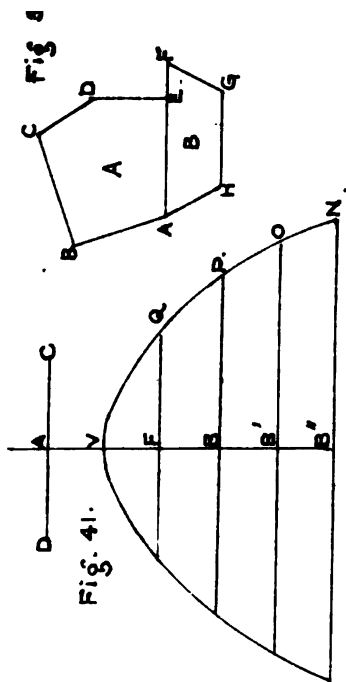


Fig. 39 & 40



५०६



Fi. 55

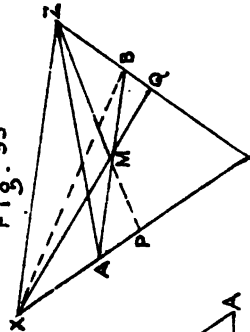


Fig. 42



Fig. 59.

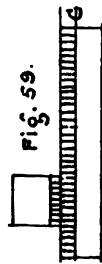


Fig. 53

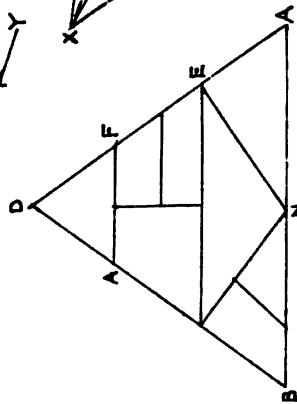
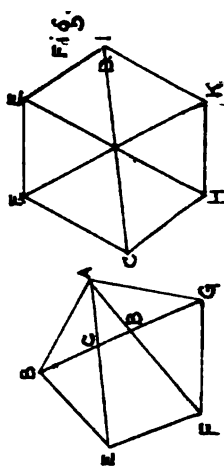
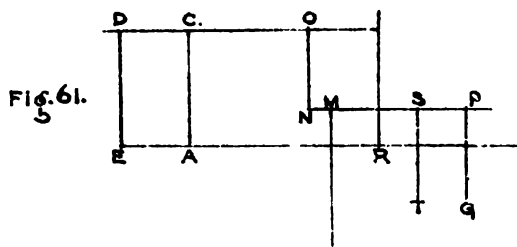
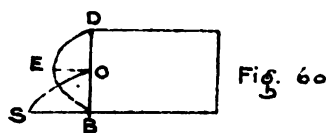


Fig. 56





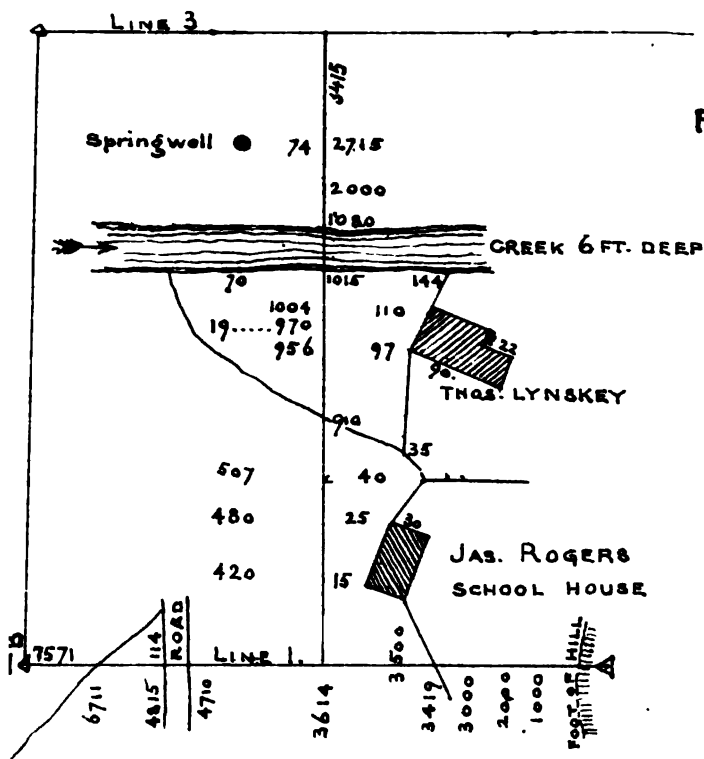


Fig. 65

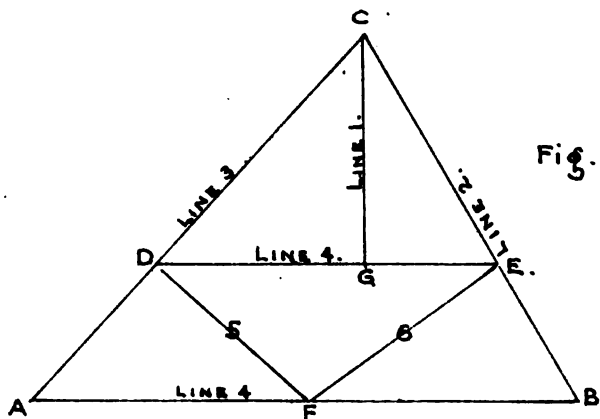


Fig. 64

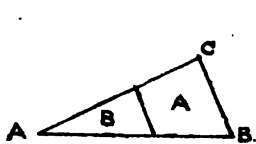


Fig. 66

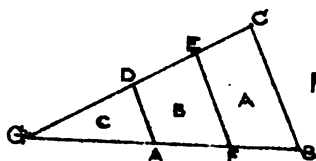
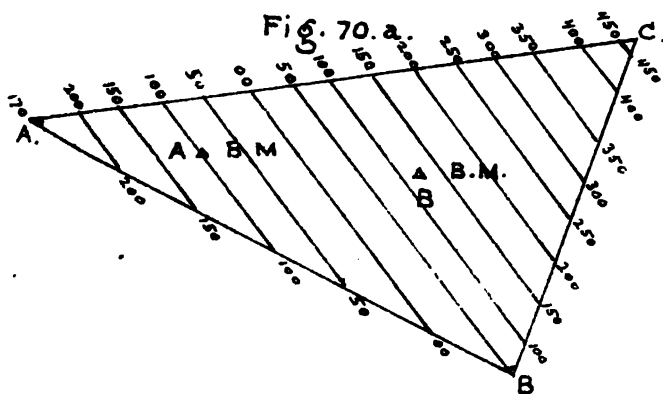
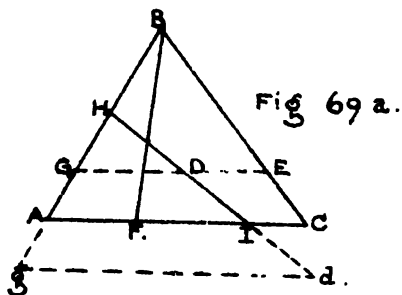
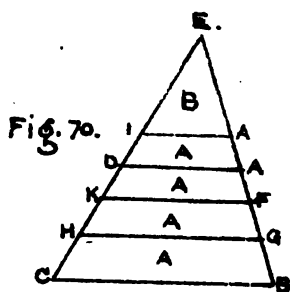
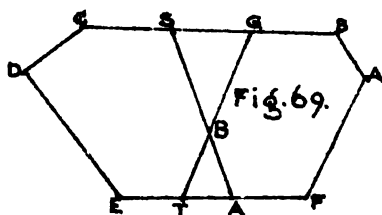
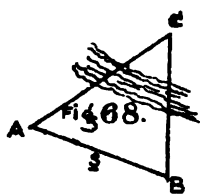


Fig. 67



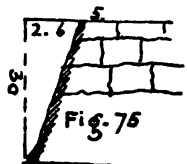
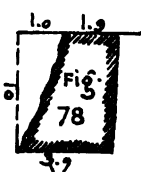
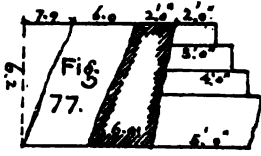
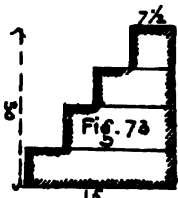
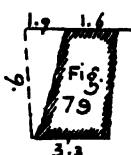
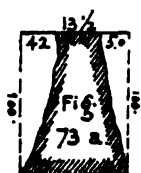
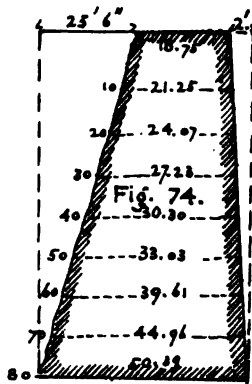
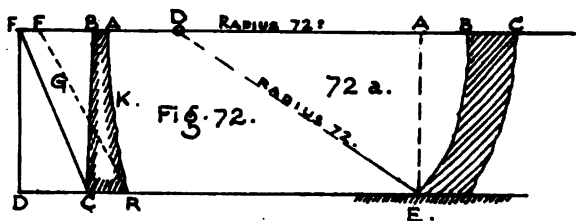
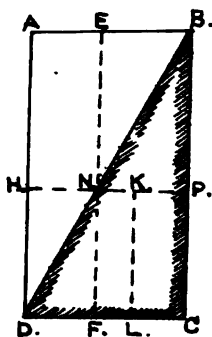
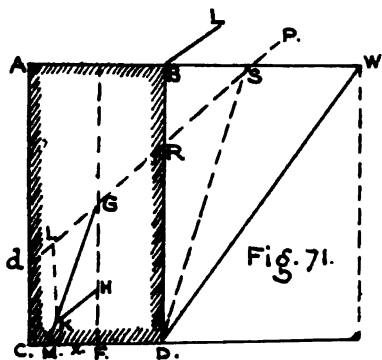


Fig. 80

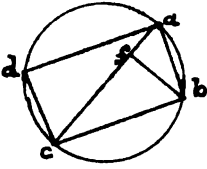
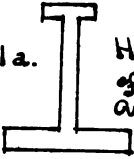


Fig. 80 a.



Threefold flanges
of equal areas.

Fig. 81 a.



Hodgkinson's section
of greatest strength.
Area of lower flang 6 times
that of its upper one.



Fig. 82



Fig. 83.

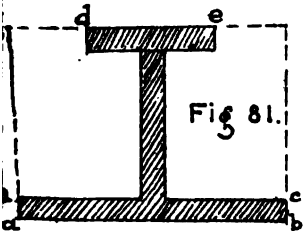
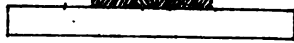


Fig. 81.

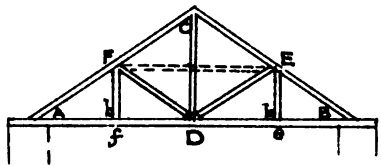


Fig. 83 a.

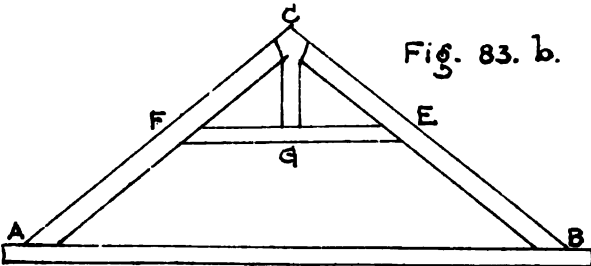
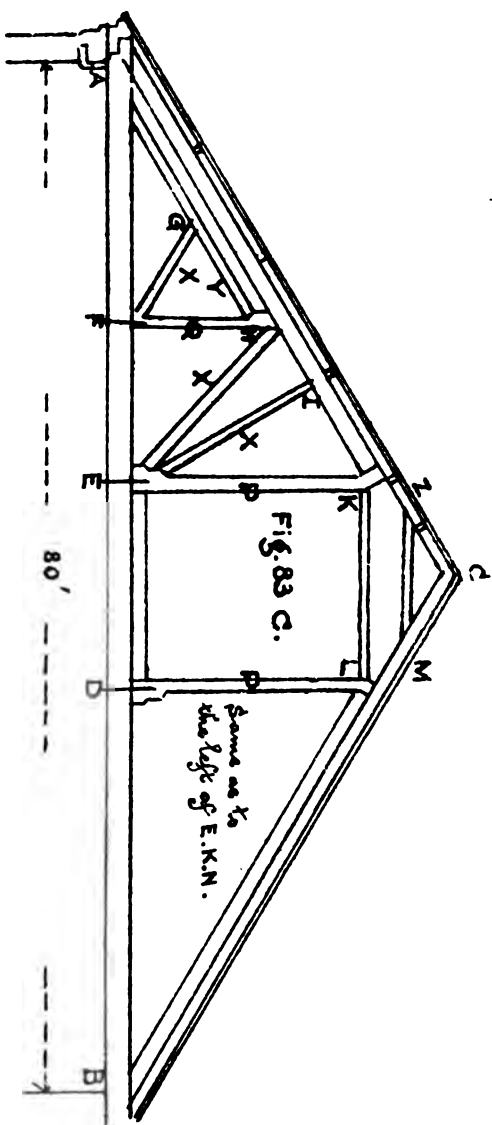
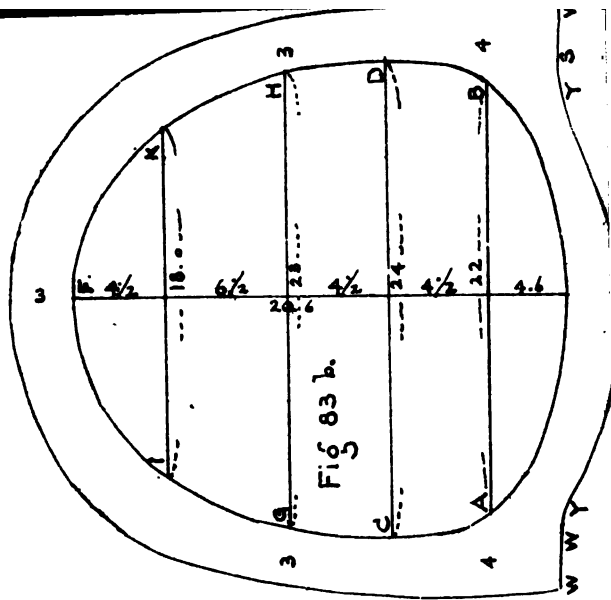
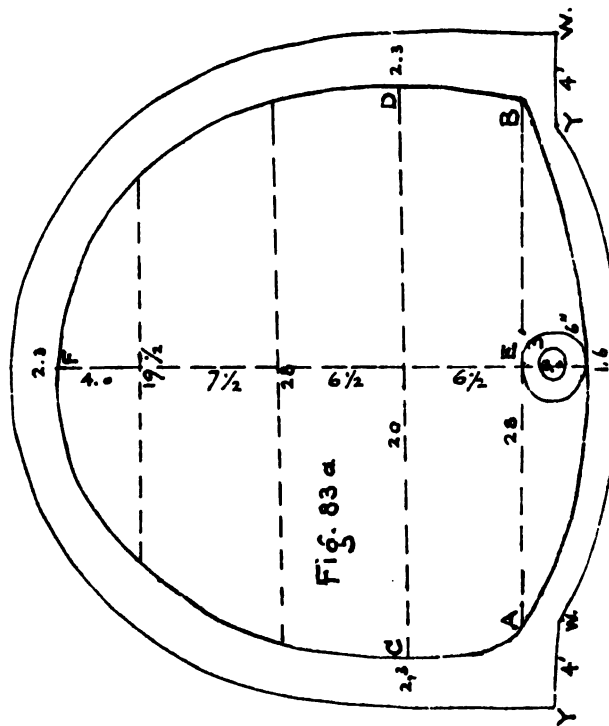


Fig. 83. b.





TUNNEL FOR ONE TRACK

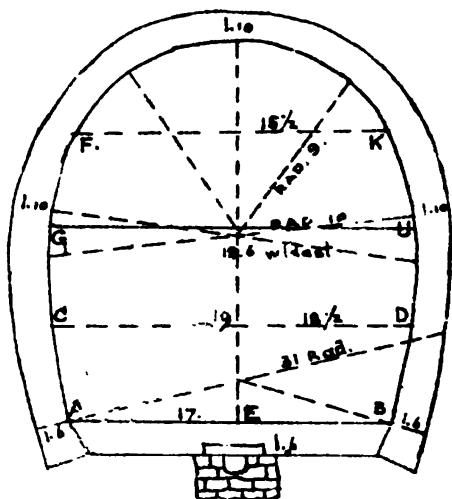


Fig. 83 d

HOOSAIC TUNNEL

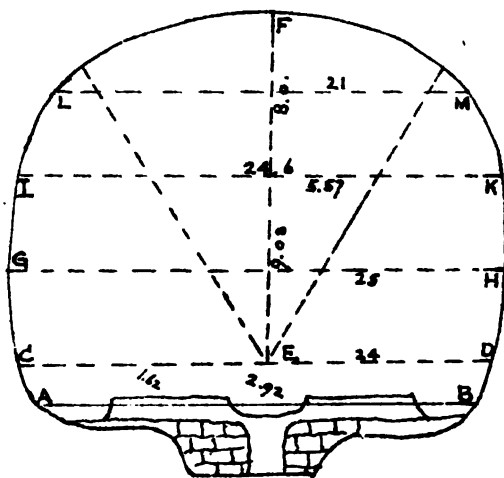


Fig. 83 c.

Fig. 85



Fig. 86

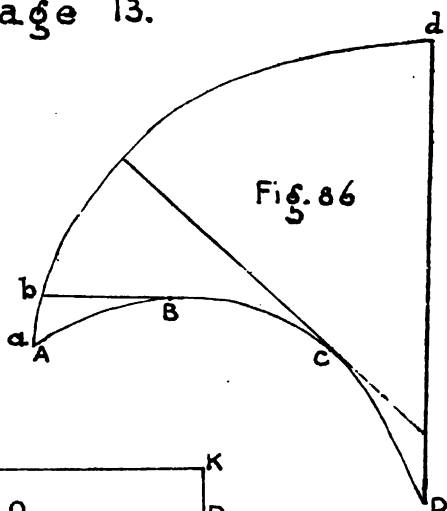


Fig. 87

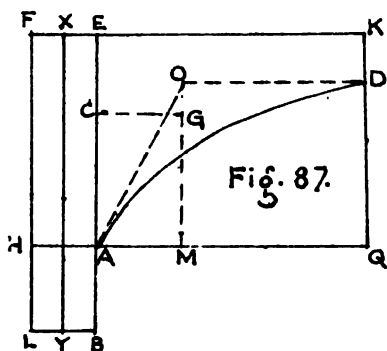


Fig. 88

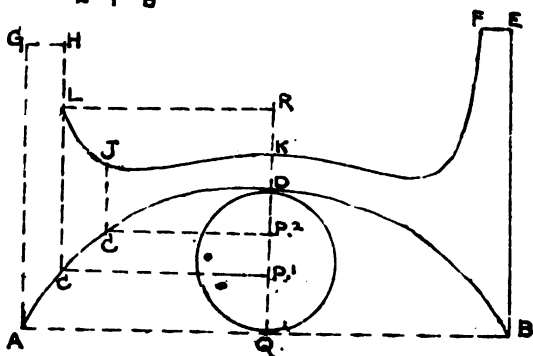
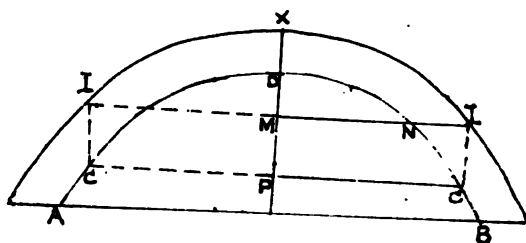


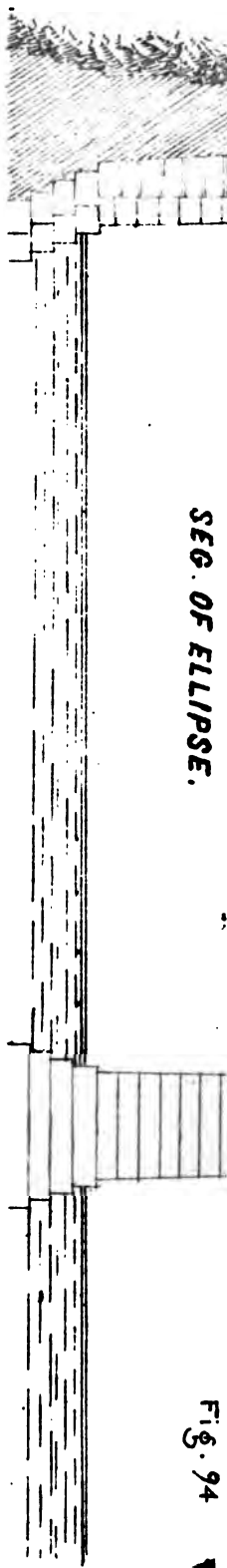
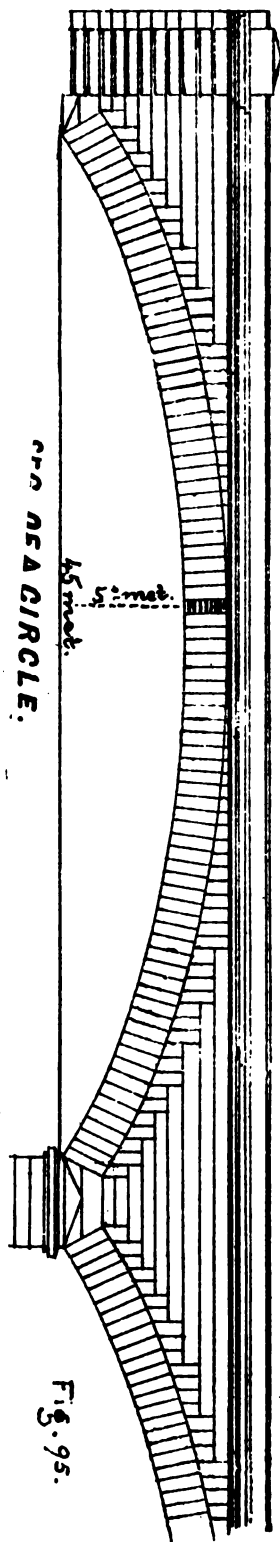
Fig. 89



DESIGN FOR THE NEW LONDON BRIDGE

Page 15.

by Joseph Gwilt, ARCHITECT. F.S.A.



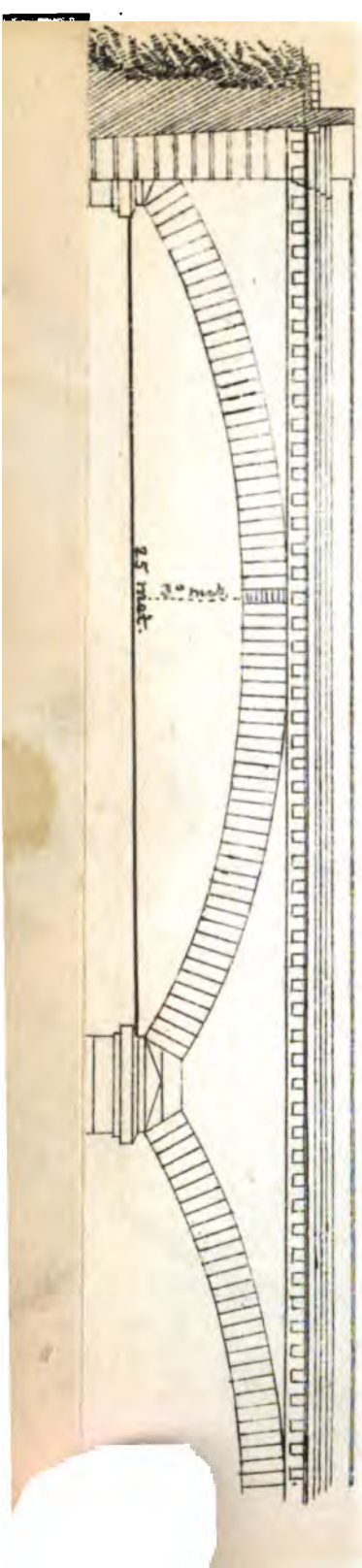
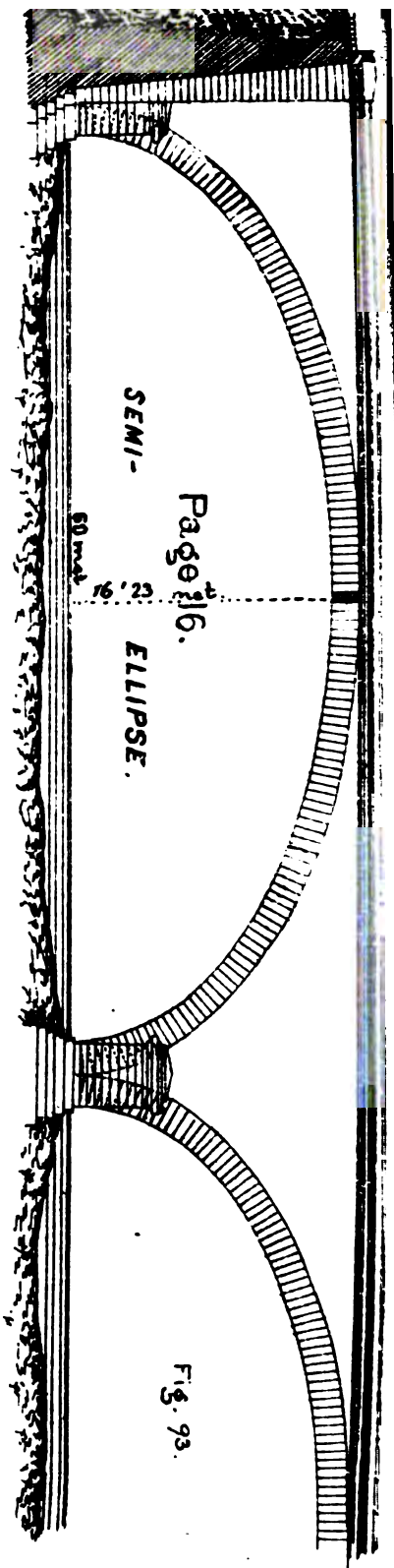


TABLE A.		NATURAL SINE.										72r*
1	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°		
0	000 00	017 45	034 90	052 84	069 76	087 16	104 53	121 87	139 17	156 50		
1	29	74	085 19	68	070 06	45	82	122 16	140 47	158 13		
2	58	018 08	48	92	84	74	105 11	124 45	143 13	161 35		
3	87	32	77	058 21	63	088 08	40	74	140 04	157 57		
4	001 16	62	086 06	50	92	81	69	128 02	146 13	164 18		
5	45	91	85	79	071 21	60	97	81	61	55		
6	75	019 20	64	054 08	50	89	106 26	125 04	143 54	162 28		
7	002 04	49	93	87	79	089 18	55	89	141 19	159 58		
8	38	78	087 28	66	072 08	47	84	124 18	142 52	160 52		
9	62	020 07	52	95	87	76	107 18	126 47	145 13	163 51		
10	91	86	81	055 24	66	090 05	42	76	142 05	160 50		
11	008 20	65	088 10	53	95	84	71	125 04	144 49	163 49		
12	49	94	89	82	078 24	63	108 00	128 29	147 18	165 48		
13	78	021 23	68	056 11	53	92	29	62	92 47	167 47		
14	004 07	52	97	40	82	091 21	58	91	143 20	169 46		
15	86	81	089 26	69	074 11	50	87	126 20	145 45	167 45		
16	65	022 11	55	98	40	79	109 16	129 49	148 44	170 44		
17	95	40	84	057 27	69	092 08	45	78	144 07	169 43		
18	005 24	69	040 18	56	98	87	73	127 06	146 42	171 42		
19	58	98	42	85	075 27	66	110 02	130 02	148 41	173 41		
20	82	028 27	71	058 14	56	95	81	64	98 40	175 40		
21	006 11	56	041 00	44	85	098 24	60	98	145 22	177 39		
22	40	85	29	78	076 14	53	89	128 22	150 18	179 38		
23	69	024 14	59	059 02	43	82	111 18	131 18	149 37	181 37		
24	98	48	88	81	72	094 11	47	80	146 08	183 36		
25	007 27	72	042 17	60	077 01	40	76	129 08	148 35	185 35		
26	56	025 01	46	89	80	69	112 05	132 05	150 34	187 34		
27	85	80	75	060 18	59	98	84	66	95 33	189 33		
28	008 14	60	048 04	47	88	095 27	63	95	147 28	191 32		
29	44	89	83	76	078 17	56	91	130 24	152 31	193 31		
30	78	026 18	62	061 05	46	85	118 20	133 20	154 30	195 30		
31	009 02	47	91	84	75	096 14	49	81	148 10	197 29		
32	81	76	044 20	63	079 04	42	78	131 10	156 28	199 28		
33	60	027 05	49	92	83	71	114 07	134 07	158 27	201 27		
34	90	84	78	062 21	62	097 00	86	68	96 26	203 26		
35	010 18	63	045 07	50	91	29	65	97	149 25	205 25		
36	47	92	86	79	080 20	58	94	132 26	159 24	207 24		
37	76	028 21	65	068 08	49	87	115 23	135 23	161 23	209 23		
38	011 05	50	94	87	78	098 16	52	83	150 11	211 22		
39	84	79	046 23	66	081 07	45	80	138 12	163 10	213 21		
40	64	029 08	53	95	86	74	116 09	141 09	165 10	215 20		
41	98	88	82	064 24	65	099 08	88	70	97 19	217 19		
42	012 22	67	047 11	53	94	82	67	99	151 28	219 18		
43	51	96	40	82	082 23	61	96	134 27	167 17	221 17		
44	80	080 25	69	065 11	52	90	117 25	137 25	169 16	223 16		
45	018 09	54	98	40	81	100 19	54	85	152 12	225 15		
46	38	83	048 27	69	083 10	48	88	135 14	171 14	227 14		
47	67	081 12	56	98	89	77	118 12	148 12	173 13	229 13		
48	96	41	85	066 27	68	101 06	40	72	99 12	231 12		
49	014 25	70	049 14	56	97	85	69	136 00	175 11	233 11		
50	54	99	43	85	084 28	64	98	29	56 10	235 10		
51	88	082 28	72	067 14	55	92	119 27	141 27	177 9	237 9		
52	015 18	57	050 01	43	84	102 21	56	87	154 14	239 8		
53	42	86	80	73	085 18	50	86	137 16	179 16	241 7		
54	71	088 16	59	068 02	42	79	120 14	144 14	181 6	243 6		
55	016 00	45	88	81	71	103 08	43	78	155 00	245 5		
56	29	74	051 17	60	086 00	87	71	138 02	183 4	247 4		
57	58	084 08	46	89	29	66	121 00	81	57 8	249 3		
58	87	82	76	069 18	58	95	29	60	86 2	251 2		
59	017 16	61	052 05	47	87	104 24	58	89	156 15	253 1		
60	45	90	84	76	087 16	53	87	139 17	185 0	255 0		
	89°	88°	87°	86°	85°	84°	83°	82°	81°			
NATURAL COSINE.												

NATURAL SINE.											TABLE A.	
72.5°	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	
0	156 48	178 65	190 81	207 91	224 95	241 92	258 82	275 64	292 37	309 00	325 59	
1	72	98	191 09	208 20	225 28	242 20	259 10	276 00	293 00	310 00	326 59	
2	157 01	174 22	88	48	52	49	88	276 20	293 58	311 00	327 58	
3	80	51	67	77	80	77	66	48	298 21	315 57	331 56	
4	58	79	95	209 05	226 08	243 05	260 00	277 04	294 04	311 04	328 03	
5	87	175 08	192 24	88	87	88	260 22	277 04	294 04	311 04	328 03	
6	158 16	87	52	62	65	62	50	81	294 04	311 04	328 03	
7	45	65	81	90	98	90	79	59	325 58	342 57	359 56	
8	78	94	198 09	210 19	227 22	244 18	261 07	278 15	295 15	312 15	329 14	
9	159 02	176 23	88	47	50	46	85	278 15	295 15	312 15	329 14	
10	81	51	66	76	78	74	63	48	295 15	312 15	329 14	
11	59	80	95	211 04	228 07	245 08	262 19	279 27	296 27	313 27	330 26	
12	88	177 08	194 23	82	85	81	262 19	279 27	296 27	313 27	330 26	
13	160 17	87	52	61	68	59	47	279 27	296 27	313 27	330 26	
14	46	66	81	89	92	87	75	55	296 27	313 27	330 26	
15	74	94	195 09	212 18	229 20	246 16	263 08	280 11	297 11	314 11	331 10	
16	161 08	178 28	88	46	48	44	81	280 11	297 11	314 11	331 10	
17	82	52	66	75	77	72	59	89	297 11	314 11	331 10	
18	60	80	96	218 08	230 05	247 00	264 15	281 23	298 23	315 23	332 22	
19	89	179 09	196 28	81	88	28	264 15	281 23	298 23	315 23	332 22	
20	162 18	87	52	60	62	56	48	281 23	298 23	315 23	332 22	
21	47	66	80	88	90	84	71	50	298 23	315 23	332 22	
22	75	95	197 09	214 17	231 18	248 18	265 00	282 06	299 06	316 06	333 05	
23	163 04	180 23	87	45	46	41	28	282 06	299 06	316 06	333 05	
24	83	52	66	74	75	69	56	84	299 06	316 06	333 05	
25	61	81	94	215 02	232 08	249 07	266 12	283 18	300 18	317 18	334 17	
26	90	181 09	198 28	80	81	249 25	266 12	283 18	300 18	317 18	334 17	
27	164 19	88	51	59	60	54	40	288 18	301 18	318 18	335 17	
28	47	66	80	87	88	82	68	46	300 18	318 18	335 17	
29	76	95	199 08	216 16	233 16	250 10	267 15	284 22	301 22	319 22	336 21	
30	165 05	182 24	87	44	45	38	267 15	284 22	301 22	319 22	336 21	
31	83	52	65	72	73	66	52	29	301 22	319 22	336 21	
32	62	81	94	217 01	234 01	251 22	268 08	285 13	302 13	320 13	337 12	
33	91	183 09	200 22	29	29	251 22	268 08	285 13	302 13	320 13	337 12	
34	166 20	88	51	58	58	51	36	285 13				
35	48	67	79	86	86	79	64	41	302 09	320 25		
36	77	95	201 08	218 14	235 14	252 07	269 20	286 25	303 25	321 25		
37	167 06	184 24	86	48	42	35	269 20	287 00	304 00	322 00		
38	84	52	65	71	71	63	48	286 25	303 25	321 25		
39	63	81	98	99	99	91	76	52	303 25	321 25		
40	92	185 09	202 22	219 28	236 27	253 20	270 04	287 08	304 08	322 08		
41	168 20	88	50	56	56	48	32	287 08	304 08	322 08		
42	49	67	79	85	84	76	60	44	304 08	322 08		
43	78	96	208 07	220 13	237 12	254 04	271 16	288 20	305 20	323 20		
44	169 06	186 24	86	41	40	32	271 16	288 20	305 20	323 20		
45	85	52	64	70	69	60	44	288 20	305 20	323 20		
46	64	81	98	98	97	88	72	47	305 20	323 20		
47	92	187 10	204 21	221 26	238 25	255 16	272 00	289 08	306 08	324 08		
48	170 21	88	50	55	58	45	28	289 08	306 08	324 08		
49	50	67	78	83	82	78	56	81	306 08	324 08		
50	78	95	205 07	222 12	239 10	256 01	273 12	290 16	307 16	325 16		
51	171 07	188 24	85	40	38	29	278 12	291 26	308 26	326 26		
52	86	52	63	68	66	57	40	290 16	308 26	326 26		
53	64	81	92	97	95	85	68	42	307 08	325 08		
54	93	189 10	206 20	223 25	240 23	257 18	274 24	291 26	308 26	326 26		
55	172 22	88	49	53	51	41	274 24	291 26	308 26	326 26		
56	50	67	77	82	79	70	52	291 26	308 26	326 26		
57	79	95	207 08	224 10	241 08	258 00	275 08	292 09	309 09	327 09		
58	178 08	190 24	84	38	36	25	275 08	292 09	309 09	327 09		
59	86	52	63	67	64	54	36	292 09	309 09	327 09		
60	65	81	91	95	92	82	64	37	309 09	327 09		
7	80°	79°	78°	77°	76°	75°	74°	73°	72°	71°	70°	
NATURAL COSINE.												

72L*		NATURAL SINE.										TABLE A.	
7	27°	28°	29°	30°	31°	32°	33°	34°	35°	36°	37°	38°	
0	453 99	469 47	484 81	500 00	516 04	529 92	544 64	559 19	573 58	586 60	599 19	611 59	
1	454 25	73 485	06 25	29 530	17 88	43 68	574 06	58 58	72 51	86 53	100 55	114 57	
2	51 99	32 50	54 64	68 79	82 91	96 10	110 18	124 26	138 34	152 42	166 50	180 58	
3	77 470	24 57	76 79	91 91	106 10	121 26	136 42	151 58	167 14	182 30	197 46	212 62	
4	455 08	50 83	501 01	516 04	531 15	546 10	561 12	576 14	591 16	606 18	621 20	636 22	
5	29 76	486 08	26 51	78 65	93 79	108 93	124 07	139 21	154 35	169 49	185 03	200 17	
6	54 471	01 34	51 63	66 78	81 93	97 07	112 21	127 35	142 49	158 03	173 17	188 31	
7	80 27	59 76	74 91	89 10	104 24	119 38	134 52	150 06	165 20	180 34	195 48	211 02	
8	456 06	53 84	502 01	517 08	532 14	547 19	562 24	577 29	592 34	607 39	622 44	637 49	
9	32 78	487 10	27 52	77 61	92 76	107 91	123 06	138 21	153 36	168 51	184 06	199 21	
10	58 472	04 35	52 53	67 68	82 83	97 98	113 13	128 28	143 43	158 58	174 13	189 28	
11	84 29	61 77	76 92	91 10	106 25	121 40	136 55	152 10	167 25	182 40	197 55	213 10	
12	457 10	55 86	503 02	518 08	533 14	548 19	563 24	578 29	593 34	608 39	623 44	638 49	
13	36 81	488 11	27 52	77 61	92 76	107 91	123 06	138 21	153 36	168 51	184 06	199 21	
14	62 473	06 37	52 53	67 68	82 83	97 98	113 13	128 28	143 43	158 58	174 13	189 28	
15	87 32	62 77	77 92	92 10	107 25	122 40	137 55	153 10	168 25	183 40	198 55	214 10	
16	458 18	58 88	504 08	519 02	534 11	549 16	564 21	579 26	594 31	609 36	624 41	639 46	
17	39 83	489 18	28 53	78 62	93 77	108 92	124 07	139 22	154 37	169 52	185 07	200 12	
18	65 474	09 38	58 79	73 94	88 10	103 25	118 40	133 55	149 10	164 25	179 40	194 55	
19	91 34	64 79	79 94	94 10	109 25	124 40	139 55	155 10	170 25	185 40	200 55	216 10	
20	459 17	60 89	505 08	520 02	535 09	550 14	565 19	580 24	595 29	610 34	625 39	640 44	
21	42 86	490 14	28 53	78 62	93 77	108 92	124 07	139 22	154 37	169 52	185 07	200 12	
22	68 475	11 40	53 51	68 66	83 81	98 96	114 11	129 26	144 41	159 56	175 11	190 26	
23	94 37	65 80	78 95	93 10	108 25	123 40	138 55	154 10	169 25	184 40	199 55	215 10	
24	460 20	62 90	506 08	521 01	536 07	551 12	566 17	581 22	596 27	611 32	626 37	641 42	
25	46 88	491 16	28 53	78 62	93 77	108 92	124 07	139 22	154 37	169 52	185 07	200 12	
26	72 476	14 41	54 51	69 66	84 81	99 96	115 11	130 26	145 41	160 56	176 11	191 26	
27	97 39	66 83	79 98	94 13	109 28	124 43	139 58	155 13	170 28	185 43	200 58	216 13	
28	461 23	65 92	507 04	522 00	537 05	552 10	567 15	582 20	597 25	612 30	627 35	642 40	
29	49 49	90 492	17 29	75 75	90 90	105 105	120 120	135 135	150 150	165 165	180 180	195 195	
30	75 477	16 42	54 50	69 65	84 80	99 95	114 100	129 115	144 130	159 145	174 160	189 175	
31	462 01	41 68	79 79	94 94	109 109	124 124	139 139	154 154	169 169	184 184	199 199	214 214	
32	26 67	93 508	04 523	00 79	15 79	30 79	45 79	60 79	75 79	90 79	105 79	120 79	
33	52 93	493 18	29 24	588 04	66 567	18 41	27 32	36 23	45 14	54 05	63 00	72 00	
34	78 478	18 44	54 49	69 54	84 59	99 64	114 69	129 74	144 79	159 84	174 89	189 94	
35	463 04	44 69	79 74	94 69	109 64	124 59	139 54	154 49	169 44	184 39	199 34	214 29	
36	30 69	94 509	04 99	77 77	92 77	107 77	122 77	137 77	152 77	167 77	182 77	197 77	
37	55 95	494 19	29 24	589 02	63 568	08 43	17 34	26 25	35 16	44 07	53 00	62 00	
38	81 479	20 45	54 48	69 53	84 58	99 63	114 68	129 73	144 78	159 83	174 88	189 93	
39	464 07	46 70	79 73	94 68	109 63	124 58	139 53	154 48	169 43	184 38	199 33	214 28	
40	33 71	95 510	04 98	75 75	90 75	105 75	120 75	135 75	150 75	165 75	180 75	195 75	
41	58 97	495 21	29 29	525 22	540 00	60 569	04 43	13 34	22 25	31 16	40 07	49 00	
42	84 480	22 46	54 47	69 52	84 57	99 62	114 67	129 72	144 77	159 82	174 87	189 92	
43	465 10	48 71	79 72	94 67	109 62	124 57	139 52	154 47	169 42	184 37	199 32	214 27	
44	36 86	73 96	511 04	97 78	33 78	48 78	63 78	78 78	93 78	108 78	123 78	138 78	
45	61 99	496 22	29 29	526 21	541 00	61 570	00 43	10 34	19 25	28 16	37 07	46 00	
46	87 481	24 47	54 46	69 51	84 56	99 61	114 66	129 71	144 76	159 81	174 86	189 91	
47	466 13	50 72	79 71	94 66	109 61	124 56	139 51	154 46	169 41	184 36	199 31	214 26	
48	38 89	75 97	512 04	96 71	31 71	46 71	61 71	76 71	91 71	106 71	121 71	136 71	
49	64 482	01 497	28 29	527 20	96 54	95 585	19 11	28 02	37 00	46 00	55 00	64 00	
50	90 26	48 54	45 542	20 78	571 19	43 10	52 01	61 00	70 00	79 00	88 00	97 00	
51	467 16	52 73	79 70	94 65	109 60	124 55	139 50	154 45	169 40	184 35	199 30	214 25	
52	42 77	98 513	04 94	69 69	84 69	99 69	114 69	129 69	144 69	159 69	174 69	189 69	
53	67 483	03 498	24 29	528 19	93 50	91 586	14 7	23 06	32 00	41 00	50 00	59 00	
54	98 28	49 54	44 543	17 75	572 15	87 6	96 00	105 00	114 00	123 00	132 00	141 00	
55	468 19	54 74	79 69	94 64	109 59	124 54	139 49	154 44	169 39	184 34	199 29	214 24	
56	44 79	99 514	04 93	66 66	81 66	96 66	111 66	126 66	141 66	156 66	171 66	186 66	
57	70 484	05 499	24 29	529 18	91 50	89 587	08 4	17 03	26 00	35 00	44 00	53 00	
58	96 80	50 54	43 544	15 71	573 10	81 2	90 00	99 00	108 00	117 00	126 00	135 00	
59	469 21	56 75	79 67	94 62	109 57	124 52	139 47	154 42	169 37	184 32	199 27	214 22	
60	47 81	500 00	515 04	92 64	559 19	58 79	67 00	76 00	85 00	94 00	103 00	112 00	
61	66°	61°	60°	59°	58°	57°	56°	55°	54°	53°	52°	51°	
NATURAL COSINE.													

TABLE A.

NATURAL SINE.

72m*

	86°	87°	88°	89°	40°	41°	42°	43°	44°	
0	587 79	601 82	615 66	629 82	642 79	656 06	669 13	682 00	694 86	60
1	588 02	602 06	616 89	630 55	643 01	656 28	669 35	682 21	695 08	59
2	26	28	616 12	77	28	50	56	42	695 08	58
3	49	51	85	680 00	46	72	78	64	29	57
4	73	74	58	22	68	94	99	85	49	56
5	96	98	81	45	90	657 16	670 21	683 06	70	55
6	589 20	603 21	617 04	631 68	644 12	657 88	671 48	684 27	697 91	54
7	43	44	26	90	35	59	64	49	696 12	53
8	67	67	49	681 13	57	81	86	70	83	52
9	90	90	72	85	79	658 08	671 07	684 01	697 54	51
10	590 14	604 14	618 95	632 58	645 01	658 25	671 29	684 12	697 75	50
11	87	87	618 18	80	24	47	51	34	96	49
12	61	60	41	682 03	46	69	72	55	697 17	48
13	84	83	64	25	68	91	94	76	87	47
14	591 08	605 06	619 87	633 48	646 01	659 18	672 15	685 07	698 58	46
15	81	29	619 09	71	646 12	85	87	685 18	79	45
16	54	53	82	93	35	56	58	39	698 00	44
17	78	76	55	683 16	57	78	80	61	21	43
18	592 01	606 99	621 78	635 88	649 79	663 00	676 01	689 82	703 42	42
19	25	606 22	620 01	61	647 01	22	23	686 03	62	41
20	48	45	24	88	23	44	44	24	88	40
21	72	68	46	684 06	46	66	66	45	699 04	39
22	95	91	69	23	68	88	87	66	25	38
23	593 18	607 14	621 92	635 51	649 10	663 10	676 09	689 88	703 46	37
24	42	38	621 15	73	648 12	81	80	687 09	66	36
25	65	61	88	96	84	58	52	30	87	35
26	89	84	60	685 18	56	75	78	51	700 08	34
27	594 12	608 07	622 83	636 40	650 78	664 97	678 95	692 72	706 29	33
28	86	80	622 06	68	649 01	662 18	675 16	688 98	702 49	32
29	59	53	29	85	23	40	38	688 14	70	31
30	82	76	51	686 08	45	62	59	85	91	30
31	595 06	609 99	624 74	638 20	652 67	666 84	680 80	694 57	708 12	29
32	29	609 22	623 97	637 58	651 89	665 06	678 02	691 78	705 32	28
33	52	45	623 20	75	650 11	27	28	99	58	27
34	76	68	42	98	33	49	45	689 20	74	26
35	99	91	65	687 20	55	71	66	41	95	25
36	596 22	610 15	624 88	639 42	653 77	667 98	681 88	695 62	709 15	24
37	46	38	624 11	65	651 00	664 14	677 09	690 83	704 36	23
38	69	61	83	87	22	36	30	690 04	57	22
39	93	84	56	688 10	44	58	52	25	77	21
40	597 16	611 07	625 79	640 32	654 66	668 80	682 78	696 46	710 98	20
41	89	80	625 02	54	88	665 01	679 95	693 67	707 19	19
42	63	53	24	77	652 10	23	678 16	88	89	18
43	86	76	47	99	32	45	37	691 09	60	17
44	598 09	612 99	627 70	641 39	655 54	669 66	683 59	697 30	711 81	16
45	82	612 22	92	44	76	88	80	51	704 01	15
46	56	45	626 15	66	98	666 10	679 01	692 72	706 22	14
47	79	68	38	89	653 20	32	23	98	43	13
48	599 02	613 91	628 60	642 11	656 42	670 53	684 44	698 14	711 63	12
49	26	613 14	83	38	64	75	65	85	84	11
50	49	37	627 06	56	86	97	87	56	705 06	10
51	72	60	28	78	654 08	667 18	680 08	693 77	707 25	9
52	95	83	51	641 00	80	40	29	98	46	8
53	600 19	614 06	74	23	52	62	51	698 19	67	7
54	42	29	96	45	74	83	72	40	87	6
55	65	51	628 19	67	96	668 05	681 93	695 61	709 08	5
56	89	74	42	90	655 18	27	681 15	82	28	4
57	601 12	615 97	630 64	644 12	658 40	672 48	686 57	699 24	712 49	3
58	85	615 20	87	84	62	70	57	24	70 2	2
59	58	43	629 09	56	84	91	79	45	90 1	1
60	82	66	82	79	656 06	669 13	682 00	694 86	707 11	0
	53°	52°	51°	50°	49°	48°	47°	46°	45°	

NATURAL COSINE.

72°*		NATURAL SINE.										TABLE A.	
46°	46°	47°	48°	49°	50°	51°	52°	53°	54°	55°	56°	57°	
0	707 11	719 84	781 85	748 14	754 71	766 04	777 15	788 01	798 64	80			
1	81	54	55	84	90	28	33	19	81	59			
2	52	74	75	58	755 09	42	51	87	99	58			
3	72	95	95	78	28	61	69	55	799	16	57		
4	98	720 15	782 15	92	47	79	88	78	84	56			
5	708 18	35	34	744 12	66	98	778 06	91	51	55			
6	84	55	54	81	85	767 17	24	789 08	68	54			
7	55	75	74	51	786 04	35	43	26	86	53			
8	75	95	94	70	23	54	61	44	800	08	52		
9	96	721 16	783 14	89	42	72	79	62	21	51			
10	709 16	36	33	745 09	61	91	97	80	33	50			
11	87	56	53	28	81	768 10	779 16	98	55	49			
12	57	76	73	48	757 00	28	34	790 16	73	48			
13	78	96	93	67	19	47	52	33	91	47			
14	98	722 16	784 13	88	38	66	70	51	801	08	46		
15	710 19	36	32	746 06	57	84	88	69	25	45			
16	89	57	52	25	75	769 03	780 07	87	43	44			
17	59	77	72	44	94	21	25	791 05	60	43			
18	80	97	91	64	758 18	40	43	22	78	42			
19	711 00	723 17	785 11	83	32	59	61	40	95	41			
20	21	37	31	747 08	51	77	79	58	802	12	40		
21	41	57	51	22	70	96	98	76	30	39			
22	62	77	70	41	89	770 14	781 16	98	47	38			
23	82	97	90	60	759 08	33	34	792 11	64	37			
24	712 03	724 17	786 10	80	27	51	52	29	82	36			
25	23	37	29	99	46	70	70	47	99	35			
26	43	57	49	748 18	65	88	88	64	803	16	34		
27	64	77	69	88	84	771 07	782 06	82	34	33			
28	84	97	88	57	760 08	25	25	793 00	51	32			
29	713 05	725 17	787 08	76	22	44	43	18	68	31			
30	25	37	28	96	41	62	61	35	86	30			
31	45	57	47	749 15	59	81	79	58	804	08	29		
32	66	77	67	84	78	99	97	71	20	28			
33	86	97	87	58	97	772 18	783 15	88	38	27			
34	714 07	726 17	788 06	78	761 16	36	38	794 06	55	26			
35	27	37	26	92	35	55	51	24	72	25			
36	47	57	46	750 11	54	78	69	41	89	24			
37	68	77	65	80	73	92	87	59	805	07	23		
38	88	97	85	50	92	773 10	784 05	77	24	22			
39	715 08	727 17	789 04	69	762 10	29	24	94	41	21			
40	29	37	24	88	29	47	42	795 12	53	20			
41	49	57	44	751 07	48	66	60	80	76	19			
42	69	77	63	26	67	84	78	47	93	18			
43	90	97	83	46	86	774 02	96	65	806	10	17		
44	716 10	728 17	740 02	65	763 04	21	785 14	83	27	16			
45	80	37	22	84	23	39	32	796 00	44	15			
46	50	57	41	752 08	42	58	50	18	62	14			
47	71	77	61	22	61	76	68	35	79	13			
48	91	97	80	41	80	94	86	53	96	12			
49	717 11	729 17	741 00	61	98	775 13	786 04	71	807	13	11		
50	82	37	20	80	764 17	31	22	88	80	10			
51	52	57	39	99	36	50	40	797 05	48	9			
52	72	76	59	753 18	55	68	58	23	65	8			
53	92	96	78	87	73	86	76	41	82	7			
54	718 13	780 16	98	56	92	776 05	94	58	99	6			
55	33	36	742 17	75	765 11	28	787 11	76	808	16	5		
56	53	56	37	95	30	41	29	98	33	4			
57	73	76	56	754 14	48	60	47	798 11	50	3			
58	94	96	76	33	87	78	65	29	67	2			
59	719 14	781 16	95	52	86	96	83	46	85	1			
60	84	35	743 14	71	766 04	777 15	788 01	64	809	02	0		
7	44°	45°	45°	41°	40°	39°	38°	37°	36°	35°	34°	33°	
NATURAL COSINE.													

TABLE A.

NATURAL SINE.

720*

	54°	55°	56°	57°	58°	59°	60°	61°	62°	
0	809 02	819 15	829 04	838 67	848 05	857 17	866 08	874 62	882 95	60
1	19	32	20	88	20	82	17	76	888	08 59
2	36	49	36	99	36	47	32	90		22 58
3	53	65	53	889	15	51	62	46	875	04 86 57
4	70	82	69	80	66	77	61	18		49 50
5	87	99	85	48	82	92	75	32		63 55
6	810 04	820 15	830 01	82	97	858	06	90	46	77 54
7	21	32	17	78	849	18	21	867	04	61 90 58
8	38	48	34	94	28	36	19	75	884	04 52
9	55	65	50	840	09	43	51	33	89	17 51
10	72	82	66	25	59	68	48	876	08	81 50
11	89	98	82	41	74	81	62	17		45 49
12	811 06	821 15	831 08	57	89	96	77	81		58 48
13	23	32	15	72	850	05	859	11	91	45 72 47
14	40	48	31	88	20	26	868	05	59	85 46
15	57	65	47	841	04	35	41	20	78	99 45
16	74	81	63	20	51	56	34	87	885	12 44
17	91	98	79	85	66	70	49	877	01	26 48
18	812 08	822 14	832 05	51	81	85	68	15		89 42
19	25	31	12	67	96	860	00	78	29	53 41
20	42	48	28	82	851	12	15	92	48	66 40
21	59	64	44	98	27	80	869	06	56	80 39
22	76	81	60	842	14	42	45	21	70	98 38
23	93	97	76	80	57	59	35	84	886	07 37
24	813 10	823 14	833 02	45	78	74	49	98		20 36
25	27	30	883	06	61	88	89	64	878	12 34 85
26	44	47	24	77	852	08	861	04	78	26 47 84
27	61	63	40	92	18	19	93	40	61	81 33
28	78	80	56	843	08	34	88	870	07	54 74 82
29	95	96	78	24	49	48	21	68		88 31
30	814 12	824 18	834 05	39	64	63	36	82	887	01 30
31	28	29	884	05	55	79	78	50	96	15 29
32	45	46	21	70	94	92	64	879	09	28 28
33	62	62	37	86	853	10	862	07	79	28 41 27
34	79	78	53	844	02	25	22	98	87	55 26
35	96	95	69	17	40	37	871	07	51	68 25
36	815 13	825 11	835 00	83	55	51	21	65		82 24
37	30	28	885	01	48	70	66	86	79	95 23
38	46	44	17	64	85	81	50	98	888	08 22
39	63	61	38	80	854	01	95	64	880	06 22 21
40	80	77	49	95	16	863	10	78	20	85 20
41	97	98	65	845	11	81	25	93	84	48 19
42	816 14	826 10	836 00	81	26	46	40	872	07	48 62 18
43	31	26	97	42	61	54	21	62		75 17
44	47	43	886	18	57	76	69	35	75	88 16
45	64	59	29	73	91	84	50	89	889	02 15
46	81	75	45	88	855	06	98	64	881	03 15 14
47	98	92	61	846	04	21	864	13	78	17 28 18
48	817 14	827 08	837 00	19	36	27	92	30		42 12
49	31	24	92	35	51	42	873	06	44	55 11
50	48	41	837	08	50	67	57	21	58	68 10
51	65	57	24	66	82	71	35	72		81 9
52	82	78	40	81	97	86	49	85		95 8
53	98	90	56	97	856	12	865	01	63	99 890 08 7
54	818 15	828 06	838 00	12	27	15	77	882	13	21 6
55	32	22	88	28	42	30	91	26		85 5
56	48	39	838	04	43	57	44	874	06	40 48 4
57	65	55	20	59	72	59	20	54		61 8
58	82	71	35	74	87	78	84	67		74 2
59	99	87	51	89	857	02	88	48	81	87 1
60	819 15	829 04	839 00	17	31	17	866	08	62	95 891 01 0
	54°	55°	56°	57°	58°	59°	60°	61°	62°	

NATURAL COSINE.

72p*		NATURAL SINE.										TABLE A.			
7	68°	64°	60°	56°	52°	48°	44°	40°	36°	32°	28°	24°	20°	16°	12°
0	891	01 898	79 906	81 918	55 920	50 927	18 933	58 939	69 945	52 952	60				
1	14	92	43	66	62	29	68	79	61	59					
2	27	899	06	55	78	40	79	89	71	58					
3	40	18	68	90	85	51	89	99	80	57					
4	53	30	80	914	02	96	62	984	00	940	09				
5	67	43	92	14	921	07	78	10	19	55					
6	80	56	907	04	25	19	84	20	29	946	09	54			
7	93	68	17	37	80	94	81	89	18	53					
8	892	06	81	29	49	41	928	06	41	49					
9	19	94	41	61	52	16	52	58	87	51					
10	82	900	07	53	72	64	27	62	68	46	50				
11	45	19	66	84	75	38	72	78	56	49					
12	59	32	78	96	86	49	88	88	65	48					
13	72	46	90	915	08	98	59	98	74	47					
14	85	57	908	02	19	922	09	70	985	03	941	08			
15	98	70	14	81	20	81	14	18	93	45					
16	898	11	82	26	43	31	92	24	27	947	02	44			
17	24	95	89	55	43	929	08	84	87	12	43				
18	37	901	08	51	66	54	18	44	47	21	42				
19	50	20	63	78	65	24	55	57	80	41					
20	63	33	75	90	76	35	65	67	40	40					
21	76	46	87	916	01	87	45	75	76	49	39				
22	89	58	89	18	99	56	85	86	58	38					
23	894	02	71	909	12	25	928	10	67	96	68	37			
24	15	83	24	86	21	78	936	06	942	06	77	36			
25	28	96	36	48	32	88	16	16	86	35					
26	41	902	08	48	60	43	99	26	25	95	34				
27	54	21	60	71	55	930	10	37	85	948	05	33			
28	67	33	72	83	66	20	47	45	14	32					
29	80	46	84	94	77	31	57	54	23	31					
30	93	59	96	917	06	88	42	67	64	30					
31	896	06	71	910	08	18	99	62	77	74					
32	19	84	20	29	924	10	63	88	84	51	28				
33	32	96	32	51	21	74	98	98	60	27					
34	45	908	09	44	52	82	84	987	08	948	03				
35	58	21	56	64	44	95	18	18	78	25					
36	71	34	68	75	55	931	06	28	22	88	24				
37	84	46	80	87	66	16	38	42	97	28					
38	97	58	92	99	77	27	48	42	949	06	22				
39	896	10	71	911	04	918	10	88	37	59	51				
40	23	83	16	22	99	48	69	61	24	20					
41	36	96	28	33	925	10	59	79	70	33	19				
42	49	904	08	40	45	21	69	89	80	48	18				
43	62	21	52	56	32	80	99	90	52	17					
44	74	33	64	68	43	90	938	09	99	61	16				
45	87	46	76	79	54	932	01	19	944	09	70	15			
46	897	00	58	88	91	65	11	29	18	79	14				
47	13	70	912	00	919	02	76	22	89	28	88	13			
48	26	83	12	14	87	32	49	88	97	12					
49	39	95	24	25	98	43	59	47	950	06	11				
50	52	905	07	36	86	926	09	53	69	57	16	10			
51	64	20	48	48	20	64	79	66	24	9					
52	77	32	60	59	31	74	89	76	33	8					
53	90	45	72	71	42	85	99	85	43	7					
54	898	08	57	83	58	95	939	09	95	52	6				
55	16	69	95	94	64	933	06	19	945	04	61	5			
56	28	82	913	07	920	05	75	16	29	14	70	4			
57	41	94	19	16	86	27	89	28	79	3					
58	54	906	06	81	28	97	87	49	38	88	2				
59	67	18	43	89	927	07	48	59	42	97	1				
60	79	81	55	50	18	58	69	52	951	06	0				
7	28°	26°	24°	22°	20°	18°	16°	14°	12°	10°	8°	6°	4°	2°	0°
NATURAL COSINE.															

TABLE A.

NATURAL SINE.

	72°	73°	74°	75°	76°	77°	78°	79°	80°
0	951 06	956 82	961 26	965 98	970 80	974 87	978 15	981 68	984
1	15	89	84	996 00	87	44	21	68	
2	24	47	42	08	44	50	27	73	984
3	88	56	50	15	51	57	88	79	
4	42	64	58	28	58	68	89	85	985
5	51	78	66	80	65	70	45	90	
6	59	81	74	88	72	76	51	96	985
7	68	90	82	45	79	88	57	982 01	
8	77	98	90	58	86	89	68	07	885
9	86	957 07	98	60	98	96	69	12	
10	95	15	962 06	67	971 00	975 02	75	18	985
11	952 04	24	14	75	08	08	81	23	
12	18	82	22	82	18	15	87	29	985
13	22	40	80	90	20	21	98	84	
14	81	49	38	97	27	28	99	40	985
15	40	57	46	967 05	84	84	979 05	45	
16	48	66	58	12	41	41	10	50	985
17	57	74	61	19	48	47	16	56	
18	66	82	69	27	55	58	22	61	985
19	75	91	77	84	62	60	28	67	
20	84	99	85	42	69	66	84	72	985
21	98	958 07	98	49	76	78	40	77	
22	958 01	16	968 01	56	82	79	46	88	985
23	10	24	08	64	89	85	52	88	
24	19	82	16	71	96	92	58	94	986
25	28	41	24	78	972 08	98	68	99	
26	37	49	82	86	10	976 04	69	988 04	
27	45	57	40	98	17	11	75	10	986
28	54	65	47	00	28	17	81	15	
29	68	74	55	968 07	80	28	87	20	986
30	72	82	68	15	87	30	92	25	
31	80	90	71	22	44	86	98	81	986
32	89	98	79	29	51	42	980 04	86	
33	98	959 07	86	87	57	48	10	41	986
34	954 07	15	94	44	64	55	16	47	
35	15	28	964 02	51	71	61	21	52	986
36	24	31	10	58	78	67	27	57	
37	38	40	17	66	84	78	38	62	986
38	41	48	25	78	91	80	89	68	
39	50	56	38	80	98	86	44	78	986
40	59	64	40	87	978 04	92	50	78	
41	67	72	48	94	11	98	56	88	986
42	76	81	56	969 02	18	977 05	61	89	
43	85	89	68	09	25	11	67	94	986
44	98	97	71	16	81	17	78	99	
45	955 02	960 05	79	28	88	28	79	984 04	987
46	11	18	86	80	45	29	84	09	
47	19	21	94	87	51	85	90	14	
48	28	29	965 02	45	58	42	96	20	987
49	36	37	09	52	65	48	981 01	25	
50	45	46	17	59	71	54	07	80	987
51	54	54	24	66	78	60	12	85	
52	62	62	32	78	84	66	18	40	987
53	71	70	40	80	91	72	24	45	
54	79	78	47	87	98	78	29	50	887
55	88	86	55	94	974 04	84	85	55	
56	96	94	62	970 01	11	91	40	61	987
57	956 05	961 02	70	08	17	97	46	66	
58	18	10	78	15	24	978 08	52	71	987
59	22	18	85	28	80	09	57	76	
60	80	26	98	80	87	15	68	81	
	17°	16°	15°	14°	13°	12°	11°	10°	

NATURAL COSINE.

NATURAL SINE.																	TABLE A.		
72°*	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°								90°	91°
0	9876	9902	7	9925	5	9945	2	9961	9	9975	6	9986	3	9993	9	9998	5	60	
1	9877	8	9903	1	8	5	9962	2	8	4	9994	0					569		
2		8		5	9926	2	8	5	9976	0	6		6				658		
3	9878	2	9	5	9946	1	7	2	7	2	7		2				657		
4		7	9904	3	9	4	9963	0	4	9	3		3				756		
5	9879	1	7	9927	2	7	2	6	9987	0	4		4				755		
6		6	9905	1	6	9947	0	5	8	2	5		5				854		
7	9880	0	5	9	9	3	7	9977	0	3	6		6				853		
8		5		9	9928	3	6	9	2	5	7		7				952		
9		9	9906	3	6	9	9964	2	4	6	8		8				951		
10	9881	4	7	9929	0	9948	2	4	6	8	9		9				950		
11		8	9907	1	8	6	7	8	9	9995	0	9999	0	49					
12	9882	8	5	7	8	9	9978	0	9988	1	1		1				048		
13		7		9	9930	0	9949	1	9965	2	2		2				147		
14	9883	2	9908	3	3	4	4	4	4	3	2		2				146		
15		6	7	7	7	4	7	6	6	5	3		3				145		
16	9884	1	9909	1	9931	0	9950	0	9	8	6		4				244		
17		5	4	4	4	8	9966	1	9979	0	8		5				243		
18		9	8	7	6	4	4	2	9	9	6		6				342		
19	9885	4	9910	2	9932	0	8	6	3	9989	0	7					341		
20		8	6	4	9951	1	8	5	2	8			8				340		
21	9886	8	9911	0	7	4	9967	1	7	8	9		9				439		
22		7	4	9933	1	7	3	9	4	9996	0						438		
23	9887	1	8	4	9952	0	6	9980	1	6	0		1				437		
24		6	9912	2	7	3	8	3	7	1			1				536		
25	9888	0	5	9934	1	6	9968	0	5	8	2		2				535		
26		4	9	4	8	3	6	9990	0	3			3				534		
27		9	9918	8	7	9953	1	5	8	1	3		3				533		
28	9889	3	7	9935	1	4	7	9981	0	2	4		4				632		
29		7	9914	1	4	7	9	2	4	5			5				631		
30	9890	2	4	7	9954	0	9969	2	3	5	6		6				630		
31		6	8	9936	0	2	4	5	6	6			6				629		
32	9891	0	9915	2	4	5	6	7	7	7			7				728		
33		4	6	7	8	9	9	9	9	9			8				627		
34		9	9916	0	9937	0	9955	1	9970	1	9982	1	9991	0	9		726		
35	9892	3	3	4	3	3	3	2	1	9			9				725		
36		7	7	7	6	5	4	2	9997	0							824		
37	9893	1	9917	1	9938	0	9	8	6	4	1		1				823		
38		6	5	8	9956	2	9971	0	7	5	2		2				822		
39	9894	0	8	6	4	2	9	6	2	2			2				821		
40		4	9918	2	9939	0	7	4	9983	1	7		3				820		
41		8	6	3	9957	0	6	3	7	4			4				819		
42	9895	3	9	6	2	9	4	9	4	9			4				918		
43		7	9919	3	9	5	9972	1	6	9992	1	5					917		
44	9896	1	7	9940	2	8	3	8	2	6			6				916		
45		5	9920	0	6	9958	0	5	9	3	6		6				915		
46		9	4	9	3	7	9984	1	4	7			7				914		
47	9897	8	8	9941	2	6	9	2	5	7			7				913		
48		8	2921	1	5	8	9973	1	4	6	8		8				912		
49	9898	2	5	8	9959	1	4	6	7	9			9				911		
50		6	9942	1	4	6	7	9					9	1000	0	10			
51	9899	0	9922	2	4	6	8	9	9998	0	9998	0					0	9	
52		4	6	8		9974	0	9985	1	1	0		0				0	8	
53		8	9923	0	9943	1	9960	2	2	2	1		1				0	7	
54	9900	2	3	4	4	4	4	4	3	2			2				0	6	
55		6	7	7	7	6	5	4	2	2			2				0	5	
56	9901	1	9924	0	9944	0	9	8	7	5	3		3				0	4	
57		5	4	3	9961	2	9975	0	8	6	3		3				0	3	
58		9	8	6	4	2	9986	0	7	4			4				0	2	
59	9902	3	9925	1	9	7	4	1	8	4			4				0	1	
60		7	5	9945	2	9	6	3	9	5			5				0	0	
7	8°	7°	6°	5°	4°	3°	2°	1°	0°	7									
NATURAL COSINE.																			

TABLE B. NATURAL TANGENT.

	0°	1°	2°	3°	4°	5°	6°	7°	8°
0	000 00	017 46	034 92	052 41	069 93	087 49	105 10	122 78	140
1	29	75	085 21	70	070 22	78	40	128 08	
2	58	018 04	50	058 00	51	088 07	69	38	141
3	87	88	79	28	80	37	99	67	
4	001 16	62	086 09	57	071 10	66	106 28	97	
5	45	91	88	87	89	96	58	124 26	142
6	75	019 20	67	054 16	68	089 25	87	56	
7	002 04	49	96	45	97	54	107 16	85	
8	83	78	087 25	74	072 27	88	46	125 15	
9	62	020 07	54	055 08	56	090 13	75	44	143
10	91	87	88	88	85	42	108 05	74	
11	008 20	66	088 12	62	078 14	71	84	126 08	
12	49	96	42	91	44	091 01	63	38	144
13	78	021 24	71	056 20	78	80	98	62	
14	004 07	53	089 00	49	074 02	59	109 22	92	
15	86	82	29	78	81	89	52	127 22	145
16	65	022 11	58	057 08	61	092 18	81	51	
17	95	40	87	87	90	47	110 11	81	
18	005 24	69	040 16	66	075 19	77	40	128 10	
19	53	98	46	95	48	098 06	70	40	146
20	82	023 28	75	058 24	78	85	99	69	
21	006 11	57	041 04	54	076 07	65	111 28	99	
22	40	86	33	88	86	94	58	129 29	147
23	69	024 15	62	059 12	65	094 23	87	58	
24	98	44	91	41	95	53	112 17	88	
25	007 27	73	042 20	70	077 24	82	46	130 17	
26	56	025 02	50	99	53	095 11	76	47	148
27	85	81	79	060 29	82	41	118 05	76	
28	008 15	60	048 08	58	078 12	70	85	131 06	
29	44	89	87	87	41	096 00	64	86	149
30	73	026 19	66	061 16	70	29	94	65	
31	009 02	48	95	45	99	58	114 28	95	
32	81	77	044 24	75	079 29	88	53	132 24	150
33	60	027 06	54	062 04	58	097 17	82	54	
34	89	85	83	38	87	46	115 11	84	
35	010 18	64	045 12	62	080 17	76	41	133 13	
36	47	98	41	91	46	098 05	70	42	151
37	76	028 22	70	063 21	75	84	116 00	72	
38	011 05	51	99	50	081 04	64	29	134 02	
39	85	81	046 28	79	84	98	59	82	152
40	64	029 10	58	064 08	63	099 23	88	61	
41	98	89	87	88	92	52	117 18	91	
42	012 22	68	047 16	67	082 22	81	47	135 21	153
43	51	97	45	96	51	100 11	77	50	
44	80	030 26	74	065 25	80	40	118 06	80	
45	013 09	55	048 03	54	083 09	69	86	136 09	
46	88	84	88	84	89	99	65	89	154
47	67	031 14	62	066 18	68	101 28	95	69	
48	96	43	91	42	97	58	119 24	98	
49	014 25	72	049 20	71	084 27	87	54	137 28	155
50	55	032 01	49	067 00	56	102 16	83	58	
51	84	80	78	80	85	46	120 18	87	
52	015 18	59	050 07	59	085 14	75	42	138 17	156
53	42	88	87	88	44	103 05	72	47	
54	71	033 17	66	068 17	78	84	121 01	76	
55	016 00	46	95	47	086 02	63	81	139 06	
56	29	76	051 24	76	82	98	60	85	157
57	58	034 05	53	069 05	61	104 22	90	65	
58	87	84	82	84	90	52	122 19	95	
59	017 16	63	052 12	63	087 20	81	49	140 24	158
60	46	92	41	93	49	105 10	78	54	
	89°	88°	87°	86°	85°	84°	83°	82°	81°

NATURAL COTANGENT.

72°*		NATURAL TANGENT.										TABLE B.	
	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°	
0	158 88	176 33	194 88	212 56	230 87	249 83	267 95	286 74	306 78	326 08	346 58	367 18	
1	68	63	68	68	231 17	64	268 26	287 06	306 06	326 06	346 58	367 18	
2	98	98	98	218 16	48	95	67	88	88	88	88	88	
3	159 28	177 28	195 29	213 47	232 09	250 26	269 20	288 01	307 00	326 56	346 58	367 18	
4	58	58	59	77	232 09	56	269 20	288 01	307 00	326 56	346 58	367 18	
5	88	88	89	214 08	40	87	51	82	82	82	82	82	
6	160 17	178 18	196 19	214 08	40	87	51	82	82	82	82	82	
7	47	48	49	69	238 01	49	270 18	289 18	308 28	327 38	346 58	367 18	
8	77	78	80	99	82	80	44	289 27	308 28	327 38	346 58	367 18	
9	161 07	179 08	197 10	215 29	68	252 11	76	58	60	61	62	63	
10	87	88	40	60	98	42	271 07	90	91	92	93	94	
11	67	68	70	90	234 24	78	88	290 21	309 28	328 38	347 48	367 58	
12	97	98	198 01	216 21	55	258 04	69	53	55	56	57	58	
13	162 26	180 28	81	51	85	85	272 01	84	87	89	91	93	
14	56	58	61	82	285 16	66	32	291 16	310 19	329 28	348 38	367 48	
15	86	88	91	217 12	47	97	68	47	51	54	57	60	
16	163 16	181 18	199 21	217 12	47	97	68	47	51	54	57	60	
17	46	48	52	78	286 08	59	278 26	292 10	311 15	330 28	349 38	368 48	
18	76	78	82	218 04	89	90	57	42	47	52	57	62	
19	164 06	182 08	200 12	218 04	89	90	57	42	47	52	57	62	
20	85	88	42	64	287 00	52	274 19	293 05	312 10	331 28	350 38	369 48	
21	65	68	78	95	81	83	51	87	92	97	102	107	
22	95	98	201 03	219 25	62	256 14	82	68	74	80	86	92	
23	165 25	183 28	83	53	98	45	275 18	294 00	313 06	332 28	351 48	370 68	
24	55	58	64	86	288 28	76	45	32	38	44	50	56	
25	85	88	94	220 17	54	257 07	76	63	70	77	84	91	
26	166 15	184 18	202 24	220 17	54	257 07	76	63	70	77	84	91	
27	45	48	54	78	289 16	69	89	295 26	314 38	333 58	353 18	372 38	
28	74	78	85	221 08	46	258 00	70	58	66	74	82	90	
29	107 04	185 04	203 15	221 08	46	258 00	70	58	66	74	82	90	
30	84	88	46	69	240 08	62	32	296 21	315 38	334 58	354 18	373 38	
31	64	68	78	222 00	89	93	64	53	62	71	80	89	
32	94	98	204 06	81	69	259 24	96	85	94	103	112	121	
33	168 24	186 28	86	61	241 00	55	278 26	297 16	316 38	335 58	355 18	374 38	
34	54	58	66	92	8	86	58	48	58	68	78	88	
35	84	88	97	223 22	62	260 17	89	80	90	100	110	120	
36	169 14	187 18	205 27	223 22	62	260 17	89	80	90	100	110	120	
37	44	46	57	83	242 28	79	52	48	54	62	72	82	
38	74	75	88	224 14	54	261 10	88	75	86	96	106	116	
39	170 04	188 06	206 18	224 14	54	261 10	88	75	86	96	106	116	
40	88	85	48	75	243 16	72	46	38	50	62	74	86	
41	68	65	79	225 05	47	262 03	77	70	82	94	106	118	
42	98	95	207 09	86	77	85	281 09	300 01	319 14	338 28	357 42	376 56	
43	171 28	189 25	89	67	244 08	66	40	38	46	54	62	70	
44	58	55	70	97	89	97	72	65	78	90	102	114	
45	88	86	208 00	226 28	70	263 28	282 03	97	320 10	339 24	358 38	377 52	
46	172 18	190 16	80	58	245 01	59	84	301 28	320 42	339 56	359 10	378 24	
47	48	46	61	89	32	90	66	60	74	88	102	116	
48	78	76	91	227 19	62	264 21	97	92	321 06	340 20	359 34	378 48	
49	173 08	191 06	209 21	50	98	52	283 29	302 24	321 38	340 52	359 66	378 80	
50	88	86	52	81	246 24	85	60	55	71	86	101	116	
51	68	66	82	228 11	55	295 16	91	87	322 03	341 17	360 31	379 45	
52	98	97	210 13	42	86	46	284 28	303 19	322 33	341 47	360 61	379 75	
53	174 28	192 27	48	72	247 17	77	54	51	67	83	99	115	
54	58	57	73	229 08	48	266 08	86	82	99	116	133	150	
55	88	87	211 04	34	78	89	285 17	304 14	323 28	342 42	361 56	380 70	
56	175 18	193 17	84	64	248 09	70	49	46	63	80	97	114	
57	48	47	64	95	40	267 01	80	78	96	114	132	150	
58	78	78	95	230 26	71	33	286 12	305 09	324 23	343 37	362 51	381 65	
59	176 08	194 08	212 25	56	289 02	64	48	41	60	79	98	117	
60	88	88	56	87	83	95	74	78	92	108	125	142	
7	80°	79°	78°	77°	76°	75°	74°	73°	72°	71°	70°	69°	
NATURAL COTANGENT.													

TABLE B.

NATURAL TANGENT.

720*

	18°	19°	20°	21°	22°	23°	24°	25°	26°	27°
0	824 92	844 88	868 97	888 86	404 08	424 47	440 28	466 81	487 78	60
1	825 24	845 24	868 80	888 20	88	82	58	66	488 09	59
2	826 56	846 56	869 68	889 53	70	425 16	98	467 02	45	58
3	828 28	848 28	870 96	890 04	405 04	51	446 27	87	81	57
4	829 59	850 59	872 68	892 20	88	85	62	73	489 17	56
5	831 31	852 31	874 96	894 53	72	426 19	97	468 08	58	55
6	832 53	854 53	876 68	896 20	87	406 06	54	447 82	48	54
7	834 25	856 25	878 96	898 53	40	88	67	79	490 26	53
8	835 57	858 57	880 68	900 20	74	427 22	448 02	469 14	62	52
9	837 29	860 29	882 96	902 04	87	407 07	57	87	50	51
10	838 61	861 61	884 96	904 21	41	91	72	85	491 84	50
11	840 13	863 13	886 40	906 54	75	428 26	449 07	470 21	70	49
12	841 45	864 45	888 68	908 20	408 09	60	42	56	492 06	48
13	842 77	866 77	890 96	910 53	43	94	77	92	42	47
14	844 9	868 9	892 24	912 20	77	429 29	450 12	471 28	78	46
15	845 31	869 31	894 52	914 53	409 11	68	47	63	493 15	45
16	846 63	870 63	896 80	916 20	45	98	82	99	51	44
17	847 95	872 95	898 68	918 53	79	430 82	451 17	472 84	87	43
18	849 27	874 27	900 96	920 20	410 18	67	52	70	494 28	42
19	850 59	876 59	902 24	922 53	47	431 01	87	478 05	59	41
20	851 91	878 91	904 52	924 20	81	86	452 22	41	95	40
21	852 23	880 23	906 80	926 53	411 15	70	57	77	495 82	39
22	853 55	881 55	908 68	928 20	49	432 05	92	474 12	68	38
23	854 87	883 87	910 96	930 53	88	39	458 27	48	496 04	37
24	855 19	885 19	912 24	932 20	412 17	74	62	83	40	36
25	856 51	886 51	914 52	934 53	51	433 08	97	475 19	77	35
26	857 83	888 83	916 80	936 20	85	43	454 82	55	497 18	34
27	858 15	889 15	918 68	938 53	413 19	78	67	90	49	33
28	859 47	890 47	920 96	940 20	58	434 12	455 02	476 26	86	32
29	860 79	891 79	922 24	942 53	87	47	88	62	498 22	31
30	861 11	892 11	924 52	944 20	414 21	81	78	98	58	30
31	862 43	893 43	926 80	946 53	55	435 16	456 08	477 83	94	29
32	863 75	894 75	928 68	948 20	90	50	48	69	499 81	28
33	864 1	895 1	930 96	950 53	415 24	85	78	478 06	67	27
34	864 33	896 33	932 24	952 20	58	436 20	457 18	40	500 04	26
35	865 65	897 65	934 52	954 53	92	54	48	78	40	25
36	866 97	898 97	936 80	956 20	416 26	89	84	479 12	76	24
37	867 29	899 29	938 68	958 53	60	437 24	458 19	48	501 18	23
38	868 61	900 61	940 96	960 20	94	58	54	84	49	22
39	869 93	901 93	942 24	962 53	417 28	98	89	480 19	85	21
40	870 25	902 25	944 52	964 20	68	438 28	459 24	55	502 22	20
41	871 57	903 57	946 80	966 53	97	62	60	91	58	19
42	872 89	904 89	948 68	968 20	418 31	97	95	481 27	96	18
43	873 21	905 21	950 96	970 53	65	439 32	460 30	68	503 81	17
44	873 53	906 53	952 24	972 20	99	66	65	98	68	16
45	874 85	907 85	954 52	974 53	419 38	440 01	461 01	482 84	504 04	15
46	875 17	908 17	956 80	976 20	68	88	86	70	41	14
47	875 49	909 49	958 68	978 53	420 02	71	71	488 06	77	13
48	876 81	910 81	960 96	980 20	36	441 05	462 06	42	505 14	12
49	877 13	911 13	962 24	982 53	70	40	42	78	50	11
50	877 45	912 45	964 52	984 20	421 05	75	77	484 14	87	10
51	878 77	913 77	966 80	986 53	89	442 10	463 12	50	506 28	9
52	879 9	914 9	968 68	988 20	78	44	48	86	60	8
53	879 41	915 41	970 96	990 53	422 07	79	83	485 21	96	7
54	879 73	916 73	972 24	992 20	42	443 14	464 18	57	507 38	6
55	880 5	917 5	974 52	994 53	76	49	54	93	69	5
56	880 37	918 37	976 80	996 20	423 10	84	89	486 29	508 06	4
57	881 69	919 69	978 68	998 53	45	444 18	465 25	65	48	3
58	882 1	920 1	980 96	1000 20	79	58	60	487 01	79	2
59	882 33	921 33	982 24	1002 53	424 18	88	95	87	509 16	1
60	882 65	922 65	984 52	1004 20	47	445 28	466 31	78	58	0
	71°	70°	69°	68°	67°	66°	65°	64°	63°	

NATURAL COTANGENT.

	27°	28°	29°	30°	31°	32°	33°	34°	35°	
1	509 58	531 71	554 81	577 85	600 88	624 87	649 41	674 51	700 21	60
2	89	582 08	609	632 18	655 27	680 27	705 82	731 93	758 59	
3	510 26	46	555 07	578 18	602 05	626 08	651 06	676 88	701 07	58
4	63	88	46	51	602 05	626 08	651 06	676 88	701 07	58
5	99	533 20	88	90	45	49	651 06	676 20	701 56	
6	511 86	58	556 21	579 29	84	89	48	63	702 38	55
7	78	95	59	68	603 24	627 30	652 31	677 05	702 54	
8	512 09	534 32	97	580 07	64	70	652 31	677 05	702 54	
9	46	70	557 86	46	604 08	628 11	72	90	703 53	
0	88	535 07	74	85	48	52	653 14	678 22	704 12	51
1	513 20	45	558 12	581 24	88	92	55	75	705 10	60
2	56	82	60	62	605 22	629 33	654 38	679 17	706 17	49
3	93	536 20	88	582 01	62	78	654 38	679 17	706 17	49
4	514 30	57	559 28	40	606 02	630 14	80	680 02	707 17	47
5	67	94	64	79	42	55	655 21	680 02	707 17	46
6	515 03	587 32	560 03	583 18	81	96	68	88	708 17	45
7	40	69	41	57	607 21	631 36	656 04	681 30	709 17	44
8	77	538 07	79	96	61	77	46	78	710 17	43
9	516 14	44	561 17	584 35	608 01	632 17	88	682 15	711 17	42
0	51	82	56	74	41	58	657 29	85	712 17	41
1	88	539 20	94	585 13	81	99	71	688 01	713 17	40
2	517 24	57	562 32	52	609 21	633 40	658 31	683 30	714 17	39
3	61	95	70	91	60	80	54	86	715 17	38
4	98	540 32	563 09	586 31	610 00	634 21	96	684 29	716 17	37
5	518 35	70	47	70	40	62	659 38	71	717 17	36
6	72	541 07	85	587 09	80	635 03	80	685 14	718 17	35
7	519 09	45	564 24	48	611 20	636 21	660 21	686 00	719 17	34
8	46	83	62	87	60	84	63	686 00	720 17	33
9	83	542 20	565 01	588 28	612 00	636 25	661 05	687 00	721 17	32
0	520 20	58	39	65	40	66	47	85	722 17	31
1	57	96	77	589 05	80	637 07	89	687 28	723 17	30
2	94	543 33	566 16	44	613 20	638 48	662 30	688 14	724 17	29
3	521 31	71	54	88	60	89	72	688 14	725 17	28
4	68	544 09	93	590 22	614 00	638 80	663 14	689 00	726 17	27
5	522 05	46	567 31	61	40	71	56	689 00	727 17	26
6	42	84	69	591 01	80	639 12	98	42	728 17	25
7	79	545 22	568 08	40	615 20	640 53	664 40	685 14	729 17	24
8	523 16	60	46	79	61	94	82	690 28	730 17	23
9	53	97	85	592 18	616 01	640 85	665 24	691 14	731 17	22
0	90	546 35	569 23	58	41	76	66	691 14	732 17	21
1	524 27	78	62	97	81	641 17	666 08	57	733 17	20
2	64	547 11	570 00	593 38	617 21	642 58	667 00	692 00	734 17	19
3	525 01	48	89	76	61	99	92	48	735 17	18
4	88	86	78	594 15	618 01	642 40	667 34	86	736 17	17
5	75	548 24	571 16	54	42	81	76	693 29	737 17	16
6	526 13	62	55	94	82	643 22	668 18	72	738 17	15
7	50	549 00	98	595 38	619 22	644 62	669 02	694 61	739 17	14
8	87	88	572 32	78	62	644 46	669 02	59	740 17	13
9	527 24	75	71	596 12	620 02	645 46	670 02	695 02	741 17	12
0	61	550 13	573 09	51	43	87	86	45	742 17	11
1	98	51	48	91	88	645 28	670 28	88	743 17	10
2	528 36	89	85	597 30	621 24	646 89	671 18	81	744 17	9
3	73	551 27	574 25	70	64	646 10	671 18	75	745 17	8
4	529 10	65	64	598 09	622 04	647 25	672 39	697 81	746 17	7
5	47	552 08	575 08	49	45	98	97	61	747 17	6
6	85	41	41	88	85	647 84	672 39	698 04	748 17	5
7	530 22	79	80	599 28	623 25	648 76	673 24	74	749 17	4
8	59	553 17	576 19	67	66	648 17	673 24	91	750 17	3
9	96	55	57	600 07	624 06	649 58	674 09	84	751 17	2
0	531 34	93	96	46	47	99	674 09	77	752 17	1
1	71	554 31	577 35	86	87	649 41	51	700 21	753 17	0

72x*		NATURAL TANGENT.								TABLE B.	
	45°	46°	47°	48°	49°	50°	51°	52°			
0	1.00000	1.08558	1.07287	1.11061	1.16087	1.19175	1.28490	1.27994	60		
1	058	618	299	126	104	246	563	1.28071	59		
2	116	674	362	191	172	316	687	148	58		
3	175	734	425	256	240	387	710	225	57		
4	233	794	487	321	308	457	784	302	56		
5	291	855	550	387	375	528	868	379	55		
6	350	915	618	452	448	599	931	456	54		
7	408	976	676	517	511	669	1.24005	533	53		
8	467	1.04086	738	582	579	740	079	610	52		
9	525	097	801	648	647	811	158	687	51		
10	583	158	864	713	715	882	227	764	50		
11	642	218	927	778	788	958	301	842	49		
12	701	279	990	844	851	1.20024	375	919	48		
13	759	340	1.08063	909	919	095	449	997	47		
14	818	401	116	975	987	166	523	1.29074	46		
15	876	461	179	1.12041	1.16056	287	597	152	45		
16	935	522	248	106	124	808	672	229	44		
17	994	583	306	172	192	879	746	307	43		
18	1.01058	644	369	238	261	451	820	385	42		
19	112	705	432	303	329	522	895	468	41		
20	170	766	496	369	398	598	969	541	40		
21	229	827	559	435	466	665	1.25044	619	39		
22	288	888	622	501	535	736	118	696	38		
23	347	949	686	567	608	808	193	775	37		
24	406	1.05010	749	633	672	879	268	853	36		
25	465	072	813	699	741	951	343	931	35		
26	524	133	876	765	809	1.21023	417	1.30009	34		
27	583	194	940	831	878	094	492	087	33		
28	642	255	1.09003	897	947	166	567	166	32		
29	702	317	067	963	1.17016	238	642	244	31		
30	761	378	131	1.18029	085	310	717	323	30		
31	820	439	195	096	154	382	792	401	29		
32	879	501	258	162	223	454	867	480	28		
33	938	562	322	228	292	526	943	558	27		
34	998	624	386	295	361	598	1.26018	637	26		
35	1.02057	685	450	361	430	670	098	716	25		
36	117	747	514	428	500	742	169	795	24		
37	176	809	578	494	569	814	244	873	23		
38	236	870	642	561	638	887	320	952	22		
39	295	932	706	627	708	959	395	1.31081	21		
40	355	994	770	694	777	1.22081	471	110	20		
41	414	1.06056	834	761	846	104	546	190	19		
42	474	117	899	828	916	176	622	269	18		
43	533	179	968	894	986	249	698	348	17		
44	598	241	1.10027	961	1.18055	321	774	427	16		
45	658	308	091	1.14028	125	394	849	507	15		
46	718	365	156	095	194	467	925	586	14		
47	772	427	220	162	264	539	1.27001	666	13		
48	832	489	285	229	384	612	077	745	12		
49	892	551	349	296	404	685	158	825	11		
50	952	613	414	363	474	758	230	904	20		
51	1.03012	676	478	430	544	831	306	984	9		
52	072	738	543	498	614	904	382	1.32064	8		
53	132	800	608	565	684	977	458	144	7		
54	192	862	672	632	754	1.23050	535	224	6		
55	252	925	737	699	824	123	611	304	5		
56	312	987	802	767	894	196	688	384	4		
57	372	1.07049	867	834	964	270	764	464	3		
58	432	112	931	902	1.19035	343	841	544	2		
59	492	174	996	969	105	416	917	624	1		
60	552	237	1.11061	1.15087	175	490	994	704	0		
7	44°	48	42°	41°	40°	38°	38°	37°	7		
NATURAL COTANGENT.											

TABLE B. NATURAL TANGENT.										72°*
	53°	54°	55°	56°	57°	58°	59°	60°		
0	1.82704	1.87638	1.42815	1.48258	1.53987	1.60038	1.66428	1.73205	60	
1	786	722	908	849	1.54085	137	538	821	59	
2	865	807	992	442	183	241	647	488	58	
3	946	891	1.43080	536	281	345	757	555	57	
4	1.83026	976	169	629	379	449	867	671	56	
5	107	1.88060	258	722	478	553	978	788	55	
6	188	145	347	816	576	657	1.67088	905	54	
7	268	229	486	909	675	761	198	1.74022	53	
8	349	314	525	1.49008	774	865	809	140	52	
9	430	399	614	097	873	970	419	257	51	
10	511	484	703	190	972	1.61074	530	375	50	
11	592	568	792	284	1.55071	179	641	492	49	
12	673	658	881	378	170	283	752	610	48	
13	754	738	970	472	269	388	863	728	47	
14	835	824	1.44060	566	368	493	974	846	46	
15	916	909	149	661	467	598	1.68085	964	45	
16	998	994	284	755	567	703	196	1.75082	44	
17	1.84079	1.89079	329	849	666	809	308	200	43	
18	160	165	418	944	766	914	419	319	42	
19	242	250	508	1.50038	866	1.62019	531	437	41	
20	323	336	598	133	966	125	643	556	40	
21	405	421	688	228	1.56065	230	754	675	39	
22	487	507	778	322	165	336	866	794	38	
23	568	593	868	417	265	442	979	918	37	
24	650	679	958	512	366	548	1.69091	1.76082	36	
25	732	764	1.45049	607	466	654	203	151	35	
26	814	850	139	702	566	760	316	271	34	
27	896	936	229	797	667	866	428	390	33	
28	978	1.40022	320	893	767	972	541	510	32	
29	1.85060	109	410	988	868	1.63079	653	630	31	
30	142	195	501	1.51084	969	185	766	749	30	
31	224	281	592	179	1.57069	292	879	869	29	
32	307	367	682	275	170	398	992	990	28	
33	389	454	773	370	271	505	1.70106	1.77110	27	
34	472	540	864	466	372	612	219	230	26	
35	554	627	955	562	474	719	332	351	25	
36	637	714	1.46046	658	575	826	446	471	24	
37	719	800	187	754	678	934	560	592	23	
38	802	887	229	850	778	1.64041	673	713	22	
39	885	974	320	946	879	148	787	834	21	
40	968	1.41061	411	1.52043	981	256	901	955	20	
41	1.86051	148	502	139	1.58083	363	1.71015	1.78077	19	
42	134	235	595	235	184	471	129	198	18	
43	217	322	686	332	286	579	244	319	17	
44	300	409	778	429	388	687	358	441	16	
45	383	497	870	525	490	795	473	563	15	
46	466	584	962	622	593	903	588	685	14	
47	549	672	1.47054	719	695	1.65011	702	807	13	
48	633	759	146	816	797	120	817	929	12	
49	716	847	238	913	900	228	932	1.79051	11	
50	800	934	330	1.58010	1.59002	337	1.72047	174	10	
51	883	1.42022	422	107	105	445	163	296	9	
52	967	110	514	205	208	554	278	419	8	
53	1.87050	198	607	302	311	663	393	542	7	
54	134	286	699	400	414	772	509	665	6	
55	218	374	792	497	517	881	625	788	5	
56	302	462	885	592	620	990	741	911	4	
57	386	550	977	693	723	1.66099	857	1.80034	3	
58	470	638	1.48070	791	826	209	973	158	2	
59	554	726	163	888	930	818	1.73099	281	1	
60	638	815	256	987	1.60033	428	205	405	0	
7	36°	36°	34°	33°	32°	31°	30°	29°	7	
NATURAL COTANGENT.										

722*	NATURAL TANGENT.								TABLE B.
	61°	62°	63°	64°	65°	66°	67°	68°	
0	1.80405	1.88073	1.96261	2.05030	2.14451	2.24604	2.35585	2.47509	60
1	529	205	402	182	614	780	776	716	59
2	658	337	544	338	777	956	967	924	58
3	777	469	685	485	940	2.26182	2.36158	2.48182	57
4	901	602	827	637	2.15104	309	349	340	56
5	1.81025	734	969	790	268	486	541	549	55
6	150	867	1.97111	942	432	668	738	758	54
7	274	1.89000	258	2.06094	596	840	925	967	53
8	399	183	395	247	760	2.26018	2.37118	2.49177	52
9	524	266	538	400	925	196	311	386	51
10	649	400	681	553	2.16090	374	504	597	50
11	774	538	823	706	255	552	697	807	49
12	899	667	966	860	420	780	891	2.50018	48
13	1.82025	801	1.98110	2.07014	585	909	2.38084	229	47
14	150	935	253	167	751	2.27088	279	440	46
15	276	1.90069	396	321	917	267	473	652	45
16	402	203	540	476	2.17038	447	668	864	44
17	528	337	684	630	249	626	863	2.51076	43
18	654	472	828	785	416	806	2.39058	289	42
19	780	607	972	939	582	987	258	502	41
20	906	741	1.99116	2.08094	749	2.28167	449	715	40
21	1.83033	876	281	250	916	848	645	929	39
22	159	1.91012	406	405	2.18084	528	841	2.52142	38
23	286	147	550	560	251	710	2.40038	357	37
24	413	282	695	716	419	891	235	571	36
25	540	418	841	872	587	2.29073	432	786	35
26	667	554	986	2.09028	755	254	629	2.53001	34
27	794	690	2.00131	184	923	437	827	217	33
28	922	826	277	341	2.19092	619	2.41025	432	32
29	1.84049	962	423	498	261	801	223	648	31
30	177	1.92098	569	654	430	984	421	865	30
31	306	285	715	811	599	2.30167	620	2.54082	29
32	433	371	862	969	769	851	819	299	28
33	561	508	2.01008	2.10126	938	534	2.42019	516	27
34	689	645	155	284	2.20108	718	218	734	26
35	818	782	302	442	278	902	418	952	25
36	946	920	449	600	449	2.31086	618	2.55170	24
37	1.85075	1.98057	596	758	619	271	819	389	23
38	204	195	743	916	790	456	2.43019	608	22
39	333	832	891	2.11075	961	641	220	827	21
40	462	470	2.02089	233	2.21182	826	422	2.56046	20
41	591	608	187	392	304	2.32012	623	266	19
42	720	746	335	552	475	197	825	487	18
43	850	885	483	711	647	333	2.44027	707	17
44	979	1.94023	631	871	819	570	230	928	16
45	1.86109	162	780	2.12080	992	756	438	2.57150	15
46	239	301	929	190	2.22164	943	636	371	14
47	369	440	2.08078	350	337	2.33130	839	538	13
48	499	579	227	511	510	317	2.45048	815	12
49	630	718	376	671	683	505	246	2.58038	11
50	760	853	526	832	857	693	451	261	10
51	891	997	675	993	2.28080	881	655	434	9
52	1.87021	1.95187	825	2.13154	204	2.34069	860	708	8
53	152	277	975	816	878	258	2.46065	932	7
54	233	417	2.04125	477	553	447	270	2.59156	6
55	415	557	276	639	727	636	476	381	5
56	546	698	426	801	902	825	682	606	4
57	677	838	577	963	2.24077	2.35015	888	831	3
58	809	979	728	2.14125	252	205	2.47095	2.60057	2
59	941	1.96120	879	288	428	395	302	283	1
60	1.88073	261	2.05030	451	604	585	509	509	0
	28°	27°	26°	25°	24°	23°	22°	21°	
NATURAL COTANGENT.									

TABLE B.		NATURAL TANGENT.						72A**	
	69°	70°	71°	72°	73°	74°	75°	76°	
0	2.60609	2.74748	2.90421	3.07768	3.27085	3.48741	3.78205	4.01078	60
1	786	997	696	8.08073	426	3.49125	640	576	59
2	963	2.75246	971	879	767	509	8.74075	4.02074	58
3	2.61190	496	2.91246	685	3.28109	894	512	574	57
4	418	746	523	991	452	3.50279	950	4.08076	56
5	646	996	799	3.09298	795	666	8.75388	578	55
6	874	2.76247	2.92076	606	3.29189	8.51053	828	4.04081	54
7	2.62103	498	854	914	483	441	8.76268	589	53
8	832	750	682	3.10223	829	829	709	4.05092	52
9	561	2.77002	910	532	8.80174	3.52219	3.77152	599	51
10	791	254	2.93189	842	521	609	595	4.06107	50
11	2.63021	507	468	3.11153	868	3.58001	8.78040	616	49
12	252	761	748	464	3.81216	898	485	4.07127	48
13	483	2.78014	2.94028	775	565	785	981	639	47
14	714	269	809	3.12087	914	3.54179	8.79878	4.08152	46
15	945	523	591	400	3.82264	578	827	666	45
16	2.64177	778	872	718	614	968	3.80276	4.09182	44
17	410	2.79033	2.95155	3.18027	965	3.55864	726	699	43
18	642	289	437	841	3.33817	761	3.81177	4.10216	42
19	875	545	721	656	670	3.56169	630	736	41
20	2.65109	802	2.96004	972	3.84023	557	3.82088	4.11256	40
21	842	2.80059	288	3.14288	877	957	587	778	39
22	576	816	573	605	732	3.57357	992	4.12301	38
23	811	574	858	922	3.85087	758	3.88449	825	37
24	2.66046	838	2.97144	3.15240	443	3.58160	906	4.13850	36
25	281	2.81091	430	558	800	562	3.84364	877	35
26	516	850	717	877	3.86158	966	824	4.14405	34
27	752	610	2.98004	3.16197	516	3.59870	3.85284	984	33
28	989	870	292	517	875	775	745	4.15465	32
29	2.67225	2.82180	580	838	3.87234	3.60181	3.86208	997	31
30	462	391	869	3.17159	594	588	671	4.16580	30
31	700	653	2.99158	481	955	996	3.87136	4.17064	29
32	937	914	447	804	3.88317	3.61405	601	600	28
33	2.68175	2.83176	738	3.18127	679	814	3.88068	4.18137	27
34	414	439	3.00028	451	3.89042	3.62224	536	675	26
35	653	702	819	775	406	636	3.89004	4.19215	25
36	892	965	611	3.19100	771	3.68048	474	756	24
37	2.69181	2.84229	903	426	3.40136	461	945	4.20298	23
38	871	494	3.01196	752	502	874	3.90417	842	22
39	612	758	489	3.20079	869	3.64289	890	4.21887	21
40	853	2.85028	783	406	3.41236	705	3.91364	933	20
41	2.70094	289	3.02077	734	604	3.65121	889	4.22481	19
42	385	555	872	3.21068	978	588	3.92316	4.23080	18
43	577	822	667	892	3.42343	957	793	580	17
44	819	2.86089	963	722	718	3.66376	3.93271	4.24132	16
45	2.71062	356	3.08260	3.22053	3.48084	796	751	685	15
46	805	624	556	884	456	3.67217	3.94232	4.25239	14
47	548	892	854	715	829	638	718	795	13
48	792	2.87161	3.04152	3.28048	3.44202	3.68061	3.95196	4.26352	12
49	2.72036	430	450	881	576	485	680	911	11
50	281	700	749	714	951	909	3.96165	4.27471	10
51	526	970	3.05049	3.24049	3.45327	3.69335	651	4.28082	9
52	771	2.88240	349	883	703	761	3.97139	595	8
53	2.78017	511	649	719	3.46080	3.70188	627	4.29159	7
54	263	783	950	3.25055	458	616	3.98117	724	6
55	509	2.89055	3.06252	392	837	3.71046	607	4.30291	5
56	756	327	544	729	3.47216	476	3.99099	860	4
57	2.74004	600	857	3.26067	596	907	592	4.31480	3
58	251	878	3.07160	406	977	3.72388	4.00086	4.32001	2
59	499	2.90147	464	745	3.48359	771	582	578	1
60	748	421	768	3.27085	741	3.73205	4.01078	4.33148	0
	20°	19°	18°	17°	16°	15°	14°	13°	
NATURAL COTANGENT.									

TRAVERSE TABLE;
OR,
LATITUDES AND DEPARTURES
TO
EVERY MINUTE:
AND
CALCULATED TO FOUR PLACES OF DECIMALS.

BY MICHAEL McDERMOTT,
CIVIL ENGINEER AND SURVEYOR.

LATITUDE 5 DEGREES.

1	2	3	4	5	6	7	8	9	10
1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000	9.0000	60
00	00	00	00	00	00	00	00	00	00 59
00	00	00	00	00	00	00	00	00	00 58
00	00	00	00	00	00	00	00	00	00 57
00	00	00	00	00	00	00	00	00	00 56
00	00	00	00	00	00	00	00	00	00 55
00	00	00	00	00	00	00	00	00	00 54
00	00	00	00	00	00	00	00	00	00 53
00	00	00	00	00	00	00	00	00	00 52
00	00	00	00	00	00	00	00	00	00 51
00	00	00	00	00	00	00	00	8.9999	50
00	00	00	00	00	5.9999	6.9999	7.9999	99	49
00	00	00	00	00	99	99	99	99	48
00	00	00	00	00	99	99	99	99	47
00	00	00	00	00	99	99	99	99	46
00	00	00	00	00	99	99	99	99	45
00	00	00	00	00	99	99	99	99	44
00	00	00	00	00	99	99	99	99	43
00	00	00	00	00	99	99	99	99	42
00	00	00	00	00	99	99	99	99	41
00	00	2.9999	3.9999	4.9999	99	99	99	98	40
00	00	99	99	99	99	99	98	98	39
00	00	99	99	99	99	99	98	98	38
00	00	99	99	99	99	99	98	98	37
00	00	99	99	99	99	99	98	98	36
00	1.9999	99	99	99	98	98	98	98	35
00	99	99	99	99	98	98	98	98	34
00	99	99	99	99	98	98	98	98	33
00	99	99	98	99	98	98	98	98	32
00	99	99	98	98	98	98	97	98	31
00	99	99	98	98	98	98	97	97	30
00	99	99	98	98	98	97	97	97	29
00	99	99	98	98	98	97	97	97	28
00	99	99	98	98	98	97	97	97	27
00	99	99	98	98	98	97	97	96	26
00	99	99	98	98	98	97	97	96	25
99	99	99	98	98	98	97	97	96	24
1.9999	99	98	98	98	97	96	97	95	23
99	99	98	98	98	97	96	96	95	22
99	99	98	98	98	97	96	96	95	21
99	99	98	97	97	97	96	96	95	20
99	99	98	97	97	97	96	96	95	19
99	99	98	97	97	97	96	95	94	18
99	98	98	97	96	95	95	95	94	17
99	98	98	97	96	95	94	94	94	16
99	98	98	97	96	95	94	93	93	15
99	98	97	96	96	95	94	93	92	14
99	98	97	96	96	94	94	93	92	13
99	98	97	96	96	94	93	92	91	12
99	98	97	96	95	94	93	92	91	11
99	98	97	96	95	93	92	91	90	10
99	98	97	96	95	93	92	91	90	9
99	98	97	96	95	93	92	91	90	8
99	98	96	95	95	93	92	90	89	7
99	97	96	95	94	93	92	90	89	6
99	97	96	95	94	92	91	90	89	5
99	97	96	95	94	92	91	90	88	4
99	97	96	94	93	92	91	89	88	3
99	97	96	94	93	92	90	89	87	2
99	97	96	94	93	91	90	89	87	1
1.9999	9.9997	2.9996	3.9994	4.9993	5.9991	6.9990	7.9988	8.9987	0
1	2	3	4	5	6	7	8	9	

DEPARTURE 80 DEGREES.

DEPARTURE 0 DEGREES.										75
1	2	3	4	5	6	7	8	9	1	
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	80	
1	03	06	09	12	15	17	20	23	26	
2	06	12	17	23	29	35	41	46	52	
3	09	17	26	35	44	52	61	70	78	
4	12	23	35	46	58	70	81	93	0.0104	
5	15	29	44	58	73	89	0.0102	0.0116	30	
6	18	35	53	70	88	0.0105	23	40	58	
7	20	41	61	82	0.0102	22	43	63	84	
8	23	47	70	93	17	40	63	86	0.0210	
9	26	52	79	0.0105	31	57	83	0.0210	36	
10	29	58	87	16	46	75	0.0204	38	62	
11	32	65	96	28	60	92	24	56	88	
12	35	70	0.0105	40	75	0.0209	44	79	0.0814	
13	38	76	13	51	89	27	65	0.0302	40	
14	41	81	22	63	0.0204	44	85	26	66	
15	44	87	31	74	18	62	0.0305	49	92	
16	47	93	40	86	33	79	26	72	0.0419	
17	50	99	48	98	48	96	47	96	46	
18	52	0.0105	57	0.0210	62	0.0314	67	0.0419	72	
19	55	11	66	21	77	82	87	42	98	
20	58	16	75	33	91	49	0.0407	66	0.0524	
21	61	22	83	44	0.0306	67	28	89	50	
22	64	28	92	56	20	84	48	0.0512	76	
23	67	34	0.0201	68	35	0.0401	68	35	0.0602	
24	70	40	09	79	49	19	89	58	28	
25	73	45	18	91	64	36	0.0509	82	54	
26	76	51	27	0.0302	78	54	29	0.0605	80	
27	79	57	36	14	93	71	50	28	0.0707	
28	81	63	44	26	0.0407	88	70	51	83	
29	84	69	53	38	22	0.5006	91	75	60	
30	87	75	62	49	37	23	0.0611	98	86	
31	89	81	71	61	51	41	31	0.0722	0.0802	
32	93	86	79	72	66	59	52	45	88	
33	96	92	88	84	80	76	72	68	64	
34	99	98	97	96	95	93	92	91	90	
35	0.0102	0.0204	0.0305	0.0407	0.0509	0.0611	0.0713	0.0814	0.0916	
36	05	09	14	19	24	28	33	38	42	
37	08	16	23	30	38	46	53	61	68	
38	11	21	32	42	53	63	74	84	95	
39	13	27	40	54	67	80	94	0.0907	0.1021	
40	16	33	49	66	82	98	0.0815	31	48	
41	19	39	58	77	97	0.0716	35	54	74	
42	22	44	67	89	0.0611	33	55	78	0.1100	
43	25	50	75	0.0500	26	51	76	0.1001	26	
44	28	56	84	12	40	68	96	24	52	
45	31	62	93	24	55	85	0.0916	47	78	
46	34	68	0.0401	35	69	0.0803	37	70	0.1204	
47	37	73	10	47	84	20	57	94	30	
48	40	79	19	58	98	38	77	0.1117	56	
49	48	85	28	70	0.0723	55	98	40	83	
50	45	91	36	82	27	72	0.1018	63	0.1309	
51	48	97	45	93	42	90	38	86	85	
52	51	0.0303	54	0.0605	57	0.0908	59	0.1210	62	
53	54	08	63	16	71	25	79	86	88	
54	57	14	71	28	86	43	0.1100	57	0.1414	
55	60	20	80	40	0.0800	60	20	80	40	
56	63	26	89	52	15	77	40	0.1303	66	
57	66	32	97	63	29	95	61	26	92	
58	69	37	0.0506	75	44	0.1012	81	50	0.1518	
59	72	43	15	86	58	29	0.1201	78	44	
60	0.0175	0.0349	0.0524	0.0698	0.0873	0.1047	0.1222	0.1396	0.1571	
1	2	3	4	5	6	7	8	9		
LATITUDE 89 DEGREES.										

LATITUDE 1 DEGREE.									
76	1	2	3	4	5	6	7	8	9
0	0.9999	1.9997	2.9996	3.9994	4.9993	5.9991	6.9990	7.9988	8.9987
1	99	97	95	94	92	91	89	88	86
2	98	97	95	94	92	90	89	87	86
3	98	97	95	93	92	90	88	86	85
4	98	97	95	93	92	90	88	86	85
5	98	96	95	93	91	89	87	86	84
6	98	96	95	93	91	89	87	86	84
7	98	96	94	92	90	88	86	85	82
8	98	96	94	92	90	88	86	84	82
9	98	96	94	92	90	87	85	83	81
10	98	96	94	92	90	87	85	83	81
11	98	96	93	91	89	87	85	82	80
12	98	96	93	91	89	87	85	82	80
13	98	95	93	91	89	86	84	82	79
14	98	95	93	91	89	86	84	82	79
15	98	95	93	90	88	86	83	81	78
16	98	95	93	90	88	86	83	81	78
17	97	95	92	90	87	85	82	80	77
18	97	95	92	90	87	85	82	79	77
19	97	95	92	89	87	84	81	79	76
20	97	95	92	89	87	84	81	79	76
21	97	94	92	89	86	83	80	78	75
22	97	94	92	89	86	83	80	78	75
23	97	94	91	88	85	82	79	76	74
24	97	94	91	88	85	82	79	76	73
25	97	94	91	88	85	81	78	75	72
26	97	94	91	88	85	81	78	75	72
27	97	93	90	87	84	81	77	74	70
28	97	93	90	87	84	80	77	74	70
29	97	93	90	86	83	80	76	73	69
30	97	93	90	86	83	80	76	73	69
31	97	93	89	86	82	79	76	72	68
32	96	93	89	86	82	78	75	71	68
33	96	93	89	85	82	78	75	71	67
34	96	93	89	85	82	78	74	70	67
35	96	92	88	84	81	77	74	70	66
36	96	92	88	84	81	77	73	69	65
37	96	92	88	84	80	76	72	68	64
38	96	92	88	84	80	75	71	67	63
39	96	92	87	83	79	75	71	67	63
40	96	92	87	83	79	75	71	66	62
41	96	91	87	83	78	74	70	66	61
42	96	91	87	82	78	74	69	65	60
43	95	91	86	82	77	73	69	64	60
44	95	91	86	82	77	72	68	63	59
45	95	90	86	81	76	72	67	63	58
46	95	90	86	81	76	71	66	62	57
47	95	90	85	80	76	71	66	62	57
48	95	90	85	80	76	71	66	61	56
49	95	90	85	80	75	70	65	60	55
50	95	90	85	80	75	69	64	59	54
51	95	90	84	79	74	69	64	59	53
52	95	89	84	79	74	68	63	58	52
53	95	89	84	78	73	67	63	57	51
54	95	89	83	78	73	67	62	56	51
55	94	89	83	77	72	66	61	55	50
56	94	89	83	77	72	66	60	54	49
57	94	89	82	76	71	65	60	54	48
58	94	88	82	76	71	65	59	53	47
59	94	88	82	76	70	64	58	52	46
60	0.9994	1.9988	2.9982	3.9976	4.9970	5.9963	6.9957	7.9951	8.9945
	1	2	3	4	5	6	7	8	9
DEPARTURE 88 DEGREES.									

DEPARTURE 1 DEGREE.

77

1	2	3	4	5	6	7	8	9	10
0	0.0175	0.0349	0.0524	0.0698	0.0873	0.1047	0.1222	0.1396	0.1571
1	77	55	32	0.0710	87	64	42	0.1419	97.69
2	80	61	41	21	0.0902	82	62	42	0.1623
3	83	66	50	83	16	99	82	66	49.57
4	86	72	59	45	81	0.1117	0.1803	90	76.56
5	89	78	67	56	46	85	24	0.1513	0.1702
6	92	84	76	68	60	52	44	36	28.54
7	95	90	85	80	75	69	64	59	54.53
8	98	96	93	91	89	87	84	82	80.52
9	0.0201	0.0401	02	0.0803	0.1004	0.1204	0.1405	0.1606	0.1806
10	04	07	11	14	18	22	25	29	32.50
11	06	13	20	26	33	39	46	52	59.49
12	09	19	28	38	47	56	66	75	85.48
13	12	25	37	49	62	74	86	98	0.1911
14	15	30	46	61	76	91	0.1506	0.1722	87.46
15	18	36	54	72	91	0.1309	27	45	68.45
16	21	43	63	84	0.1106	27	48	69	90.44
17	24	48	72	96	20	44	68	92	0.2016
18	27	54	81	0.0908	35	61	88	0.1815	42.42
19	30	60	89	19	49	80	0.1609	38	68.41
20	33	65	98	31	64	96	29	62	94.40
21	36	71	0.0707	42	78	0.1414	49	85	0.2120
22	38	77	16	54	98	81	70	0.1908	47.88
23	41	83	24	66	0.1207	48	90	81	78.37
24	44	89	33	77	22	66	0.1710	54	99.86
25	47	94	42	89	36	83	80	78	0.2225
26	50	0.0500	50	0.1000	51	0.1501	51	0.2001	51.34
27	53	06	59	12	65	18	71	24	77.33
28	56	12	68	24	80	36	92	48	0.2304
29	59	18	77	36	95	53	0.1808	71	30.31
30	62	24	85	47	0.1809	61	83	94	56.30
31	65	29	94	59	24	88	53	0.2118	82.29
32	68	35	0.0803	70	88	0.1606	73	41	0.2408
33	71	41	12	82	53	23	93	64	85.27
34	74	47	20	94	67	40	0.1914	87	61.26
35	76	53	29	0.1105	82	58	84	0.2210	87.25
36	79	58	38	17	96	75	54	84	0.2513
37	82	64	46	28	0.1411	98	74	57	89.23
38	85	70	55	40	25	0.1710	95	80	66.22
39	88	76	64	52	40	27	0.2015	0.2303	91.22
40	91	82	72	63	54	45	86	26	0.2617
41	94	88	81	75	69	63	57	50	44.19
42	97	93	90	87	84	80	77	74	70.18
43	0.0300	99	99	98	98	98	97	97	96.17
44	03	0.0605	0.0908	0.1210	0.1513	0.1815	0.2118	0.2420	0.2722
45	05	11	62	22	27	32	38	43	49.15
46	08	17	25	33	42	50	58	66	75.14
47	11	22	34	45	56	67	78	90	0.2801
48	14	28	42	56	71	85	99	0.2513	27.12
49	17	34	51	68	85	0.1902	0.2219	36	53.11
50	20	40	60	80	0.1600	19	39	59	79.10
51	23	46	68	91	14	37	60	82	0.2905
52	26	51	77	0.1303	29	54	80	0.2606	81.8
53	29	57	86	14	43	72	0.2300	29	57.7
54	32	63	95	26	58	90	21	53	84.6
55	35	69	0.1004	38	73	0.2007	42	76	0.3011
56	37	75	12	50	87	24	62	99	87.4
57	40	81	21	61	0.1702	42	82	0.2722	63.8
58	43	86	30	73	16	59	0.2402	46	89.2
59	46	92	38	84	81	77	23	69	0.3115
60	0.0349	0.0698	0.1047	0.1396	0.1745	0.2094	0.2443	0.2792	0.3141
1	2	3	4	5	6	7	8	9	

LATITUDE 88 DEGREES.

1	2	3	4	5	6	7	8	9	10
0	0.9986	1.9973	2.9959	3.9945	4.9932	5.9918	6.9904	7.9890	8.9877
1	86	72	58	44	31	17	03	89	75
2	86	72	58	44	30	16	02	88	74
3	86	72	57	43	29	15	01	86	72
4	86	71	57	43	29	14	00	86	71
5	86	71	57	42	28	13	6.9899	84	70
6	85	71	56	42	27	12	98	83	69
7	85	70	55	41	26	11	96	82	67
8	85	70	55	40	26	11	96	81	66
9	85	70	54	40	25	09	94	79	64
10	85	69	54	39	24	08	93	78	62
11	85	69	54	38	23	08	92	77	61
12	84	69	53	38	22	06	91	75	60
13	84	69	53	37	21	05	89	74	58
14	84	68	52	36	21	05	89	73	57
15	84	68	52	36	20	03	87	71	55
16	84	68	51	35	19	03	87	70	54
17	84	67	51	34	18	02	85	69	52
18	83	67	50	34	17	00	84	67	51
19	83	67	50	33	17	5.9900	83	66	50
20	83	66	49	32	16	5.9899	82	65	48
21	83	66	49	32	15	97	80	63	46
22	82	65	48	31	14	96	79	62	44
23	82	65	48	30	13	96	78	61	42
24	82	65	47	30	12	94	77	59	42
25	82	64	47	29	11	93	75	58	40
26	82	64	46	28	11	93	75	57	39
27	82	64	46	28	10	91	73	55	37
28	82	63	45	27	09	90	72	54	35
29	82	63	45	26	08	88	70	52	34
30	81	63	44	25	07	88	69	50	32
31	81	62	44	25	06	87	68	50	31
32	81	62	43	24	05	86	67	48	29
33	81	62	42	23	04	85	66	46	27
34	81	61	42	22	03	84	64	45	25
35	80	61	41	22	02	82	63	43	24
36	80	61	41	21	02	82	62	42	23
37	80	60	40	20	01	81	61	41	21
38	80	60	40	20	00	79	59	39	19
39	80	59	39	19	4.9899	78	58	38	17
40	80	59	39	18	98	77	57	36	16
41	79	59	38	17	97	76	55	34	14
42	79	58	38	17	96	75	54	34	13
43	79	58	37	16	95	74	53	32	11
44	79	58	36	15	94	73	52	30	09
45	79	57	36	14	93	72	50	29	07
46	78	57	35	14	92	70	49	27	06
47	78	56	35	13	91	69	47	26	04
48	78	56	34	12	90	68	46	24	02
49	78	56	33	11	89	67	45	22	00
50	78	55	33	10	88	66	43	21	8.9798
51	77	55	32	10	87	64	42	19	97
52	77	54	32	09	86	63	40	18	95
53	77	54	31	08	85	62	39	16	93
54	77	54	30	07	84	61	38	14	91
55	77	53	30	06	83	60	36	13	89
56	76	53	29	06	82	58	35	11	88
57	76	52	29	05	81	57	33	10	86
58	76	52	28	04	80	56	32	08	84
59	76	52	27	03	79	55	31	06	82
60	0.9976	1.9951	2.9927	3.9902	4.9878	5.9854	6.9829	7.9806	8.9780

DEPARTURE 8 DEGREES.									
	1	2	3	4	5	6	7	8	81
0	0.0523	0.1047	0.1570	0.2094	0.2617	0.3140	0.3664	0.4187	0.4711
1	26	53	79	0.2105	82	58	84	0.4210	87
2	29	58	88	17	46	75	0.3704	84	63
3	32	64	96	28	61	93	25	57	89
4	35	70	0.1605	40	75	0.3210	45	80	0.4815
5	38	76	14	52	90	27	65	0.4308	41
6	41	82	22	63	0.2704	45	86	26	67
7	44	77	31	75	19	62	0.3806	50	93
8	47	98	40	86	33	80	26	78	0.4919
9	50	99	49	98	48	97	47	96	46
10	52	0.1105	57	0.2210	62	0.3314	67	0.4419	72
11	55	11	66	21	77	22	87	42	98
12	58	16	75	33	88	91	0.3907	66	0.5024
13	61	22	83	44	0.2806	67	28	89	50
14	64	28	92	56	20	84	48	0.4512	76
15	67	34	0.1701	67	35	0.3401	68	35	0.5102
16	70	40	09	79	49	19	89	58	18
17	73	45	18	81	64	36	0.4009	82	44
18	76	51	27	0.2302	78	54	29	0.4605	80
19	79	57	36	14	93	71	50	28	0.5207
20	81	63	44	26	0.2907	88	70	51	83
21	84	69	53	38	22	0.3506	91	75	60
22	87	75	62	49	37	24	0.4111	98	86
23	90	80	71	61	51	41	81	0.4722	0.5312
24	93	86	79	72	66	59	52	25	88
25	96	92	88	84	80	76	72	48	64
26	99	98	97	96	95	93	92	91	90
27	0.0602	0.1204	0.1805	0.2407	0.3009	0.3611	0.4213	0.4814	0.5416
28	05	09	14	19	24	28	33	38	42
29	08	15	23	30	38	46	53	61	68
30	11	21	32	42	53	63	74	84	95
31	18	27	40	54	67	80	94	0.4907	0.5521
32	16	38	49	65	82	98	0.4314	30	47
33	19	38	58	77	96	0.3715	34	54	73
34	22	44	66	88	0.3111	33	55	77	99
35	24	50	75	0.2500	25	50	75	0.5000	0.5625
36	28	55	84	12	40	67	95	23	51
37	31	62	92	28	54	85	0.4416	46	77
38	34	67	0.1901	35	69	0.3802	36	70	0.5703
39	37	73	10	46	83	20	56	93	29
40	40	79	19	58	98	37	77	0.5116	56
41	42	85	27	70	0.3212	54	97	39	82
42	45	91	36	81	27	72	0.4517	62	0.5808
43	48	96	45	92	41	89	37	86	34
44	51	0.1302	53	0.2604	56	0.3907	58	0.5209	60
45	54	08	62	16	71	24	78	32	86
46	57	14	71	28	85	41	98	55	0.5912
47	60	20	79	39	99	69	0.4619	78	38
48	63	25	88	51	0.3314	76	89	0.5302	64
49	66	31	97	62	28	94	59	25	80
50	69	37	0.2006	74	43	0.4011	80	48	0.6017
51	71	43	14	86	57	28	0.4700	71	43
52	74	49	23	97	72	46	20	94	69
53	77	55	31	0.2709	87	64	40	0.5422	96
54	80	60	41	21	0.3401	81	61	42	0.6122
55	83	66	49	32	16	99	82	65	48
56	86	72	58	44	30	0.4116	0.4302	88	74
57	89	78	67	56	45	38	23	0.5511	0.6200
58	92	84	75	67	59	51	48	34	26
59	95	89	84	79	74	78	68	58	52
60	0.0698	0.1395	0.2093	0.2790	0.3488	0.4183	0.4883	0.5551	0.6278
	1	2	3	4	5	6	7	8	9

LATITUDE 86 DEGREES.

r	1	2	3	4	5	6	7	8	9	0
0	0.9976	1.1951	2.9927	3.9902	3.9878	5.9854	6.9829	7.9805	8.9780	60
1	75	51	26	02	77	52	28	03	79	59
2	75	50	26	01	76	51	26	02	77	58
3	75	50	25	00	75	50	25	00	75	57
4	75	50	24	3.9899	74	49	24	7.9798	73	56
5	75	49	24	98	73	48	22	97	71	55
6	74	49	23	98	72	46	21	95	70	54
7	74	48	22	97	71	45	19	94	68	53
8	74	48	21	96	70	44	18	92	66	52
9	74	48	21	95	69	43	17	90	64	51
10	74	47	20	94	68	42	15	89	62	50
11	73	47	20	94	67	40	14	87	51	49
12	73	46	19	92	66	39	12	85	58	48
13	73	46	19	92	65	37	10	83	56	47
14	73	46	18	91	64	36	09	82	54	46
15	73	45	18	90	63	35	08	80	53	45
16	72	45	17	89	62	34	06	78	51	44
17	72	44	16	88	61	33	05	77	49	43
18	72	44	16	88	60	31	03	75	47	42
19	72	43	14	86	58	30	01	73	44	41
20	71	43	14	86	57	28	00	71	43	40
21	71	42	13	85	56	27	6.9798	70	41	39
22	71	42	12	84	55	26	97	68	39	38
23	71	42	12	83	54	25	96	66	37	37
24	71	41	11	82	53	23	94	64	35	36
25	70	41	10	81	52	22	92	62	33	35
26	70	40	10	80	51	21	91	61	31	34
27	0.9970	40	09	80	40	19	89	59	29	33
28	70	40	08	78	48	18	87	57	26	32
29	70	39	08	78	47	16	86	55	25	31
30	69	38	08	77	46	15	84	54	23	30
31	69	38	07	76	45	13	82	51	20	29
32	69	37	06	75	44	12	81	50	18	28
33	69	37	06	74	43	11	80	48	17	27
34	68	37	05	73	42	10	78	46	15	26
35	68	36	04	72	40	08	76	44	12	25
36	68	36	03	71	39	07	75	42	10	24
37	68	35	03	70	38	06	73	41	08	23
38	68	35	02	69	37	04	71	38	06	22
39	67	34	01	68	36	03	70	37	04	21
40	67	34	00	67	34	01	68	34	01	20
41	66	33	00	66	33	00	66	33	8.9699	19
42	66	33	2.9899	66	32	5.9798	65	31	98	18
43	66	32	98	64	31	97	63	29	95	17
44	66	32	98	64	30	95	61	27	93	16
45	66	31	97	63	29	94	60	26	91	15
46	65	31	96	62	27	92	58	23	89	14
47	65	30	96	61	26	91	57	22	87	13
48	65	30	95	60	25	89	54	19	85	12
49	65	29	94	59	24	87	53	18	82	11
50	64	29	93	58	22	86	51	15	80	10
51	64	28	93	57	21	85	49	14	78	9
52	64	28	92	56	20	83	47	11	75	8
53	64	27	91	55	19	82	46	10	73	7
54	64	27	91	54	18	81	45	08	72	6
55	63	26	90	53	16	79	42	06	69	5
56	63	26	89	52	15	78	41	04	67	4
57	63	25	88	51	14	76	39	02	64	3
58	63	25	88	50	13	75	38	00	63	2
59	62	24	87	49	11	73	35	7.9698	60	1
60	0.9962	1.9924	2.9886	3.9848	4.9810	5.9771	6.9733	7.9695	8.9657	0
	1	2	3	4	5	6	7	8	9	0

DEPARTURE 4 DEGREES.										88
1	2	3	4	5	6	7	8	9	10	
0	0.0698	0.1395	0.2093	0.2790	0.3488	0.4186	0.4883	0.5581	0.6278	60
1	0.0701	0.1401	0.2102	0.2802	0.3503	0.4203	0.4904	0.5604	0.6305	59
2	03	07	10	14	17	20	24	27	31	58
3	06	13	19	25	32	38	44	50	57	57
4	09	18	28	37	46	55	64	74	83	56
5	12	24	36	48	61	73	85	97	0.6409	55
6	15	30	45	60	75	90	0.5005	0.5720	85	54
7	18	36	54	72	89	0.4307	25	43	61	53
8	21	42	62	83	0.3604	25	46	66	87	52
9	24	47	71	95	19	42	66	90	0.6513	51
10	27	53	80	0.2906	83	60	86	0.5813	89	50
11	30	58	89	18	48	77	0.5107	86	66	49
12	32	65	97	30	62	94	27	59	92	48
13	35	71	0.2206	41	77	0.4412	47	82	0.6618	47
14	38	76	15	53	91	29	67	0.5906	44	46
15	41	82	23	64	0.3706	47	88	29	70	45
16	44	88	32	76	20	64	0.5208	52	96	44
17	47	94	41	88	35	81	28	75	0.6722	43
18	50	0.1500	49	99	49	99	49	98	48	42
19	53	05	58	0.3001	64	0.4516	69	0.6022	74	41
20	56	11	67	22	78	34	89	45	0.6800	40
21	59	17	76	34	93	51	0.5309	68	27	39
22	61	23	84	46	0.3807	68	30	91	53	38
23	64	29	93	57	22	85	50	0.6111	78	37
24	67	34	0.2302	69	36	0.4603	70	88	0.6905	36
25	70	40	10	80	51	21	91	61	31	35
26	73	46	19	92	65	38	0.5411	84	57	34
27	76	52	28	0.3104	80	55	31	0.6207	83	33
28	79	58	36	15	94	73	52	30	0.7009	32
29	82	63	45	27	0.8909	90	72	54	35	31
30	85	69	54	38	23	0.4708	92	77	61	30
31	88	75	63	50	38	25	0.5513	0.6300	98	29
32	90	81	71	62	52	42	33	23	0.7114	28
33	93	87	80	73	67	60	53	46	40	27
34	96	92	89	85	81	77	73	70	66	26
35	99	98	97	96	96	95	94	93	92	25
36	0.0802	0.1604	0.2406	0.3208	0.4010	0.4812	0.5614	0.6416	0.7218	24
37	05	10	15	20	25	29	34	39	44	23
38	08	16	23	31	39	47	55	62	70	22
39	11	21	32	43	54	64	75	86	96	21
40	14	27	41	54	68	82	95	0.6509	0.7322	20
41	17	33	50	66	83	99	0.5716	82	49	19
42	19	39	58	78	97	0.4916	86	55	75	18
43	22	45	67	89	0.4112	34	56	78	0.7401	17
44	25	50	76	0.3301	26	51	76	0.6602	27	16
45	28	56	84	12	41	69	97	25	53	15
46	31	62	93	24	55	86	0.5817	48	79	14
47	34	68	0.2502	36	70	0.5008	37	71	0.7505	13
48	37	74	10	47	84	21	58	94	31	12
49	40	79	19	59	99	38	78	0.6718	57	11
50	43	85	28	70	0.4213	56	98	41	83	10
51	46	91	37	82	28	73	0.5919	64	0.7610	9
52	48	97	45	94	42	90	39	87	36	8
53	51	0.1703	54	0.3405	57	0.5108	59	0.6810	62	7
54	54	08	63	17	71	25	79	34	88	6
55	57	14	71	28	86	43	0.6000	57	0.7714	5
56	60	20	80	40	0.4300	60	20	80	40	4
57	63	26	89	52	15	17	40	0.6908	66	3
58	66	32	98	63	29	95	61	26	92	2
59	69	37	0.2606	75	44	0.5212	81	50	0.7818	1
60	0.0872	0.1740	0.2615	0.3436	0.4358	0.5230	0.6101	0.6973	0.7844	0
1	2	3	4	5	6	7	8	9		
LATITUDE 85 DEGREES.										

1	2	3	4	5	6	7	8	9	10
0.9962	1.4924	2.9886	3.9848	4.9810	5.9771	6.9733	7.9695	8.9657	60
1	62	28	85	47	09	69	31	94	55
2	61	28	84	46	07	68	30	91	53
3	61	22	84	45	06	67	28	90	51
4	61	22	83	44	06	65	26	87	48
5	60	21	82	43	04	64	24	86	46
6	60	21	81	42	02	62	23	83	44
7	60	20	81	41	01	61	21	82	42
8	60	20	80	40	00	59	19	79	39
9	60	19	79	38	4.9798	58	17	77	37
10	59	19	78	38	97	56	16	75	35
11	59	18	77	36	96	55	14	73	32
12	59	18	76	35	94	53	12	70	29
13	59	17	76	34	93	52	10	69	27
14	58	17	75	33	92	50	08	66	25
15	58	16	74	32	90	48	06	64	22
16	58	16	73	31	89	47	05	62	20
17	58	15	73	30	88	45	08	60	18
18	57	14	72	29	86	43	6.9600	58	15
19	57	14	71	28	85	42	99	56	13
20	57	13	70	27	84	40	97	54	10
21	56	13	69	26	82	38	95	52	08
22	56	12	69	25	81	37	93	50	06
23	56	12	68	24	80	35	91	47	03
24	56	11	67	22	78	34	89	45	00
25	56	11	66	21	77	32	88	42	8.9598
26	55	10	65	20	76	31	86	41	96
27	55	00	64	19	74	29	84	48	93
28	55	09	64	18	73	27	82	86	91
29	55	08	63	17	71	25	79	84	88
30	54	08	62	16	70	24	78	82	86
31	54	07	61	15	69	22	76	80	83
32	53	07	60	14	67	20	74	27	81
33	53	06	59	12	66	19	72	25	78
34	53	06	58	11	64	17	70	22	75
35	53	05	58	10	63	16	68	20	73
36	52	05	57	09	62	14	66	18	71
37	52	04	56	08	60	12	64	16	68
38	52	03	55	07	59	10	62	14	65
39	52	03	54	06	57	08	60	11	63
40	51	02	53	04	56	07	58	09	60
41	51	02	52	03	54	05	56	07	57
42	51	01	52	02	53	04	54	06	55
43	51	01	51	01	52	02	52	02	53
44	50	00	50	00	50	00	50	00	50
45	50	1.9899	59	3.9799	49	8.9698	48	7.9598	47
46	49	99	58	98	47	96	46	95	45
47	49	98	57	96	46	95	44	93	42
48	49	98	56	95	44	93	42	90	39
49	48	97	56	94	43	91	40	88	37
50	48	96	55	93	41	89	37	86	35
51	48	96	53	92	40	87	35	83	33
52	48	95	53	90	38	86	33	81	31
53	48	95	52	89	37	84	31	78	29
54	47	94	51	88	35	82	29	76	27
55	47	93	50	87	34	80	27	74	25
56	46	93	49	86	32	78	25	71	23
57	46	92	48	84	31	77	23	69	21
58	46	92	47	83	29	75	21	66	19
59	46	91	47	82	28	73	19	64	17
60	0.9945	1.9890	2.9836	3.9781	4.9726	5.9671	6.9616	7.9562	8.9507
1	2	3	4	5	6	7	8	9	10

DEPARTURE 5 DEGREES.										85
1	2	3	4	5	6	7	8	9	10	
0	0.0872	0.1743	0.2615	0.3486	0.4358	0.5230	0.6101	0.6973	0.7844	60
1	75	49	24	98	73	47	22	96	71	59
2	77	55	32	0.3510	87	64	42	0.7019	97	58
3	80	61	41	21	0.4402	82	62	42	0.7928	57
4	83	66	49	32	16	99	82	65	48	56
5	85	72	58	34	80	0.5316	0.6202	88	74	55
6	89	78	67	56	45	83	22	0.7111	0.8000	54
7	92	84	75	67	59	51	43	84	26	53
8	95	89	84	79	74	68	63	58	52	52
9	98	95	93	90	88	86	88	81	78	51
10	0.0901	0.1801	0.2702	0.3602	0.4503	0.5403	0.6304	0.7204	0.8105	50
11	03	07	10	14	17	20	24	27	31	49
12	06	13	19	25	32	38	44	50	57	48
13	09	18	28	37	46	55	64	74	83	47
14	12	24	36	48	61	73	85	97	0.8209	46
15	15	30	45	60	75	90	0.6405	0.7320	85	45
16	18	36	54	72	90	0.5507	25	43	61	44
17	21	42	62	83	0.4604	25	46	66	87	43
18	24	47	71	95	19	42	66	90	0.8313	42
19	27	53	80	0.3706	88	60	86	0.7418	89	41
20	30	59	88	18	48	77	0.6507	86	66	40
21	32	65	97	20	62	94	27	59	92	39
22	35	71	0.3806	41	77	0.5612	47	82	0.8418	38
23	38	76	15	53	91	29	67	0.7606	44	37
24	41	82	23	64	0.4706	47	88	29	70	36
25	44	88	32	76	20	64	0.6608	52	96	35
26	47	94	41	88	35	81	28	75	0.8522	34
27	50	0.1900	49	99	49	0.5709	49	98	48	33
28	53	05	58	0.3811	64	16	69	0.7622	74	32
29	56	11	67	22	78	34	89	45	0.8600	31
30	59	17	76	34	93	51	0.6710	68	27	30
31	61	23	84	46	0.4807	85	80	91	58	29
32	64	28	93	57	21	0.5808	49	0.7714	78	28
33	67	34	0.2901	68	36	20	70	87	0.8704	27
34	70	40	10	80	50	37	90	60	80	26
35	73	46	19	92	65	55	0.6810	83	56	25
36	75	52	27	0.3903	79	72	81	0.7806	82	24
37	79	57	36	15	94	81	51	80	0.8808	23
38	82	63	45	26	0.4908	90	71	53	84	22
39	85	69	54	38	23	0.5907	92	76	61	21
40	87	75	62	50	37	24	0.6912	99	87	20
41	90	81	71	61	52	42	82	0.7922	0.8913	19
42	93	86	80	73	66	59	52	46	89	18
43	96	92	88	84	81	77	78	69	65	17
44	99	98	97	96	95	94	98	92	91	16
45	0.1002	0.2004	0.3006	0.4008	0.5010	0.6011	0.7013	0.8015	0.9017	15
46	05	10	14	19	24	29	34	38	43	14
47	08	15	23	31	39	46	54	62	69	13
48	11	21	32	42	53	64	74	85	95	12
49	14	27	41	54	68	81	96	0.8108	0.9122	11
50	16	33	49	66	82	98	0.7115	81	48	10
51	19	38	58	77	96	0.6115	34	54	73	9
52	22	44	66	89	0.5111	33	55	77	99	8
53	25	50	75	0.4100	25	50	75	0.8200	0.9225	7
54	28	56	84	12	40	67	95	23	51	6
55	31	62	92	23	54	85	0.7216	46	77	5
56	34	67	0.3101	35	69	0.6202	36	70	0.9802	4
57	37	73	10	46	83	20	56	93	29	3
58	40	79	19	58	98	37	77	0.8316	56	2
59	42	85	27	70	0.5212	54	97	89	82	1
60	0.1045	0.2091	0.3136	0.4181	0.5226	0.6272	0.7317	0.8362	0.9408	0
1	2	3	4	5	6	7	8	9	10	
LATITUDE 84 DEGREES.										

LATITUDE 6 DEGREES.									
1	2	3	4	5	6	7	8	9	10
0	0.9945	1.9890	2.9836	3.9781	4.9726	5.9671	6.9616	7.9562	8.9507
1	45	90	33	79	24	69	14	59	04
2	45	89	34	78	23	68	12	57	01
3	44	89	33	77	21	66	10	54	8.9499
4	44	88	32	76	20	64	08	52	96
5	43	88	31	75	18	62	06	50	93
6	43	87	30	74	17	60	04	47	91
7	43	87	29	72	15	59	02	45	98
8	43	86	28	71	14	57	00	42	85
9	42	85	27	69	12	54	6.9597	39	82
10	42	84	26	68	11	53	95	37	79
11	41	83	25	67	09	51	93	34	76
12	41	83	25	66	08	49	91	32	74
13	41	82	24	65	06	47	88	30	71
14	41	82	23	64	05	45	86	27	68
15	40	80	22	62	03	43	84	25	65
16	40	80	21	61	01	41	81	22	62
17	40	79	20	59	4.9699	39	79	19	59
18	40	79	19	58	98	38	77	17	56
19	39	78	18	57	96	36	75	14	54
20	38	78	17	56	95	34	73	12	51
21	38	77	16	54	93	32	70	09	48
22	38	77	15	53	92	30	68	06	45
23	38	76	14	52	90	28	66	04	42
24	38	75	13	51	89	26	64	02	39
25	37	74	12	49	87	24	62	7.9499	36
26	37	74	11	48	85	22	59	96	33
27	36	73	10	47	83	20	57	94	30
28	36	73	09	46	82	18	55	91	28
29	36	72	08	44	80	16	52	88	24
30	36	71	07	43	79	14	50	86	21
31	35	70	06	41	77	12	48	83	19
32	35	70	05	40	76	11	46	81	16
33	35	69	04	39	74	08	43	78	13
34	34	69	03	38	72	06	41	75	10
35	34	68	02	36	70	04	38	73	07
36	34	67	01	35	69	02	36	70	03
37	33	66	00	33	67	00	34	67	01
38	33	66	2.9799	32	66	5.9599	32	65	8.9398
39	33	65	98	31	64	96	29	62	95
40	32	65	97	30	62	94	27	59	92
41	32	64	96	28	60	92	24	56	88
42	32	63	95	27	59	90	22	54	85
43	31	62	94	25	57	88	20	51	82
44	31	62	93	24	55	86	17	48	79
45	31	61	92	22	53	84	15	46	76
46	30	61	91	21	52	82	12	42	73
47	30	60	90	20	50	80	10	40	70
48	30	59	89	19	49	78	08	38	67
49	29	58	88	17	47	76	05	34	64
50	29	58	87	16	45	74	03	32	61
51	29	57	86	14	43	72	00	29	58
52	28	57	85	13	42	70	6.9498	26	55
53	28	56	84	11	40	68	95	23	51
54	28	55	83	10	38	66	93	21	48
55	27	54	82	09	36	63	90	18	45
56	27	54	81	08	35	61	88	15	42
57	27	53	80	06	33	59	86	14	39
58	26	52	79	05	31	57	83	10	36
59	26	51	78	03	29	55	81	06	33
60	0.9926	1.9851	2.9777	3.9702	4.9628	5.9553	6.9479	7.9404	8.9330
1	2	3	4	5	6	7	8	9	
DEPARTURE 83 DEGREES.									

DEPARTURE 6 DEGREES.

87

	1	2	3	4	5	6	7	8	9	
0	0.1045	0.2091	0.3136	0.4181	0.5227	0.6272	0.7317	0.8362	0.9408	60
1	0.0748	97	43	93	41	89	37	86	34	59
2	51	0.2102	53	0.4204	56	0.6307	58	0.8409	60	58
3	54	08	62	16	70	24	78	32	86	57
4	57	14	71	28	85	41	98	55	0.9512	56
5	60	19	79	39	99	58	0.7418	78	87	55
6	63	25	88	50	0.5313	76	38	0.8501	63	54
7	66	31	97	62	28	93	59	24	90	53
8	68	37	0.3205	74	42	0.6410	79	47	0.9616	52
9	71	43	14	85	57	28	99	70	42	51
10	74	48	23	97	71	35	0.7519	94	68	50
11	77	54	31	0.4308	86	53	40	0.8617	94	49
12	80	60	40	20	0.5400	80	60	40	0.9720	48
13	83	66	49	32	15	0.6507	80	63	46	47
14	86	72	57	43	29	15	0.7601	86	72	46
15	88	77	66	55	44	32	21	0.8710	98	45
16	92	83	75	66	58	60	41	33	0.9824	44
17	95	89	94	78	73	77	62	56	51	43
18	97	95	92	89	87	84	81	78	76	42
19	0.1100	0.2200	81	0.4401	0.5501	0.6601	0.7701	0.8802	0.9902	41
20	03	06	0.8309	12	16	19	22	25	28	40
21	06	12	18	24	30	36	42	48	54	39
22	09	18	27	36	45	53	62	71	80	38
23	12	24	35	43	59	71	83	94	1.0006	37
24	15	29	44	59	74	88	0.7803	0.8918	32	36
25	18	35	53	70	88	0.6706	23	41	58	35
26	21	41	62	82	0.5603	23	44	64	85	34
27	23	47	70	94	17	40	64	87	1.0111	33
28	26	53	79	0.4505	32	58	84	0.9010	87	32
29	29	58	87	16	46	75	0.7904	33	62	31
30	32	64	96	28	60	92	24	56	88	30
31	35	70	0.3405	40	75	0.6809	44	79	1.0214	29
32	38	76	13	51	89	27	65	0.9102	40	28
33	41	81	22	63	0.5704	44	85	26	66	27
34	44	87	31	74	18	62	0.8005	69	92	26
35	47	93	40	86	32	79	26	92	1.0319	25
36	49	99	48	98	47	96	46	95	45	24
37	52	0.2305	57	0.4609	62	0.6914	66	0.9218	71	23
38	55	10	66	21	76	31	86	42	97	22
39	58	16	74	32	90	48	0.8106	64	1.0422	22
40	61	22	83	44	0.5805	65	26	87	48	20
41	64	28	91	55	19	83	47	0.9310	71	19
42	67	33	0.3500	67	34	0.7000	67	34	1.0500	18
43	70	39	09	78	48	18	87	57	86	17
44	73	45	18	90	63	35	0.8208	80	53	16
45	75	51	26	0.4702	77	52	28	0.9408	79	15
46	78	57	35	13	92	70	48	26	1.0605	14
47	81	62	44	25	0.5906	87	68	50	81	13
48	84	68	52	36	20	0.7104	88	72	56	12
49	87	74	61	48	35	21	0.8308	95	82	11
50	90	80	69	59	49	89	29	0.9518	1.0708	10
51	93	85	78	71	64	56	49	42	81	9
52	96	91	87	82	78	74	69	65	60	8
53	99	97	96	94	93	91	90	88	87	7
54	0.1201	0.2403	0.3604	0.4806	0.6007	0.7208	0.8410	0.9611	1.0813	6
55	04	09	13	17	22	26	30	34	39	5
56	07	14	21	28	36	43	50	57	64	4
57	10	20	30	40	50	60	70	80	90	3
58	13	25	39	52	65	77	90	0.9708	1.0916	2
59	16	31	47	63	79	95	21	26	42	1
60	0.1219	0.2437	0.3656	0.4875	0.6094	0.7312	0.8531	0.9750	1.0968	0
	1	2	3	4	5	6	7	8	9	

LATITUDE 88 DEGREES.

LATITUDE 7 DEGREES.

	1	2	3	4	5	6	7	8	9	10
0	0.9926	1.9851	2.9777	3.9702	4.9628	5.9553	6.9479	7.9404	8.9330	9.9255
1	25	50	75	00	25	51	76	01	26	51
2	25	50	74	3.9699	24	49	74	7.9888	23	58
3	24	49	73	97	22	46	71	95	19	57
4	24	48	72	96	20	44	68	92	16	56
5	23	48	70	94	18	42	65	89	13	55
6	23	47	70	93	17	40	63	86	10	54
7	28	46	69	91	15	38	60	83	06	53
8	23	45	68	90	13	36	58	81	03	52
9	22	44	67	89	11	33	55	78	00	51
0	22	44	66	88	10	31	53	75	8.9297	50
1	21	43	64	86	08	29	50	72	93	49
2	21	42	63	84	06	27	48	69	90	48
3	20	41	62	83	03	24	45	66	87	47
4	20	41	61	82	02	22	43	63	84	46
5	20	40	60	80	00	20	40	61	80	45
6	20	39	59	79	4.9599	18	38	58	77	44
7	19	39	58	77	97	15	35	64	73	43
8	19	38	57	76	95	13	32	61	70	42
9	18	37	56	74	93	11	29	48	66	41
0	18	36	55	73	91	09	27	46	64	40
1	18	35	54	71	89	07	25	43	61	39
2	18	35	53	70	88	05	23	40	58	38
3	17	34	51	68	86	03	20	37	54	37
4	17	33	50	67	84	02	17	34	50	36
5	16	33	49	65	82	5.9498	14	31	47	35
6	16	32	48	64	80	96	12	28	44	34
7	15	30	47	62	78	93	09	25	40	33
8	15	30	46	61	76	91	06	22	37	32
9	14	29	44	59	74	88	08	18	36	31
0	14	29	43	58	72	86	01	15	30	30
1	14	28	42	56	70	84	6.9398	12	26	29
2	14	27	41	55	69	82	96	10	23	28
3	13	26	40	53	67	80	93	06	19	27
4	13	26	39	52	65	78	90	03	16	26
5	12	24	38	50	63	75	87	00	13	25
6	12	24	37	49	61	73	85	7.9298	10	24
7	11	23	35	47	59	70	82	94	06	23
8	11	23	34	46	57	68	80	91	03	22
9	11	22	33	44	55	66	77	88	8.9199	21
0	11	21	32	42	53	64	74	85	95	20
1	10	20	30	40	51	61	71	81	91	19
2	10	20	29	39	49	59	69	78	88	18
3	09	19	28	37	48	57	66	75	85	17
4	09	18	27	36	46	55	64	73	82	16
5	08	17	26	34	44	52	61	69	78	15
6	08	17	25	33	42	50	59	66	74	14
7	08	16	24	31	40	47	55	63	71	13
8	08	16	23	30	38	45	53	60	68	12
9	07	14	21	28	36	42	50	57	64	11
0	07	13	20	27	34	40	47	54	61	10
1	06	12	19	25	32	37	44	51	58	9
2	06	12	18	24	30	35	41	48	55	8
3	05	11	16	22	28	33	38	44	50	7
4	05	10	15	20	26	31	36	41	46	6
5	04	09	14	18	24	28	33	37	42	5
6	04	09	13	17	22	26	30	34	38	4
7	04	08	12	15	20	23	27	31	35	3
8	04	07	11	14	18	21	25	28	32	2
9	03	06	09	12	16	18	22	25	28	1
0	0.9908	1.9805	2.9708	3.9611	4.9514	5.9416	6.9319	7.9222	8.9124	9.9026
	1	2	3	4	5	6	7	8	9	10

DEPARTURE 82 DEGREES.

DEPARTURE 7 DEGREES.

80

	1	2	3	4	5	6	7	8	9	
0	0.1219	0.2437	0.3656	0.4875	0.6094	0.7312	0.8531	0.9750	1.0968	60
1	22	43	65	86	0.6108	29	51	73	94	59
2	25	49	74	98	23	47	72	96	1.1021	58
3	27	55	82	0.4910	37	64	92	0.9819	47	57
4	30	60	91	21	51	81	0.8611	42	72	56
5	33	66	99	32	66	99	32	65	98	55
6	36	72	0.3708	44	80	0.7416	52	88	1.1124	54
7	39	78	17	56	95	33	72	0.9911	50	53
8	42	84	25	67	0.6209	51	93	34	76	52
9	45	89	34	79	24	68	0.8713	58	1.1202	51
10	48	95	43	90	38	86	33	81	28	50
11	50	0.2501	51	0.5004	52	0.7502	53	1.0003	54	49
12	53	07	60	13	67	20	78	26	80	48
13	56	12	69	25	81	37	93	50	1.1306	47
14	59	18	77	36	96	55	0.8814	73	32	46
15	62	24	86	48	0.6810	72	84	96	58	45
16	65	30	95	60	25	89	54	1.0119	84	44
17	68	36	0.3803	71	39	0.7607	75	42	1.1410	43
18	71	41	12	82	53	24	94	65	35	42
19	74	47	21	94	68	41	0.8915	88	62	41
20	76	53	29	0.5106	82	58	35	1.0211	88	40
21	79	59	38	17	97	76	55	34	1.1514	39
22	82	64	47	29	0.6411	93	75	58	40	38
23	85	70	55	30	26	0.7711	96	81	66	37
24	88	76	64	52	40	28	0.9016	1.0304	92	36
25	91	82	72	63	54	45	36	26	1.1617	35
26	94	87	81	75	69	62	56	50	43	34
27	97	93	90	86	83	80	76	73	69	33
28	0.1300	98	99	98	98	97	97	96	96	32
29	02	0.2605	0.3907	0.5210	0.6512	0.7814	0.9117	1.0419	1.1722	31
30	05	11	16	21	27	32	37	42	48	30
31	08	16	24	32	41	49	57	65	73	29
32	11	22	33	44	55	66	77	88	99	28
33	14	28	42	56	70	88	97	1.0511	1.1825	27
34	17	34	50	67	84	0.7901	0.9218	84	51	26
35	20	39	59	79	99	18	38	58	77	25
36	23	45	68	90	0.6613	36	58	81	1.1903	24
37	25	51	76	0.5302	27	52	78	1.0603	29	23
38	28	57	85	13	42	70	98	26	55	22
39	31	62	94	25	56	87	0.9318	50	81	21
40	34	68	0.4002	36	71	0.8005	89	73	1.2007	20
41	37	74	11	48	85	22	59	96	33	19
42	40	80	20	60	0.6700	39	79	1.0719	59	18
43	43	85	28	71	14	56	99	42	84	17
44	46	91	37	82	28	74	0.9419	65	1.2110	16
45	49	97	46	94	43	91	40	88	37	15
46	51	0.2703	54	0.5406	57	0.8108	60	1.0811	63	14
47	54	09	68	17	72	26	80	34	89	13
48	57	14	72	29	86	48	0.9500	58	1.2215	12
49	60	20	80	40	0.6800	60	20	80	40	11
50	63	26	89	52	15	77	40	1.0903	66	10
51	66	32	97	63	29	95	61	26	92	9
52	69	37	0.4106	75	44	0.8212	81	50	1.2318	8
53	72	43	15	86	58	80	0.9601	73	44	7
54	74	49	23	98	72	46	21	95	70	6
55	77	55	32	0.5509	87	64	41	1.1018	96	5
56	80	60	41	21	0.6901	81	61	42	1.2422	4
57	83	66	49	32	16	99	82	65	48	3
58	86	72	58	44	30	0.8816	0.9702	88	74	2
59	89	78	67	56	45	38	22	1.1111	1.2500	1
60	0.1892	0.2783	0.4175	0.5567	0.6959	0.8351	0.9742	1.1134	1.2525	0
	1	2	3	4	5	6	7	8	9	

LATITUDE 82 DEGREES.

	1	2	3	4	5	6	7	8	9	
0	0.9903	1.9805	2.9708	3.9611	4.9514	5.9416	6.9319	7.9222	8.9124	00
1	02	05	07	09	12	14	16	18	21	59
2	02	04	06	08	09	11	13	15	17	58
3	01	03	05	06	08	09	11	12	14	57
4	01	02	03	04	06	07	08	09	10	56
5	00	01	02	02	03	04	04	05	06	55
6	00	00	01	01	01	01	01	02	02	54
7	0.9899	00	2.9699	3.9599	4.9499	5.9399	6.9299	7.9198	8.9098	53
8	99	1.9799	98	98	97	96	96	95	95	52
9	99	98	97	96	95	94	93	92	91	51
10	99	97	96	94	93	92	90	89	87	50
11	98	96	95	93	91	89	87	86	84	49
12	98	96	93	91	89	87	85	82	80	48
13	97	95	92	89	87	84	81	78	77	47
14	97	94	91	88	85	81	78	75	72	46
15	97	93	90	86	83	79	75	72	69	45
16	96	92	88	84	81	77	73	69	65	44
17	96	91	87	83	79	74	70	66	61	43
18	95	91	86	81	76	72	67	62	58	42
19	95	90	84	79	74	69	64	58	53	41
20	94	89	83	78	72	66	61	55	50	40
21	94	88	82	76	70	64	58	52	46	39
22	94	87	81	74	68	62	55	49	42	38
23	93	86	79	72	66	59	52	45	38	37
24	93	85	78	71	64	56	49	42	34	36
25	92	85	77	69	62	54	46	38	31	35
26	92	84	76	68	60	51	43	35	27	34
27	91	83	74	67	57	48	40	31	23	33
28	91	82	73	64	55	46	37	28	19	32
29	91	81	72	66	53	44	34	25	15	31
30	90	80	71	61	51	41	31	22	12	30
31	90	79	69	59	49	38	28	18	08	29
32	89	79	68	57	47	36	25	14	04	28
33	89	78	67	56	44	33	22	10	00	27
34	88	77	65	54	42	30	19	07	8.8996	26
35	88	76	64	52	40	28	16	04	92	25
36	88	75	63	50	38	26	13	01	88	24
37	87	74	61	48	36	23	10	7.9094	84	23
38	87	73	60	47	34	20	07	94	80	22
39	86	73	59	45	32	17	04	90	77	21
40	86	72	57	43	29	15	01	86	72	20
41	85	71	56	42	27	12	6.9198	83	69	19
42	85	70	55	40	25	09	94	79	64	18
43	84	69	54	38	23	07	92	76	61	17
44	84	68	52	36	21	05	89	73	57	16
45	83	67	51	34	18	02	85	69	53	15
46	83	66	50	33	16	5.9299	82	66	49	14
47	82	65	48	31	14	96	79	62	44	13
48	82	65	47	29	12	94	76	58	41	12
49	82	64	45	27	09	91	73	54	36	11
50	81	63	44	26	07	88	70	51	33	10
51	81	62	43	24	05	85	66	47	28	9
52	81	61	42	22	03	83	64	44	26	8
53	80	60	40	20	00	80	60	40	20	7
54	80	59	39	18	4.9398	78	57	37	16	6
55	79	58	37	16	96	75	54	33	12	5
56	79	57	36	15	94	72	51	30	08	4
57	78	56	35	13	91	69	47	26	04	3
58	78	56	33	11	89	67	45	22	00	2
59	77	55	32	09	87	64	41	18	8.8896	1
60	0.9877	1.9754	2.9631	3.9508	4.9385	5.9261	6.9138	7.9015	8.8892	0
	1	2	3	4	5	6	7	8	9	

DEPARTURE 8 DEGREES.

91

	1	2	3	4	5	6	7	8	9	
0	0.1892	0.2783	0.4175	0.5567	0.6959	0.8350	0.9742	1.1134	1.2525	60
1	96	89	84	78	73	68	62	57	51	59
2	98	95	94	90	88	85	83	80	78	58
3	0.1400	0.2801	91	0.5802	0.7002	0.8402	0.9803	1.1203	1.2604	57
4	08	07	0.4210	13	17	20	23	26	31	56
5	06	12	18	24	31	37	43	49	55	55
6	09	18	27	36	45	54	63	72	81	54
7	12	24	36	48	60	71	83	95	1.2707	53
8	15	30	44	59	74	89	0.9904	1.1818	33	52
9	18	35	53	71	89	0.8508	24	42	59	51
10	21	41	62	82	0.7103	28	44	64	85	50
11	23	47	70	94	17	40	64	87	1.2811	49
12	26	53	79	0.5705	32	58	84	1.1410	37	48
13	29	58	88	17	46	75	1.0004	34	63	47
14	32	64	96	28	60	92	24	56	88	46
15	35	70	0.4805	40	75	0.8609	44	79	1.2914	45
16	38	76	13	51	89	27	65	1.1502	40	44
17	41	81	22	63	0.7204	44	85	26	66	43
18	44	87	31	74	18	62	1.0105	49	92	42
19	46	98	39	86	32	78	25	71	1.3018	41
20	49	99	48	97	47	96	45	94	44	40
21	52	0.2904	57	0.5809	61	0.8713	65	1.1618	70	39
22	55	10	65	20	76	31	86	41	96	38
23	58	16	74	32	90	48	1.0206	64	1.3122	37
24	61	22	82	43	0.7304	65	28	86	47	36
25	64	27	91	55	19	82	46	1.1770	78	35
26	67	33	0.4400	66	33	0.8800	66	33	99	34
27	70	39	09	78	48	17	86	56	1.3226	33
28	72	45	17	89	62	34	1.0806	78	51	32
29	75	50	26	0.5901	76	51	26	1.1802	77	31
30	78	56	34	12	91	69	47	25	1.3803	30
31	81	62	43	24	0.7405	86	67	48	29	29
32	84	68	51	35	19	0.8903	87	70	54	28
33	87	73	60	47	34	20	1.0405	84	80	27
34	90	79	69	58	48	38	27	1.1917	1.3406	26
35	93	85	78	70	63	55	48	40	33	25
36	96	91	86	82	77	72	68	63	59	24
37	98	96	95	93	91	89	87	86	84	23
38	0.1501	0.3002	0.4503	0.6004	0.7506	0.9007	1.0508	1.2009	1.3510	22
39	04	08	12	16	20	24	28	32	36	21
40	07	13	21	28	35	41	48	55	62	20
41	10	19	29	39	49	58	68	78	87	19
42	13	25	38	50	63	76	88	1.2101	1.3613	18
43	16	31	47	62	78	93	1.0609	24	40	17
44	18	37	55	74	92	0.9110	29	47	66	16
45	21	42	64	85	0.7806	27	48	70	91	15
46	24	48	72	96	21	45	69	93	1.3717	14
47	27	54	81	0.6108	35	62	89	1.2216	43	13
48	30	60	90	20	50	79	1.0709	39	69	12
49	33	65	98	31	64	96	29	62	94	11
50	36	71	0.4607	42	78	0.9214	49	85	1.3820	10
51	39	77	16	54	93	81	70	1.2308	47	9
52	41	83	24	66	0.7707	48	90	21	78	8
53	44	88	33	77	21	65	1.0809	44	98	7
54	47	94	41	88	36	83	80	77	1.3924	6
55	50	0.3100	50	0.6200	50	0.9300	50	1.2400	50	5
56	53	06	59	12	65	17	70	23	76	4
57	56	11	67	23	79	34	90	46	1.4001	3
58	59	17	76	34	93	52	1.0910	69	27	2
59	62	23	85	46	0.7808	69	81	92	54	1
60	0.1564	0.3129	0.4698	0.6257	0.7822	0.9386	1.0950	1.2514	1.4079	0
	1	2	3	4	5	6	7	8	9	

LATITUDE 81 DEGREES.

LATITUDE 9 DEGREES.										
1	2	3	4	5	6	7	8	9	10	11
0	0.9877	1.9754	2.9631	3.9508	4.9385	5.9261	6.9138	7.9015	8.8892	9.8769
1	76	53	29	06	82	58	35	11	88	59
2	76	52	28	04	80	56	32	08	84	58
3	75	51	27	02	78	53	29	04	80	57
4	75	50	25	00	76	51	26	01	76	56
5	74	49	24	3.9498	73	48	22	7.8997	71	55
6	74	48	22	96	71	45	19	93	67	54
7	73	47	21	95	69	42	15	90	63	53
8	73	46	20	93	66	39	12	86	59	52
9	72	46	18	91	64	37	00	82	55	51
10	72	45	17	89	62	34	06	78	51	50
11	71	44	15	87	59	31	03	74	46	49
12	71	43	14	86	57	28	00	71	43	48
13	70	42	13	84	55	25	6.9096	67	38	47
14	70	41	11	82	52	22	93	66	34	46
15	70	40	10	79	50	20	90	60	30	45
16	70	39	09	78	48	17	87	56	26	44
17	69	38	07	76	46	14	83	52	21	43
18	69	37	06	74	43	12	80	49	17	42
19	68	36	04	72	41	09	76	45	13	41
20	68	35	03	70	38	06	73	41	08	40
21	67	34	01	68	36	08	70	37	04	39
22	67	33	00	67	34	00	67	34	00	38
23	66	32	1.9599	65	31	5.9197	63	30	8.8796	37
24	66	31	97	63	29	94	60	26	91	36
25	65	30	96	61	26	91	57	22	87	35
26	65	30	94	59	24	89	54	18	83	34
27	64	29	93	57	22	86	50	14	78	33
28	64	28	91	55	19	83	47	10	74	32
29	63	27	90	53	17	80	43	06	70	31
30	63	26	89	52	15	77	40	03	66	30
31	62	25	87	50	12	74	36	7.8899	62	29
32	62	24	86	48	10	71	33	95	57	28
33	61	23	84	46	07	68	29	91	53	27
34	61	22	83	44	05	65	26	87	48	26
35	60	21	81	42	02	62	23	83	44	25
36	60	20	80	40	00	60	20	80	40	24
37	59	19	78	38	4.9298	57	16	76	35	23
38	59	18	77	36	95	54	13	72	31	22
39	58	17	76	34	93	51	09	68	27	21
40	58	16	74	32	90	48	06	64	22	20
41	57	15	73	30	88	45	02	60	18	19
42	57	14	71	28	85	42	6.8999	56	13	18
43	56	13	70	26	83	39	96	52	09	17
44	56	12	68	24	81	37	93	49	05	16
45	55	11	66	22	78	34	89	45	00	15
46	55	10	65	20	76	31	86	41	8.8696	14
47	54	09	63	18	73	28	82	37	91	13
48	54	08	62	16	71	25	79	33	87	12
49	53	07	61	14	68	22	75	29	83	11
50	53	06	59	12	66	19	72	25	78	10
51	52	05	58	10	63	16	68	21	73	9
52	52	04	56	08	61	13	65	17	69	8
53	51	03	55	06	58	10	61	13	64	7
54	51	02	53	04	56	07	58	09	60	6
55	50	01	52	02	53	04	54	05	55	5
56	50	00	50	00	51	01	51	01	51	4
57	49	1.9699	49	3.9398	48	5.9098	47	7.8797	46	3
58	49	98	47	96	46	95	44	93	42	2
59	48	97	46	94	43	92	40	89	37	1
60	0.9848	1.9696	2.9544	3.9392	4.9241	5.9089	6.8937	7.8785	8.8633	0
	1	2	3	4	5	6	7	8	9	10
DEPARTURE 80 DEGREES.										

DEPARTURE 9 DEGREES.										93
1	2	3	4	5	6	7	8	9	1	
0	0.1564	0.3129	0.4693	0.6257	0.7822	0.9386	1.0950	1.2514	1.4079	60
1	67	84	0.4702	69	86	0.9408	70	88	1.4105	59
2	70	40	10	80	51	21	91	61	81	58
3	78	46	19	92	65	88	1.1011	84	57	57
4	76	52	27	0.6803	79	55	81	1.2606	82	56
5	79	57	36	15	94	72	49	80	1.4208	55
6	82	63	45	26	0.7908	90	71	53	84	54
7	85	69	54	38	23	0.9507	92	76	61	53
8	87	75	62	49	87	24	1.1111	98	86	55
9	90	80	71	61	51	41	81	1.2722	1.4812	51
10	93	86	79	72	66	59	52	45	38	50
11	96	92	88	84	80	75	71	67	63	49
12	99	98	96	95	94	93	92	90	89	48
13	0.1602	0.8203	0.4805	0.6407	0.8009	0.9610	1.1212	1.2814	1.4415	47
14	05	09	14	18	23	28	32	37	41	46
15	07	15	22	30	37	44	52	59	67	45
16	10	21	31	41	52	62	72	82	98	44
17	13	26	40	58	66	79	92	1.2906	1.4519	43
18	16	32	48	64	81	97	1.1818	29	45	42
19	19	38	57	76	95	0.9718	32	51	70	41
20	22	44	65	87	0.8109	81	58	74	96	40
21	25	49	77	98	23	48	73	97	1.4620	39
22	28	55	83	0.6510	38	65	98	1.8020	48	38
23	30	61	91	22	52	82	1.1413	43	74	37
24	33	67	0.4900	33	67	0.9800	33	66	1.4700	36
25	36	72	08	44	81	17	53	89	25	35
26	39	78	17	56	95	34	73	1.3112	51	34
27	42	84	26	68	0.8210	51	98	85	77	33
28	45	89	34	79	24	68	1.1518	58	1.4802	32
29	48	95	43	90	38	86	33	81	28	31
30	51	0.3801	52	0.6602	53	0.9903	54	1.3204	55	30
31	53	07	60	13	67	20	78	26	80	29
32	56	12	69	25	81	37	93	50	1.4906	28
33	59	18	77	36	96	55	1.1614	78	82	27
34	62	24	86	48	0.8810	72	84	96	58	26
35	65	30	94	59	24	89	54	1.3318	88	25
36	68	35	0.5003	71	39	1.0006	74	42	1.5009	24
37	70	41	12	82	53	24	94	65	85	23
38	73	47	20	94	67	40	1.1714	87	61	22
39	76	53	29	0.6705	82	58	84	1.3410	87	22
40	79	58	38	17	96	75	54	84	1.5118	20
41	82	64	46	28	0.8410	92	74	56	88	19
42	85	70	55	40	25	1.0109	94	79	64	18
43	88	76	63	51	39	27	1.1815	1.8502	90	17
44	91	81	72	62	53	44	84	25	1.5215	16
45	94	87	81	74	68	61	55	48	42	15
46	96	93	89	86	82	78	75	72	68	14
47	99	98	98	97	96	95	94	94	98	13
48	0.1702	0.8404	0.5106	0.6808	0.8511	1.0213	1.1915	1.8617	1.5819	12
49	05	10	15	20	25	30	35	40	45	11
50	08	16	23	31	39	47	55	62	70	10
51	11	21	32	43	54	64	75	86	96	9
52	14	27	41	54	68	82	95	1.8709	1.5422	8
53	16	33	49	66	82	98	1.2015	81	48	7
54	19	39	58	77	97	1.0316	35	54	74	6
55	22	44	67	89	0.8611	33	55	78	1.5500	5
56	25	50	75	0.6900	25	50	75	1.3800	25	4
57	28	56	84	12	40	67	95	23	51	3
58	31	62	92	23	54	85	1.2118	46	77	2
59	34	67	0.5201	34	68	1.0402	35	68	1.5602	1
60	0.1736	0.3473	0.5210	0.6946	0.8683	1.0419	1.2156	1.3892	1.5629	0
1	2	3	4	5	6	7	8	9		
LATITUDE 80 DEGREES.										

	1	2	3	4	5	6	7	8	9	10
0	0.9848	1.9696	2.9544	3.9392	4.9241	5.9089	6.8937	7.8785	8.8633	9.8481
1	48	95	43	90	38	86	33	81	28	59
2	47	94	41	88	36	83	30	77	24	58
3	47	93	39	86	33	80	26	73	19	57
4	46	92	38	84	31	77	23	69	15	56
5	46	91	37	82	28	73	19	64	10	55
6	45	90	35	80	25	70	15	60	05	54
7	45	89	34	78	23	67	12	56	01	53
8	44	88	32	76	20	64	08	52	8.8596	52
9	44	87	31	74	18	61	05	48	92	51
10	43	86	29	72	15	58	01	44	87	50
11	43	85	28	70	13	55	6.8898	40	83	49
12	42	84	26	68	10	52	94	36	78	48
13	41	83	24	66	07	48	90	31	73	47
14	41	82	22	64	05	45	86	27	68	46
15	40	81	21	62	02	42	83	23	64	45
16	40	80	20	60	00	39	79	19	59	44
17	39	79	18	58	1.9197	36	76	15	55	43
18	39	78	17	56	95	33	72	11	50	42
19	38	77	15	53	92	30	68	06	45	41
20	38	76	13	51	89	27	65	02	40	40
21	37	75	11	49	87	24	61	7.8698	35	39
22	37	74	10	47	84	21	58	94	31	38
23	36	72	09	45	81	17	54	90	26	37
24	36	71	07	43	79	14	50	86	21	36
25	35	70	06	41	76	11	46	82	17	35
26	35	69	04	39	74	08	43	78	12	34
27	34	68	02	36	71	05	39	73	07	33
28	34	67	01	34	68	02	35	69	02	32
29	33	66	2.9499	32	66	5.8999	32	65	8.8498	31
30	33	65	98	30	63	95	28	60	93	30
31	32	64	96	28	60	92	24	56	88	29
32	32	63	95	26	58	89	21	52	84	28
33	31	62	93	24	55	86	17	48	79	27
34	30	61	91	22	52	82	13	43	74	26
35	30	60	90	20	50	79	09	39	69	25
36	29	59	88	18	47	76	06	35	65	24
37	29	58	86	15	44	73	02	30	60	23
38	28	57	85	13	42	70	6.8798	26	55	22
39	28	56	83	11	39	66	94	22	50	21
40	27	54	82	09	36	63	90	18	45	20
41	27	53	80	06	35	60	87	14	40	19
42	26	52	78	04	31	57	83	09	35	18
43	26	51	77	02	28	54	79	05	30	17
44	25	50	75	00	25	50	75	00	25	16
45	25	49	74	3.9298	23	47	72	7.8596	20	15
46	24	48	72	96	20	44	68	92	16	14
47	23	47	70	94	17	40	64	87	10	13
48	23	46	69	92	15	37	60	83	06	12
49	22	45	67	89	12	34	56	78	01	11
50	22	44	65	87	09	31	53	74	8.8496	10
51	21	42	64	85	06	27	48	70	91	9
52	21	41	62	83	04	24	45	66	86	8
53	20	40	60	80	01	21	41	61	81	7
54	20	39	59	78	4.9098	18	37	57	76	6
55	19	38	57	76	95	14	33	52	71	5
56	19	37	56	74	93	11	30	48	67	4
57	18	36	54	72	90	07	26	43	61	3
58	17	35	52	70	87	04	22	39	57	2
59	17	34	50	67	84	01	18	34	51	1
60	0.9816	1.9633	2.9449	3.9275	4.9082	5.8898	6.8714	7.8530	8.8346	9.8162
	1	2	3	4	5	6	7	8	9	10

DEPARTURE 10 DEGREES.

95

1	2	3	4	5	6	7	8	9	10
0	0.1787	0.8478	0.5210	0.6946	0.8688	1.0419	1.2158	1.8892	1.5629
1	89	89	18	57	97	86	75	1.8914	54 59
2	42	84	27	69	0.8711	58	95	88	80 58
3	45	90	85	80	25	71	1.2216	61	1.5706
4	48	96	44	92	40	87	85	88	81 56
5	51	0.8502	52	0.7008	64	1.0505	56	1.4006	57 55
6	54	07	61	15	69	22	76	80	88 54
7	57	18	70	26	83	89	96	52	1.5809
8	59	19	78	88	97	56	1.2816	75	85 52
9	62	25	87	49	0.8812	74	86	98	61 51
10	65	30	95	60	26	91	56	1.4121	86 50
11	68	36	0.5804	72	40	1.0608	76	54	1.5912
12	71	42	12	88	54	27	96	66	87 48
13	74	47	21	95	69	42	1.2416	90	68 47
14	77	53	80	0.7106	83	60	86	1.4218	89 46
15	79	59	88	18	97	76	56	85	1.6015
16	82	65	47	29	0.8912	94	76	58	41 44
17	85	70	56	41	26	1.0711	96	82	67 48
18	88	76	64	52	40	28	1.2516	1.4304	92 42
19	91	82	72	68	55	45	86	27	1.6118
20	94	87	81	76	69	62	56	50	48 40
21	97	93	90	87	83	80	76	73	69 39
22	0.1800	99	99	98	98	97	97	96	96 88
23	02	0.8605	0.5407	0.7209	0.9012	1.0814	1.2616	1.4418	1.6221
24	05	10	16	21	26	81	86	42	47 86
25	08	16	24	32	41	49	57	65	73 85
26	11	22	33	44	55	65	76	87	98 84
27	14	28	41	55	69	83	97	1.4510	1.6325
28	17	33	50	66	83	1.0900	1.2716	88	49 82
29	20	39	59	78	98	17	87	56	76 81
30	22	45	67	90	0.9112	84	57	79	1.6402
31	25	50	76	0.7301	26	51	76	1.4602	27 29
32	28	56	84	12	41	69	97	25	53 28
33	31	62	92	24	55	85	1.2816	47	78 27
34	34	68	0.5501	85	69	1.1008	87	70	1.6504
35	37	73	10	47	84	20	57	94	80 25
36	40	79	19	88	98	87	77	1.4716	56 24
37	42	85	27	70	0.9212	54	97	89	82 23
38	45	90	35	81	26	71	1.2916	62	1.6607
39	48	96	44	92	41	89	87	85	83 21
40	51	0.8702	58	0.7404	55	1.1105	56	1.4807	58 20
41	54	08	61	15	69	23	77	80	84 19
42	57	18	70	27	84	40	97	54	1.6710
43	60	19	78	38	98	57	1.8017	76	86 17
44	62	25	87	50	0.9812	74	87	99	62 16
45	65	30	96	61	26	91	56	1.4922	87 15
46	68	36	0.5504	72	41	1.1209	77	45	1.6813
47	71	42	18	84	55	26	97	68	89 13
48	74	48	21	95	69	48	1.8117	90	64 12
49	77	53	80	0.7507	84	60	87	1.5014	90 11
50	80	59	89	18	98	77	57	86	1.6916
51	82	65	47	80	0.9412	94	77	59	42 9
52	85	70	56	41	26	1.1811	96	82	67 8
53	88	76	64	52	41	29	1.8217	1.5105	98 7
54	91	82	73	64	55	46	87	28	1.7019
55	94	88	81	75	69	68	57	50	44 5
56	97	93	90	87	84	80	77	74	70 4
57	0.1900	99	99	98	98	97	97	96	96 3
58	02	0.8805	0.5707	10	0.9512	1.1414	1.8317	1.5219	1.7122
59	05	10	15	21	26	81	86	42	47 1
60	0.1908	0.8816	0.5724	0.7682	0.9541	1.1449	1.8357	1.5265	1.7178
1	1	2	3	4	5	6	7	8	9

LATITUDE 79 DEGREES.

LATITUDE 11 DEGREES.

1	2	3	4	5	6	7	8	9	10
0.9816	1.9633	2.9449	3.9265	4.9082	5.8898	6.8714	7.8530	8.8347	60
16	81	47	63	89	94	10	26	41	59
15	80	46	61	76	91	06	22	37	58
15	29	44	58	73	88	02	27	31	57
14	28	42	56	70	84	6.8698	12	26	56
14	27	41	54	88	81	95	08	22	55
18	26	39	52	65	77	90	03	16	54
12	25	37	50	62	74	87	7.8599	12	53
12	24	35	47	59	71	83	94	06	52
11	22	34	45	56	67	78	90	01	51
11	21	32	43	54	64	75	86	8.8296	50
10	20	30	40	51	61	71	81	91	49
10	19	29	38	48	58	67	77	86	48
09	18	27	36	45	54	63	72	81	47
08	17	25	34	42	50	59	67	76	46
08	16	26	32	40	47	55	63	71	45
07	15	22	29	37	44	51	58	66	44
07	13	20	27	34	40	57	54	60	43
06	12	18	24	31	37	43	49	55	42
06	11	17	22	28	34	39	45	50	41
05	10	15	20	25	30	35	40	45	40
04	09	13	18	22	26	33	35	40	39
04	08	12	16	20	23	27	31	35	38
03	07	10	13	17	20	23	26	30	37
03	05	08	11	14	16	15	22	24	36
02	04	06	08	11	13	18	17	20	35
02	03	05	06	08	10	11	13	14	34
01	02	03	04	05	06	07	08	09	33
00	01	01	01	02	02	03	03	04	32
00	00	2.9399	3.9199	4.8999	5.8799	6.8599	7.8398	8.8198	31
0.9799	1.9598	98	97	96	95	94	94	93	30
99	97	96	95	94	92	91	90	88	29
98	96	94	92	91	89	87	85	83	28
98	95	92	90	88	85	83	80	78	27
97	94	91	88	85	81	78	75	72	26
96	93	89	85	82	78	74	70	67	25
96	92	87	83	79	75	71	66	62	24
95	90	86	81	76	71	66	62	57	23
95	89	84	78	73	68	62	57	51	22
94	88	82	76	70	64	58	52	46	21
93	87	80	74	67	60	54	47	41	20
93	86	78	71	64	57	50	42	35	19
92	84	77	69	61	53	45	38	30	18
92	83	75	66	58	50	41	33	24	17
91	82	73	64	55	46	37	28	19	16
91	81	72	62	53	43	34	24	15	15
90	80	70	60	50	39	29	19	09	14
89	79	68	57	47	36	25	14	04	13
89	77	66	55	44	32	21	10	00	12
88	76	64	52	41	29	17	05	8.8098	11
88	75	63	50	38	25	13	00	93	10
87	74	61	48	35	22	08	7.8295	88	09
86	73	59	45	32	18	04	90	82	08
86	71	57	43	29	14	00	86	77	07
85	70	55	40	26	11	6.8496	81	71	06
85	69	54	38	23	07	92	76	66	05
84	68	52	36	20	03	87	71	61	04
83	67	50	33	17	00	83	66	55	03
83	65	48	31	14	5.8696	79	62	50	02
82	64	46	28	11	83	75	57	39	01
0.9782	1.9563	2.9345	3.9126	4.8908	5.8689	6.8471	7.8252	8.8034	0
1	2	3	4	5	6	7	8	9	

DEPARTURE 78 DEGREES.

DEPARTURE 11 DEGREES.

97

1	2	3	4	5	6	7	8	9
0	0.1908	0.8816	0.5724	0.7632	0.9541	1.1449	1.8857	1.5265
1	11	22	33	44	55	65	76	88
2	14	28	41	55	69	83	97	1.5310
3	17	33	50	67	84	1.1500	1.8417	84
4	20	39	59	78	98	17	37	56
5	22	45	67	90	0.9612	34	57	79
6	25	50	76	0.7701	26	51	76	1.5402
7	28	56	84	12	41	69	97	25
8	31	62	93	24	55	85	1.8516	47
9	34	68	0.5801	35	69	1.1603	87	70
10	37	73	10	46	83	20	56	98
11	40	79	19	58	98	37	77	1.5516
12	42	85	27	69	0.9712	54	96	38
13	45	90	36	81	26	71	1.8616	62
14	48	96	44	92	41	89	87	85
15	51	0.3902	53	0.7804	55	1.1705	57	1.5607
16	54	08	61	15	69	23	77	80
17	57	13	70	26	88	40	96	58
18	60	19	79	38	98	57	1.8717	76
19	62	25	87	49	0.9812	74	39	98
20	65	30	96	61	26	91	56	1.5722
21	68	36	0.5904	72	40	1.1808	76	44
22	71	42	13	84	55	26	96	67
23	74	47	21	95	69	42	1.8816	90
24	77	53	30	0.7906	83	60	36	1.5813
25	79	59	38	18	97	76	56	35
26	82	65	47	29	0.9912	94	76	58
27	85	70	55	40	26	1.1911	96	81
28	88	76	64	52	40	28	1.8916	1.5904
29	91	82	72	63	54	45	36	26
30	94	87	81	75	69	62	56	50
31	97	93	90	86	83	79	76	72
32	99	99	98	98	97	96	96	95
33	0.2002	0.4004	0.6007	0.8009	1.0011	1.2013	1.4016	1.6018
34	05	10	15	20	26	31	36	41
35	08	16	24	32	40	47	56	63
36	11	22	32	48	54	65	76	86
37	14	27	41	54	68	82	96	1.6109
38	17	33	50	66	83	99	1.4116	32
39	19	39	58	77	97	1.2116	35	55
40	22	44	67	89	1.0111	33	55	78
41	25	50	75	90	25	50	75	00
42	28	56	84	0.8112	40	67	95	1.6223
43	31	61	92	23	54	84	1.4215	46
44	34	67	0.6101	34	17	1.2202	85	69
45	36	73	09	46	32	18	55	92
46	39	79	18	57	97	36	75	1.6314
47	42	84	26	68	1.0211	53	95	87
48	45	90	35	80	25	70	1.4315	60
49	48	96	43	91	39	87	35	82
50	51	0.4101	52	0.8203	54	1.2304	55	1.6406
51	54	07	61	14	68	21	75	28
52	56	13	69	25	82	38	94	50
53	59	18	78	37	96	55	1.4414	74
54	62	24	86	48	1.0310	72	34	96
55	65	30	95	60	25	89	54	1.6519
56	68	35	0.6203	71	34	1.2406	74	42
57	71	41	12	82	53	24	94	65
58	73	47	20	94	37	40	1.4514	87
59	76	53	29	0.8305	52	58	34	1.6610
60	0.2079	0.4158	0.6237	0.8316	1.0396	1.2475	1.4554	1.6633
	1	2	3	4	5	6	7	8
								9

LATITUDE 78 DEGREES.

	1	2	3	4	5	6	7	8	9	10
0	0.9782	0.9668	2.9845	3.9126	4.8908	5.8689	6.8471	7.8252	8.8034	60
1	81	62	43	24	06	85	66	47	28	59
2	80	61	41	21	02	82	62	42	23	58
3	80	59	39	19	4.8899	78	58	38	17	57
4	79	58	37	16	95	75	54	33	12	56
5	78	57	35	14	92	71	49	27	06	55
6	78	56	33	11	89	67	45	22	00	54
7	77	54	32	09	86	63	40	18	8.7995	53
8	77	53	30	06	83	60	36	13	89	52
9	76	52	28	04	80	56	32	08	84	51
10	75	51	26	02	77	52	28	03	79	50
11	75	50	24	3.9099	74	49	24	7.8199	73	49
12	74	48	23	97	71	45	19	94	68	48
13	74	47	21	94	68	41	15	88	62	47
14	73	46	19	92	65	37	10	83	56	46
15	72	45	17	89	62	34	06	79	51	45
16	72	43	15	87	59	30	02	74	45	44
17	71	42	13	84	56	26	6.8398	69	40	43
18	71	41	12	82	53	23	94	64	35	42
19	70	40	10	79	49	19	89	59	28	41
20	69	38	08	77	46	15	84	54	23	40
21	69	37	06	74	43	11	80	49	17	39
22	68	36	04	72	40	08	76	44	12	38
23	67	35	02	69	37	04	71	38	06	37
24	67	33	00	67	34	00	67	34	00	36
25	66	32	2.9298	64	31	5.8597	63	29	8.7895	35
26	66	31	97	62	28	93	59	24	90	34
27	65	30	94	59	24	89	54	18	83	33
28	64	28	93	57	21	85	49	14	78	32
29	64	27	91	54	18	82	45	09	72	31
30	63	26	89	52	15	78	41	04	67	30
31	62	25	87	49	12	74	36	7.8098	61	29
32	62	23	85	47	09	70	32	94	55	28
33	61	22	83	44	06	67	28	89	50	27
34	60	21	81	42	02	62	23	83	44	26
35	60	20	79	39	4.8799	58	18	78	38	25
36	59	18	78	37	96	55	14	74	33	24
37	59	17	76	34	93	51	09	68	27	23
38	58	16	74	32	90	47	05	63	21	22
39	57	15	72	29	86	44	00	58	16	21
40	57	13	70	26	83	40	6.8296	53	09	20
41	56	12	68	24	80	36	92	47	04	19
42	55	11	66	21	77	32	87	42	8.7798	18
43	55	09	64	18	74	28	83	38	92	17
44	54	08	62	16	71	25	79	33	87	16
45	53	07	60	14	67	21	74	27	81	15
46	53	06	58	11	64	17	70	22	76	14
47	52	04	56	08	61	13	65	17	69	13
48	52	03	55	06	58	09	61	12	64	12
49	51	02	52	08	54	05	56	07	58	11
50	50	00	51	01	51	01	51	02	52	10
51	49	0.9499	49	3.8998	48	5.8498	47	7.7998	46	9
52	49	98	47	96	45	93	42	91	40	8
53	48	96	45	93	42	90	38	86	37	7
54	48	95	43	90	38	86	35	83	34	6
55	47	94	41	88	35	82	32	80	31	5
56	46	93	39	85	32	78	28	76	29	4
57	46	91	37	83	28	74	24	72	26	3
58	45	90	35	80	25	70	20	69	24	2
59	44	88	33	77	22	66	15	66	22	1
60	0.9744	1.9487	2.9281	3.8975	4.8719	5.8462	6.8206	7.7950	8.7694	0

DEPARTURE 77 DEGREES.

DEPARTURE 12 DEGREES.

99

1	2	3	4	5	6	7	8	9	10
0	0.2079	0.4158	0.6287	0.8316	1.0396	1.2475	1.4554	1.6633	1.8712
1	82	64	46	28	1.0410	92	74	56	38
2	85	70	54	39	24	1.2509	94	78	63
3	88	75	63	51	39	26	1.4614	1.6702	89
4	91	81	72	62	53	43	84	24	1.8815
5	93	87	80	73	67	60	53	46	40
6	96	92	89	85	81	77	73	70	66
7	99	98	97	96	95	94	93	92	91
8	0.2102	0.4204	0.6306	0.8408	1.0510	1.2611	1.4713	1.6815	1.8917
9	05	09	14	19	24	28	33	38	42
10	08	15	23	30	38	46	53	61	68
11	10	21	31	42	54	62	73	83	94
12	18	26	40	53	66	79	92	1.6906	1.9019
13	16	32	48	64	81	97	1.4813	29	45
14	19	38	57	76	95	1.2713	32	51	70
15	22	44	65	87	1.0609	31	53	74	96
16	25	49	74	98	23	48	72	97	1.9121
17	28	55	83	0.8510	38	65	92	1.7020	48
18	30	61	91	21	52	82	1.4912	42	78
19	33	66	99	32	66	99	32	65	98
20	36	72	0.6408	44	80	2816	52	88	1.9224
21	39	78	16	55	94	33	72	1.7110	49
22	42	83	25	67	1.0709	50	92	34	75
23	45	89	34	78	23	67	1.5012	56	1.9801
24	47	95	42	90	37	84	82	79	27
25	50	0.4300	51	0.8601	51	1.2901	51	1.7202	52
26	53	06	59	12	65	18	71	24	77
27	56	12	68	24	80	35	91	47	98
28	59	17	76	35	94	52	1.5111	70	1.9428
29	62	23	85	46	1.0808	70	81	93	54
30	64	29	93	58	22	86	51	1.7815	80
31	67	34	0.6502	69	36	1.3003	70	38	1.9505
32	70	40	10	80	51	21	91	61	81
33	73	46	19	92	65	37	10	83	56
34	76	52	29	0.8703	79	55	31	1.7406	82
35	79	57	36	14	93	72	50	29	1.9607
36	81	63	44	26	1.0907	88	70	51	38
37	84	69	53	37	23	1.3106	90	74	59
38	87	74	61	48	36	23	1.5310	97	84
39	90	80	70	60	50	39	29	1.7519	1.9709
40	93	86	78	71	64	57	50	42	35
41	96	91	87	82	78	74	69	65	60
42	99	97	96	94	93	91	90	88	87
43	0.2201	0.4403	0.6604	0.8805	1.1007	1.3208	1.5409	1.7610	1.9812
44	04	08	12	16	21	25	29	33	37
45	07	14	21	28	35	42	49	56	63
46	10	20	29	39	49	59	69	78	88
47	13	25	38	40	63	76	88	1.7701	1.9913
48	16	31	47	62	78	93	1.5509	24	40
49	18	37	55	73	92	1.3810	28	46	65
50	21	42	64	85	1.1106	27	48	70	91
51	24	48	72	96	20	44	68	92	2.0016
52	27	54	80	07	34	61	88	1.7814	41
53	30	59	89	0.8919	49	78	1.5608	38	67
54	33	65	98	30	63	95	28	60	98
55	35	71	0.6706	41	77	1.3412	47	82	2.0118
56	38	76	15	53	91	29	67	1.7906	44
57	41	82	23	64	1.1205	46	87	28	69
58	44	88	31	75	19	63	1.5707	50	94
59	47	93	40	87	34	80	27	74	2.0220
60	0.2250	0.4489	0.6749	0.8998	1.1248	1.3497	1.5747	1.7996	2.0246
1	1	2	3	4	5	6	7	8	9

LATITUDE 77 DEGREES.

	1	2	3	4	5	6	7	8	9	10
0	0.9744	1.9487	2.9231	3.8975	4.8719	5.8462	6.8206	7.7950	8.7693	9.7436
1	40	86	29	72	15	58	01	44	87	59
2	42	85	27	70	12	54	6.8197	39	82	58
3	42	83	25	67	09	50	92	34	75	57
4	41	82	23	64	06	47	88	29	70	56
5	40	81	21	62	02	42	83	23	64	55
6	40	80	19	59	4.8699	39	79	18	58	54
7	39	78	17	56	96	35	74	13	52	53
8	38	77	15	54	92	30	69	07	46	52
9	38	76	13	51	89	27	65	02	40	51
0	37	74	11	48	86	23	60	7.7897	84	50
1	36	73	09	46	82	18	55	91	28	49
2	36	72	07	43	79	15	51	86	22	48
3	35	70	05	40	76	11	46	81	16	47
4	35	69	04	38	73	07	42	76	11	46
5	34	68	01	35	69	03	37	70	04	45
6	33	66	2.9199	32	66	5.8399	32	65	08	44
7	33	65	98	30	63	95	28	60	8.7594	43
8	32	64	95	27	59	91	23	54	86	42
9	31	62	93	24	56	87	18	49	80	41
0	30	61	91	22	52	82	13	43	74	40
1	30	60	89	19	49	79	09	38	68	39
2	29	58	87	16	46	75	04	33	62	38
3	28	57	85	14	42	70	6.8099	27	56	37
4	28	56	83	11	39	67	95	22	50	36
5	27	54	81	08	36	63	90	17	44	35
6	26	53	79	06	32	58	85	11	38	34
7	26	51	77	03	29	54	80	06	32	33
8	25	50	75	00	26	51	76	01	26	32
9	24	49	73	3.8898	22	46	71	7.7795	19	31
0	24	47	71	94	19	42	66	90	13	30
1	23	46	69	92	15	38	61	84	07	29
2	22	45	67	89	12	34	56	78	01	28
3	22	42	65	87	09	30	52	74	8.7495	27
4	21	42	63	84	05	26	47	68	89	26
5	20	41	61	81	02	22	42	62	83	25
6	20	39	59	78	4.8598	18	37	57	76	24
7	19	38	57	76	95	13	32	51	70	23
8	18	36	55	73	91	09	27	46	64	22
9	18	35	53	70	88	06	23	41	58	21
0	17	34	51	68	85	01	18	35	52	20
1	16	32	49	65	81	5.8297	13	30	46	19
2	16	31	47	62	78	93	09	24	40	18
3	15	30	44	59	74	89	04	18	33	17
4	14	28	42	56	71	85	6.7999	13	27	16
5	13	27	40	54	67	80	94	07	21	15
6	13	25	38	51	64	76	89	02	14	14
7	12	24	36	48	60	72	84	7.7696	08	13
8	11	23	34	45	57	68	79	90	02	12
9	11	21	32	42	53	64	74	85	8.7395	11
0	10	20	30	40	50	60	70	80	90	10
1	09	19	28	37	46	56	65	74	84	9
2	09	17	26	34	43	52	60	69	77	8
3	08	16	24	32	40	47	55	63	71	7
4	07	15	22	29	36	43	50	58	65	6
5	07	13	20	26	33	39	46	52	59	5
6	06	12	19	23	29	35	41	46	52	4
7	05	10	15	20	26	31	36	41	46	3
8	04	09	13	18	22	26	31	35	40	2
9	04	07	11	15	19	22	26	30	34	1
0	0.9703	1.9406	2.9109	3.8812	4.8515	5.8218	6.7921	7.7624	8.7327	9.7030
	1	2	3	4	5	6	7	8	9	

DEPARTURE 18 DEGREES.

101

1	2	3	4	5	6	7	8	9	10
0	0.2250	0.4499	0.6749	0.8998	1.1248	1.3497	1.5747	1.7996	2.0246
1	52	0.4505	57	0.9009	62	1.8514	66	1.8018	71
2	55	10	66	21	76	81	86	42	97
3	58	16	74	32	94	48	1.5806	64	2.0822
4	61	22	82	48	1.1804	65	26	86	47
5	64	27	91	55	19	82	46	1.8110	78
6	67	33	0.6800	66	33	99	66	82	99
7	69	39	08	77	46	1.8616	85	54	2.0424
8	72	44	17	89	61	83	1.5905	78	50
9	75	50	25	0.9100	75	50	25	1.8200	75
10	78	56	33	11	89	67	45	22	2.0500
11	81	61	42	23	1.1404	85	65	46	26
12	84	67	52	34	18	1.8701	85	67	52
13	86	73	69	45	82	18	1.6004	90	77
14	89	79	68	57	46	35	24	1.8314	2.0608
15	92	84	76	68	60	52	44	86	28
16	95	90	84	79	74	69	64	58	58
17	98	95	93	91	89	86	84	82	79
18	0.2301	0.4601	0.6902	0.9202	1.1503	1.3803	1.6104	1.8404	2.0705
19	03	07	10	13	17	20	23	26	30
20	06	12	19	25	31	37	43	50	56
21	09	18	27	36	45	54	63	72	81
22	12	24	35	47	59	71	83	94	2.0806
23	15	29	44	58	73	88	1.6206	1.8517	2.0817
24	18	35	53	70	88	1.3905	23	40	58
25	20	41	61	81	1.1602	22	42	62	83
26	23	46	69	92	16	39	62	85	2.0908
27	26	52	78	0.9804	30	56	82	1.8608	84
28	29	58	86	15	44	78	1.6802	80	60
29	31	63	95	26	58	90	21	53	84
30	35	69	0.7004	38	73	1.4007	42	1.8676	2.1011
31	37	75	12	49	87	24	61	98	86
32	40	80	20	60	1.1701	41	81	1.8721	61
33	43	86	29	72	15	57	1.6400	48	86
34	46	92	37	83	29	75	21	66	2.1112
35	49	97	46	94	43	92	40	89	37
36	51	0.4703	54	0.9406	57	1.4108	60	1.8811	63
37	54	08	63	17	71	25	79	84	88
38	57	14	71	28	86	48	1.6500	57	2.1214
39	60	20	80	40	1.1800	59	19	79	89
40	63	25	88	51	14	76	39	1.8902	64
41	66	31	97	62	28	94	59	25	90
42	68	37	0.7105	74	42	1.4210	79	47	2.1316
43	71	42	14	85	56	27	98	70	41
44	74	48	22	96	70	44	1.6618	92	66
45	77	54	31	0.9508	85	61	38	1.9015	92
46	80	59	39	19	99	78	58	38	2.1417
47	83	65	48	30	1.1913	95	78	60	48
48	85	71	56	41	27	1.4312	97	82	68
49	88	76	65	53	41	29	1.6717	1.9106	94
50	91	82	73	64	55	46	37	28	2.1519
51	94	88	81	75	69	63	57	50	44
52	97	93	90	86	83	80	76	73	69
53	0.2400	99	99	98	1.2000	97	97	96	96
54	02	0.4805	0.7207	0.9609	12	1.4414	1.6816	1.9218	2.1621
55	05	10	15	20	26	31	36	41	46
56	08	16	24	32	40	47	55	63	71
57	11	22	32	43	54	65	76	86	97
58	14	27	41	54	68	82	95	1.9809	2.1722
59	16	33	49	65	82	98	1.6915	81	48
60	0.2419	0.4838	0.7258	0.9677	1.2096	1.4515	1.6934	1.9354	2.1773
1	2	3	4	5	6	7	8	9	

LATITUDE 76 DEGREES.

	1	2	3	4	5	6	7	8	9	10
0	0.9703	1.9406	2.9109	3.8812	4.8515	5.8218	6.7921	7.7624	8.7327	60
1	02	06	07	09	12	14	16	18	21	59
2	02	03	05	06	08	09	11	12	14	58
3	01	02	02	03	04	05	06	06	07	57
4	00	00	00	00	01	01	01	01	01	56
5	0.9699	1.9399	2.9098	3.8798	4.8497	5.8196	6.7896	7.7595	8.7295	55
6	99	97	96	95	94	92	91	90	88	54
7	98	96	94	92	90	88	86	84	82	53
8	97	95	92	89	87	84	81	78	76	52
9	97	93	90	86	83	80	76	73	69	51
10	96	92	88	84	80	75	71	67	63	50
11	95	90	86	81	76	71	66	62	57	49
12	95	89	84	78	73	67	62	56	51	48
13	94	87	81	75	69	62	56	50	43	47
14	93	86	79	72	65	58	51	44	37	46
15	92	85	77	69	62	54	46	39	31	45
16	92	83	75	66	58	50	41	33	24	44
17	91	82	73	64	55	45	36	27	19	43
18	90	80	71	61	51	41	31	22	12	42
19	89	79	68	58	47	36	26	15	06	41
20	89	77	66	55	44	32	21	10	8.7198	40
21	88	76	64	52	40	28	16	04	92	39
22	87	75	62	49	37	24	11	7.7498	86	38
23	87	73	60	46	33	20	06	93	79	37
24	86	72	57	43	29	15	01	86	72	36
25	85	70	55	41	26	11	6.7796	81	66	35
26	84	69	53	38	22	06	91	75	60	34
27	84	67	51	35	18	02	86	70	53	33
28	83	66	49	32	15	5.8097	80	63	46	32
29	82	64	47	29	11	93	75	58	40	31
30	82	63	45	26	08	89	71	52	34	30
31	81	61	42	23	04	84	65	46	27	29
32	80	60	40	20	00	80	60	40	20	28
33	79	59	38	17	4.8397	76	55	34	14	27
34	79	57	36	14	93	72	50	29	07	26
35	78	56	33	11	89	67	45	22	00	25
36	77	54	31	08	86	63	40	17	8.7094	24
37	76	53	29	06	82	58	35	11	88	23
38	76	51	27	02	78	54	29	05	80	22
39	75	50	25	3.8699	75	49	24	7.7399	74	21
40	74	48	23	97	71	45	19	94	68	20
41	73	47	20	95	67	40	14	87	61	19
42	73	45	18	91	64	36	09	82	54	18
43	72	44	16	88	60	31	03	76	47	17
44	71	42	14	85	56	27	6.7698	70	41	16
45	71	41	11	82	53	23	94	64	35	15
46	70	39	09	79	49	18	88	58	27	14
47	69	38	07	76	45	14	83	52	21	13
48	68	36	05	73	41	09	77	46	14	12
49	68	35	03	70	38	05	73	40	08	11
50	67	33	00	67	34	00	67	34	00	10
51	66	32	2.8998	64	30	5.7996	62	28	8.6994	9
52	65	31	96	61	27	92	57	22	88	8
53	65	39	94	58	24	87	52	16	81	7
54	64	28	91	55	19	83	47	10	74	6
55	63	26	89	52	15	78	41	04	67	5
56	62	25	87	49	12	74	36	7.7298	61	4
57	62	23	85	46	08	69	31	92	54	3
58	61	22	82	43	05	65	26	86	47	2
59	60	20	80	40	00	60	20	80	40	1
60	0.9659	1.9319	2.8978	3.8637	4.8297	5.7956	6.7615	7.7274	8.6934	0
	1	2	3	4	5	6	7	8	9	

DEPARTURE 14 DEGREES.

103

1	2	3	4	5	6	7	8	9	10
0	0.2419	0.4838	0.7258	0.9677	1.2096	1.4516	1.6934	1.9354	2.1773
1	22	44	66	88	1.2110	82	54	76	98
2	25	50	75	0.9700	25	49	74	99	2.1824
3	28	55	83	11	39	66	94	1.9422	49
4	31	61	92	22	53	83	1.7014	44	75
5	33	67	0.7800	83	67	1.4600	83	66	2.1900
6	36	72	09	45	81	17	58	.90	26
7	39	78	17	56	95	34	73	1.9512	51
8	42	84	25	67	1.2209	51	98	84	76
9	45	89	34	78	83	68	1.7112	57	2.2001
10	47	95	42	90	87	84	82	79	27
11	50	0.4901	51	0.9801	52	7.4702	52	1.9602	53
12	53	06	59	12	66	19	72	25	78
13	56	12	68	24	80	35	91	47	2.2103
14	59	17	76	35	94	52	1.7211	70	28
15	62	28	85	46	1.2808	69	81	92	54
16	64	29	93	58	22	86	51	1.9715	80
17	67	34	0.7402	69	86	1.4803	70	88	2.2205
18	70	40	10	80	50	20	90	60	30
19	73	46	18	91	64	37	1.7810	82	55
20	76	51	27	0.9902	78	54	29	1.9805	80
21	78	57	35	14	92	70	49	27	2.2806
22	81	63	44	25	1.2407	88	69	50	82
23	84	68	52	36	21	1.4905	89	73	57
24	87	74	61	47	35	21	1.7408	95	82
25	90	79	69	59	49	38	29	1.9918	2.2407
26	93	85	78	70	63	55	48	40	83
27	96	91	86	81	77	72	67	62	58
28	98	96	95	93	91	89	87	86	84
29	0.2501	0.5002	0.7503	1.0004	1.2505	1.5006	1.7507	2.0008	2.2509
30	04	08	11	15	19	23	27	30	34
31	07	13	20	26	33	40	46	53	59
32	09	19	28	38	47	56	66	75	85
33	12	24	37	49	61	73	85	98	2.2610
34	15	30	45	60	76	91	1.7806	2.0121	86
35	18	36	54	72	90	1.5107	25	48	61
36	21	41	62	83	1.2604	24	45	66	86
37	24	47	71	94	18	41	65	88	2.2712
38	26	53	79	1.0105	32	58	84	2.0210	87
39	29	58	87	16	46	75	1.7704	33	62
40	32	64	96	28	60	92	24	56	88
41	35	70	0.7604	39	74	1.5209	44	78	2.2813
42	38	75	13	50	88	26	63	2.0301	88
43	40	81	21	62	1.2702	42	83	23	64
44	43	86	30	73	16	59	1.7802	46	89
45	46	92	38	84	30	76	22	68	2.2914
46	49	98	46	95	44	93	42	90	89
47	52	0.5103	55	1.0206	58	1.5310	61	2.0418	64
48	55	09	64	18	73	27	82	86	91
49	57	15	72	29	87	44	1.7901	58	2.3016
50	60	20	80	40	1.2801	61	21	81	41
51	63	26	89	52	15	77	40	2.0508	66
52	66	31	97	63	29	94	60	26	91
53	69	37	0.7706	74	48	1.5411	80	48	2.3117
54	71	43	14	85	57	28	99	70	42
55	74	48	22	96	71	45	1.8019	98	67
56	77	54	31	1.0308	85	61	38	2.0615	92
57	80	60	39	19	99	79	59	38	2.3218
58	83	65	48	30	1.2918	96	78	61	43
59	85	71	56	42	27	1.5512	98	83	69
60	0.2568	0.5176	0.7765	1.0353	1.2941	1.5529	1.8117	2.0706	2.3294
1	1	2	3	4	5	6	7	8	9

LATITUDE 75 DEGREES.

	1	2	3	4	5	6	7	8	9	10
0	0.9659	1.9819	2.8978	3.8637	4.8297	5.7956	6.7616	7.7274	8.6934	9.6593
1	59	17	86	34	93	51	10	68	27	59
2	58	16	73	31	89	47	05	62	20	58
3	57	14	71	28	85	42	6.7599	56	18	57
4	56	12	69	25	81	37	93	50	06	56
5	56	11	67	22	76	33	88	44	00	55
6	55	08	64	19	74	28	83	38	8.6892	54
7	54	07	62	16	70	24	78	32	86	53
8	53	06	60	13	66	19	72	26	79	52
9	52	05	57	10	62	14	67	19	72	51
10	52	03	55	07	59	10	62	14	65	50
11	51	02	53	04	55	06	56	07	58	49
12	50	00	51	01	51	01	51	02	52	48
13	49	1.9399	48	3.8598	47	5.7896	46	7.7195	45	47
14	49	97	46	94	43	92	40	89	37	46
15	48	96	44	92	40	87	35	83	31	45
16	47	94	41	88	36	83	30	77	24	44
17	46	93	39	85	32	78	24	70	17	43
18	46	91	37	82	28	74	19	65	10	42
19	45	90	34	79	24	69	14	58	03	41
20	44	88	32	76	20	64	08	52	8.6796	40
21	43	87	30	73	17	60	03	46	90	39
22	43	85	28	70	13	55	6.7498	40	83	38
23	42	83	25	67	09	50	92	34	75	37
24	41	82	23	64	05	46	87	28	69	36
25	40	80	21	61	01	41	81	22	62	35
26	39	79	18	58	4.8197	36	76	15	55	34
27	39	77	16	54	93	32	70	09	47	33
28	38	76	14	52	90	27	65	08	41	32
29	37	74	11	48	86	23	60	7.7096	34	31
30	36	73	09	45	82	18	54	90	27	30
31	36	71	06	42	78	13	48	84	20	29
32	35	69	04	39	74	08	43	78	12	28
33	34	68	02	36	70	04	38	72	06	27
34	33	66	00	33	66	5.7799	32	66	8.6699	26
35	32	65	2.8897	30	62	94	27	59	92	25
36	32	63	95	26	58	90	21	53	84	24
37	31	62	92	23	54	85	16	46	78	23
38	30	60	90	20	51	81	11	41	71	22
39	29	59	88	17	47	76	05	34	64	21
40	29	57	86	14	43	71	00	28	57	20
41	28	55	83	11	39	66	6.7394	22	49	19
42	27	54	81	08	35	61	88	15	42	18
43	26	52	78	04	31	57	83	09	35	17
44	25	51	76	01	27	52	77	02	28	16
45	25	49	73	3.8498	23	48	72	7.6997	21	15
46	24	48	71	95	19	43	67	90	14	14
47	23	46	69	92	15	38	61	84	07	13
48	22	44	67	89	11	33	55	78	00	12
49	21	43	64	86	07	28	50	71	8.6593	11
50	21	41	62	82	03	24	44	65	85	10
51	20	40	59	79	4.8099	19	39	58	78	9
52	19	38	57	76	95	14	33	52	71	8
53	18	36	55	73	91	09	27	46	64	7
54	17	35	52	70	87	04	22	39	57	6
55	16	33	50	66	83	00	16	33	49	5
56	16	32	47	63	79	5.7695	11	26	42	4
57	15	30	45	60	75	90	05	20	35	3
58	14	28	43	57	71	85	6.7299	14	28	2
59	13	27	40	53	67	80	94	07	21	1
60	0.9618	1.9225	2.8838	3.8450	4.8063	5.7676	6.7288	7.6901	8.6513	0
	1	2	3	4	5	6	7	8	9	

DEPARTURE 15 DEGREES.

105

/	1	2	3	4	5	6	7	8	9	/
0	0.2588	0.6176	0.7765	1.0358	1.2941	1.5529	1.8117	2.0706	2.3294	60
1	91	82	73	64	55	46	37	28	2.3819	59
2	94	88	81	75	69	63	57	50	44	58
3	97	93	90	86	83	80	76	73	69	57
4	99	99	98	98	97	96	96	95	95	56
5	0.2602	0.5204	0.7807	1.0409	1.3011	1.5618	1.8215	2.0818	2.3420	55
6	05	10	15	20	25	30	35	40	45	54
7	08	16	24	32	40	47	55	63	71	53
8	11	21	32	43	54	64	75	86	96	52
9	14	27	42	54	68	81	95	2.0908	2.3522	51
10	16	38	49	65	82	98	1.8314	30	47	50
11	19	38	57	76	95	1.5715	34	53	72	49
12	22	44	66	88	1.8110	31	53	75	97	48
13	25	49	74	99	24	48	73	• 98	2.3622	47
14	28	55	83	1.0510	38	65	93	2.1020	48	46
15	30	61	91	21	52	82	1.8412	42	73	45
16	33	66	99	32	66	99	32	65	98	44
17	36	72	0.7908	44	80	1.5815	61	87	2.8723	43
18	39	77	16	55	94	32	71	2.1110	48	42
19	42	83	25	66	1.8208	49	91	32	74	41
20	44	89	33	77	22	66	1.8510	54	99	40
21	47	94	41	88	36	83	30	77	2.3824	39
22	50	0.5300	50	1.0600	50	1.5900	50	2.1200	50	38
23	53	07	58	11	64	17	70	22	75	37
24	55	11	67	22	78	34	89	45	2.8900	36
25	58	18	75	34	84	92	50	1.8609	67	35
26	61	22	84	45	1.3306	67	28	90	51	34
27	66	28	92	56	20	84	48	2.1312	76	33
28	67	34	0.8000	67	84	1.6001	68	34	2.4001	32
29	70	39	09	78	48	18	87	57	26	31
30	72	45	17	90	62	34	1.8707	79	52	30
31	75	50	26	1.0701	76	51	26	2.1402	77	29
32	78	56	34	12	90	68	46	24	2.4102	28
33	81	62	42	23	1.8404	85	66	46	27	27
34	84	67	51	34	18	1.6102	85	69	52	26
35	86	73	59	46	32	18	1.8805	91	78	25
36	89	78	68	57	46	35	24	2.1514	2.4203	24
37	92	84	76	68	60	52	44	36	28	23
38	95	90	84	79	74	69	64	58	53	22
39	98	95	93	90	78	86	83	81	78	21
40	0.2700	0.5401	0.8101	1.0802	1.3502	1.6202	1.8908	2.1603	2.4304	20
41	03	06	10	13	16	19	22	26	29	19
42	06	12	18	24	30	36	42	48	54	18
43	09	18	26	35	44	53	62	70	79	17
44	12	23	35	46	58	70	81	93	2.4404	16
45	14	29	43	58	72	86	1.9001	2.1715	80	15
46	17	34	52	69	86	1.6303	20	38	55	14
47	20	40	60	80	1.3600	20	40	60	80	13
48	23	46	68	91	14	37	60	82	2.4505	12
49	26	51	77	1.0902	28	54	79	2.1805	80	11
50	28	57	85	14	42	70	99	27	56	10
51	31	62	94	25	56	87	1.9118	50	81	9
52	34	68	0.8202	36	70	1.6404	38	72	2.4606	8
53	37	74	10	47	84	21	58	94	31	7
54	40	79	19	58	98	38	77	2.1917	56	6
55	42	85	27	70	1.3712	54	97	89	82	5
56	45	90	36	81	26	71	1.9216	62	2.4707	4
57	48	96	44	92	44	88	36	84	32	3
58	51	0.5502	52	1.1008	54	1.6505	56	2.2006	57	2
59	54	07	61	14	68	22	75	29	82	1
60	0.2756	0.5513	0.8269	1.1026	1.3782	1.6588	1.9295	2.2051	2.4808	0
	1	2	3	4	5	6	7	8	9	

LATITUDE 74 DEGREES.

	1	2	3	4	5	6	7	8	9
0	0.9618	1.9225	2.8838	3.8450	4.8063	5.7676	6.7288	7.6901	8.6513
1	12	24	35	47	59	71	83	7.6894	06.59
2	11	22	33	44	55	66	77	89	8.6199
3	10	20	31	41	51	61	71	82	92.57
4	09	19	28	38	47	56	66	75	85.56
5	09	17	26	34	43	52	60	69	77.55
6	08	16	23	31	39	47	55	62	70.54
7	07	14	21	28	35	42	49	56	63.53
8	06	12	19	25	31	37	43	50	56.52
9	05	11	16	22	27	32	38	43	49.51
10	05	09	14	18	23	28	32	37	41.50
11	04	07	11	15	19	22	26	30	33.49
12	03	06	09	12	15	17	20	23	26.48
13	02	04	06	08	11	13	15	17	19.47
14	01	03	04	05	07	08	09	10	12.46
15	00	01	02	02	03	03	04	04	05.45
16	00	1.9199	2.8799	3.8399	4.7999	5.7598	6.7198	7.6798	8.6397
17	0.9599	98	97	96	95	93	92	91	90.43
18	98	96	94	92	91	89	87	85	83.42
19	97	94	92	89	86	83	80	78	75.41
20	96	93	89	87	82	78	75	71	68.40
21	96	91	87	83	78	74	69	65	60.39
22	95	90	84	79	74	69	64	58	53.38
23	94	88	82	76	70	64	58	52	46.37
24	93	86	79	72	66	59	52	45	38.36
25	92	85	77	69	62	54	46	38	31.35
26	92	83	75	66	58	49	41	32	24.34
27	91	81	72	62	54	44	35	26	16.33
28	90	80	69	59	49	39	29	18	08.32
29	89	78	67	56	45	34	23	12	01.31
30	88	76	65	53	41	29	17	06	8.6294
31	87	75	62	50	37	24	11	7.6699	87.25
32	87	78	60	46	33	19	06	92	79.24
33	86	71	57	43	29	14	00	86	71.23
34	85	70	55	40	25	09	6.7094	79	64.22
35	84	68	52	36	20	05	89	73	57.21
36	83	66	50	33	16	5.7499	82	66	49.20
37	82	65	47	30	12	94	77	59	41.19
38	82	63	45	26	08	90	71	53	34.18
39	81	61	42	23	04	84	65	46	26.17
40	80	60	40	20	00	79	59	39	18.16
41	79	58	37	16	4.7895	75	54	33	11.15
42	78	56	35	13	91	69	47	26	04.14
43	77	55	32	10	87	64	42	19	8.6197
44	76	53	30	06	83	60	36	13	89.16
45	76	51	27	03	79	54	30	06	81.15
46	75	50	25	00	75	49	24	7.6599	74.14
47	74	48	22	3.8296	70	44	18	92	66.13
48	73	46	20	99	66	39	12	86	59.12
49	72	45	17	90	62	34	07	79	52.11
50	72	43	15	86	58	29	01	72	44.10
51	71	41	12	83	54	24	6.6995	66	36.09
52	70	40	09	79	49	19	89	58	28.08
53	69	38	07	76	45	14	83	52	21.07
54	68	36	04	72	41	09	77	45	13.06
55	67	34	02	69	36	03	70	38	05.05
56	66	33	2.8699	66	32	5.7398	65	31	8.6098
57	65	31	97	62	28	93	59	24	90.97
58	65	29	94	59	24	88	53	18	82.96
59	64	28	91	55	19	83	47	10	74.95
60	0.9563	1.9126	2.8689	3.8252	4.7815	5.7878	6.6941	7.6504	8.6067
	1	2	3	4	5	6	7	8	9

DEPARTURE 16 DEGREES.

107

	1	2	3	4	5	6	7	8	9	10
0	0.2756	0.5513	0.8269	1.1026	1.3782	1.6538	1.9295	2.2051	2.4808	60
1	59	18	78	37	96	55	1.9314	74	33	59
2	62	24	86	48	1.3810	72	34	96	58	58
3	65	30	94	59	24	89	54	2.2118	83	57
4	68	35	0.8303	70	38	1.6606	73	41	2.4908	56
5	70	41	11	82	52	22	93	63	34	55
6	73	46	19	92	66	39	1.9412	85	58	54
7	76	52	28	1.1104	80	55	31	2.2207	83	53
8	79	57	36	15	94	72	51	30	2.5008	52
9	82	63	45	26	1.3908	89	71	52	34	51
10	84	69	53	37	22	1.6706	90	74	59	50
11	87	74	61	48	36	23	1.9510	97	84	49
12	90	80	70	60	50	89	29	2.2319	2.5109	48
13	93	85	78	71	64	56	49	32	34	47
14	96	91	87	82	78	73	69	64	60	46
15	98	97	95	92	94	88	86	84	82	45
16	0.2801	0.5602	0.8403	1.1204	1.4006	1.6807	1.9608	2.2409	2.5210	44
17	04	08	12	16	30	23	27	31	35	43
18	07	13	20	27	34	40	47	54	60	42
19	10	19	29	38	48	57	67	76	86	41
20	12	25	37	49	62	74	86	98	2.5311	40
21	15	30	45	60	75	90	1.9705	2.2510	35	39
22	18	36	53	71	89	1.6907	25	42	60	38
23	21	41	62	82	1.4103	24	44	65	86	37
24	23	47	70	94	17	40	64	87	2.5411	36
25	26	52	79	1.1305	31	57	83	2.2600	86	35
26	29	58	87	16	45	74	1.9803	32	61	34
27	32	64	95	27	59	91	23	54	86	33
28	35	69	0.8504	38	73	1.7008	42	77	2.5511	32
29	37	75	12	50	87	25	62	99	87	31
30	40	80	21	61	1.4201	41	81	2.2722	62	30
31	43	86	29	72	15	58	1.9900	43	86	29
32	46	91	37	83	29	75	20	66	2.5611	28
33	49	97	46	94	43	91	40	88	87	27
34	51	0.5703	54	1.1405	57	1.7108	59	2.2810	62	26
35	54	08	62	16	71	25	79	33	87	25
36	57	14	71	28	85	41	98	55	2.5712	24
37	60	19	79	39	99	58	2.0018	78	87	23
38	63	25	88	50	1.4313	75	88	2.2900	63	22
39	65	31	96	61	26	91	56	22	87	22
40	68	36	0.8604	72	40	1.7208	76	44	2.5812	20
41	71	42	12	83	54	25	96	66	37	19
42	74	47	21	94	68	42	2.0115	89	62	18
43	77	53	29	1.1508	82	61	38	2.3015	82	17
44	79	58	38	17	96	75	54	34	2.5913	16
45	82	64	46	28	1.4420	92	74	56	88	15
46	85	69	54	39	24	1.7308	93	78	62	14
47	88	75	63	50	38	25	2.0213	2.3100	88	13
48	90	81	71	61	52	42	32	22	2.6013	12
49	93	86	79	72	66	59	52	45	38	11
50	96	92	87	84	80	75	71	67	63	10
51	99	97	96	95	94	92	91	90	88	9
52	0.2902	0.5803	0.8705	1.1606	1.4508	1.7409	2.0311	2.3212	2.6114	8
53	04	09	13	17	21	25	29	34	38	7
54	07	14	21	28	35	42	49	56	63	6
55	10	20	29	39	49	59	69	78	88	5
56	13	25	38	50	63	76	88	2.3301	2.6213	4
57	15	31	46	62	77	92	2.0408	23	39	3
58	18	36	55	73	91	1.7509	27	46	64	2
59	21	42	63	84	1.4605	25	46	67	88	1
60	0.2924	0.5847	0.8771	1.1695	1.4619	1.7542	2.0466	2.3390	2.6313	0
	1	2	3	4	5	6	7	8	9	

LATITUDE 73 DEGREES.

	1	2	3	4	5	6	7	8	9	
0	0.9563	1.9126	2.8689	3.8652	4.7815	5.7378	6.6941	7.6504	8.6067	60
1	02	24	87	49	11	73	35	7.6498	60	59
2	61	23	84	45	07	68	29	90	52	58
3	61	21	82	41	03	63	24	84	45	57
4	60	19	79	38	4.7798	58	17	77	36	56
5	59	18	76	35	94	53	12	70	29	55
6	58	16	74	32	90	47	05	63	21	54
7	57	14	71	28	86	43	00	57	14	53
8	56	12	69	25	81	36	6.6893	49	06	52
9	55	11	66	22	77	32	88	43	8.5999	51
10	55	09	64	18	73	27	82	36	91	50
11	54	07	61	14	68	22	75	29	82	49
12	53	06	58	11	64	17	70	22	75	48
13	52	04	56	08	60	11	63	15	67	47
14	51	02	53	04	56	07	58	09	60	46
15	50	00	51	01	51	01	51	01	52	45
16	49	1.9099	48	3.8197	47	5.7296	45	7.6394	44	44
17	49	97	46	94	43	91	40	88	37	43
18	48	95	43	90	38	86	33	81	28	42
19	47	93	40	87	34	80	27	74	20	41
20	46	92	38	84	30	75	21	67	13	40
21	45	90	35	80	25	70	15	60	05	39
22	44	88	32	76	21	65	09	53	8.5897	38
23	43	86	30	73	17	60	03	46	90	37
24	42	85	27	70	12	54	6.6797	39	82	36
25	42	83	25	66	08	49	91	32	74	35
26	41	81	22	63	04	44	85	25	66	34
27	40	79	19	59	4.7699	39	79	18	58	33
28	39	78	17	56	95	33	72	11	50	32
29	38	76	14	52	90	28	66	04	42	31
30	37	74	12	49	86	23	60	7.6297	35	30
31	36	73	09	45	82	18	54	90	27	29
32	35	71	06	42	77	12	48	83	19	28
33	35	69	04	38	73	07	42	76	11	27
34	34	67	01	35	69	02	36	69	03	26
35	33	66	2.8598	31	64	5.7197	30	62	8.5795	25
36	32	64	96	28	60	91	23	55	47	24
37	31	62	93	24	55	86	17	48	79	23
38	30	60	90	20	51	81	11	41	71	22
39	29	59	88	17	47	76	05	34	64	21
40	28	57	85	14	42	70	6.6699	27	56	20
41	28	55	83	10	38	65	93	20	48	19
42	27	53	80	06	33	60	86	13	39	18
43	26	51	77	03	29	54	80	05	31	17
44	25	50	74	3.8099	24	49	74	7.6198	23	16
45	24	48	72	96	20	44	68	92	16	15
46	23	46	69	92	16	39	62	85	08	14
47	22	44	67	89	11	32	55	77	00	13
48	21	43	64	85	07	28	49	70	8.5692	12
49	20	41	61	82	02	22	43	63	84	11
50	20	39	59	78	4.7598	17	37	56	76	10
51	19	37	56	74	93	12	30	49	67	9
52	18	35	53	71	89	06	24	41	59	8
53	17	32	50	67	84	01	18	34	51	7
54	16	34	48	64	80	5.7095	11	27	43	6
55	15	30	45	60	75	90	05	20	35	5
56	14	28	43	57	71	85	6.6599	13	28	4
57	13	27	40	53	67	80	93	06	20	3
58	12	25	37	50	62	74	87	7.6099	12	2
59	12	23	35	46	58	69	81	92	04	1
60	0.9511	1.9021	2.8532	3.8042	4.7553	5.7064	6.6574	7.6085	8.5595	0
	1	2	3	4	5	6	7	8	9	

DEPARTURE 72 DEGREES.

DEPARTURE 17 DEGREES.

109

1	2	3	4	5	6	7	8	9	10
0	0.2924	0.5847	0.8771	1.1695	1.4619	1.7542	2.0466	2.3390	2.6318
1	27	53	80	1.1706	33	59	86	2.3412	39 59
2	29	59	88	17	47	76	2.0505	34	64 58
3	32	64	96	28	61	93	25	57	89 57
4	35	70	0.8804	39	74	1.7609	44	78	2.6413
5	38	75	13	50	88	26	63	2.3501	38 55
6	40	81	21	62	1.4702	42	83	23	64 54
7	43	86	30	73	16	59	2.0602	46	89 53
8	46	92	38	84	30	76	22	68	2.6514
9	49	97	46	95	44	92	41	90	38 51
10	52	0.5903	55	1.1806	58	1.7709	61	2.3612	64 50
11	54	09	63	17	72	26	80	34	2.6609
12	57	14	71	28	86	43	2.0700	57	14 48
13	60	20	80	40	1.4800	59	19	79	39 47
14	63	25	88	50	13	76	38	2.3701	63 46
15	65	31	96	62	27	92	58	23	89 45
16	68	36	0.8905	73	41	1.7809	77	46	2.6714
17	71	42	13	84	55	26	97	68	39 43
18	74	47	21	95	69	42	2.0816	90	63 42
19	77	53	30	1.1906	83	59	36	2.3812	89 41
20	79	59	38	17	97	76	55	34	2.6814
21	82	64	46	28	1.4911	93	75	57	39 39
22	85	70	55	40	25	1.7909	94	79	64 38
23	88	75	63	50	38	26	2.0913	2.3901	88 37
24	90	81	71	62	52	40	33	23	2.6914
25	93	86	80	73	66	59	52	46	39 35
26	96	92	88	84	80	76	72	68	64 34
27	99	97	96	95	94	92	91	90	88 33
28	0.3002	0.6003	0.9005	1.2006	1.5008	1.8009	2.1011	2.4012	2.7014
29	04	09	13	17	22	26	30	34	39 31
30	07	14	21	28	34	43	50	57	64 30
31	10	20	29	39	49	59	69	78	88 29
32	13	25	38	50	63	76	88	2.4101	2.7113
33	15	31	46	68	77	92	2.1108	23	89 27
34	18	36	55	73	91	1.8109	27	46	64 26
35	21	42	63	84	1.5105	25	46	67	88 25
36	24	47	71	95	19	42	66	90	2.7213
37	27	53	80	1.2106	33	59	86	2.4212	89 23
38	29	58	88	17	46	75	2.1204	34	63 22
39	32	64	96	28	60	92	24	56	88 21
40	35	70	0.9104	39	74	1.8209	44	78	2.7313
41	38	75	13	50	88	26	63	91	38 19
42	40	81	21	61	1.5202	42	82	2.4322	63 18
43	43	86	29	72	16	59	2.1302	45	88 17
44	46	92	38	84	30	75	21	67	2.7413
45	49	97	46	94	43	92	40	89	37 15
46	51	0.6103	54	1.2206	57	1.8308	60	2.4411	64 14
47	54	08	63	17	71	25	79	34	88 13
48	57	14	71	28	85	42	99	56	2.7513
49	60	19	79	39	99	58	2.1418	78	37 11
50	63	25	88	50	1.5313	75	38	2.4500	63 10
51	65	31	96	61	27	92	57	22	87 9
52	68	36	0.9204	72	40	1.8408	76	44	2.7612
53	71	42	12	83	54	25	96	66	37 7
54	74	47	21	94	68	42	2.1515	89	62 6
55	76	53	29	1.2305	82	58	34	2.4610	87 5
56	79	58	37	16	96	75	64	13	2.7712
57	82	64	46	28	1.5410	91	78	35	37 3
58	85	69	54	38	23	1.8508	92	77	61 2
59	87	75	62	50	47	24	2.1612	99	87 1
60	0.3090	0.6180	0.9271	1.2361	1.5451	1.8541	2.1631	1.4722	2.7812
1	1	2	3	4	5	6	7	8	9

LATITUDE 72 DEGREES.

LATITUDE 18 DEGREES.

1	2	3	4	5	6	7	8	9	10
1.9511	1.9021	2.8532	3.8042	4.7553	5.7064	6.6574	7.6085	8.5596	9.5107
10	19	29	39	49	58	68	78	87	96
09	18	26	35	44	53	62	70	79	88
08	16	24	32	40	47	55	63	71	79
07	14	21	28	35	42	49	56	63	70
06	12	18	24	31	37	43	49	55	61
05	10	16	21	26	31	36	42	47	52
04	09	13	17	22	26	30	34	39	43
03	07	10	13	17	20	23	26	30	33
02	05	07	10	12	14	17	19	22	24
02	03	05	06	08	09	11	12	14	15
01	01	02	02	03	04	04	05	05	06
00	1.8999	2.8499	3.7999	4.7499	5.6998	6.6498	7.5998	8.5497	9.4997
99	98	96	95	94	93	92	90	89	87
98	96	94	92	90	87	86	83	81	78
97	94	91	88	85	82	79	76	73	69
96	92	88	84	81	77	73	69	65	61
95	90	86	81	76	71	66	62	57	53
94	89	83	77	72	66	60	54	49	44
93	87	80	73	67	60	53	46	40	34
92	85	77	70	62	56	47	39	32	25
92	83	75	66	58	49	41	32	23	14
91	81	72	62	53	44	34	25	16	07
90	79	69	59	49	38	28	18	07	-03
89	78	66	55	44	33	22	10	-02	-10
88	76	63	51	39	27	15	02	-10	-18
87	74	61	48	35	21	08	-05	-12	-20
86	72	58	44	30	16	02	-08	-15	-22
85	70	55	40	26	11	-03	-10	-17	-24
84	68	53	37	21	05	-09	-16	-23	-30
83	66	50	33	16	-01	-08	-15	-22	-29
82	65	47	29	12	-04	-11	-18	-25	-32
81	63	44	26	07	-08	-15	-22	-29	-36
81	61	42	22	03	-11	-18	-25	-32	-39
80	59	39	18	-01	-08	-15	-22	-29	-36
79	57	36	14	-04	-11	-18	-25	-32	-39
78	55	33	11	-07	-14	-21	-28	-35	-42
77	54	30	07	-10	-17	-24	-31	-38	-45
76	52	27	03	-13	-20	-27	-34	-41	-48
75	50	25	00	-16	-23	-30	-37	-44	-51
74	48	22	-01	-08	-15	-22	-29	-36	-43
73	46	19	-04	-11	-18	-25	-32	-39	-46
72	44	16	-07	-14	-21	-28	-35	-42	-49
71	42	14	-10	-17	-24	-31	-38	-45	-52
70	40	11	-13	-20	-27	-34	-41	-48	-55
69	39	08	-16	-23	-30	-37	-44	-51	-58
68	37	05	-19	-26	-33	-40	-47	-54	-61
67	35	02	-22	-29	-36	-43	-50	-57	-64
67	33	00	-25	-32	-39	-46	-53	-60	-67
66	31	-01	-28	-35	-42	-49	-56	-63	-70
65	29	-04	-31	-38	-45	-52	-59	-66	-73
64	27	-07	-34	-41	-48	-55	-62	-69	-76
63	25	-10	-37	-44	-51	-58	-65	-72	-79
62	24	-13	-40	-47	-54	-61	-68	-75	-82
61	22	-16	-43	-50	-57	-64	-71	-78	-85
60	20	-19	-46	-53	-60	-67	-74	-81	-88
59	18	-22	-49	-56	-63	-70	-77	-84	-91
58	16	-25	-52	-59	-66	-73	-80	-87	-94
57	14	-28	-55	-62	-69	-76	-83	-90	-97
56	12	-31	-58	-65	-72	-79	-86	-93	-100
0.9455	1.8910	2.8366	3.7821	4.7276	5.6731	6.6186	7.5642	8.5097	9.4552

DEPARTURE 71 DEGREES.

DEPARTURE 18 DEGREES.

111

1	2	3	4	5	6	7	8	9	10
0	0.3090	0.6180	0.9271	1.2361	1.5451	1.8541	2.1631	2.4722	2.7812
1	93	86	79	72	65	57	50	43	36
2	96	91	87	83	79	74	70	66	61
3	99	97	96	94	93	91	90	88	87
4	0.8101	0.6202	0.9304	1.2405	1.5506	1.8607	2.1708	2.4810	2.7911
5	04	08	12	16	20	24	28	32	36
6	07	14	20	27	34	41	48	54	61
7	10	19	29	38	48	57	67	76	86
8	12	25	37	49	62	74	86	98	2.8011
9	15	30	45	60	76	91	2.1806	2.4921	36
10	18	36	53	71	89	1.8707	24	42	60
11	21	41	62	82	1.5603	24	44	65	85
12	23	47	70	93	17	40	63	86	2.8110
13	26	52	78	1.2504	31	57	83	2.5009	35
14	29	58	87	16	45	73	2.1902	31	60
15	32	63	95	26	58	90	21	53	84
16	34	69	0.9403	88	72	1.8806	41	75	2.8210
17	37	74	12	49	86	23	60	98	85
18	40	80	20	60	1.5700	39	79	2.5119	59
19	43	85	28	71	14	56	99	42	84
20	45	91	36	82	27	72	2.2018	63	2.8309
21	48	96	45	93	41	89	37	86	34
22	51	0.6302	53	1.2604	55	1.8906	57	2.5208	59
23	54	07	61	15	69	22	76	30	83
24	57	18	70	26	83	39	96	52	2.8409
25	59	19	78	37	97	56	2.2115	74	34
26	62	24	86	48	1.5810	72	84	96	59
27	65	30	94	59	24	89	54	2.5318	83
28	68	35	0.9503	70	38	1.9005	73	40	2.8608
29	70	41	11	81	52	22	92	62	33
30	73	46	19	92	65	38	2.2211	84	57
31	76	52	27	1.2703	79	55	81	2.5406	82
32	79	57	36	14	93	72	50	29	2.8607
33	81	63	44	25	1.5907	88	69	50	32
34	84	68	52	36	21	1.9105	89	73	56
35	87	74	60	47	34	21	2.2308	94	81
36	90	79	69	58	48	38	27	2.5517	2.8706
37	92	85	77	69	62	54	46	38	81
38	95	90	85	80	76	71	66	61	56
39	98	96	94	92	90	87	85	83	81
40	0.8201	0.6401	0.9602	1.2802	1.6003	1.9204	2.2404	2.5605	2.8805
41	03	07	10	14	17	20	24	27	31
42	06	12	18	24	31	37	43	49	55
43	09	18	27	36	45	53	62	71	80
44	12	23	35	46	58	70	81	93	2.8904
45	14	29	43	58	72	86	2.2501	2.5715	80
46	17	34	51	68	86	1.9303	20	37	54
47	20	40	60	80	1.6105	19	39	59	79
48	23	45	68	91	14	36	59	82	2.9004
49	25	51	76	1.2902	27	52	78	2.5803	29
50	28	58	85	13	41	69	97	26	54
51	31	62	93	24	55	85	2.2616	47	78
52	34	67	0.9701	35	69	1.9402	36	70	2.9103
53	36	73	09	46	82	18	55	91	28
54	39	78	18	57	96	35	74	2.5904	53
55	42	84	26	68	1.6210	51	93	35	77
56	45	89	34	79	24	68	2.2713	57	2.9202
57	47	95	42	90	37	84	32	79	27
58	50	0.6500	51	1.3001	51	1.9501	51	2.6002	52
59	53	06	59	12	65	17	70	23	76
60	0.8256	0.6511	0.9767	1.8023	1.6279	1.9534	2.2790	2.6046	2.9301
	1	2	3	4	5	6	7	8	9

LATITUDE 71 DEGREES.

	1	2	3	4	5	6	7	8	9	10
0	0.9455	1.8910	2.8366	3.7821	4.7276	5.6731	6.6186	7.5642	8.5097	9.4552
1	54	08	63	17	71	25	79	34	89	59
2	53	07	60	13	67	20	73	26	80	58
3	52	05	57	09	62	14	66	18	71	57
4	51	03	54	06	57	08	60	11	63	56
5	50	01	51	02	52	02	53	03	54	55
6	50	1.8899	49	3.7798	48	5.6697	47	7.5596	46	54
7	49	97	46	94	43	91	40	88	37	53
8	48	95	43	90	38	86	33	81	28	52
9	47	93	40	86	33	80	26	78	19	51
10	46	91	37	83	29	74	20	66	11	50
11	45	89	34	79	24	68	13	58	08	49
12	44	88	31	75	19	63	07	50	8.4994	48
13	43	86	28	71	14	57	00	42	85	47
14	42	84	25	67	09	51	6.6093	34	76	46
15	41	82	23	64	05	45	86	27	68	45
16	40	80	20	60	00	39	79	19	59	44
17	39	78	17	56	4.7193	34	73	12	51	43
18	38	76	14	52	90	28	66	04	42	42
19	37	74	11	48	85	22	59	7.5496	33	41
20	36	72	08	44	81	17	53	89	25	40
21	35	70	05	40	76	11	46	81	16	39
22	34	68	03	37	71	05	39	74	08	38
23	33	66	00	33	66	5.6599	32	66	8.4899	37
24	32	64	2.8297	29	61	93	25	58	90	36
25	31	63	94	25	57	88	19	50	82	35
26	30	61	91	21	52	82	12	42	73	34
27	29	59	88	18	47	76	05	34	64	33
28	28	57	85	14	42	70	5.5999	27	56	32
29	27	55	82	10	37	64	92	19	47	31
30	26	53	79	06	32	58	85	11	38	30
31	25	51	76	02	27	52	78	03	39	29
32	25	49	74	3.7698	23	47	72	7.5396	21	28
33	24	47	71	94	18	41	65	88	12	27
34	23	45	68	90	13	35	58	80	03	26
35	22	43	65	86	08	29	51	72	8.4794	25
36	21	41	62	82	03	24	44	65	35	24
37	20	39	59	78	4.7098	18	37	57	76	23
38	19	37	56	74	93	12	30	49	67	22
39	18	35	53	70	88	06	23	41	58	21
40	17	33	50	67	84	00	17	34	50	20
41	16	31	47	63	79	5.6494	10	26	41	19
42	15	29	44	59	74	88	03	18	32	18
43	14	27	41	55	69	82	6.5896	10	23	17
44	13	25	38	51	64	76	89	02	14	16
45	12	24	35	47	59	71	83	7.5294	06	15
46	11	22	32	43	54	65	76	86	8.4697	14
47	10	20	29	39	49	59	69	78	38	13
48	09	18	26	35	44	53	62	70	79	12
49	08	16	23	31	39	47	55	62	70	11
50	07	14	20	27	34	41	48	54	61	10
51	06	12	17	23	29	35	41	46	52	9
52	05	10	15	20	25	29	34	39	44	8
53	04	08	12	16	20	23	27	31	35	7
54	03	06	09	12	15	17	20	23	26	6
55	02	04	06	08	10	11	13	15	17	5
56	01	02	03	04	05	05	06	07	08	4
57	00	00	00	00	00	5.6299	6.5799	7.5199	8.4599	8
58	0.9399	1.8798	2.8197	3.7596	4.6995	93	92	91	90	2
59	98	96	94	92	90	87	85	83	81	1
60	0.9397	1.8794	2.8191	3.7588	4.6985	5.6381	6.5778	7.5175	8.4572	0
	1	2	3	4	5	6	7	8	9	

DEPARTURE 70 DEGREES.

DEPARTURE 19 DEGREES.										113
1	2	3	4	5	6	7	8	9	10	
0	0.3256	0.6511	0.9767	1.3023	1.6278	1.9534	2.2790	2.6046	2.9801	60
1	58	17	75	34	92	50	2.2809	67	26	59
2	61	22	84	45	1.6306	67	28	90	51	58
3	64	28	92	56	20	83	47	2.6111	75	57
4	67	33	0.9800	67	84	1.9600	67	84	2.9400	56
5	69	39	08	78	47	16	86	55	25	55
6	72	44	17	89	61	33	2.2905	78	50	54
7	75	50	25	1.3100	75	50	24	99	74	53
8	78	55	33	11	89	66	44	2.6222	99	52
9	80	60	41	22	1.6402	82	63	48	2.9524	51
10	83	66	50	33	16	99	82	66	49	50
11	86	72	58	44	30	1.9715	2.3001	87	78	49
12	89	77	66	55	44	32	21	2.6310	98	48
13	91	83	74	66	57	48	40	31	2.9623	47
14	94	88	83	77	71	65	59	54	48	46
15	97	94	91	88	85	81	78	75	72	45
16	0.3300	99	99	99	99	98	98	98	97	44
17	02	0.6605	0.9907	1.3210	1.6512	1.9814	2.3197	2.6419	2.6722	48
18	05	10	15	20	26	31	36	41	46	42
19	08	16	24	32	40	47	55	63	71	41
20	11	21	32	42	53	64	74	85	95	40
21	13	27	40	56	67	80	94	2.6507	2.9821	39
22	16	32	48	64	81	97	2.3213	29	45	38
23	19	38	57	76	95	1.9913	32	51	70	37
24	22	43	65	86	1.6608	30	51	73	94	36
25	24	49	73	98	22	46	71	96	2.9920	35
26	27	54	81	1.3308	36	63	90	2.6617	44	34
27	30	60	89	19	49	79	2.3309	38	68	33
28	33	65	98	30	63	96	28	61	98	32
29	35	71	1.0006	41	77	2.0012	47	82	8.0018	31
30	38	76	14	52	91	29	67	2.6705	48	30
31	41	82	22	63	1.6704	45	86	26	67	29
32	44	87	31	74	18	62	2.3405	49	92	28
33	46	93	39	85	32	78	24	70	3.0117	27
34	49	98	47	96	45	94	43	92	41	26
35	52	0.6704	55	1.3407	59	2.0111	63	2.6814	66	25
36	55	09	64	18	73	27	82	36	91	24
37	57	15	72	29	87	44	2.3501	58	3.0216	23
38	60	20	80	40	1.6800	60	20	80	40	22
39	63	25	88	51	14	76	39	2.6902	64	21
40	66	31	97	62	28	93	59	24	90	20
41	68	36	1.0105	73	41	2.0209	77	46	3.0314	19
42	71	42	13	84	55	26	97	68	39	18
43	74	47	21	95	69	42	2.3616	90	63	17
44	76	53	29	1.3506	82	58	35	2.7011	88	16
45	79	59	38	17	96	75	54	34	3.0413	15
46	82	64	46	28	1.6910	91	73	55	37	14
47	85	69	54	38	23	2.0308	92	77	61	13
48	87	75	62	50	37	24	2.3712	99	87	12
49	90	80	70	60	51	41	31	2.7121	3.0511	11
50	93	86	79	72	65	57	50	48	36	10
51	96	91	87	82	78	74	69	65	60	9
52	98	97	95	93	92	90	88	86	85	8
53	0.3401	0.6802	1.0203	1.3604	1.7006	2.0407	1.3808	2.7209	3.0610	7
54	04	08	11	15	19	23	27	30	34	6
55	07	13	20	26	33	39	46	52	59	5
56	09	19	28	37	47	56	65	74	84	4
57	12	24	36	48	60	72	84	96	3.0708	3
58	15	29	44	59	74	88	2.3903	2.7318	32	2
59	18	35	53	70	88	2.0505	23	40	58	1
60	0.3420	0.6840	1.2661	1.3681	1.7101	2.0521	2.3941	2.7362	3.0782	0
1	2	3	4	5	6	7	8	9	10	
LATITUDE 70 DEGREES.										

LATITUDE 20 DEGREES.

1	2	3	4	5	6	7	8	9	1
.9397	1.8794	2.8191	3.7588	4.6985	5.6381	6.5778	7.5175	8.4572	60
96	92	88	84	80	75	71	67	63	59
95	90	85	80	75	69	64	59	54	58
94	88	82	76	70	63	57	51	45	57
93	86	79	72	65	57	50	43	36	56
92	84	76	68	60	51	43	35	27	55
91	82	73	64	55	45	36	27	18	54
90	80	70	60	50	39	29	19	09	53
89	78	67	56	45	33	22	11	00	52
88	76	64	52	40	37	15	03	8.4491	51
87	74	61	48	35	21	08	7.5095	82	50
86	72	58	44	30	15	01	87	73	49
85	70	55	40	25	09	6.5694	79	64	48
84	68	52	36	20	03	87	71	55	47
83	66	49	32	15	5.6297	80	63	46	46
82	64	46	28	10	91	73	55	37	45
81	62	43	24	05	85	66	47	28	44
80	60	40	20	00	79	59	39	19	43
79	58	37	16	4.6895	73	52	31	10	42
78	56	34	12	90	67	45	23	01	41
77	54	31	08	85	61	38	15	8.4392	40
76	52	28	04	79	55	32	07	83	39
75	50	24	3.7499	74	49	24	7.4998	73	38
74	48	21	95	69	43	17	90	64	37
73	46	18	91	64	37	10	82	55	36
72	44	15	87	59	31	02	74	46	35
71	42	12	83	54	25	6.5596	66	37	34
70	40	09	79	49	19	89	58	28	33
69	38	06	75	44	13	82	50	19	32
68	36	03	71	39	07	75	42	10	31
67	33	00	67	34	00	67	34	00	30
66	31	2.8097	63	29	5.6194	60	26	8.4291	29
65	29	94	59	24	88	53	18	82	28
64	27	91	55	19	82	45	09	72	27
63	25	88	50	13	76	38	01	63	26
62	23	85	46	08	70	31	7.4893	54	25
61	21	82	42	03	64	24	85	45	24
60	19	79	38	4.6798	58	17	77	36	23
59	17	76	34	93	51	10	68	27	22
58	15	73	30	89	45	03	60	18	21
57	13	70	26	83	39	6.5496	52	09	20
56	11	66	22	78	33	89	44	00	19
55	09	63	18	72	26	81	35	8.4190	18
54	07	60	14	67	20	74	27	81	17
53	05	57	10	62	14	67	19	72	16
52	03	54	06	57	08	60	11	63	15
51	01	51	01	52	02	52	02	53	14
50	01	51	01	52	02	52	02	53	14
49	1.8699	48	3.7897	47	5.6096	45	7.4794	44	13
48	97	45	93	42	90	88	86	35	12
47	94	42	89	36	83	80	78	25	11
46	92	39	85	31	77	28	70	16	10
45	90	36	81	26	71	16	62	07	9
44	88	32	76	21	65	09	53	8.4097	8
43	86	29	72	16	59	02	46	88	7
42	84	26	68	10	52	6.5894	36	78	6
41	82	23	64	05	46	87	28	69	5
40	80	20	60	00	40	80	20	60	4
39	78	18	56	4.6695	33	72	11	50	3
38	76	14	52	90	27	65	03	41	2
37	74	10	47	84	21	58	7.4694	31	1
36	1.8672	2.8007	3.7343	4.6679	5.6015	6.5851	7.4686	8.4022	0
2	3	4	5	6	7	8	9		

DEPARTURE 69 DEGREES.

DEPARTURE 20 DEGREES.										115
1	2	3	4	5	6	7	8	9	10	
0	0.8420	0.6840	1.0261	1.8681	1.7101	2.0521	2.8941	2.7862	8.0782	06
1	28	46	69	92	15	87	60	88	8.0806	59
2	26	51	77	1.8708	29	54	80	2.7404	81	58
3	28	57	85	14	42	70	99	27	56	57
4	31	62	98	24	56	87	2.4018	49	80	56
5	34	68	1.0302	36	70	2.0608	87	71	3.0905	55
6	37	78	10	46	88	20	56	98	29	54
7	39	79	18	57	97	88	75	2.7514	54	58
8	42	84	26	68	1.7211	58	95	37	79	52
9	45	90	34	79	24	69	2.4114	58	8.1008	51
10	48	95	43	90	38	85	83	80	28	50
11	50	0.6901	51	1.8801	52	2.0702	52	2.7602	58	49
12	53	06	59	12	65	18	71	24	77	48
13	56	11	67	23	79	84	90	46	8.1101	47
14	58	17	75	34	92	50	2.4209	67	26	46
15	61	22	84	45	1.7806	67	28	90	51	45
16	64	28	92	56	20	83	47	2.7711	75	44
17	67	33	1.0400	66	38	2.0800	66	33	99	43
18	69	39	08	78	47	16	86	58	8.1225	42
19	72	44	16	88	61	33	2.4305	77	49	41
20	75	50	24	99	74	49	24	98	78	40
21	78	55	33	1.8910	88	65	48	2.7820	98	39
22	80	61	41	21	1.7402	82	62	42	8.1828	38
23	83	66	49	32	15	98	81	64	47	37
24	86	71	57	43	29	2.0914	2.4400	86	71	36
25	88	77	65	54	42	80	19	2.7907	96	35
26	91	82	74	65	56	47	38	80	8.1421	34
27	94	88	82	76	70	63	57	51	45	33
28	97	93	90	86	88	80	76	78	69	32
29	99	99	98	97	97	96	95	94	94	31
30	0.8502	0.7004	1.0606	1.4008	1.7511	2.1018	2.4515	2.8017	8.1519	30
31	06	10	14	19	24	29	34	38	48	29
32	08	15	23	30	38	45	58	60	68	28
33	10	20	31	41	51	61	71	82	92	27
34	13	26	39	52	65	78	91	2.8104	8.1617	26
35	16	31	47	63	79	94	2.4610	26	41	25
36	18	37	55	74	92	2.1110	29	47	66	24
37	21	42	63	84	1.7606	27	48	69	90	23
38	24	48	72	96	20	48	67	91	8.1715	22
39	27	53	80	1.4106	38	60	86	2.8218	89	21
40	29	59	88	17	47	76	2.4705	84	64	20
41	32	64	96	28	60	92	24	56	88	19
42	35	69	1.0604	39	74	2.1208	43	78	8.1812	18
43	38	75	13	50	87	25	63	2.8300	88	17
44	40	80	21	61	1.7701	41	81	22	62	16
45	43	86	29	72	15	57	2.4800	43	86	15
46	46	91	37	82	28	74	19	65	8.1910	14
47	48	97	45	94	42	90	89	87	86	13
48	51	0.7102	53	1.4204	56	2.1307	58	2.8409	60	12
49	54	08	61	15	69	23	77	80	84	11
50	57	13	70	26	83	39	96	52	8.2009	10
51	59	18	78	37	96	55	2.4914	74	83	9
52	62	24	86	48	1.7810	71	83	95	57	8
53	65	29	94	59	24	88	53	2.8518	82	7
54	67	35	1.0702	70	37	2.1404	72	89	8.2107	6
55	70	40	10	80	51	21	91	61	81	5
56	73	46	18	91	64	37	2.5010	82	55	4
57	76	51	27	1.4802	78	53	29	2.8604	80	3
58	79	56	35	13	91	69	47	26	8.2204	2
59	81	62	43	24	1.7905	86	67	48	29	1
60	0.8584	0.7167	1.0751	1.4335	1.7919	2.1502	2.5086	2.8670	8.2258	0
1	2	3	4	5	6	7	8	9		
LATITUDE 69 DEGREES.										

	1	2	3	4	5	6	7	8	9	10
0	0.9336	1.8672	2.8007	3.7343	4.6679	5.6015	6.5351	7.4686	8.4022	60
1	35	70	04	39	74	09	44	78	13	59
2	34	67	01	35	69	02	36	70	03	58
3	33	65	2.7998	81	64	5.5996	29	61	8.3994	57
4	32	63	95	26	58	90	21	53	84	56
5	31	61	92	22	53	84	14	44	75	55
6	30	59	89	18	48	77	07	36	66	54
7	29	57	86	14	43	71	6.5299	28	57	53
8	27	55	82	10	37	64	92	19	47	52
9	26	53	79	06	32	58	84	11	38	51
10	24	51	76	01	27	52	77	02	28	50
11	24	49	73	3.7297	22	46	69	7.4594	19	49
12	23	46	70	93	16	39	62	86	09	48
13	22	44	67	89	11	33	55	78	00	47
14	21	42	63	84	06	27	48	69	8.3890	46
15	20	40	60	80	01	21	41	61	81	45
16	19	38	57	76	4.6595	14	33	52	71	44
17	18	36	54	72	90	08	26	44	62	43
18	17	34	51	68	85	01	18	35	52	42
19	16	32	48	64	80	5.5895	11	26	43	41
20	15	30	44	59	74	89	04	18	33	40
21	14	27	41	55	69	82	6.5196	10	23	39
22	13	25	38	51	64	76	89	02	14	38
23	12	23	35	46	58	70	81	7.4493	04	37
24	11	21	32	42	53	64	74	85	8.3795	36
25	10	19	29	38	48	57	67	76	86	35
26	08	17	25	34	42	50	59	67	76	34
27	07	15	22	30	37	44	52	59	67	33
28	06	13	19	25	32	38	44	50	57	32
29	05	10	16	21	26	31	36	42	47	31
30	04	08	13	17	21	25	29	34	38	30
31	03	06	09	12	16	19	22	25	28	29
32	02	04	06	08	10	12	14	16	18	28
33	01	02	03	04	05	05	07	08	09	27
34	00	00	00	00	00	5.5799	6.5099	7.4399	8.3699	26
35	0.9299	1.8598	2.7896	3.7195	4.6494	93	92	90	89	25
36	98	96	93	91	89	87	85	82	80	24
37	97	93	90	87	84	80	77	74	70	23
38	96	91	87	82	78	74	69	65	60	22
39	95	89	84	78	73	67	62	56	51	21
40	94	87	81	74	68	61	55	48	42	20
41	92	85	77	70	62	54	47	39	32	19
42	91	83	74	65	57	48	39	30	22	18
43	90	80	71	61	51	41	31	22	12	17
44	89	78	68	57	46	35	24	14	03	16
45	88	76	64	52	40	29	17	05	8.3593	15
46	87	74	61	48	35	22	09	7.4296	83	14
47	86	72	58	44	30	15	01	87	73	13
48	85	70	55	40	25	09	6.4994	79	64	12
49	84	68	51	35	19	03	87	70	54	11
50	83	65	48	31	14	5.5696	79	62	44	10
51	82	63	45	26	08	90	71	53	84	9
52	81	61	42	22	03	83	64	44	25	8
53	79	59	38	18	4.6397	76	56	35	16	7
54	78	57	35	14	92	70	49	27	06	6
55	77	55	32	09	87	64	41	18	8.3496	5
56	76	52	29	05	81	57	33	10	86	4
57	75	50	25	00	76	51	26	02	76	3
58	74	48	22	3.7096	70	44	18	7.4192	66	2
59	73	46	19	91	65	87	10	83	56	1
60	0.9272	1.8544	2.7815	3.7087	4.6359	5.5631	6.4903	7.4174	8.3446	0
	1	2	3	4	5	6	7	8	9	

DEPARTURE 21 DEGREES.

117

/	1	2	3	4	5	6	7	8	9	/
0	0.8584	0.7167	1.0751	1.4385	1.7919	2.1502	2.5086	2.8670	3.2253	60
1	86	73	59	46	32	18	2.5105	91	78	59
2	89	78	67	56	46	35	24	2.8713	3.2202	58
3	92	84	75	67	59	51	43	34	26	57
4	95	89	84	78	73	67	62	56	51	56
5	97	95	92	89	87	84	81	78	76	55
6	0.8600	0.7200	1.0800	1.4400	1.8000	2.1600	2.5200	2.8800	3.2300	54
7	03	05	08	11	14	16	19	22	24	53
8	05	11	16	21	27	32	38	43	49	52
9	08	16	24	32	41	49	57	65	73	51
10	11	22	32	43	54	65	76	86	97	50
11	14	27	41	54	68	81	95	2.8908	3.2422	49
12	16	32	49	65	81	97	2.5813	30	46	48
13	19	38	57	76	95	2.1714	83	52	71	47
14	22	43	65	87	1.8109	00	52	74	95	46
15	24	49	73	98	22	46	71	95	3.2520	45
16	27	54	81	1.4508	36	63	90	2.9017	44	44
17	30	60	89	19	49	79	2.5409	38	68	43
18	33	65	98	30	63	95	28	60	93	42
19	35	70	1.0906	41	76	2.1811	46	82	3.2617	41
20	38	76	14	52	90	27	65	2.9103	41	40
21	41	81	22	62	1.8203	44	84	25	65	39
22	43	87	30	74	17	60	2.5504	57	91	38
23	46	92	38	84	31	77	23	69	3.2715	37
24	49	98	46	95	44	98	42	90	89	36
25	52	0.7808	55	1.4606	58	2.1909	61	2.9212	64	35
26	54	08	63	17	71	25	79	34	88	34
27	57	14	71	28	85	41	98	55	3.2812	33
28	60	19	79	38	98	58	2.5617	77	86	32
29	62	25	87	49	1.8312	74	36	98	61	31
30	65	30	95	60	25	90	55	2.9320	3.2985	30
31	68	35	1.1003	71	39	2.2006	74	42	3.3019	29
32	70	41	10	82	52	22	93	63	34	28
33	73	46	19	92	66	39	2.5712	85	58	27
34	76	52	27	1.4703	79	55	31	2.9406	82	26
35	79	57	36	14	93	71	50	28	3.3107	25
36	81	62	43	25	1.8406	87	68	50	31	24
37	84	68	52	36	20	2.2103	87	71	55	23
38	87	73	60	47	34	20	2.5807	94	80	22
39	89	79	68	58	47	36	26	2.9515	3.3205	21
40	92	84	76	68	61	53	45	37	29	20
41	95	90	84	79	74	69	64	58	53	19
42	98	95	93	90	88	85	83	80	78	18
43	0.8700	0.7400	1.1101	1.4801	1.8501	2.2201	2.5901	2.9602	3.3302	17
44	03	06	09	12	15	17	20	23	26	16
45	06	11	17	22	28	34	39	45	50	15
46	08	17	25	33	42	50	58	66	75	14
47	11	22	33	44	55	66	77	88	99	13
48	14	27	41	55	69	82	96	2.9710	3.3423	12
49	16	33	49	66	82	98	2.6015	31	48	11
50	19	38	57	76	96	2.2315	84	53	72	10
51	22	44	65	87	1.8609	31	53	74	96	9
52	25	49	74	98	23	47	72	96	3.3521	8
53	27	54	82	1.4909	36	63	90	2.9818	45	7
54	30	60	90	20	50	79	2.6109	39	69	6
55	33	65	98	30	63	96	28	61	93	5
56	35	71	1.1206	41	77	2.2412	47	82	3.3618	4
57	38	76	14	52	90	28	66	2.9904	62	3
58	41	81	22	63	1.8704	44	85	26	66	2
59	43	87	30	74	17	60	2.6204	47	91	1
60	0.2746	0.7492	1.1238	1.4984	1.8731	2.2477	2.6223	2.9967	3.3715	0
	1	2	3	4	5	6	7	8	9	

LATITUDE 68 DEGREES.

	1	2	3	4	5	6	7	8	9	
0	0.9272	1.8544	2.7815	3.7087	4.6359	5.5631	6.4903	7.4174	8.3446	60
1	71	41	12	83	54	24	6.4895	66	36	59
2	70	39	09	79	49	18	88	58	27	58
3	69	37	06	74	43	12	80	49	17	57
4	68	35	03	70	38	05	73	40	08	56
5	66	33	2.7799	66	32	5.5598	65	31	8.3398	55
6	65	31	96	61	27	92	57	22	88	54
7	64	28	93	56	21	85	49	14	78	53
8	63	26	89	52	16	79	42	05	68	52
9	62	24	86	48	10	72	34	7.4096	58	51
10	61	22	83	44	05	65	26	87	48	50
11	60	20	79	39	00	59	19	78	38	49
12	59	17	76	35	4.6294	52	11	70	28	48
13	58	15	73	30	88	46	03	61	18	47
14	57	13	70	26	83	39	6.4796	52	09	46
15	55	11	66	22	77	33	88	43	8.3299	45
16	54	09	63	17	72	26	80	34	89	44
17	53	06	60	12	66	19	72	25	79	43
18	52	04	56	08	61	13	65	17	69	42
19	51	02	53	04	55	06	57	08	59	41
20	50	00	50	00	50	5.5499	49	7.3999	49	40
21	49	1.8498	46	3.6995	46	93	42	90	39	39
22	48	95	43	91	39	86	34	82	29	38
23	47	93	40	86	33	80	27	73	19	37
24	46	91	37	82	28	73	19	64	10	36
25	44	89	33	78	22	66	11	55	00	35
26	43	86	30	73	16	59	02	46	8.3189	34
27	42	84	26	68	11	53	6.4695	37	79	33
28	41	82	23	64	05	46	87	28	69	32
29	40	80	20	60	00	39	79	19	59	31
30	39	78	16	55	4.6194	33	72	10	49	30
31	38	75	13	51	89	26	64	02	39	29
32	37	73	10	46	83	20	56	7.3893	29	28
33	36	71	07	42	78	13	49	84	20	27
34	34	69	03	37	72	06	40	74	09	26
35	33	66	00	32	66	5.5399	32	66	8.3099	25
36	32	64	2.7696	28	61	93	25	57	89	24
37	31	62	93	24	55	86	17	48	79	23
38	30	60	90	20	50	79	09	39	69	22
39	29	57	86	15	44	72	01	30	58	21
40	28	55	83	10	38	66	6.4593	21	48	20
41	27	53	80	06	33	59	86	12	39	19
42	25	51	76	02	27	52	78	03	29	18
43	24	49	73	3.6897	22	46	70	7.3794	19	17
44	23	46	69	92	16	39	62	85	08	16
45	22	44	66	88	10	32	54	76	8.2998	15
46	21	42	63	84	05	25	46	67	88	14
47	20	40	59	79	4.6099	19	39	58	78	13
48	19	37	56	74	93	12	30	49	67	12
49	18	35	53	70	88	05	23	40	58	11
50	16	33	49	66	82	5.5298	15	31	48	10
51	15	30	46	61	76	91	06	22	37	9
52	14	28	42	56	71	85	6.4499	13	27	8
53	13	26	39	52	65	78	91	06	17	7
54	12	24	36	48	60	71	83	7.3695	07	6
55	11	21	32	43	54	64	75	86	8.2896	5
56	10	19	29	38	48	58	67	77	86	4
57	09	17	26	34	43	51	60	68	77	3
58	07	15	22	29	37	44	51	58	66	2
59	06	12	19	25	31	37	43	50	56	1
60	0.9205	1.8410	2.7615	3.6820	4.6025	5.5230	6.4435	7.3640	8.2845	0

1	2	3	4	5	6	7	8	9	10
0	0.3746	0.7492	1.1238	1.4984	1.8781	2.2477	2.6223	2.9969	3.3715
1	49	98	46	95	44	93	42	90	39
2	52	0.7508	55	1.5006	58	2.2509	61	3.0012	64
3	54	08	68	17	71	25	79	34	88
4	57	14	71	28	85	41	98	55	3.3812
5	60	19	79	38	98	57	2.6317	76	36
6	62	24	87	49	1.8811	73	85	98	60
7	65	30	95	60	25	89	54	3.0119	84
8	68	35	1.1308	70	38	2.2606	78	41	3.3908
9	70	41	11	81	52	22	92	62	33
10	73	46	19	92	65	38	2.6411	84	57
11	76	51	27	1.5103	79	54	80	3.0206	81
12	78	57	35	14	92	70	49	27	3.4006
13	81	62	43	24	1.8906	87	68	49	30
14	84	68	51	35	19	2.2703	87	70	54
15	87	73	60	46	38	19	2.6506	92	79
16	89	78	68	57	46	35	24	3.0314	3.4108
17	92	84	76	68	60	51	43	35	27
18	95	89	84	78	78	68	62	57	51
19	97	95	92	89	87	84	81	78	76
20	0.3800	0.7600	1.1400	1.5200	1.9000	99	99	99	99
21	03	05	08	10	13	2.2816	2.6618	3.0421	3.4223
22	05	11	16	21	27	32	37	42	48
23	08	16	24	32	40	48	56	64	72
24	11	21	32	43	54	64	75	86	96
25	13	27	40	54	67	80	94	3.0507	3.4321
26	16	32	48	64	81	97	2.6718	29	45
27	19	38	56	75	94	2.2913	32	50	69
28	22	43	65	86	1.9108	29	51	72	94
29	24	48	72	96	21	45	69	98	3.4417
30	27	54	80	1.5307	34	61	88	3.0614	41
31	30	59	89	18	48	77	2.6807	36	66
32	32	64	97	29	61	93	25	58	90
33	35	70	1.1505	40	75	2.8009	44	79	3.4514
34	38	75	13	50	88	26	63	3.0701	88
35	40	81	21	61	1.9202	42	82	22	63
36	43	86	29	72	15	58	2.6901	44	87
37	46	91	37	82	28	74	19	65	3.4610
38	48	97	45	93	42	90	38	86	85
39	51	0.7702	53	1.5404	55	2.8106	57	3.0808	59
40	54	07	61	15	69	22	76	80	83
41	56	13	69	26	82	38	95	51	3.4708
42	59	18	77	36	96	55	2.7014	72	82
43	62	23	85	47	1.9809	70	82	94	55
44	64	29	98	58	22	86	51	3.0916	80
45	67	34	1.1601	68	36	2.3203	70	37	3.4804
46	70	40	09	79	49	19	89	58	28
47	73	45	18	90	63	35	2.7108	80	53
48	75	50	26	1.5501	76	57	26	3.1002	77
49	78	56	33	11	89	67	45	22	3.4900
50	81	61	42	22	1.9403	83	64	44	25
51	83	66	50	33	16	99	82	66	49
52	86	72	58	44	30	2.3315	2.7201	87	73
53	89	77	66	54	43	32	20	3.1109	97
54	91	82	74	65	56	47	38	30	3.5021
55	94	88	82	76	70	63	57	51	45
56	97	93	90	86	83	80	76	78	69
57	99	99	98	97	97	96	95	94	94
58	0.3902	0.7804	1.1706	1.5608	1.9510	2.3412	2.7314	3.1216	3.5118
59	05	09	14	18	23	28	32	37	41
60	0.3907	0.7815	1.1722	1.5629	1.9537	2.3444	2.7351	3.1258	3.5166
1	2	3	4	5	6	7	8	9	

LATITUDE 23 DEGREES.									
120	1	2	3	4	5	6	7	8	9
0	0.9205	1.8410	2.7615	3.6820	4.6025	5.5230	6.4435	7.3640	8.2845
1	04	08	12	16	20	23	27	31	35
2	03	06	08	11	14	17	20	22	25
3	02	03	05	06	08	10	11	13	14
4	01	01	02	02	03	03	04	04	05
5	0.9199	00	2.7598	3.6798	4.5997	5.5196	6.4396	7.3595	8.2795
6	98	1.8396	95	93	91	89	87	86	84
7	97	94	91	88	86	83	80	77	74
8	96	92	88	84	80	75	71	67	63
9	95	90	84	79	74	69	64	58	53
10	94	87	81	74	68	62	55	49	42
11	93	85	78	70	63	55	48	40	33
12	91	83	74	66	57	48	40	31	23
13	90	80	71	61	51	41	31	22	12
14	89	78	67	56	46	35	24	13	02
15	88	76	64	52	40	28	15	08	8.2691
16	87	74	60	47	34	21	08	7.3494	81
17	86	71	57	42	28	14	00	85	70
18	85	69	54	38	23	07	6.4292	76	61
19	83	67	50	33	17	00	83	67	50
20	82	64	47	29	11	5.5093	75	58	40
21	81	62	43	24	05	86	67	48	29
22	80	60	40	20	00	79	59	39	19
23	79	57	36	15	4.5894	72	51	30	08
24	78	55	33	10	88	65	43	20	8.2598
25	76	53	29	06	82	58	35	11	88
26	75	50	26	01	76	51	26	02	77
27	74	48	22	3.6695	71	45	19	7.3393	67
28	73	46	19	92	65	37	10	83	56
29	72	44	15	87	59	31	03	74	46
30	71	41	12	82	53	24	6.4194	65	35
31	69	39	08	78	47	16	86	55	25
32	68	37	05	73	42	10	78	46	15
33	67	34	01	68	36	03	70	37	04
34	66	32	2.7498	64	30	5.4996	62	28	8.2494
35	65	30	94	59	24	89	54	18	83
36	64	27	91	54	18	82	45	09	72
37	63	25	88	50	13	75	38	00	63
38	61	23	84	45	07	68	29	7.3290	52
39	60	20	80	40	01	61	21	81	41
40	59	18	77	36	4.5795	54	13	72	31
41	58	16	73	31	89	47	05	62	20
42	57	13	70	26	83	40	6.4096	53	09
43	56	11	67	22	78	33	89	44	00
44	54	09	63	17	72	26	80	34	8.2389
45	53	06	59	12	66	19	72	25	78
46	52	04	56	08	60	11	63	15	67
47	51	02	52	03	54	05	56	06	57
48	50	1.8299	49	3.6598	48	5.4898	47	7.3197	46
49	48	97	45	94	42	90	39	87	36
50	47	94	42	89	36	83	30	78	25
51	46	92	38	84	31	77	23	69	15
52	45	90	35	80	25	69	14	59	04
53	44	87	31	75	19	62	06	50	8.2293
54	43	85	28	70	13	55	6.3998	40	83
55	41	83	24	65	07	48	90	31	73
56	40	80	21	61	01	41	81	22	62
57	39	78	17	56	4.5695	34	73	12	51
58	38	76	13	51	89	27	65	02	40
59	37	73	10	46	83	20	56	7.8093	29
60	0.9186	1.8271	2.7407	3.6542	4.5678	5.4813	6.3949	7.3084	8.2220
	1	2	3	4	5	6	7	8	9
DEPARTURE 66 DEGREES.									

DEPARTURE 23 DEGREES.

121

<i>j</i>	1	2	3	4	5	6	7	8	9	<i>j</i>
0	0.3907	0.7815	1.1722	1.5629	1.9537	2.3444	2.7351	3.1258	3.5166	60
1	10	20	30	40	50	60	70	80	90	59
2	13	25	38	51	63	76	89	3.1302	3.5214	58
3	15	31	46	61	77	92	2.7407	22	38	57
4	18	36	54	72	90	2.3508	26	44	62	56
5	21	41	62	83	1.9604	24	45	66	86	55
6	23	47	70	94	17	40	64	87	3.5311	54
7	26	52	78	1.5704	30	56	82	3.1408	34	53
8	29	57	86	15	44	72	2.7501	30	58	52
9	31	63	94	26	57	88	20	51	83	51
10	34	68	1.1802	36	71	2.3605	39	73	3.5407	50
11	37	73	10	47	84	20	57	94	30	49
12	39	79	18	58	97	36	76	3.1515	55	48
13	42	84	26	68	1.9711	52	95	37	79	47
14	45	90	34	79	24	69	2.7614	58	3.5508	46
15	47	95	42	90	37	84	32	79	27	45
16	50	0.7900	50	1.5800	51	2.3701	51	3.1601	51	44
17	53	06	58	11	64	17	70	22	75	43
18	56	11	67	22	78	33	89	44	3.5600	42
19	58	16	74	32	91	49	2.7707	65	23	41
20	61	22	82	43	1.9804	65	26	86	47	40
21	64	27	91	54	18	81	45	3.1708	72	39
22	66	32	98	64	31	97	63	29	95	38
23	69	38	1.1906	75	44	2.3813	82	50	3.5719	37
24	72	43	15	86	58	29	2.7801	72	44	36
25	74	48	22	96	71	45	19	93	67	35
26	77	54	30	1.5907	84	61	38	3.1814	91	34
27	80	59	39	18	98	77	57	36	3.5816	33
28	82	64	47	29	1.9911	93	75	59	40	32
29	85	70	54	39	24	2.3909	94	78	63	31
30	88	75	63	50	38	25	2.7913	3.1900	88	30
31	90	80	71	61	51	41	31	22	3.5912	29
32	93	86	78	71	64	57	50	42	35	28
33	96	91	87	82	78	73	69	64	60	27
34	98	96	95	93	91	89	87	86	84	26
35	0.4001	0.8002	1.2002	1.6003	2.0004	2.4005	2.8006	3.2006	3.6007	25
36	04	07	11	14	18	21	25	28	32	24
37	06	12	19	25	31	37	43	50	56	23
38	09	18	26	35	44	53	62	70	79	22
39	12	23	35	46	58	69	81	92	3.6104	21
40	14	28	42	56	71	85	99	3.2113	27	20
41	17	34	50	67	84	2.4101	2.8118	34	51	19
42	20	39	59	78	98	17	37	56	76	18
43	22	44	66	88	2.0111	33	55	77	99	17
44	25	50	74	99	24	49	74	98	3.6223	16
45	28	55	83	1.6110	38	65	93	3.2220	48	15
46	30	60	90	20	51	81	2.8211	41	71	14
47	33	66	98	31	64	97	30	62	95	13
48	36	71	1.2107	42	78	2.4218	49	84	3.6320	12
49	38	76	14	52	91	29	67	3.2305	43	11
50	41	82	22	63	2.0204	45	86	26	67	10
51	43	87	30	74	17	61	2.8304	47	91	9
52	46	92	38	84	31	77	23	69	3.6414	8
53	49	98	46	95	44	93	42	90	39	7
54	51	0.8103	54	1.6206	57	2.4308	60	3.2411	63	6
55	54	08	62	16	71	25	79	33	87	5
56	57	13	70	27	84	40	97	54	3.6510	4
57	59	19	78	38	97	56	2.8416	75	35	3
58	62	24	86	48	2.0311	73	35	97	59	2
59	65	29	94	59	24	88	53	3.2518	82	1
60	0.4067	0.8135	1.2202	1.6270	2.0337	2.4404	2.8472	3.2539	3.6607	0
	1	2	3	4	5	6	7	8	9	

LATITUDE 66 DEGREES.

	1	2	3	4	5	6	7	8	9	10
0	0.9186	1.8271	2.7407	3.6542	4.5678	5.4813	6.3949	7.3084	8.2220	60
1	84	69	03	37	72	06	40	74	09	59
2	33	66	2.7399	32	66	5.4799	32	65	8.2198	58
3	32	64	96	28	60	91	23	55	87	57
4	31	61	92	23	54	84	15	46	76	56
5	30	59	89	18	48	77	07	36	66	55
6	28	57	85	13	42	70	6.3898	26	55	54
7	27	54	82	09	36	63	90	18	45	53
8	26	52	78	04	30	56	82	08	34	52
9	25	50	74	3.6499	24	49	74	7.2998	23	51
10	24	47	71	94	18	42	65	89	12	50
11	22	45	67	90	12	34	57	79	02	49
12	21	42	64	85	06	27	48	70	8.2091	48
13	20	40	60	80	00	20	40	60	80	47
14	19	38	56	75	4.5594	13	32	50	69	46
15	18	35	53	70	88	06	23	41	58	45
16	16	33	49	66	82	5.4698	15	31	48	44
17	15	30	46	61	76	91	06	22	37	43
18	14	28	42	56	70	84	6.3798	12	26	42
19	13	26	38	51	64	77	90	02	15	41
20	12	23	35	46	58	70	81	7.2893	04	40
21	10	21	31	42	52	62	73	83	8.1994	39
22	09	18	28	37	46	55	64	74	83	38
23	08	16	24	32	40	48	56	64	72	37
24	07	14	20	27	34	41	48	54	61	36
25	06	11	17	22	28	34	39	45	51	35
26	04	09	13	18	22	26	31	35	40	34
27	03	06	10	13	16	19	22	26	29	33
28	02	04	06	08	10	12	14	16	18	32
29	01	02	03	03	04	05	06	06	07	31
30	00	1.8199	2.7299	3.6398	4.5498	5.4598	6.3697	7.2797	8.1896	30
31	0.9098	97	95	94	92	90	89	87	86	29
32	97	94	92	89	86	83	80	78	75	28
33	96	92	88	84	80	76	72	68	64	27
34	95	90	84	79	74	69	64	58	53	26
35	94	87	81	74	68	62	55	49	42	25
36	92	85	77	70	62	54	47	39	32	24
37	91	82	73	64	56	47	38	29	20	23
38	90	80	70	60	50	39	29	19	09	22
39	89	77	66	55	44	32	21	10	8.1798	21
40	88	75	63	50	38	25	13	00	88	20
41	86	73	59	45	32	18	04	7.2690	77	19
42	85	70	55	40	26	11	6.3596	81	66	18
43	84	67	52	36	20	08	87	71	55	17
44	83	65	48	30	13	5.4496	78	61	43	16
45	81	63	44	26	07	88	70	51	33	15
46	80	60	41	21	01	81	61	42	22	14
47	79	58	37	16	4.5395	74	53	32	11	13
48	78	56	33	11	89	67	45	22	00	12
49	77	53	30	06	83	60	36	13	8.1689	11
50	75	51	26	01	77	52	27	02	78	10
51	74	48	22	3.6296	71	45	19	7.2593	67	9
52	73	46	19	92	65	37	10	83	56	8
53	72	43	15	87	59	30	02	74	45	7
54	70	41	11	82	52	22	6.3493	63	34	6
55	69	38	08	77	46	15	84	54	23	5
56	68	36	04	72	40	08	76	44	12	4
57	67	34	00	67	34	01	68	34	01	3
58	66	31	2.7197	62	28	5.4393	59	24	8.1590	2
59	64	29	93	57	22	86	50	14	79	1
60	0.9063	1.8126	2.7189	3.6252	4.5316	5.4379	6.3442	7.2505	8.1568	0
	1	2	3	4	5	6	7	8	9	

DEPARTURE 24 DEGREES.

123

1	1	2	3	4	5	6	7	8	9	1
0	0.4067	0.8185	1.2202	1.6270	2.0387	2.4404	2.8472	3.2539	3.6607	60
1	70	40	10	80	50	20	90	60	30	59
2	73	45	18	91	64	36	2.8507	82	54	58
3	75	51	26	1.6301	77	52	27	3.2602	78	57
4	78	56	34	12	90	68	46	24	3.6702	56
5	81	61	42	22	2.0408	84	64	45	25	55
6	83	67	50	83	17	2.4500	83	66	51	54
7	86	72	58	44	80	16	2.8602	88	74	53
8	89	77	66	54	48	82	20	3.2709	97	52
9	91	83	74	65	57	48	89	80	3.6822	51
10	94	88	92	76	70	68	57	51	45	50
11	97	93	90	86	83	80	76	73	69	49
12	99	98	98	97	97	95	94	94	98	48
13	0.4102	0.8204	1.2306	1.6408	2.0510	2.4611	2.8718	3.2815	3.6917	47
14	05	09	14	18	23	27	32	36	41	46
15	07	14	22	29	36	43	50	58	65	45
16	10	20	29	39	49	59	69	78	88	44
17	13	25	38	50	63	75	88	3.2900	3.7018	43
18	15	30	45	60	76	91	2.8806	21	86	42
19	18	36	53	71	89	2.4707	25	42	60	41
20	20	41	61	82	2.0602	22	43	63	84	40
21	23	46	69	92	16	39	62	85	3.7108	39
22	26	51	77	1.8503	29	54	80	3.3006	31	38
23	28	57	85	14	42	70	99	27	56	37
24	31	62	93	24	55	86	2.8917	48	79	36
25	34	67	1.2401	35	69	2.4802	86	70	3.7208	35
26	36	73	09	45	82	18	54	90	27	34
27	39	78	17	56	95	34	73	3.3112	51	33
28	42	83	25	66	2.0708	50	91	33	74	32
29	44	89	33	77	22	66	2.9010	54	99	31
30	47	94	41	88	35	81	28	75	3.7322	30
31	50	99	49	98	48	98	47	97	46	29
32	52	0.8804	57	1.6609	61	2.4913	65	3.8218	70	28
33	55	10	65	20	75	29	84	39	94	27
34	58	15	73	30	88	45	2.9108	60	3.7418	26
35	60	20	81	41	2.0801	61	21	82	42	25
36	63	21	88	51	14	77	40	3.8302	65	24
37	66	31	97	62	28	93	59	24	90	23
38	68	36	1.2504	72	41	2.5009	77	45	3.7518	22
39	71	41	12	83	54	24	95	66	36	22
40	73	47	20	94	67	40	2.9214	87	61	20
41	76	52	28	1.6704	80	56	32	3.8408	84	19
42	79	57	36	15	94	72	51	30	3.7608	18
43	81	63	44	25	2.0907	88	69	50	32	17
44	84	68	52	36	20	2.5104	88	72	56	16
45	87	73	60	46	33	20	2.9306	93	79	15
46	89	78	68	57	46	35	24	3.8514	3.7708	14
47	92	84	76	68	60	51	43	35	27	13
48	95	89	84	78	73	67	62	56	51	12
49	97	94	92	89	86	83	80	78	75	11
50	0.4200	0.8400	99	99	99	99	99	98	98	10
51	02	05	1.2607	1.6810	2.1012	2.5214	2.9417	3.3619	3.7822	9
52	05	10	15	20	26	31	36	41	46	8
53	08	15	23	31	39	46	54	62	69	7
54	10	21	31	42	52	62	73	83	94	6
55	13	26	39	52	65	78	91	3.8704	3.7917	5
56	16	31	47	62	78	94	2.9509	25	40	4
57	18	37	55	73	92	2.5310	28	46	65	3
58	21	42	63	84	2.1105	25	46	67	88	2
59	24	47	71	94	18	41	65	88	3.8012	1
60	0.4226	0.8452	1.2679	1.6905	2.1181	2.5357	2.9583	3.3810	3.8036	0
	1	2	3	4	5	6	7	8	9	

LATITUDE 65 DEGREES.

1	2	3	4	5	6	7	8	9	10
0	0.4226	0.8452	1.2679	1.6905	2.1131	2.5357	2.9583	3.3810	3.8036
1	29	58	86	15	44	73	2.9602	30	59
2	32	63	95	26	58	89	21	52	84
3	34	68	1.2702	36	71	2.5405	39	73	3.8107
4	37	73	10	47	84	20	57	94	30
5	39	79	18	58	97	36	76	3.3915	55
6	42	84	26	68	2.1211	52	94	36	78
7	45	89	34	78	23	68	2.9712	57	3.8201
8	47	95	42	89	37	84	31	78	26
9	50	0.8500	50	1.7000	50	99	49	99	49
10	53	05	58	10	63	2.5515	68	3.4020	73
11	55	10	66	21	76	31	86	42	97
12	58	16	73	31	89	47	2.9805	62	3.8320
13	60	21	81	42	2.1302	62	23	83	44
14	63	26	89	52	16	79	42	3.4105	68
15	66	31	97	63	29	94	60	26	91
18	68	37	1.2805	73	42	2.5610	78	46	3.8415
17	71	42	13	84	55	25	96	67	38
18	74	47	21	94	68	42	2.9915	89	62
19	76	52	29	1.7105	81	57	33	3.4210	86
20	79	58	36	15	94	73	52	30	3.8599
21	82	63	45	26	2.1408	89	71	52	34
22	84	68	52	36	21	2.5705	89	73	57
23	87	73	60	47	34	20	3.0007	94	80
24	89	79	68	58	47	36	26	3.4315	3.8605
25	92	84	76	68	60	52	44	36	28
26	95	89	84	78	73	68	62	57	51
27	97	94	92	89	86	83	80	78	75
28	0.4300	0.8600	1.2900	1.7200	2.1500	99	99	99	99
29	03	06	08	10	13	2.5815	3.0118	3.4420	3.8723
30	05	10	15	20	26	31	36	41	46
31	08	15	23	31	39	46	54	62	69
32	10	21	31	42	52	62	73	83	94
33	13	26	39	52	65	78	91	3.4504	3.8817
34	16	31	47	62	78	94	3.0209	25	40
35	18	36	55	73	91	2.5909	27	46	64
36	21	42	63	84	2.1605	25	46	67	88
37	24	47	71	94	18	41	65	88	3.8912
38	26	52	78	1.7304	31	57	83	3.4609	35
39	29	57	86	15	44	72	3.0301	30	58
40	31	63	94	25	57	88	19	50	82
41	34	68	1.3002	36	70	2.6004	38	72	3.9006
42	37	73	10	46	83	20	56	93	29
43	39	78	18	57	96	35	74	3.4714	53
44	42	84	25	67	2.1709	51	93	34	76
45	45	89	34	78	23	67	3.0412	56	3.9101
46	47	94	41	88	36	83	30	77	24
47	50	99	49	99	49	98	48	98	47
48	52	0.8705	57	1.7409	62	2.6114	66	3.4819	71
49	55	10	65	20	75	29	84	39	94
50	58	15	73	30	88	45	3.0503	60	3.9218
51	60	20	81	41	2.1801	61	21	82	42
52	63	26	88	51	14	77	40	3.4902	65
53	65	31	96	62	27	92	58	23	89
54	68	36	1.3104	72	40	2.6208	76	44	3.9312
55	71	41	12	82	53	24	94	65	35
55	73	47	20	93	57	40	3.0213	86	60
57	76	52	28	1.7504	80	55	31	3.5007	83
58	79	57	36	14	93	71	50	28	3.9407
59	81	62	43	24	2.1906	87	68	49	30
60	0.4384	0.8767	1.3151	1.7535	2.1919	2.6302	3.0686	3.5070	3.9453
1	2	3	4	5	6	7	8	9	

	1	2	3	4	5	6	7	8	9	
0	0.8988	1.7976	2.6964	3.5952	4.4940	5.3927	6.2915	7.1903	8.0891	60
1	87	73	60	47	33	20	70	7.1894	80	59
2	85	71	56	42	27	12	6.2898	83	69	58
3	83	68	52	36	21	05	89	73	57	57
4	82	66	48	31	14	5.3897	80	62	45	56
5	81	63	45	26	08	90	71	53	34	55
6	80	61	41	21	02	82	62	42	23	54
7	79	58	37	16	4.4895	74	53	32	11	53
8	78	55	33	11	89	66	44	22	8.0799	52
9	76	53	29	06	82	58	35	11	88	51
10	75	50	26	01	76	51	26	02	77	50
11	74	48	22	3.5896	70	43	17	7.1791	65	49
12	73	45	18	90	63	36	08	81	53	48
13	71	43	14	85	57	28	6.2799	70	42	47
14	70	40	10	80	50	20	90	60	30	46
15	69	37	06	75	44	12	81	50	18	45
16	67	35	02	70	37	04	72	39	07	44
17	66	32	2.6899	65	31	5.3797	63	30	8.0696	43
18	65	30	95	60	25	89	54	19	84	42
19	64	27	91	54	18	82	45	09	72	41
20	62	25	87	49	12	74	36	7.1698	61	40
21	61	22	83	44	05	66	27	88	49	39
22	60	19	79	39	4.4799	58	18	78	37	38
23	58	17	75	34	92	50	09	67	25	37
24	57	14	71	28	86	43	00	57	14	36
25	56	12	68	23	79	35	6.2691	46	02	35
26	55	09	64	18	73	27	82	36	8.0591	34
27	53	06	60	13	66	19	72	26	79	33
28	52	04	56	08	60	11	63	15	67	32
29	51	01	52	02	53	04	54	05	55	31
30	49	1.7899	48	3.5797	47	5.3696	45	7.1594	44	30
31	48	96	44	92	40	88	36	84	32	29
32	47	93	40	87	34	80	27	74	20	28
33	45	91	36	82	27	72	18	63	09	27
34	44	88	32	76	21	65	09	53	97	26
35	43	86	28	71	14	57	00	42	8.0485	25
36	42	83	25	66	08	49	6.2591	32	74	24
37	40	80	21	61	01	41	81	22	62	23
38	39	78	17	56	4.4695	33	72	11	50	22
39	38	75	13	50	88	26	68	01	38	21
40	36	73	09	45	82	18	54	7.1490	27	20
41	35	70	05	40	75	10	45	80	15	19
42	34	67	01	35	69	02	36	70	03	18
43	32	65	2.6797	30	62	5.3594	27	59	8.0392	17
44	31	62	93	24	56	87	18	49	80	16
45	30	60	89	19	49	79	09	38	68	15
46	29	57	85	14	43	71	00	28	57	14
47	27	54	82	09	36	63	6.2490	18	45	13
48	26	52	78	04	30	55	81	07	33	12
49	25	49	74	3.5698	23	47	71	7.1396	21	11
50	23	46	70	93	16	39	62	86	09	10
51	22	44	66	88	10	32	53	75	8.0297	9
52	21	41	62	82	03	24	44	65	85	8
53	19	39	58	77	4.4597	16	35	54	74	7
54	18	36	54	72	90	08	26	44	62	6
55	17	33	50	67	84	00	17	34	50	5
56	15	31	46	61	77	5.3492	07	23	38	4
57	14	28	42	56	70	84	6.2398	12	26	3
58	13	25	38	51	64	76	89	02	14	2
59	11	23	34	46	57	68	80	7.1291	03	1
60	0.8910	1.7820	2.6780	3.5640	4.4551	5.3461	6.2371	7.1281	8.0191	0
	1	2	3	4	5	6	7	8	9	

DEPARTURE 26 DEGREES.

127

1	2	3	4	5	6	7	8	9	10
0	0.4384	0.8767	1.3151	1.7535	2.1919	2.6302	3.0686	3.5070	3.9453
1	86	73	59	45	32	18	3.0704	90	77
2	89	78	67	56	45	33	22	3.5111	3.9500
3	92	83	75	66	58	50	41	33	24
4	94	88	83	77	71	65	59	54	48
5	97	94	90	87	84	81	80	74	71
6	99	99	98	98	97	96	96	95	95
7	0.4402	0.8804	1.3206	1.7608	2.2010	2.6412	3.0814	3.5216	3.9618
8	05	09	14	18	23	28	32	37	41
9	07	14	22	29	36	43	50	58	65
10	10	20	29	39	49	59	69	78	88
11	12	25	37	50	62	74	87	99	3.9712
12	15	30	45	60	76	91	3.0906	3.5321	36
13	18	35	53	71	89	2.6506	24	42	57
14	20	41	61	81	2.2102	22	42	62	88
15	23	46	69	92	15	37	60	83	3.9806
16	26	51	77	1.7702	28	53	79	3.5404	30
17	28	56	84	12	41	69	97	25	53
18	31	61	92	23	54	84	3.1015	46	76
19	33	67	1.3300	33	67	2.6600	33	66	3.9900
20	36	72	08	44	80	15	51	87	23
21	39	77	16	54	93	31	70	3.5508	47
22	41	82	23	64	2.2206	47	88	29	70
23	44	87	31	75	19	62	3.1106	50	93
24	46	93	39	86	32	78	25	71	4.0018
25	49	98	47	96	45	94	43	92	41
26	52	0.8903	55	1.7806	58	2.6710	61	3.5613	64
27	54	08	63	17	71	25	79	34	88
28	57	14	70	27	84	41	98	54	4.0111
29	59	19	78	38	97	56	3.1216	75	85
30	62	24	86	48	2.2310	72	34	96	58
31	65	29	94	58	23	88	52	3.5717	81
32	67	34	1.3402	69	36	2.6808	70	38	4.0205
33	70	40	09	79	49	19	09	58	28
34	72	45	17	90	62	34	3.1307	79	52
35	75	50	25	1.7900	75	50	25	3.5800	75
36	78	55	33	10	88	66	43	18	98
37	80	60	41	21	2.2401	81	61	42	4.0322
38	83	66	48	31	14	97	80	62	45
39	85	71	56	42	27	2.6912	98	83	69
40	88	76	64	52	40	28	3.1416	3.5904	92
41	91	81	72	62	53	44	34	25	4.0415
42	93	86	80	73	66	59	52	46	89
43	96	92	87	83	79	75	71	66	62
44	98	97	95	94	92	90	89	87	86
45	0.4501	0.9002	1.3503	1.8004	2.2505	2.7006	3.1507	3.6008	4.0509
46	04	07	11	14	18	22	25	29	32
47	06	12	19	25	31	37	43	50	56
48	09	18	26	35	44	53	62	70	79
49	11	23	34	46	57	68	80	91	4.0603
50	14	28	42	56	70	84	98	3.6112	26
51	17	33	50	66	83	2.7100	3.1616	33	49
52	19	38	58	77	96	15	34	54	73
53	22	44	65	87	2.2609	31	53	74	96
54	24	49	73	97	22	46	70	94	4.0719
55	27	54	81	1.8108	35	61	88	3.6215	42
56	30	59	89	18	48	77	3.1707	26	66
57	32	64	96	28	61	93	25	57	89
58	35	69	1.3604	39	74	2.7208	43	78	4.0812
59	37	75	12	49	87	24	61	98	86
60	0.4510	0.9080	1.3620	1.8160	2.2700	2.7239	3.1779	3.6319	4.0859
1	2	3	4	5	6	7	8	9	10

LATITUDE 68 DEGREES.

	1	2	3	4	5	6	7	8	9
0	0.8910	1.7820	2.6730	3.5640	4.4551	5.3461	6.2371	7.1281	8.0191
1	09	17	26	35	44	52	61	70	78.59
2	07	15	22	30	37	44	52	59	67.58
3	06	12	18	24	31	37	43	49	55.57
4	05	10	14	19	24	29	34	38	43.56
5	04	07	10	14	18	21	25	28	32.55
6	02	04	06	08	11	13	15	17	19.54
7	01	02	2.6602	03	04	05	06	06	07.53
8	00	1.7799	99	3.5598	4.4498	5.3397	6.2297	7.1196	8.0096.52
9	0.8898	96	94	92	91	89	87	85	83.51
10	97	94	90	87	84	81	78	74	71.50
11	96	91	87	82	78	73	69	64	60.49
12	94	88	83	77	71	65	60	54	48.48
13	93	86	78	71	64	57	50	42	35.47
14	92	83	75	66	58	49	41	32	24.46
15	90	80	71	61	51	41	31	22	12.45
16	89	78	66	55	44	33	22	10	7.0999.44
17	88	75	63	50	38	25	13	01	88.43
18	86	72	59	45	31	17	03	7.1090	76.42
19	85	70	54	39	24	09	6.2194	78	63.41
20	84	67	51	34	18	01	85	68	52.40
21	82	64	47	29	11	5.3293	75	58	40.39
22	81	62	42	23	04	85	66	46	27.38
23	80	59	39	18	4.4398	77	57	36	16.37
24	78	56	35	13	91	69	47	26	04.36
25	77	54	30	07	84	61	38	14	7.9891.35
26	76	51	27	02	78	53	29	04	80.34
27	74	48	22	3.5496	71	45	19	7.0993	67.33
28	73	46	18	91	64	37	10	82	55.32
29	72	43	15	86	58	29	01	72	44.31
30	70	40	10	80	51	21	6.2091	61	31.30
31	69	38	06	75	44	13	82	50	19.29
32	67	35	02	70	37	04	72	39	07.28
33	66	32	2.6598	64	31	5.3197	63	29	7.9795.27
34	65	29	94	59	24	88	53	18	82.26
35	63	27	90	54	17	80	44	07	71.25
36	62	24	86	48	10	72	34	7.0896	58.24
37	61	21	82	43	04	64	25	86	46.23
38	59	19	78	37	4.4297	56	15	74	34.22
39	58	16	74	32	90	48	06	64	22.21
40	57	13	70	26	83	40	6.1996	53	09.20
41	55	11	66	21	77	32	87	42	7.9698.19
42	54	08	62	16	70	23	77	31	85.18
43	53	05	58	10	63	16	68	21	73.17
44	51	02	54	05	56	07	58	10	61.16
45	50	00	50	00	50	5.3099	49	7.0799	49.15
46	49	1.7697	46	3.5394	43	91	40	88	37.14
47	47	94	42	89	36	83	30	78	25.13
48	46	92	37	83	29	75	21	66	12.12
49	45	89	34	78	23	67	12	56	01.11
50	43	86	29	72	16	59	02	45	7.9588.10
51	42	83	25	67	09	50	6.1892	34	75.9
52	40	81	21	62	02	42	83	23	64.8
53	39	78	17	56	4.4195	34	73	12	51.7
54	38	75	13	51	89	26	64	02	39.6
55	36	73	09	45	82	18	54	7.0690	27.5
56	35	70	05	40	75	09	44	79	14.4
57	34	67	01	34	68	02	35	69	02.3
58	32	64	2.6497	29	61	5.2993	25	58	7.9490.2
59	31	62	92	23	54	85	16	46	77.1
60	0.8830	1.7650	2.6489	3.5318	4.4148	5.2977	6.1807	7.0636	7.9466.0
	1	2	3	4	5	6	7	8	9

DEPARTURE 27 DEGREES.										129
1	2	3	4	5	6	7	8	9	10	
0	0.4540	0.9080	1.3620	1.8160	2.2700	2.7239	3.1779	3.6319	4.0859	60
1	43	85	28	70	13	55	98	40	83	59
2	45	90	35	80	26	71	3.1816	61	4.0906	58
3	48	95	43	91	39	86	34	82	29	57
4	50	0.9101	51	1.8201	52	2.7302	52	3.6402	53	56
5	53	06	59	12	65	17	70	23	76	55
6	55	11	66	22	77	32	88	43	99	54
7	58	16	74	32	90	48	3.1906	64	4.1022	53
8	61	21	82	42	2.2803	64	24	85	45	52
9	63	26	90	53	16	79	42	3.6506	69	51
10	66	32	97	63	29	95	61	26	92	50
11	68	37	1.3705	74	42	2.7410	79	47	4.1116	49
12	71	42	13	84	55	26	97	68	39	48
13	74	47	21	94	68	42	3.2015	89	62	47
14	76	52	29	1.8305	81	57	33	3.6610	86	46
15	79	57	36	15	94	72	51	30	4.1208	45
16	81	63	44	25	2.2907	88	69	50	32	44
17	84	68	52	36	20	2.7503	87	71	55	43
18	87	73	60	46	33	19	3.2105	92	79	42
19	89	78	67	56	46	35	24	3.6713	4.1302	41
20	92	83	75	67	59	50	42	34	25	40
21	94	88	83	77	71	65	59	54	48	39
22	97	94	90	87	84	81	78	74	71	38
23	99	99	98	98	97	96	96	95	95	37
24	0.4602	0.9204	1.3806	1.8408	2.3010	2.7612	3.2214	3.6816	4.1418	36
25	05	09	14	18	23	28	32	37	41	35
26	07	14	22	29	36	43	50	58	65	34
27	10	19	29	39	49	58	68	78	87	33
28	12	25	37	49	62	74	86	98	4.1511	32
29	15	30	45	60	75	89	3.2304	3.6919	34	31
30	18	35	53	70	88	2.7705	23	40	58	30
31	20	40	60	80	2.3101	21	41	61	81	29
32	23	45	68	90	13	36	58	81	4.1603	28
33	25	50	76	1.8501	26	51	76	3.7002	27	27
34	28	56	83	11	39	67	95	22	50	26
35	30	61	91	22	52	82	3.2413	43	74	25
36	33	66	99	32	65	98	31	64	97	24
37	36	71	1.3907	42	78	2.7813	49	84	4.1720	23
38	38	76	14	52	91	29	67	3.7105	43	22
39	41	81	22	63	2.3204	44	85	26	66	21
40	43	87	30	73	17	60	3.2503	46	91	20
41	46	92	37	83	29	75	21	66	4.1812	19
42	48	97	45	94	42	90	39	87	36	18
43	51	0.9302	53	1.8604	55	2.7906	57	3.7208	59	17
44	54	07	61	14	68	22	75	29	82	16
45	56	12	68	24	81	37	93	49	4.1905	15
46	59	17	76	35	94	52	3.2611	70	28	14
47	61	23	84	45	2.3307	68	29	90	52	13
48	64	28	92	56	20	83	47	3.7311	75	12
49	66	33	99	66	32	98	65	31	98	11
50	69	38	1.4007	76	45	2.8014	83	52	4.2021	10
51	72	43	15	86	58	30	3.2701	73	44	9
52	74	48	23	97	71	45	19	94	68	8
53	77	53	30	1.8707	84	60	37	3.7414	90	7
54	79	59	38	17	97	76	55	34	4.2114	6
55	82	64	46	28	2.3410	91	73	55	37	5
56	84	69	53	38	22	2.8106	91	75	60	4
57	87	74	61	48	35	22	3.2809	96	83	3
58	90	79	70	58	48	38	27	3.7517	4.2206	2
59	92	84	76	68	61	53	45	37	29	1
60	0.4695	0.9389	1.4084	1.8779	2.3474	2.8168	3.2863	3.7558	4.2252	0
1	2	3	4	5	6	7	8	9		
LATITUDE 62 DEGREES.										

0

LATITUDE 28 DEGREES.

1	2	3	4	5	6	7	8	9	10
0.8830	1.7659	2.6489	3.5318	4.4148	5.2977	6.1807	7.0636	7.9466	60
28	56	84	12	41	69	6.1797	25	58	59
27	58	80	07	84	60	87	14	40	58
25	51	76	02	27	52	78	03	29	57
24	48	72	3.5296	20	44	68	7.0592	16	56
23	45	68	90	13	86	58	81	03	55
21	43	64	85	07	28	49	70	7.9391	54
20	40	60	80	00	19	39	59	79	53
19	37	56	74	4.4098	11	30	48	67	52
17	34	52	69	86	03	21	38	55	51
16	32	47	63	79	5.2895	11	26	42	50
14	29	43	58	72	86	01	15	30	49
13	26	39	52	65	78	6.1691	04	17	48
12	23	35	47	59	70	82	7.0494	05	47
10	21	31	41	52	62	72	82	7.9293	46
09	18	27	36	45	53	62	71	80	45
08	15	23	30	38	45	53	60	68	44
06	12	19	25	31	37	43	50	56	43
05	10	14	19	24	29	34	38	43	42
03	07	10	14	17	20	24	27	31	41
02	04	06	08	10	12	14	16	18	40
01	01	02	02	03	04	04	05	05	39
0.8799	1.7599	2.6398	3.5197	4.3997	5.2796	6.1595	7.0394	7.9194	38
98	96	94	92	90	87	85	83	81	37
97	93	90	86	83	79	76	72	69	36
95	90	85	80	76	70	66	61	56	35
94	87	81	75	69	62	56	50	43	34
92	85	77	69	63	53	46	38	31	33
91	82	73	64	55	45	36	27	18	32
90	79	69	58	48	38	27	17	06	31
88	76	65	53	41	29	17	06	7.9094	30
87	74	60	47	34	21	08	7.0294	81	29
85	71	56	42	27	12	6.1498	83	69	28
84	68	52	36	20	04	88	72	56	27
83	65	48	30	13	5.2696	78	61	43	26
81	62	44	25	06	87	68	50	31	25
80	60	39	19	00	79	59	38	18	24
78	57	35	14	4.8892	70	49	27	06	23
77	54	31	08	85	62	39	16	7.8993	22
76	52	27	02	78	54	29	05	80	21
74	49	23	3.5097	72	46	20	7.0194	69	20
73	46	19	92	65	37	10	83	56	19
72	43	15	86	58	29	01	72	44	18
70	40	10	80	51	21	6.1391	61	31	17
69	37	06	75	44	12	81	50	18	16
67	35	02	69	37	04	71	38	06	15
66	32	2.6298	64	30	5.2595	61	27	7.8893	14
65	29	94	58	23	87	52	16	81	13
63	26	89	52	16	79	41	05	68	12
62	23	85	47	09	70	32	7.0094	55	11
60	21	81	41	02	62	22	82	43	10
59	18	77	36	4.3795	53	12	71	30	9
58	15	73	30	88	45	03	60	18	8
56	12	68	24	81	37	6.1293	49	05	7
55	09	64	18	73	28	82	37	7.8791	6
53	06	60	13	66	19	72	26	79	5
52	04	55	07	59	11	63	14	66	4
50	01	51	02	52	02	53	08	54	3
49	98	47	3.4996	45	5.2494	43	6.9992	41	2
48	1.7495	43	90	38	86	33	81	28	1
0.8746	1.7492	2.6239	3.4985	4.3731	5.2477	6.1223	6.9970	7.8716	0
1	2	3	4	5	6	7	8	9	10

DEPARTURE 61 DEGREES.

DEPARTURE 28 DEGREES.										181
1	2	3	4	5	6	7	8	9	10	11
0	0.4695	0.9389	1.4084	1.8779	2.3474	2.8168	3.2863	3.7558	4.2252	60
1	97	95	92	89	87	84	81	78	76	59
2	0.4700	0.9400	1.4100	1.8800	2.3500	2.8200	3.2900	3.7600	4.2300	58
3	02	05	07	10	12	14	17	20	23	57
4	05	10	15	20	25	30	35	40	45	56
5	08	15	23	30	38	46	53	61	68	55
6	10	20	30	40	51	61	71	81	91	54
7	13	25	38	51	64	76	89	102	115	53
8	15	31	46	61	77	92	107	122	137	52
9	18	36	53	71	89	107	125	143	161	51
10	20	41	61	82	102	122	143	163	183	50
11	23	46	69	92	115	137	160	183	206	49
12	26	51	77	102	128	153	179	204	229	48
13	28	56	84	112	141	169	197	225	253	47
14	31	61	92	122	153	184	214	245	276	46
15	33	66	100	133	166	199	232	265	298	45
16	36	72	107	143	179	214	249	284	319	44
17	38	77	115	153	192	230	268	306	344	43
18	41	82	123	164	207	245	286	327	368	42
19	43	87	130	174	217	256	298	341	384	41
20	46	92	138	184	227	266	310	354	398	40
21	49	97	146	194	237	276	320	365	410	39
22	51	0.9502	153	1.9004	247	287	332	378	423	38
23	54	07	161	15	69	22	76	30	83	37
24	56	12	169	25	81	37	93	50	106	36
25	59	18	176	35	94	53	106	70	120	35
26	61	23	184	46	107	68	120	91	134	34
27	64	28	192	56	120	83	135	106	149	33
28	67	33	1.4300	66	133	99	150	122	163	32
29	69	38	07	76	145	114	166	137	177	31
30	72	43	15	86	158	130	181	150	190	30
31	74	48	22	96	171	145	197	166	206	29
32	77	53	30	1.0107	184	160	214	183	222	28
33	79	59	38	17	197	176	230	197	238	27
34	82	64	45	27	2.3909	191	247	214	256	26
35	84	69	53	38	22	2.8706	214	230	277	25
36	87	74	61	48	35	21	3.3508	255	306	24
37	90	79	69	58	48	37	27	3.8316	331	23
38	92	84	76	68	60	52	44	36	368	22
39	95	89	84	78	73	68	62	57	405	21
40	97	94	91	88	86	83	80	77	442	20
41	0.4800	99	99	99	99	98	98	98	97	19
42	02	0.9604	1.4407	1.9209	2.4011	2.8813	3.3615	3.8418	4.3220	18
43	05	10	14	19	24	29	34	38	43	17
44	07	15	22	29	37	44	51	58	66	16
45	10	20	30	40	50	59	69	79	89	15
46	12	25	37	50	62	74	87	99	112	14
47	15	30	45	60	75	90	105	120	135	13
48	18	35	53	70	88	105	123	140	158	12
49	20	40	60	80	101	121	141	161	181	11
50	22	45	68	90	113	136	159	183	206	10
51	25	50	76	1.9301	126	151	176	201	226	9
52	28	55	83	11	39	66	94	122	150	8
53	30	61	91	21	52	82	112	142	172	7
54	33	66	98	31	64	97	128	159	190	6
55	35	71	1.4506	42	77	112	148	183	218	5
56	38	76	14	52	90	127	165	203	241	4
57	41	81	22	62	103	143	184	225	266	3
58	43	86	29	72	116	158	201	244	287	2
59	46	91	37	82	128	174	219	265	311	1
60	0.4848	0.9696	1.4544	1.9392	2.4241	2.9089	3.3937	3.8785	4.3633	0
1	2	3	4	5	6	7	8	9		
LATITUDE 61 DEGREES.										

	1	2	3	4	5	6	7	8	9	10
0	0.8746	1.7492	2.6239	3.4985	4.3731	5.2477	6.1223	6.9970	7.8716	8.7460
1	45	90	34	79	24	69	14	58	03	59
2	43	87	30	74	17	60	04	47	7.8691	58
3	42	84	26	68	10	52	6.1194	36	78	57
4	41	81	22	62	03	44	84	25	65	56
5	39	78	17	56	4.3696	35	74	13	52	55
6	38	75	13	51	89	26	64	02	39	54
7	36	73	09	45	82	18	54	6.9890	27	53
8	35	70	05	40	75	09	44	79	14	52
9	33	67	01	34	68	01	35	68	02	51
10	32	64	2.6196	28	61	5.2393	25	57	7.8589	50
11	31	61	92	22	53	84	14	45	75	49
12	29	58	88	17	46	75	04	34	63	48
13	28	56	83	11	39	67	6.1095	22	50	47
14	26	53	79	06	32	58	85	11	38	46
15	25	50	75	00	25	50	75	6.9700	25	45
16	24	47	71	3.4894	18	41	65	88	12	44
17	22	44	66	88	11	33	55	77	7.8499	43
18	21	41	62	83	04	24	45	66	86	42
19	19	39	58	77	4.3597	16	35	54	74	41
20	18	36	53	71	89	07	25	42	60	40
21	16	33	49	66	82	5.2298	15	31	48	39
22	15	30	45	60	75	90	05	20	35	38
23	14	27	41	54	68	82	6.0995	09	22	37
24	12	24	36	48	61	73	85	6.9697	09	36
25	11	21	32	43	54	64	75	86	7.8396	35
26	09	19	28	37	47	55	65	74	84	34
27	08	16	24	32	40	47	55	63	71	33
28	06	13	19	26	32	38	45	51	58	32
29	05	10	15	20	25	30	35	40	45	31
30	04	07	11	14	18	22	25	29	32	30
31	02	04	06	08	11	13	15	17	19	29
32	01	01	02	03	04	04	05	06	06	28
33	0.8699	1.7399	2.6098	3.4797	4.3497	5.2196	6.0895	6.9594	7.8294	27
34	98	96	93	91	89	87	85	82	80	26
35	96	93	89	86	82	78	75	71	68	25
36	95	90	85	80	75	69	64	59	54	24
37	94	87	81	74	68	61	55	48	42	23
38	92	84	76	68	61	53	45	37	29	22
39	91	81	72	62	53	44	34	25	16	21
40	89	78	68	57	46	35	24	14	03	20
41	88	76	63	51	39	27	15	02	7.8190	19
42	86	73	59	45	32	18	04	6.9490	77	18
43	85	70	55	40	25	09	6.0794	79	64	17
44	83	67	50	34	17	00	84	67	51	16
45	82	64	46	28	10	5.2092	74	56	38	15
46	81	61	42	22	03	83	64	44	25	14
47	79	58	37	16	4.3396	75	54	33	12	13
48	78	55	33	11	89	66	44	22	7.8099	12
49	76	52	29	05	81	57	33	20	86	11
50	75	50	24	3.4699	74	49	24	6.9398	73	10
51	73	47	20	93	67	40	13	86	60	9
52	72	44	16	88	60	31	03	75	47	8
53	70	41	11	82	52	22	6.0693	63	34	7
54	69	38	07	76	45	14	83	52	21	6
55	68	35	03	70	38	05	73	40	08	5
56	66	32	2.5998	64	31	5.1997	63	29	7.7995	4
57	65	29	94	58	23	88	52	17	81	3
58	63	26	90	53	16	79	42	06	69	2
59	62	23	85	47	09	70	32	94	55	1
60	0.8660	1.7321	2.5981	3.4641	4.3302	5.1962	6.0622	6.9282	7.7943	0
	1	2	3	4	5	6	7	8	9	

DEPARTURE 60 DEGREES.

DEPARTURE 29 DEGREES.

133

	1	2	3	4	5	6	7	8	9	10
0	0.4848	0.9696	1.4544	1.9392	2.4241	2.9089	3.3937	3.8785	4.3633	60
1	51	0.9701	52	1.9402	53	2.9104	54	3.8805	55	59
2	53	06	60	18	66	19	72	26	79	58
3	56	11	67	23	79	84	90	46	4.8701	57
4	58	17	75	33	92	50	3.4008	66	25	56
5	61	22	82	43	2.4804	65	26	86	47	55
6	63	27	90	54	17	80	44	3.8907	71	54
7	66	32	98	64	30	95	61	27	93	53
8	68	37	1.4605	74	42	2.9210	79	47	4.8816	52
9	71	42	13	84	55	26	97	68	39	51
10	74	47	21	94	68	41	3.4115	88	62	50
11	76	52	28	1.9504	81	57	33	3.9009	85	49
12	79	57	37	14	98	72	50	29	4.8907	48
13	81	62	43	24	2.4406	87	68	49	30	47
14	84	67	51	35	19	2.9302	86	70	53	46
15	86	72	59	45	31	17	3.4203	90	76	45
16	89	78	66	55	44	38	22	3.9110	99	44
17	91	83	74	65	57	48	39	30	4.4022	43
18	94	88	81	75	69	63	57	50	44	42
19	96	93	89	86	82	78	75	71	68	41
20	99	98	97	96	95	93	92	91	90	40
21	0.4901	0.9803	1.4704	1.9606	2.4507	2.9408	3.4310	3.9211	4.4113	39
22	04	08	12	16	20	24	28	32	36	38
23	07	13	20	26	33	39	46	52	59	37
24	09	18	27	36	45	54	63	72	81	36
25	12	23	35	46	58	70	81	93	4.4204	35
26	14	28	42	56	71	85	99	3.9313	27	34
27	17	33	50	66	83	3.4416	80	38	49	33
28	19	38	58	77	96	2.9515	34	54	78	32
29	22	43	65	87	2.4609	30	52	74	96	31
30	24	48	73	97	21	45	69	94	4.4318	30
31	27	54	80	1.9707	84	61	88	3.9414	41	29
32	29	59	88	17	47	76	3.4505	34	64	28
33	32	64	95	27	59	91	23	54	86	27
34	34	69	1.4803	38	72	2.9606	41	75	4.4410	26
35	37	74	11	48	85	21	58	95	82	25
36	39	79	18	58	97	36	76	3.9515	55	24
37	42	84	26	68	2.4710	51	93	35	77	23
38	45	89	34	78	23	67	3.4612	56	4.4501	22
39	47	94	41	88	35	82	29	76	23	22
40	50	99	49	98	48	97	27	96	46	20
41	52	0.9904	56	1.9808	61	2.9713	65	3.9617	69	19
42	55	09	64	18	73	28	82	37	91	18
43	57	14	71	28	86	43	3.4700	57	4.4614	17
44	60	19	79	38	98	58	17	77	86	16
45	62	24	87	49	2.4811	73	35	98	60	15
46	65	29	94	59	24	88	53	3.9718	82	14
47	67	34	1.4902	69	36	2.9803	70	38	4.4705	13
48	70	39	09	79	49	18	88	58	27	12
49	72	45	17	89	62	34	3.4806	78	51	11
50	75	50	24	99	74	49	24	98	73	10
51	77	55	32	1.9909	87	64	41	3.9818	96	9
52	80	60	39	19	99	79	59	38	4.4818	8
53	82	65	47	30	2.4912	94	77	59	42	7
54	85	70	55	40	25	2.9909	94	79	64	6
55	87	75	62	50	37	24	3.4912	99	87	5
56	90	80	70	60	50	39	29	3.9919	4.4909	4
57	92	85	77	70	62	54	47	39	32	3
58	95	90	85	80	75	70	65	60	55	2
59	98	95	93	90	88	85	83	80	78	1
60	0.5000	1.0000	1.5000	2.0000	2.5000	3.0000	3.5000	4.0000	4.5000	0
	1	2	3	4	5	6	7	8	9	

LATITUDE 60 DEGREES.

	1	2	3	4	5	6	7	8	9	10
0	0.8660	1.7321	2.5981	3.4641	4.3302	5.1962	6.0622	6.9282	7.7943	8.6603
1	59	18	76	35	4.3294	53	12	70	29	59
2	57	15	72	29	87	44	01	58	16	58
3	56	12	68	24	80	35	6.0591	47	03	57
4	54	09	63	18	72	26	81	35	7.7890	56
5	53	06	59	12	65	18	71	24	77	55
6	52	03	55	06	58	09	61	12	64	54
7	50	00	50	00	51	01	51	01	51	53
8	49	1.7297	46	3.4594	43	5.1892	40	6.9189	37	52
9	47	94	41	88	36	83	30	77	24	51
10	46	91	37	83	29	74	20	66	11	50
11	44	88	33	77	21	65	09	54	7.7798	49
12	43	85	28	71	14	56	6.0499	42	84	48
13	41	83	24	65	07	48	89	30	72	47
14	40	80	19	59	4.3199	39	79	18	58	46
15	38	77	15	54	92	30	69	07	46	45
16	37	74	11	48	85	21	58	6.9095	32	44
17	35	71	06	42	77	12	48	83	19	43
18	34	68	02	36	70	04	38	72	06	42
19	33	65	2.5898	30	63	5.1795	28	60	7.7693	41
20	31	62	93	24	55	80	17	48	79	40
21	30	59	89	18	48	77	07	86	66	39
22	28	56	81	12	41	69	6.0397	25	53	38
23	27	53	80	06	33	60	86	13	39	37
24	25	50	75	00	26	51	76	01	26	36
25	24	47	71	3.4495	19	42	66	6.8990	13	35
26	22	44	67	89	11	33	55	78	00	34
27	21	41	62	83	04	24	45	66	7.7596	33
28	19	38	58	77	4.3096	15	34	54	73	32
29	18	36	53	71	89	07	25	42	60	31
30	16	33	49	65	82	5.1698	14	30	47	30
31	15	30	44	59	74	89	04	18	33	29
32	13	27	40	53	67	80	6.0293	06	20	28
33	12	24	36	48	60	71	83	6.8895	07	27
34	10	21	31	42	52	62	73	83	7.7494	26
35	09	18	27	36	45	53	62	71	80	25
36	07	15	22	30	37	44	52	59	67	24
37	06	12	18	24	30	35	41	47	53	23
38	05	09	14	18	23	27	32	38	41	22
39	03	06	09	12	15	18	21	24	27	21
40	02	03	05	06	08	09	11	12	14	20
41	00	00	00	00	00	00	00	00	00	19
42	0.8599	1.7197	2.5796	3.4394	4.2993	5.1591	6.0190	6.8788	7.7387	8.6018
43	97	94	91	88	85	82	79	76	73	17
44	96	91	87	82	78	74	69	65	60	16
45	94	88	82	76	71	65	59	53	47	15
46	93	85	78	70	63	56	48	41	33	14
47	91	82	73	64	56	47	37	29	20	13
48	90	79	69	58	48	38	27	17	06	12
49	88	76	64	52	41	29	17	05	7.7298	11
50	87	73	60	46	33	20	06	6.8693	79	10
51	85	70	55	40	25	11	6.0096	81	66	9
52	84	67	51	34	18	02	85	69	52	8
53	82	64	46	28	10	5.1493	75	57	39	7
54	81	61	42	22	03	84	64	45	25	6
55	79	58	38	17	4.2895	75	54	34	13	5
56	78	55	33	11	88	66	44	22	7.7199	4
57	76	52	29	05	81	57	33	10	86	3
58	75	49	24	3.4299	74	48	23	98	72	2
59	73	46	20	93	66	39	12	6.8586	59	1
60	0.8572	1.7143	2.5715	3.4287	4.2859	5.1430	6.0002	6.8574	7.7145	8.6000

DEPARTURE 30 DEGREES.

135

1	1	2	3	4	5	6	7	8	9	1
0	0.5000	1.0000	1.5000	2.0000	2.5000	3.0000	3.5000	4.0000	4.5000	60
1	03	05	08	10	13	15	18	20	23	59
2	05	11	15	21	25	30	35	40	45	58
3	08	15	23	30	38	46	53	61	68	57
4	10	20	30	40	51	61	71	81	91	56
5	13	25	38	50	63	76	88	4.0101	4.5118	55
6	15	30	45	60	76	91	3.5106	21	36	54
7	18	35	53	70	88	3.0106	23	41	58	53
8	20	40	60	80	2.5101	21	41	61	81	52
9	23	45	68	91	14	36	56	82	4.5204	51
10	25	50	76	2.0101	26	51	76	4.0202	27	50
11	28	55	83	11	39	66	94	22	49	49
12	30	60	91	21	51	81	3.5211	42	72	48
13	33	65	98	31	64	96	29	62	94	47
14	35	70	1.5106	41	76	3.0211	46	82	4.5817	46
15	38	75	13	51	89	26	64	4.0802	39	45
16	40	81	21	61	2.5202	42	82	22	63	44
17	43	86	28	71	14	57	00	42	85	43
18	45	91	36	81	27	72	3.5317	62	4.5408	42
19	48	96	43	91	39	87	35	82	30	41
20	50	1.0101	51	2.0201	52	3.0303	52	4.0402	53	40
21	53	06	58	11	64	17	70	22	75	39
22	55	11	66	21	77	32	87	42	98	38
23	58	16	73	31	89	47	3.5407	62	4.5520	37
24	60	21	81	41	2.5802	62	22	82	43	36
25	63	26	88	51	14	75	40	4.0502	65	35
26	65	31	96	62	27	92	58	23	89	34
27	68	36	1.5204	72	40	3.0408	75	43	4.5611	33
28	70	41	11	82	52	22	98	63	34	32
29	73	46	19	92	65	87	3.5503	88	56	31
30	75	51	26	2.0302	77	52	28	4.0603	79	30
31	78	56	34	12	90	67	45	23	4.5701	29
32	80	61	41	22	2.5402	82	63	43	24	28
33	83	66	49	32	15	97	80	63	46	27
34	85	71	56	42	27	3.0512	98	88	69	26
35	88	76	64	52	40	27	3.5615	4.0703	91	25
36	90	81	71	62	52	42	33	23	4.5814	24
37	93	86	79	72	65	57	50	43	36	23
38	95	91	86	82	77	72	68	63	59	22
39	98	96	94	92	90	87	85	88	81	21
40	0.5100	1.0201	1.5301	2.0402	2.5502	3.0602	3.5703	4.0803	4.5904	20
41	03	08	09	12	15	17	20	23	26	19
42	05	11	16	22	27	32	35	43	49	18
43	08	16	24	32	40	47	55	63	71	17
44	10	21	31	42	52	62	73	83	94	16
45	13	26	39	52	65	77	90	4.0903	4.6016	15
46	15	31	46	62	77	92	3.5808	23	39	14
47	18	36	54	72	90	3.0707	25	43	61	13
48	20	41	61	82	2.5602	22	43	63	84	12
49	23	46	69	92	15	37	60	83	4.6106	11
50	25	51	76	2.0502	27	52	78	4.1003	29	10
51	28	56	84	12	40	67	95	28	51	9
52	30	61	91	22	52	82	3.5913	48	74	8
53	33	66	99	32	65	97	80	63	96	7
54	35	71	1.5406	42	77	3.0812	48	83	4.6219	6
55	38	76	14	52	90	87	65	4.1103	41	5
56	40	81	21	62	2.5702	42	83	23	64	4
57	43	86	29	72	15	57	3.6000	43	86	3
58	45	91	36	82	27	72	18	63	4.6309	2
59	48	96	44	92	40	87	35	88	31	1
60	0.5150	1.0301	1.5451	2.0602	2.5752	3.0902	3.6053	4.1203	4.6354	0
	1	2	3	4	5	6	7	8	9	

LATITUDE 59 DEGREES.

	1	2	3	4	5	6	7	8	9	10
0	0.8572	1.7143	2.5715	3.4287	4.2859	5.1430	6.0002	6.8574	7.7145	8.5716
1	70	40	11	81	51	21	5.9991	62	32	59
2	69	37	06	75	44	12	81	50	18	58
3	67	34	02	69	36	03	70	38	05	57
4	66	31	2.5697	63	29	5.1394	60	26	7.7091	56
5	64	28	93	57	21	85	49	14	78	55
6	63	25	88	51	14	76	39	02	64	54
7	61	22	84	45	06	67	28	6.8490	51	53
8	60	19	79	39	4.2799	58	18	78	37	52
9	58	16	75	33	91	49	07	66	24	51
10	57	13	70	27	84	40	5.9897	54	10	50
11	55	10	65	20	76	31	86	41	7.6996	49
12	54	07	61	14	68	22	75	29	82	48
13	52	04	56	08	61	13	65	17	69	47
14	51	01	52	02	53	04	54	05	55	46
15	49	1.7098	47	3.4196	46	5.1295	44	6.8393	42	45
16	48	95	43	90	38	86	33	81	28	44
17	46	92	38	84	31	77	23	69	15	43
18	45	89	34	78	23	68	12	57	01	42
19	43	86	29	72	16	59	02	45	7.6888	41
20	42	83	25	66	08	50	5.9791	33	74	40
21	40	80	20	60	01	41	81	21	61	39
22	39	77	16	54	4.2693	31	70	08	47	38
23	37	74	11	48	85	22	59	6.8296	33	37
24	36	71	07	42	78	13	49	84	20	36
25	34	68	02	36	70	04	38	72	06	35
26	33	65	2.5598	30	63	5.1195	28	60	7.6793	34
27	31	62	93	24	55	86	17	48	79	33
28	29	59	88	18	47	76	06	35	65	32
29	28	56	84	12	40	67	5.9695	23	51	31
30	26	53	79	06	32	58	85	11	38	30
31	25	50	75	00	25	49	74	6.8199	24	29
32	23	47	70	3.4094	17	40	64	87	11	28
33	22	44	65	87	09	31	53	74	7.6696	27
34	20	41	61	81	02	22	42	62	83	26
35	19	38	56	75	4.2594	13	32	50	69	25
36	17	35	52	69	87	04	21	38	56	24
37	16	31	47	63	79	5.1094	10	26	41	23
38	14	28	43	57	71	85	5.9599	14	28	22
39	13	25	38	51	64	76	89	02	14	21
40	11	22	34	45	56	67	78	6.8090	01	20
41	10	19	29	38	48	58	67	77	7.6586	19
42	08	16	24	32	41	49	57	65	73	18
43	07	13	20	26	33	40	46	53	59	17
44	05	10	15	20	26	31	36	41	46	16
45	04	07	11	14	18	21	25	28	32	15
46	02	04	06	08	10	12	14	16	18	14
47	01	01	02	02	03	03	04	04	05	13
48	0.8499	1.6998	2.5497	3.3996	4.2495	5.0993	5.9492	6.7991	7.6490	12
49	97	95	92	90	87	84	82	79	77	11
50	96	92	88	84	80	75	71	67	63	10
51	94	89	83	77	72	66	60	54	49	9
52	93	86	78	71	64	57	50	42	35	8
53	91	83	74	65	57	48	39	30	22	7
54	90	79	69	59	49	38	28	18	07	6
55	88	76	65	53	41	29	17	06	7.6394	5
56	87	73	60	46	33	20	06	6.7893	79	4
57	85	70	55	40	26	11	5.9396	81	66	3
58	84	67	51	34	18	02	85	69	52	2
59	82	64	46	28	10	5.0892	74	56	38	1
60	0.8481	1.6961	2.5442	3.3922	4.2403	5.0883	5.9364	6.7844	7.6325	0

DEPARTURE 31 DEGREES.

137

	1	2	3	4	5	6	7	8	9	
0	0.5150	1.0801	1.5451	2.0602	2.5752	3.0902	3.6053	4.1203	4.6354	60
1	58	06	59	12	65	17	70	23	76	59
2	55	11	66	22	77	32	88	43	99	58
3	58	16	74	32	90	47	3.6105	63	4.6421	57
4	60	21	81	42	2.5802	62	23	88	44	56
5	68	26	88	51	14	77	40	4.1802	65	55
6	65	31	96	61	27	92	57	22	88	54
7	68	36	1.5503	71	39	3.1007	74	42	4.6510	58
8	70	41	11	81	52	22	92	62	33	52
9	73	46	18	91	64	37	3.6209	82	55	51
10	75	51	26	2.0701	77	52	27	4.1402	78	60
11	78	56	33	11	89	67	44	22	9.6600	49
12	80	61	41	21	2.5902	82	62	42	23	48
13	83	66	48	31	14	97	80	62	45	47
14	85	70	56	41	26	3.1111	96	82	67	46
15	88	75	63	51	39	26	3.6314	4.1502	89	45
16	90	80	71	61	51	41	81	22	4.6712	44
17	98	85	78	71	64	56	49	42	84	48
18	95	90	86	81	76	71	66	62	57	42
19	98	95	93	91	89	86	84	82	79	41
20	0.5200	00	1.5601	2.0801	2.6001	3.1201	3.6401	4.1602	4.6802	40
21	03	1.0405	08	10	13	16	18	21	23	39
22	05	10	15	20	26	31	36	41	46	88
23	08	15	23	30	38	46	53	61	68	87
24	10	20	30	40	51	61	71	81	91	86
25	13	25	08	50	63	76	88	4.1701	4.6913	35
26	15	30	45	60	76	91	3.6506	21	36	84
27	18	35	53	70	88	3.1305	23	40	58	88
28	20	40	60	80	2.6100	20	40	60	80	82
29	28	45	60	90	13	35	58	80	4.7003	81
30	25	50	75	2.0900	25	50	75	4.1800	25	80
31	28	55	83	10	38	65	93	20	47	29
32	30	60	90	20	50	79	3.6609	39	69	28
33	32	65	97	30	63	94	27	59	92	27
34	35	70	1.5705	40	75	3.1409	44	79	4.7114	26
35	37	75	12	50	87	24	62	99	37	25
36	40	80	20	60	2.6200	39	79	4.1919	59	24
37	43	85	27	70	12	53	96	38	81	23
38	45	90	34	79	24	69	2.6714	58	4.7203	22
39	47	95	42	89	37	84	31	78	26	21
40	50	1.0500	49	99	49	99	49	98	48	20
41	52	04	57	2.1010	61	3.1514	65	4.2018	70	19
42	55	09	64	19	74	28	83	38	92	18
43	57	14	72	29	86	43	2.6800	58	4.7315	17
44	60	19	79	29	99	58	18	78	37	16
45	62	24	86	49	2.6311	73	35	98	59	15
46	65	29	94	48	23	88	52	4.2117	81	14
47	67	34	1.5801	68	36	3.1603	69	37	4.7404	13
48	70	39	09	78	48	18	87	57	26	12
49	72	44	16	88	60	32	2.6904	76	48	11
50	75	49	24	98	73	47	22	96	71	10
51	77	54	31	2.1108	85	62	39	4.2216	93	9
52	79	59	38	18	97	76	56	35	4.7515	8
53	82	64	46	28	2.6410	91	73	55	37	7
54	84	69	53	38	22	3.1706	91	75	60	6
55	87	74	61	48	35	21	2.7008	95	82	5
56	89	79	68	37	47	36	25	4.2314	4.7604	4
57	92	84	75	67	59	51	43	34	26	3
58	94	89	83	77	72	66	60	54	49	2
59	97	92	90	87	84	80	77	74	70	1
60	0.5299	1.0598	1.5898	2.1197	2.6496	3.1795	3.7094	4.2394	4.7698	0
	1	2	3	4	5	6	7	8	9	

LATITUDE 58 DEGREES.

1	2	3	4	5	6	7	8	9	10
0 0.8481	1.6961	2.5442	3.8922	4.2408	5.0883	5.9364	6.7844	7.6325	8.4806
1 79	58	87	16	4.2395	73	52	31	10	59
2 77	55	82	10	87	64	42	19	7.6297	58
3 76	52	28	04	80	55	31	07	88	57
4 74	49	23	8.8897	72	46	20	6.7794	69	56
5 73	46	18	91	64	37	10	82	55	55
6 71	42	14	85	56	27	5.9298	70	41	54
7 70	39	09	79	49	18	88	58	27	53
8 68	36	04	72	41	09	77	45	13	52
9 67	33	00	66	33	00	66	33	7.6199	51
10 65	20	2.5395	60	25	5.0790	55	20	85	50
11 64	27	91	54	18	81	45	08	72	49
12 62	24	86	48	10	71	33	6.7695	67	48
13 60	21	81	42	02	62	23	83	44	47
14 59	18	76	35	4.2294	53	12	70	29	46
15 57	15	72	29	87	44	01	58	16	45
16 56	11	67	23	79	34	5.9190	46	01	44
17 54	08	63	17	71	25	79	34	7.6088	43
18 53	05	58	10	63	16	68	21	73	42
19 51	02	53	04	56	06	58	09	60	41
20 50	1.6899	49	8.8798	48	5.0697	47	6.7596	46	40
21 48	96	44	92	40	88	36	84	82	39
22 46	93	39	86	32	78	25	71	18	38
23 45	90	34	79	24	69	14	58	03	37
24 43	87	30	73	17	60	08	46	7.5990	36
25 42	83	25	67	09	50	5.9092	34	75	35
26 40	80	21	61	01	41	81	22	62	34
27 39	77	16	54	4.2193	32	70	09	47	33
28 37	74	11	48	85	22	59	6.7496	83	32
29 36	71	07	42	78	13	49	84	20	31
30 34	68	02	36	70	08	37	71	06	30
31 32	65	2.5297	30	62	5.0594	27	59	7.5892	29
32 31	62	92	23	54	85	16	46	77	28
33 29	58	88	17	46	75	04	34	63	27
34 28	55	83	11	39	66	5.8994	22	49	26
35 26	52	78	04	31	57	83	09	35	25
36 25	49	74	8.8698	23	47	72	6.7896	21	24
37 23	46	69	92	15	38	61	84	07	23
38 21	43	64	86	07	28	50	71	7.5793	22
39 20	40	59	79	4.2099	19	39	58	78	21
40 18	36	55	73	91	09	27	46	64	20
41 17	33	50	67	84	00	17	34	50	19
42 15	30	45	60	76	5.0491	06	21	36	18
43 14	27	41	54	68	81	5.8895	08	22	17
44 12	24	36	48	60	72	84	6.7296	08	16
45 10	21	31	42	52	62	73	88	7.5694	15
46 09	18	26	85	44	53	62	70	79	14
47 07	14	22	29	36	43	50	58	65	13
48 06	11	17	23	29	34	40	46	51	12
49 04	08	12	16	21	25	29	38	87	11
50 03	05	08	10	13	15	18	20	23	10
51 01	02	03	04	05	05	06	07	08	9
52 0.8399	1.6799	2.5198	4.1997	5.0396	5.8796	6.7195	7.5595	8	8
53 98	96	93	91	89	87	85	82	80	7
54 96	92	89	85	81	77	73	70	66	6
55 95	89	84	78	73	68	62	57	51	5
56 93	86	79	72	65	58	51	44	37	4
57 92	83	75	66	58	49	41	32	24	3
58 90	80	70	60	50	39	29	19	09	2
59 88	77	65	53	42	30	18	08	7.5495	1
60 0.8387	1.6773	2.5160	8.3547	4.1934	5.0820	5.8707	6.7094	7.5480	0
1	2	3	4	5	6	7	8	9	10

DEPARTURE 57 DEGREES.

DEPARTURE 82 DEGREES.

189

1	2	3	4	5	6	7	8	9	10
0	0.5299	1.0598	1.5898	2.1197	2.6496	3.1795	3.7094	4.2394	4.7693
1	0.5302	1.0603	1.5905	2.1207	2.6509	3.1810	3.7112	4.2414	4.7715
2	04	08	12	16	21	25	29	33	37
3	07	18	20	26	33	40	46	53	59
4	09	18	27	36	46	55	64	73	82
5	12	23	35	46	58	69	81	92	4.7804
6	14	28	42	56	70	84	98	4.2512	26
7	16	33	49	66	82	98	3.7215	81	48
8	19	38	57	76	95	3.1918	82	51	70
9	21	43	64	86	2.6607	28	50	71	98
10	24	48	71	95	19	48	67	90	4.7614
11	26	53	79	2.1805	32	58	84	4.2610	87
12	29	58	86	15	44	78	3.7802	80	59
13	31	62	94	25	56	87	18	50	81
14	34	67	1.6001	35	69	3.2002	36	70	4.8004
15	36	72	08	44	81	17	53	89	24
16	39	77	16	54	98	32	70	4.2709	47
17	41	82	23	64	2.6708	47	88	29	70
18	44	87	31	74	18	61	3.7405	48	92
19	46	92	38	84	30	76	22	68	4.8114
20	48	97	45	94	42	90	39	87	36
21	51	1.0702	53	5.1404	55	3.2105	58	4.2807	58
22	53	07	60	14	67	20	74	27	81
23	56	12	67	24	79	35	91	46	4.8202
24	58	17	75	33	92	50	3.7508	66	25
25	61	21	82	43	2.6804	64	25	86	46
26	63	26	90	53	16	79	42	4.2906	69
27	66	31	97	62	28	94	59	25	90
28	68	36	1.6104	72	41	3.2209	77	45	4.8313
29	71	41	12	82	53	28	94	64	85
30	73	46	19	92	65	38	3.7611	84	57
31	75	51	26	2.1502	87	52	28	4.8008	79
32	78	56	34	12	90	67	45	23	4.8401
33	80	61	41	22	2.6902	82	63	48	24
34	83	66	48	31	14	97	80	62	45
35	85	71	56	41	27	3.2312	97	82	68
36	88	76	63	51	39	26	3.7714	4.8102	89
37	90	80	71	61	51	41	81	22	4.8512
38	93	85	78	70	63	56	48	41	88
39	95	90	85	80	76	71	66	61	56
40	98	95	93	90	88	85	83	80	78
41	0.5400	1.0800	1.6200	2.1600	2.7000	3.2400	3.7800	4.3200	4.8600
42	02	05	07	10	12	14	17	19	22
43	05	10	15	20	25	29	34	39	44
44	07	15	22	29	37	44	51	58	66
45	10	19	29	39	49	58	68	78	87
46	12	24	37	49	61	73	85	98	4.8710
47	15	29	44	58	73	88	3.7902	4.8217	81
48	17	34	51	68	86	3.2508	20	37	54
49	20	39	59	78	98	17	37	56	76
50	22	44	66	88	2.7110	32	54	76	98
51	24	49	73	98	22	46	71	95	4.8820
52	27	54	81	2.1708	35	61	88	4.8415	42
53	29	59	88	17	47	76	3.8005	34	64
54	32	63	95	27	59	90	22	54	85
55	34	68	1.6308	37	71	3.2605	39	74	4.8908
56	37	72	10	46	88	20	56	98	29
57	39	78	17	56	96	35	74	4.8518	52
58	42	83	25	66	2.7208	49	91	32	74
59	44	88	32	76	22	64	3.1808	52	96
60	0.5446	1.0893	1.6339	2.1786	2.7232	3.2678	3.8125	4.8571	1.9018
	1	2	3	4	5	6	7	8	9

LATITUDE 57 DEGREES.

1	2	3	4	5	6	7	8	9	10
0	0.8887	1.6778	2.5160	3.3547	4.1984	5.0320	5.8707	6.7094	7.5480
1	85	70	55	40	26	11	5.8696	81	66 59
2	84	67	51	34	18	01	85	68	52 58
3	82	64	46	28	10	5.0291	73	55	37 57
4	80	61	41	22	02	82	63	43	24 56
5	79	58	36	15	4.1994	73	52	30	09 55
6	77	54	32	09	86	63	40	18	7.5895 54
7	76	51	27	02	78	55	29	05	80 53
8	74	48	22	3.3496	70	44	18	6.6992	66 52
9	72	45	17	90	62	34	07	79	52 51
10	71	42	12	83	54	25	5.8596	66	37 50
11	69	38	08	77	46	15	84	54	23 49
12	68	35	03	70	38	06	73	41	08 48
13	66	32	2.5098	64	30	5.0196	62	28	7.5294 47
14	65	29	94	58	23	87	51	16	81 46
15	63	26	89	52	15	77	40	08	66 45
16	61	23	84	45	07	68	28	6.6890	52 44
17	60	19	79	39	4.1899	58	17	78	37 43
18	58	16	74	32	91	49	06	65	23 42
19	57	13	70	26	83	39	5.8495	52	09 41
20	55	10	65	20	75	29	84	39	7.5194 40
21	53	07	60	13	67	20	73	26	30 39
22	52	03	55	07	59	10	66	14	65 38
23	50	00	50	00	51	01	51	01	51 37
24	49	1.6697	46	3.3394	43	5.0091	40	6.6788	37 36
25	47	94	41	88	35	87	28	75	22 35
26	45	91	37	81	27	72	17	62	08 34
27	44	87	31	75	19	62	06	50	7.5093 33
28	42	84	26	68	11	53	5.8395	37	79 32
29	41	81	22	62	03	43	84	24	65 31
30	39	78	17	56	4.1795	33	72	11	50 30
31	37	75	12	49	87	24	61	6.6698	36 29
32	36	71	07	42	78	14	49	85	20 28
33	34	68	02	36	70	04	38	72	06 27
34	32	65	2.4997	30	62	4.9994	27	59	7.4992 26
35	31	62	92	23	54	85	16	46	77 25
36	29	58	88	17	46	75	04	34	63 24
37	28	55	83	10	38	66	5.8293	21	48 23
38	26	52	78	04	30	56	82	08	34 22
39	24	49	73	3.3298	22	46	71	6.6595	20 21
40	23	46	68	91	14	37	60	82	06 20
41	21	42	64	85	06	27	48	70	7.4891 19
42	20	39	59	78	4.1698	17	37	56	76 18
43	18	36	54	72	90	07	25	43	61 17
44	16	33	49	65	82	4.9898	14	30	47 16
45	15	29	44	59	74	88	03	18	32 15
46	13	26	39	52	66	79	5.8192	05	18 14
47	12	23	35	46	58	69	81	6.6492	04 13
48	10	20	29	39	49	59	69	78	7.4788 12
49	08	16	25	33	41	49	57	66	74 11
50	07	13	20	26	33	40	46	53	59 10
51	05	10	15	20	25	30	35	40	45 9
52	03	07	10	14	17	20	24	27	31 8
53	02	03	05	07	09	10	12	14	15 7
54	00	00	00	00	01	01	01	01	01 6
55	0.8299	1.6597	3.4896	3.3194	4.1593	4.9791	5.8090	6.6388	7.4687 5
56	97	94	91	88	85	81	78	75	72 4
57	96	91	86	80	77	72	67	62	58 3
58	94	87	81	74	68	62	55	49	42 2
59	92	84	76	68	60	52	44	36	28 1
60	0.8290	1.6581	3.4871	3.3162	4.1452	4.9742	5.8083	6.6323	7.4614 0
1	2	3	4	5	6	7	8	9	

DEPARTURE 38 DEGREES.										141
1	2	3	4	5	6	7	8	9	1	
0	0.5446	1.0898	1.6389	2.1786	2.7282	3.2678	3.8125	4.3571	4.9018	60
1	49	98	46	95	44	93	42	90	89	59
2	51	1.0908	54	2.1805	57	3.2708	59	4.3610	62	58
3	54	07	61	15	69	22	76	80	83	57
4	56	12	68	24	81	37	93	49	4.9105	56
5	59	17	76	34	93	52	3.8210	69	27	55
6	61	22	83	44	2.7805	66	27	88	49	54
7	64	27	91	54	18	81	45	4.3708	72	53
8	66	32	98	64	30	95	61	27	93	52
9	68	37	1.6405	73	42	3.2810	78	46	4.9215	51
10	71	42	12	83	54	25	96	66	87	50
11	73	46	20	98	66	39	3.8812	86	59	49
12	76	51	27	2.1902	78	54	29	4.3805	80	48
13	78	56	34	12	91	69	47	25	4.9808	47
14	81	61	42	22	2.7408	83	64	44	25	46
15	83	66	49	32	15	97	80	63	46	45
16	85	71	56	42	27	3.2912	98	83	69	44
17	88	76	63	51	39	27	3.8415	4.3902	90	43
18	90	80	71	61	51	41	81	22	4.9412	42
19	93	85	78	72	64	56	49	44	84	41
20	95	90	85	80	76	71	66	61	56	40
21	98	95	93	90	88	85	83	80	78	39
22	0.5500	1.1000	00	2.2000	2.7500	99	99	99	99	38
23	02	05	1.8507	10	12	3.8014	3.8517	4.4019	4.9522	37
24	05	10	14	19	24	29	34	38	43	36
25	07	14	22	29	36	43	50	58	65	35
26	10	19	29	39	49	58	68	78	87	34
27	12	24	36	48	61	73	85	97	4.9609	33
28	15	29	44	58	73	87	3.8602	4.4116	81	32
29	17	34	51	68	85	3.3101	18	35	52	31
30	19	39	58	78	97	16	36	55	75	30
31	22	44	65	87	2.7609	81	58	74	96	29
32	24	48	73	97	21	45	69	94	4.9718	28
33	27	53	80	2.2106	38	60	86	4.4218	89	27
34	29	58	87	16	46	75	3.8704	38	62	26
35	31	63	95	26	58	89	21	52	84	25
36	34	68	1.6602	36	70	3.3208	37	71	4.9805	24
37	36	73	09	45	82	18	54	90	27	23
38	39	78	16	55	94	33	72	4.4310	49	22
39	41	82	24	65	2.7706	47	88	30	71	21
40	44	87	31	74	18	62	3.8805	49	92	20
41	46	92	38	84	30	76	22	68	4.9914	19
42	48	97	45	94	42	90	39	87	86	18
43	51	1.1102	53	2.2204	55	3.3805	56	4.4407	58	17
44	53	07	60	18	67	20	78	26	80	16
45	56	11	67	23	79	34	90	46	5.0001	15
46	58	16	74	32	91	49	3.8907	65	23	14
47	61	21	82	42	2.7808	68	24	84	45	13
48	63	26	89	52	15	78	41	4.4504	67	12
49	65	31	96	62	27	92	58	23	89	11
50	68	36	1.6703	71	39	3.8407	75	42	5.0110	10
51	70	40	11	81	51	21	91	62	82	9
52	73	45	18	90	63	36	3.9008	81	53	8
53	75	50	25	2.2300	75	50	25	4.4600	75	7
54	78	55	33	10	88	65	43	20	98	6
55	80	60	40	20	2.7900	79	59	39	5.0219	5
56	82	65	47	29	12	94	76	58	41	4
57	85	69	54	39	24	3.8508	93	78	62	3
58	87	74	61	48	36	23	3.9110	97	84	2
59	90	79	69	58	48	37	27	4.4716	5.0306	1
60	0.5592	1.1184	1.6776	2.2368	2.7960	3.8551	3.9148	4.4785	5.0327	0
1	2	3	4	5	6	7	8	9		
LATITUDE 56 DEGREES.										

LATITUDE 84 DEGREES.									
1	2	3	4	5	6	7	8	9	10
0	0.8290	1.6581	2.4871	3.3162	4.1452	4.9742	5.8033	6.6323	7.4614
1	89	77	66	55	44	32	21	10	7.4598
2	87	74	61	48	36	23	10	6.6297	84.58
3	86	71	57	42	28	18	5.7999	84	70.67
4	84	68	52	36	20	08	87	71	55.56
5	82	64	47	29	11	4.9698	75	58	40.55
6	81	61	42	22	08	84	64	45	25.54
7	79	58	37	16	4.1895	74	53	32	11.53
8	77	55	32	09	87	64	41	18	7.4496
9	76	51	27	08	79	54	80	06	81.51
10	74	48	22	8.8096	71	45	19	5.6198	67.50
11	72	45	17	90	62	34	07	79	52.49
12	71	42	12	83	54	25	5.7896	66	37.48
13	69	08	08	77	46	15	84	54	23.47
14	68	35	08	70	38	05	73	40	08.48
15	66	32	2.4798	64	30	4.9595	61	27	7.4894
16	64	29	93	57	22	86	50	14	79.44
17	63	25	88	50	18	76	38	01	64.43
18	61	22	83	44	05	66	27	6.6088	49.42
19	59	19	78	37	4.1297	56	15	74	34.41
20	58	15	73	31	89	46	04	62	19.40
21	56	12	68	24	81	37	2.7798	49	05.39
22	54	09	63	18	72	26	81	35	7.4290
23	53	06	58	11	64	17	70	22	75.87
24	51	02	53	04	56	07	50	09	60.86
25	50	1.6499	49	3.2998	48	4.9497	47	6.5996	46.35
26	48	96	43	91	39	87	85	82	80.34
27	46	92	39	85	31	77	23	70	16.33
28	45	89	34	78	23	68	12	57	01.33
29	43	86	29	72	15	58	00	43	7.4186
30	41	83	24	65	07	48	5.7639	30	72.30
31	40	79	19	58	4.1198	38	77	17	56.29
32	38	76	14	52	90	28	66	04	42.28
33	36	73	09	45	82	18	54	6.5890	27.27
34	35	69	04	39	74	08	43	78	12.26
35	33	66	2.4699	32	65	4.9898	31	64	7.4097
36	31	63	94	26	57	88	20	51	83.24
37	30	59	89	19	49	78	08	38	67.23
38	28	56	84	12	41	69	5.7597	25	53.22
39	26	53	79	06	32	58	85	11	38.21
40	25	50	74	3.2899	24	49	74	6.5798	23.20
41	23	46	69	92	16	39	62	85	08.19
42	21	43	64	86	07	28	50	71	7.8998
43	20	40	59	79	4.1099	19	39	58	78.17
44	18	36	54	72	91	09	27	45	68.16
45	17	33	50	66	83	4.9299	16	32	49.15
46	15	30	44	59	74	89	04	18	33.14
47	13	26	40	53	66	79	5.7492	06	19.18
48	12	23	35	46	58	69	81	6.5692	04.12
49	10	20	29	39	49	59	69	78	7.8888
50	08	16	25	33	41	49	57	66	74.10
51	07	13	20	26	32	39	46	52	59.9
52	05	10	14	19	24	29	34	38	43.8
53	03	06	10	13	16	19	22	26	29.7
54	02	03	05	06	08	09	11	12	14.6
55	00	00	00	00	00	4.9199	5.7899	6.5599	7.8799
56	0.8198	1.6395	2.4595	3.2793	4.0991	89	87	86	84.4
57	97	93	90	86	83	79	76	72	69.3
58	95	90	85	80	75	69	64	59	54.2
59	93	86	80	73	66	59	52	46	39.1
60	0.8192	1.6383	2.4575	3.2766	4.0958	4.9149	5.7841	6.5532	7.8724
	1	2	3	4	5	6	7	8	9
DEPARTURE 55 DEGREES.									

DEPARTURE 84 DEGREES.

143

	1	2	3	4	5	6	7	8	9	
0	0.5592	1.1184	1.6776	2.2368	2.7960	3.3551	3.9143	4.4735	5.0327	60
1	94	89	88	77	72	66	60	54	49	59
2	97	94	90	87	84	81	78	74	71	58
3	99	98	98	97	96	95	94	94	93	57
4	0.5602	1.1203	1.6806	2.2406	1.8008	3.3610	3.9211	4.4813	5.0414	56
5	04	08	12	16	20	24	28	32	36	55
6	06	13	19	26	32	38	45	51	58	54
7	09	18	26	35	44	53	62	70	79	53
8	11	22	34	45	56	67	78	90	5.0501	52
9	14	27	41	54	68	82	95	4.4909	22	51
10	16	32	48	64	80	96	3.9812	28	44	50
11	18	37	55	74	92	3.8710	29	47	66	49
12	21	42	62	83	2.8104	25	46	66	87	48
13	23	46	70	98	16	39	62	86	5.0609	47
14	26	51	77	2.2502	28	54	79	4.5005	30	46
15	28	56	84	12	40	68	96	24	52	45
16	31	61	92	22	53	88	3.9414	44	75	44
17	33	66	99	32	65	97	30	63	96	43
18	35	71	1.6906	41	77	3.8812	47	82	5.0718	42
19	38	75	13	51	89	26	64	4.5102	89	41
20	40	80	20	60	2.8201	41	81	21	61	40
21	43	85	28	70	18	55	98	40	88	39
22	45	90	35	80	25	69	3.9514	59	5.0804	38
23	47	95	42	89	37	84	31	78	26	37
24	50	99	49	99	49	98	48	98	47	36
25	52	1.1804	56	2.2608	61	3.8918	65	4.5217	69	35
26	55	09	64	18	78	27	82	36	91	34
27	57	14	71	28	85	41	98	55	5.0912	33
28	59	19	78	37	97	56	3.9615	74	84	32
29	62	23	85	47	2.8309	70	32	94	55	31
30	64	28	92	56	21	85	49	4.5313	77	30
31	67	33	1.7000	66	38	99	66	32	99	29
32	69	38	07	76	45	3.4013	82	51	5.1020	28
33	71	43	14	85	57	28	99	70	42	27
34	74	47	21	94	68	42	3.9715	89	62	26
35	76	52	28	2.2704	80	56	32	4.5408	84	25
36	78	57	35	14	92	70	49	27	5.1106	24
37	81	62	42	23	2.8404	85	66	46	27	23
38	83	66	50	33	16	99	82	66	49	22
39	86	71	57	42	28	3.4114	99	85	70	21
40	88	76	64	52	40	28	3.9816	4.5504	92	20
41	90	81	71	62	52	42	33	28	3.1214	19
42	93	86	78	71	64	57	50	42	35	18
43	95	90	86	81	76	71	66	62	57	17
44	98	95	93	90	88	86	88	81	78	16
45	0.5700	1.1400	1.7100	2.2800	2.8500	3.4200	3.9900	4.5600	5.1300	15
46	02	05	07	10	12	14	17	19	22	14
47	05	09	14	19	24	28	33	38	42	13
48	07	14	21	28	36	43	50	57	64	12
49	10	19	29	38	48	57	67	76	86	11
50	12	24	36	48	60	71	83	95	5.1407	10
51	14	29	43	57	72	86	4.0000	4.5714	29	9
52	17	33	50	67	84	3.4300	17	34	50	8
53	20	38	57	76	96	15	34	58	72	7
54	22	43	65	86	2.8608	29	51	72	94	6
55	24	48	71	95	19	48	67	90	5.1514	5
56	26	52	79	2.2905	81	57	83	4.5810	36	4
57	29	57	86	14	43	72	4.0100	29	57	3
58	31	62	93	24	55	86	17	48	79	2
59	33	67	1.7200	34	67	3.4400	34	67	5.1601	1
60	0.5736	1.1472	1.7207	2.2943	2.8679	3.4415	4.0151	4.5886	5.1622	0
	1	2	3	4	5	6	7	8	9	

LATITUDE 55 DEGREES.

1	2	3	4	5	6	7	8	9	10
0	0.8192	1.6383	2.4575	3.2766	4.0958	4.9149	5.7341	6.5532	7.3722
1	90	80	70	60	50	39	29	19	0
2	88	76	65	53	41	29	17	06	7.369
3	87	73	59	46	33	19	06	6.5492	7
4	85	70	54	39	24	09	5.7294	78	6
5	83	66	50	33	16	4.9099	82	66	4
6	82	63	45	26	08	89	71	62	3
7	80	60	39	19	4.0899	79	59	38	1
8	78	56	34	12	91	69	47	25	0
9	77	53	30	06	83	59	36	12	7.368
10	75	50	24	2.2699	74	49	24	6.5398	7
11	73	46	19	92	66	39	12	85	6
12	71	43	14	86	57	32	00	71	4
13	70	40	09	79	49	19	5.7189	58	2
14	68	36	04	72	41	09	77	45	1
15	66	33	2.4499	66	82	4.8994	65	31	7.3497
16	65	29	94	59	24	88	53	18	82
17	63	26	89	52	16	76	42	06	68
18	61	23	84	46	07	68	30	6.5291	53
19	60	19	79	39	4.0799	58	18	77	37
20	58	16	74	32	90	48	06	64	22
21	56	13	69	25	82	38	5.7094	50	07
22	55	09	64	18	73	28	82	37	7.3391
23	53	06	59	12	65	18	71	43	77
24	51	03	54	05	97	08	59	10	62
25	50	1.6299	49	2.2698	48	4.8898	47	6.5196	46
26	48	96	44	92	40	87	35	83	31
27	46	92	39	85	31	77	23	69	16
28	45	89	34	78	23	67	12	56	01
29	43	86	28	71	14	57	00	43	7.3285
30	41	82	24	65	06	47	5.6988	30	71
31	40	79	19	58	4.0698	27	77	16	56
32	38	76	13	51	89	27	65	02	40
33	36	72	08	44	81	17	53	6.5089	25
34	34	69	03	38	72	06	41	75	10
35	33	65	2.4398	31	64	4.8796	29	62	7.8194
36	31	62	93	24	55	86	17	48	79
37	29	59	88	17	47	76	05	34	64
38	28	55	83	10	38	66	5.6898	21	48
39	26	52	78	04	30	55	81	07	33
40	24	48	73	2.2497	21	45	69	6.4994	18
41	23	45	68	90	13	35	58	80	03
42	21	42	62	83	04	25	46	68	7.3087
43	19	38	57	76	4.0596	15	34	53	72
44	17	35	52	70	87	04	22	39	57
45	16	31	47	63	79	4.8694	10	26	41
46	14	28	42	56	70	84	5.6798	12	26
47	14	25	37	49	62	74	86	6.4898	11
48	11	21	32	42	53	64	74	85	7.2995
49	09	17	26	34	43	51	62	68	7
50	07	14	22	29	36	43	50	58	6
51	06	11	17	22	28	33	39	44	5
52	04	08	11	15	19	23	27	30	4
53	02	04	06	08	11	13	15	17	3
54	00	01	01	02	02	02	03	03	2
55	0.8099	1.6197	2.4296	2.2395	4.0494	4.8592	5.6691	6.4790	7.289
56	97	94	91	88	85	82	79	76	73
57	95	91	86	81	77	72	67	62	58
58	94	87	81	74	68	62	55	49	53
59	92	84	76	68	60	51	43	35	48
60	0.8090	1.6180	2.4271	3.2361	4.0451	4.8541	5.6631	6.4722	7.281
1	2	3	4	5	6	7	8	9	10

DEPARTURE 54 DEGREES.

DEPARTURE 35 DEGREES.

145

1	2	3	4	5	6	7	8	9	10
0	0.5736	1.1472	1.7207	2.2943	2.8679	3.4415	4.0151	4.5886	5.1622
1	38	76	14	52	91	29	67	4.5905	43 59
2	41	81	22	62	2.8703	43	84	24	65 58
3	43	86	29	72	15	57	4.0200	43	86 57
4	45	91	36	81	27	72	17	62	5.1708
5	48	95	48	91	39	86	34	82	29 55
6	50	1.1500	50	2.3000	51	3.4501	51	4.6001	51 54
7	52	05	57	10	62	14	67	19	72 53
8	55	10	64	19	74	29	84	38	93 52
9	57	14	72	29	86	43	4.0300	57	5.1815
10	60	19	79	38	98	58	17	77	86 50
11	62	24	86	48	2.8810	71	33	95	57 49
12	64	29	93	57	22	86	50	4.6114	79 48
13	67	33	1.7300	67	34	3.4600	67	84	5.1900
14	69	38	07	76	46	15	84	53	22 46
15	72	43	15	86	58	39	4.0401	72	44 45
16	74	48	21	95	69	43	17	90	64 44
17	76	52	29	2.3105	81	57	33	4.6210	86 43
18	79	57	36	14	98	72	50	29	5.2007
19	81	62	43	24	2.8905	85	66	47	28 41
20	83	67	50	33	17	3.4700	83	66	50 40
21	86	71	57	43	29	14	4.0500	86	71 39
22	88	76	64	52	41	29	17	4.6305	93 38
23	90	81	71	62	52	42	33	23	5.2114
24	93	86	78	71	64	57	50	42	35 36
25	95	90	86	81	76	71	66	62	57 35
26	98	95	93	90	88	86	83	81	78 34
27	0.5800	1.1600	1.7400	2.3200	2.9000	99	99	99	99 33
28	02	05	07	09	12	3.4814	4.0616	4.6418	5.2221
29	05	09	14	19	24	28	33	38	42 31
30	07	14	21	28	35	42	49	56	63 30
31	09	19	28	38	47	56	66	75	85 29
32	12	24	35	47	59	71	83	94	5.2306
33	14	28	42	56	71	85	99	4.6513	27 27
34	17	33	50	66	83	99	4.0716	32	49 26
35	19	38	57	76	95	3.4913	82	51	70 25
36	21	42	64	85	2.9106	27	48	70	91 24
37	24	47	71	94	18	42	65	89	5.2412
38	26	52	78	2.3304	30	56	82	4.6608	84 23
39	28	57	85	13	42	70	98	26	55 21
40	31	61	92	23	54	84	4.0815	46	76 20
41	33	66	99	32	65	98	81	64	97 19
42	35	71	1.7506	42	77	3.5012	48	83	5.2519
43	38	76	13	51	89	27	65	4.6702	40 17
44	40	80	20	60	2.9201	41	82	21	61 16
45	43	85	28	70	13	55	98	40	83 15
46	45	90	35	80	25	69	4.0914	59	5.2604
47	47	94	42	89	36	83	80	78	25 18
48	50	99	49	98	48	98	57	97	46 12
49	52	1.1704	56	2.3408	60	3.5111	63	4.6815	67 11
50	54	09	63	17	72	26	80	34	89 10
51	57	13	70	27	84	40	97	54	5.2710
52	59	18	77	36	95	54	4.1013	72	31 8
53	61	23	84	46	2.9307	68	30	91	53 7
54	64	27	91	55	19	82	46	4.6910	73 6
55	66	32	98	64	31	97	63	29	95 5
56	68	37	05	74	42	3.5210	79	47	5.2816
57	71	42	12	83	54	25	96	66	37 3
58	73	46	19	92	66	39	4.1112	85	58 2
59	76	51	27	2.3502	78	53	29	4.7004	80 1
60	0.5878	1.1756	1.7684	2.3612	2.9390	3.5267	4.1145	4.7023	5.2901
	1	2	3	4	5	6	7	8	9

LATITUDE 54 DEGREES.

1	2	3	4	5	6	7	8	9	10
0	0.8090	1.6180	2.4271	3.2361	4.0451	4.8541	5.6631	6.4722	7.2812
1	89	77	69	54	43	31	19	08	7.2797
2	87	73	64	47	34	20	07	6.4694	80.58
3	85	70	55	40	25	10	5.6595	80	65.57
4	83	67	50	33	17	00	83	66	50.56
5	82	63	45	26	08	4.8490	71	53	34.55
6	80	60	40	22	00	79	59	39	19.54
7	78	56	35	13	4.0391	69	47	26	04.53
8	77	53	30	06	83	59	36	12	7.2689
9	75	50	24	3.2299	74	49	24	6.4598	78.51
10	73	46	19	92	65	38	11	84	57.50
11	71	43	14	85	57	28	5.6499	70	42.49
12	70	39	09	78	48	18	87	57	26.48
13	68	36	04	72	40	07	75	43	11.47
14	66	32	2.4199	65	31	4.8397	63	30	7.2596
15	64	29	93	58	22	86	51	15	80.45
16	63	25	88	51	14	76	39	02	64.44
17	61	22	83	44	06	66	27	6.4488	49.43
18	59	19	78	37	4.0297	56	15	74	34.42
19	58	16	73	30	88	46	03	61	18.41
20	56	12	67	23	79	35	5.6391	46	02.40
21	54	08	62	16	71	25	79	33	7.2487
22	52	05	57	10	62	14	67	19	72.38
23	51	01	52	03	54	04	55	06	56.37
24	49	1.6098	47	3.2196	45	4.8293	42	6.4391	40.36
25	47	94	42	89	37	83	30	78	25.35
26	46	91	37	82	28	73	19	64	10.34
27	44	88	31	75	19	63	07	50	7.2394
28	42	84	26	68	10	52	5.6294	36	78.32
29	40	81	21	61	02	42	82	22	63.31
30	39	77	16	54	4.0193	32	70	09	47.30
31	37	74	10	47	84	21	58	6.4294	31.29
32	35	70	05	40	76	11	46	81	16.28
33	33	67	00	34	67	00	34	67	01.27
34	32	63	2.4095	26	58	4.8190	21	53	7.2284
35	30	60	90	20	50	79	09	39	69.25
36	28	56	85	13	41	69	5.6197	26	54.24
37	26	53	79	06	32	58	85	11	38.23
38	25	49	74	3.2099	24	48	73	6.4198	22.22
39	23	46	69	92	15	38	61	84	07.21
40	21	42	64	85	06	27	48	70	7.2191
41	20	39	59	78	4.0098	17	37	56	76.19
42	18	36	53	71	89	07	25	42	60.18
43	16	32	48	64	81	4.8096	12	28	44.17
44	14	29	43	57	72	86	00	14	29.16
45	13	25	38	50	63	75	5.6088	00	13.15
46	11	22	32	43	54	65	76	6.4086	7.2097
47	09	18	27	36	46	55	64	73	82.13
48	07	15	22	29	37	44	51	58	66.12
49	06	11	17	22	28	34	39	45	50.11
50	04	08	11	15	19	23	27	30	34.10
51	02	04	06	08	11	13	15	17	19.9
52	00	01	01	01	02	02	02	02	03.8
53	0.7999	1.5997	2.3996	3.1994	3.9993	4.7992	5.5990	6.3989	7.1987
54	97	94	90	87	84	81	78	74	71.6
55	95	90	85	81	76	71	66	61	56.5
56	93	87	80	74	67	60	54	47	41.4
57	92	83	75	67	58	50	41	33	24.8
58	90	80	70	60	50	39	29	19	09.2
59	88	76	64	52	41	29	17	05	7.1893
60	0.7986	1.5973	2.3959	3.1946	3.9932	4.7918	5.5905	6.3891	7.1878
	1	2	3	4	5	6	7	8	9

DEPARTURE 36 DEGREES.

147

	1	2	3	4	5	6	7	8	9	
0	0.5878	1.1756	1.7634	2.3512	2.9390	3.5267	4.1145	4.7023	5.2901	60
1	80	60	41	21	2.9401	81	61	42	22	59
2	83	65	48	30	13	96	78	61	43	58
3	85	70	55	40	25	3.5309	94	79	64	57
4	87	75	62	49	37	24	4.1211	98	86	56
5	90	79	69	58	48	38	27	4.7117	5.3006	55
6	92	84	76	68	60	52	44	36	28	54
7	94	89	83	77	72	66	60	54	49	53
8	97	93	90	87	84	80	77	74	70	52
9	99	98	97	96	95	94	93	92	91	51
10	0.5901	1.1803	1.7704	2.3606	2.9507	3.5408	4.1310	4.7211	4.3113	50
11	04	07	11	15	19	22	26	30	33	49
12	06	12	18	24	31	37	43	49	55	48
13	08	17	25	34	42	50	59	67	76	47
14	11	21	32	43	54	64	75	86	96	46
15	18	26	39	52	66	79	92	4.7805	4.3218	45
16	15	31	46	62	77	92	4.1408	28	89	44
17	18	36	53	71	89	3.5507	25	42	59	43
18	20	40	60	80	2.9601	21	41	61	81	42
19	23	45	68	90	18	35	58	80	4.3308	41
20	25	50	74	99	24	49	74	98	23	40
21	27	54	82	2.3709	36	63	90	4.7418	45	39
22	30	59	89	18	48	77	4.1507	86	66	38
23	32	64	95	27	59	91	23	54	86	37
24	34	68	1.7803	37	71	3.5605	39	74	4.3408	36
25	37	73	10	46	83	19	56	92	29	35
26	39	78	17	56	95	33	72	4.7511	50	34
27	41	82	24	65	2.9706	47	88	30	71	33
28	44	87	31	74	18	61	4.1605	48	92	32
29	46	92	38	84	80	75	21	67	4.3513	31
30	48	96	45	93	41	89	87	86	34	30
31	51	01	52	2.3802	53	3.5704	54	4.7605	55	29
32	53	1.1906	59	12	65	17	70	23	76	28
33	55	10	66	21	76	31	86	42	97	27
34	58	15	73	30	88	46	4.1703	61	4.3618	26
35	60	20	80	39	2.9800	59	19	80	39	25
36	62	24	87	49	11	73	35	98	60	24
37	64	29	94	58	23	88	52	4.7717	81	23
38	67	34	1.7901	68	35	3.5801	68	85	4.3702	22
39	69	39	08	77	47	16	85	54	24	21
40	72	43	15	86	58	30	4.1801	73	44	20
41	74	48	22	96	70	43	17	91	65	19
42	76	53	29	2.3905	82	58	34	4.7810	87	18
43	79	58	37	16	95	73	52	31	4.3810	17
44	81	62	43	24	2.9905	85	66	47	28	16
45	83	66	50	33	16	99	82	66	49	15
46	86	71	57	42	28	3.5914	99	85	70	14
47	88	76	64	52	40	27	4.1915	4.7903	91	13
48	90	80	71	61	51	41	31	22	6.3912	12
49	93	85	78	70	63	56	48	41	83	11
50	95	90	85	80	75	69	64	59	54	10
51	97	94	92	89	86	83	80	78	75	9
52	0.6000	99	99	98	98	97	97	96	96	8
53	02	1.2004	1.8006	2.4008	3.0010	3.6011	42.013	4.8015	5.4017	7
54	04	08	13	17	21	25	29	34	38	6
55	07	13	20	26	33	39	46	52	59	5
56	09	18	27	36	45	53	62	71	80	4
57	11	22	34	45	56	67	78	90	01	3
58	14	27	41	54	68	81	4.2195	4.8108	5.4122	2
59	16	32	47	63	79	95	11	26	42	1
60	0.6018	1.2036	1.8054	2.4072	3.0091	3.6106	4.2127	4.8145	5.4163	0

LATITUDE 53 DEGREES.

	1	2	3	4	5	6	7	8	9	
0	0.7986	1.5973	2.3959	3.1946	3.9932	4.7918	5.5905	6.3891	7.1878	60
1	85	69	54	38	23	08	5.5892	77	61	59
2	83	66	49	32	15	4.7897	80	63	46	58
3	81	62	43	24	06	87	68	49	30	57
4	79	59	38	17	3.9897	76	55	34	14	56
5	78	55	33	10	88	66	43	21	7.1798	55
6	76	52	27	03	79	55	31	06	82	54
7	74	48	22	3.1896	71	45	19	5.8793	67	53
8	72	45	17	89	62	34	06	78	51	52
9	71	41	12	82	53	24	5.5794	65	35	51
10	69	38	06	75	44	13	82	50	19	50
11	67	34	01	68	36	03	70	37	04	49
12	65	31	2.3896	61	27	4.7792	57	22	7.1688	48
13	64	27	91	54	18	82	45	08	72	47
14	62	24	85	47	09	71	33	6.3694	56	46
15	60	20	80	48	00	60	20	80	40	45
16	58	17	75	33	3.9792	50	08	66	25	44
17	57	13	70	26	83	39	2.5696	52	09	43
18	55	09	64	19	74	28	83	38	7.1592	42
19	53	06	59	12	65	18	71	24	77	41
20	51	02	54	05	56	07	58	10	61	40
21	49	1.5899	48	3.1798	47	4.7696	46	6.3595	45	39
22	48	95	43	91	39	86	34	82	29	38
23	46	92	38	84	30	75	21	67	23	37
24	44	88	32	76	21	65	09	53	7.1497	86
25	42	85	27	70	12	54	5.5597	89	82	35
26	41	81	22	62	03	44	34	25	65	34
27	39	78	16	55	3.9694	33	72	10	49	33
28	37	74	11	48	86	23	60	6.3497	84	32
29	35	71	06	41	77	12	47	82	18	31
30	34	67	01	34	68	01	35	68	02	30
31	32	64	2.3795	27	59	4.7591	28	54	7.1386	29
32	30	60	90	20	50	80	10	40	70	28
33	28	56	85	13	41	69	5.5497	26	54	27
34	26	53	79	06	32	58	85	11	38	26
35	25	49	74	3.1699	24	48	73	6.3398	22	25
36	23	46	69	92	15	37	60	83	06	24
37	21	42	63	84	06	27	48	69	7.1290	23
38	19	39	58	77	8.9597	16	35	54	74	22
39	18	35	53	70	88	06	23	41	58	21
40	16	32	47	63	79	4.7495	11	26	42	20
41	14	28	42	56	70	84	5.5398	12	26	19
42	12	24	37	49	61	73	85	6.3298	10	18
43	11	21	32	42	53	63	74	84	7.1195	17
44	09	17	26	35	44	52	61	70	78	16
45	07	14	21	28	35	41	48	55	62	15
46	05	10	15	20	26	31	36	41	46	14
47	03	07	10	13	17	20	23	26	30	13
48	02	03	05	06	08	09	11	12	14	12
49	00	00	2.3699	3.1599	3.9499	4.7399	5.5299	6.3198	7.1098	11
50	0.7898	1.5796	94	92	90	88	86	84	82	10
51	96	92	89	85	81	77	73	70	66	9
52	94	89	83	78	72	66	51	55	50	8
53	93	85	78	70	63	56	48	41	38	7
54	91	82	72	63	54	45	36	26	17	6
55	89	78	67	56	46	35	24	13	02	5
56	87	75	62	49	37	24	11	6.3098	7.0986	4
57	86	71	57	42	28	18	5.5199	84	70	3
58	84	67	51	35	19	02	86	70	53	2
59	82	64	45	28	10	4.7291	73	55	47	1
60	0.7880	1.5760	2.3640	3.1520	3.9401	4.7281	5.5161	6.3041	7.0921	0
	1	2	3	4	5	6	7	8	9	

DEPARTURE 87 DEGREES.

149

1	2	3	4	5	6	7	8	9	10
0	0.6018	1.2036	1.8054	2.4072	3.0091	3.6109	4.2127	4.8145	5.4163
1	21	41	62	81	3.0103	23	44	64	85
2	23	46	68	91	14	37	60	82	5.4205
3	25	50	75	2.4100	26	51	76	4.8201	26
4	27	55	82	10	37	64	92	19	47
5	30	60	89	19	49	79	4.2209	88	68
6	32	64	96	28	61	93	25	57	89
7	34	69	1.8103	38	72	3.6206	41	75	5.4310
8	37	73	10	47	84	21	57	94	30
9	39	78	17	56	95	34	73	4.8313	51
10	41	83	24	66	3.0207	48	90	31	73
11	44	87	31	75	19	62	4.2306	50	93
12	46	92	38	84	30	76	22	68	5.4414
13	48	97	45	93	42	90	38	86	35
14	51	1.2101	52	2.4202	53	3.6304	54	4.8405	55
15	53	06	59	12	65	17	70	23	76
16	55	11	66	21	77	32	87	42	98
17	58	15	73	30	88	46	4.2403	61	5.4518
18	60	20	80	40	3.0300	59	19	79	89
19	62	25	87	49	11	73	35	98	60
20	65	29	94	58	23	87	52	4.8516	81
21	67	34	1.8201	67	34	3.6401	68	35	5.4601
22	69	38	07	76	46	15	84	53	22
23	71	43	14	86	57	28	4.2500	72	43
24	74	48	21	95	69	43	17	90	64
25	76	52	28	5.4804	81	57	33	4.8609	85
26	78	57	35	14	92	70	49	27	5.4706
27	81	61	42	23	3.0404	84	65	46	27
28	83	66	49	32	15	98	81	64	47
29	85	71	56	41	27	3.6512	97	88	68
30	88	79	63	50	38	26	4.2613	4.8701	79
31	90	80	70	60	50	39	29	20	5.4809
32	92	85	77	69	61	53	45	38	30
33	95	89	84	78	73	67	62	56	51
34	97	94	90	87	84	81	78	74	71
35	99	98	97	96	96	95	94	93	92
36	0.6102	1.2203	1.8305	2.4406	3.0508	3.6609	4.2711	4.8812	5.4914
37	04	08	11	15	19	23	27	31	34
38	06	12	18	24	31	37	43	49	55
39	08	17	25	34	42	50	59	68	76
40	11	21	32	43	54	64	75	86	96
41	13	26	39	52	65	78	91	4.8905	5.5017
42	15	31	46	61	77	92	4.2807	22	38
43	18	35	53	70	88	3.6706	23	41	58
44	20	40	60	80	3.0600	19	39	59	79
45	22	45	67	89	11	33	55	78	5.5100
46	25	49	74	98	23	47	72	96	21
47	27	54	80	2.4507	34	61	88	4.9014	41
48	29	58	87	16	45	75	4.2904	88	62
49	31	63	94	26	57	88	20	51	83
50	34	67	1.8401	35	69	3.6802	36	70	5.5203
51	36	72	08	44	80	16	52	88	24
52	38	77	15	53	92	30	68	4.9106	45
53	41	81	22	62	3.0703	44	84	25	65
54	43	86	29	72	15	57	4.3000	43	86
55	45	90	35	80	26	71	16	62	5.5306
56	47	95	42	90	37	84	32	79	37
57	50	99	49	99	49	98	48	98	58
58	52	1.2304	56	2.4608	60	3.6912	64	4.9216	65
59	54	09	63	17	72	26	80	35	86
60	1.0157	1.2313	1.8470	2.4626	3.0783	3.6940	4.3096	4.9252	5.5409
1	1	2	3	4	5	6	7	8	9

• LATITUDE 52 DEGREES.

1	2	3	4	5	6	7	8	9	10
0	0.7880	1.5760	2.3640	3.1520	3.9401	4.7281	5.5161	6.3041	7.0921
1	78	57	35	13	3.9392	70	48	26	05 59
2	77	53	30	06	83	59	36	12	7.0889
3	75	50	24	99	74	48	23	6.2998	72 57
4	73	46	19	3.1492	65	37	10	83	59 56
5	71	42	13	84	56	27	5.5098	69	40 55
6	69	39	08	77	47	16	85	54	24 54
7	68	35	03	70	38	06	73	41	08 53
8	66	32	2.8597	63	29	4.7195	61	26	7.0792
9	64	28	92	56	20	84	48	12	76 51
10	62	24	87	49	11	73	35	6.2798	60 50
11	60	21	81	42	02	62	23	88	44 49
12	59	17	76	34	3.9293	52	10	69	27 48
13	57	14	70	27	84	41	5.4998	54	11 47
14	55	10	65	20	75	30	85	40	7.0695
15	53	06	60	13	66	19	72	26	79 45
16	51	03	54	06	57	08	60	11	63 44
17	50	00	49	3.1398	48	4.7098	47	6.2697	46 43
18	48	1.5696	43	91	39	87	35	82	30 42
19	46	92	38	84	30	76	22	68	14 41
20	44	88	33	77	21	65	09	54	7.0698
21	42	85	27	70	12	54	5.4897	39	82 39
22	41	81	22	62	3.9103	43	84	24	65 38
23	39	77	16	55	94	32	71	10	48 37
24	37	74	11	48	85	21	58	6.2595	32 36
25	35	71	05	40	76	11	46	81	16 35
26	33	67	00	33	67	00	33	66	00 34
27	32	63	2.3495	26	58	4.6989	21	52	7.0484
28	30	59	89	19	49	78	08	38	67 32
29	28	56	84	12	40	67	5.4795	23	51 31
30	26	52	78	04	31	57	83	09	35 30
31	24	49	73	3.1297	22	46	70	6.2494	19 29
32	23	45	68	90	13	35	58	80	03 28
33	21	41	62	82	03	24	44	65	7.0885
34	19	38	56	75	3.9094	13	32	50	69 26
35	17	34	51	68	85	02	19	36	53 25
36	15	30	46	61	76	4.6891	06	22	37 24
37	13	27	40	54	67	80	5.4694	07	21 23
38	12	23	35	46	58	70	81	6.2393	04 22
39	10	20	29	39	49	59	69	78	7.0288
40	08	16	24	32	40	47	55	63	71 20
41	06	12	18	24	31	37	43	49	55 19
42	04	09	13	17	22	26	30	34	39 18
43	03	05	08	10	13	15	18	20	23 17
44	01	01	02	03	04	04	05	06	06 16
45	0.7799	1.5598	2.3396	3.1195	3.8994	4.6793	5.4592	6.2290	7.0189
46	97	94	91	88	85	82	79	76	73 14
47	95	91	86	81	76	71	66	62	57 13
48	93	87	80	74	67	60	54	47	41 12
49	92	83	75	66	58	50	41	03	24 11
50	90	79	69	59	49	38	28	18	07 10
51	88	76	64	52	40	27	15	03	7.0091
52	86	72	58	44	31	17	03	6.2189	75 8
53	84	69	53	37	22	06	5.4490	74	59 7
54	82	65	47	30	12	4.6694	77	59	42 6
55	81	61	41	22	03	84	64	45	25 5
56	79	58	36	15	94	73	52	30	09 4
57	77	54	31	08	85	61	38	15	6.9992
58	75	50	25	00	76	51	26	01	76 2
59	73	47	20	3.1093	3.8867	40	13	86	80 1
60	0.7772	1.5543	2.3315	3.1086	3.8858	4.6629	5.4401	6.2172	6.9944
	1	2	3	4	5	6	7	8	9

DEPARTURE 58 DEGREES.

151

	1	2	3	4	5	6	7	8	9	
0	0.6157	1.2313	1.8470	2.4626	3.0783	3.6940	4.3096	4.9253	5.5409	60
1	59	18	77	86	95	53	4.3112	71	80	59
2	61	22	84	45	3.0806	67	28	90	51	58
3	64	27	91	54	18	81	45	4.9808	72	57
4	66	32	97	63	29	95	61	26	92	56
5	68	36	1.8504	72	41	3.7009	77	45	5.5513	55
6	70	41	11	82	52	22	93	63	84	54
7	73	45	18	90	63	36	4.8208	81	53	53
8	75	50	25	2.4700	75	49	24	4.9400	74	52
9	77	54	32	09	86	63	40	18	95	51
10	80	59	39	18	98	77	57	86	5.5616	50
11	82	64	45	27	3.0909	91	73	54	86	49
12	84	68	52	36	21	3.7105	89	73	57	48
13	86	73	59	46	32	18	4.3805	91	78	47
14	89	77	66	55	44	32	21	4.9510	98	46
15	91	82	73	64	56	45	36	27	5.5718	45
16	93	86	80	73	66	59	52	46	31	44
17	96	91	87	82	78	73	69	64	61	43
18	98	96	93	91	89	87	85	82	80	42
19	0.6200	1.2400	1.8600	2.4800	3.1001	3.7201	4.3401	4.9601	5.5801	41
20	02	05	07	10	12	14	17	19	22	40
21	05	09	14	18	23	28	32	37	41	39
22	07	14	21	28	35	41	48	55	62	38
23	09	18	28	37	46	55	64	74	83	37
24	12	23	35	46	58	69	81	92	5.5904	36
25	14	28	41	55	69	83	97	4.9710	24	35
26	16	32	48	64	80	96	4.3512	28	44	34
27	18	37	55	73	92	3.7810	28	46	65	33
28	21	41	62	82	3.1103	24	44	65	85	32
29	23	46	69	92	15	37	60	83	5.6006	31
30	25	50	75	2.4900	26	51	76	4.9801	26	30
31	27	55	82	10	37	64	92	19	47	29
32	30	59	89	19	49	78	4.3608	38	67	28
33	32	64	96	28	60	92	24	56	88	27
34	34	68	1.8703	37	71	3.7405	39	74	5.6108	26
35	37	73	10	46	83	19	56	92	29	25
36	39	78	16	55	94	33	72	4.9910	49	24
37	41	82	23	64	3.1206	47	88	29	70	23
38	43	87	30	73	17	60	4.3703	46	90	22
39	46	91	37	82	28	74	19	65	5.6210	21
40	48	96	44	92	40	87	35	83	31	20
41	50	1.2500	51	2.5001	51	3.7501	51	5.0002	52	19
42	52	05	57	10	62	14	67	19	72	18
43	55	09	64	19	74	28	83	38	92	17
44	57	14	71	28	86	42	99	56	5.6313	16
45	59	18	78	37	96	55	4.3814	74	33	15
46	62	23	85	46	3.1308	69	31	92	54	14
47	64	28	91	55	19	83	47	5.0110	74	13
48	66	32	98	64	30	96	62	28	94	12
49	68	37	05	73	42	3.7610	78	46	5.6415	11
50	71	41	1.8812	82	53	24	94	65	35	10
51	73	46	18	91	65	37	4.3910	82	55	9
52	75	50	25	2.5100	76	51	26	5.0201	76	8
53	77	55	32	10	87	64	42	19	97	7
54	80	59	39	18	98	78	57	37	5.6516	6
55	82	64	46	28	3.1410	91	73	55	87	5
56	84	68	53	37	21	3.7705	89	74	58	4
57	86	73	59	46	32	18	4.4005	91	78	3
58	89	77	66	55	44	32	21	5.0310	98	2
59	91	82	73	64	55	45	36	27	5.6618	1
60	0.6293	1.2586	1.8880	2.5173	3.1446	3.7759	4.4052	5.0346	5.6639	0
	1	2	3	4	5	6	7	8	9	

LATITUDE 51 DEGREES.

	1	2	3	4	5	6	7	8	9	10
0	0.7772	1.5543	2.3315	3.1086	3.8858	4.6629	5.4401	6.2172	6.9944	60
1	70	39	09	78	48	18	87	57	26	59
2	68	36	03	71	39	07	5.4375	42	10	58
3	66	32	2.3298	64	30	4.6596	62	28	6.9894	57
4	64	28	92	56	21	85	59	13	77	56
5	62	25	87	49	12	74	36	6.2098	61	55
6	61	21	82	42	03	63	24	84	45	54
7	59	17	76	34	3.8793	52	10	69	27	53
8	57	14	70	27	84	41	5.4298	54	11	52
9	55	10	65	20	76	30	85	40	6.9795	51
10	53	06	59	12	66	19	72	25	78	50
11	51	03	54	05	77	08	59	10	62	49
12	49	1.5499	48	3.0998	47	4.6496	46	6.1995	45	48
13	48	95	43	90	38	86	33	81	28	47
14	46	92	37	83	29	75	21	66	12	46
15	44	88	32	76	20	63	07	51	6.9695	45
16	42	84	26	68	11	53	5.4195	37	79	44
17	40	80	21	61	01	41	81	22	62	43
18	38	77	15	54	3.8692	30	69	07	46	42
19	37	73	10	46	83	20	56	6.1893	29	41
20	35	69	04	39	74	08	43	78	12	40
21	33	66	2.3199	32	65	4.6397	30	63	6.9596	39
22	31	62	93	24	55	86	17	48	79	38
23	29	59	88	17	46	75	04	34	63	37
24	27	55	82	09	37	64	5.4091	18	46	36
25	26	51	77	02	28	53	79	04	30	35
26	24	47	71	3.0894	18	42	65	6.1789	12	34
27	22	44	65	87	09	31	53	74	6.9496	33
28	20	40	60	80	00	19	39	59	79	32
29	18	36	54	72	3.8591	09	27	45	63	31
30	16	32	49	65	81	4.6297	13	30	46	30
31	14	29	43	58	72	86	01	15	30	29
32	13	25	38	50	63	75	5.3983	00	13	28
33	11	21	32	43	54	64	75	6.1685	6.9396	27
34	09	18	26	35	44	53	62	70	79	26
35	07	14	21	28	35	42	49	56	63	25
36	05	10	15	20	26	31	36	41	46	24
37	03	07	10	13	16	20	23	26	30	23
38	01	03	04	06	07	08	10	11	13	22
39	00	1.5399	2.3099	3.0798	3.8498	4.6198	5.3897	6.1597	6.9266	21
40	0.7698	95	93	91	89	86	84	82	79	20
41	96	92	88	84	80	75	71	67	63	19
42	94	88	82	76	70	64	58	52	46	18
43	92	84	76	68	61	53	45	37	29	17
44	90	81	71	61	52	42	32	22	13	16
45	88	77	65	54	42	30	19	07	6.9196	15
46	87	73	60	46	33	20	06	6.1493	79	14
47	85	69	54	39	24	08	5.3793	78	62	13
48	83	66	48	31	14	4.6097	80	62	45	12
49	81	62	43	24	05	86	67	48	29	11
50	79	58	37	16	96	75	54	33	12	10
51	77	54	32	09	3.8386	63	41	18	6.9095	9
52	75	51	26	02	77	52	28	03	79	8
53	74	47	21	2.0694	68	41	15	6.1388	62	7
54	72	43	15	87	59	30	02	74	45	6
55	70	40	09	79	49	19	5.8689	58	23	5
56	68	36	04	72	40	07	75	43	11	4
57	66	32	98	64	31	4.5997	63	29	6.8995	3
58	64	28	93	57	21	85	49	14	78	2
59	62	25	87	49	12	74	36	6.1298	61	1
60	0.7660	1.5321	2.2981	3.0642	3.8302	4.5962	5.3623	6.1283	6.8944	0

DEPARTURE 39 DEGREES.

153

	1	2	3	4	5	6	7	8	9	10
0	0.6293	1.2586	1.8880	2.5173	3.1466	3.7759	4.4052	5.0346	5.6639	60
1	96	91	87	82	78	73	69	64	59	59
2	98	95	93	91	89	86	84	82	79	58
3	0.6300	1.2600	1.8900	2.5200	3.1500	3.7800	4.4100	5.0400	5.6700	57
4	02	04	07	09	11	13	15	18	20	56
5	05	09	14	18	23	27	32	36	41	55
6	07	14	20	27	34	41	48	54	61	54
7	09	18	27	36	45	54	63	72	81	53
8	11	23	34	45	57	68	79	90	5.6802	52
9	14	27	41	54	68	81	95	5.0508	22	51
10	16	32	47	63	79	95	4.4211	26	42	50
11	18	36	54	72	90	3.7908	26	44	62	49
12	20	41	61	81	3.1602	22	42	62	83	48
13	23	45	68	90	13	35	58	80	5.6903	47
14	25	50	74	99	24	49	74	98	23	46
15	27	54	81	2.5308	36	63	90	5.0617	44	45
16	29	60	88	17	47	76	4.4305	84	64	44
17	32	63	95	26	58	90	21	53	84	43
18	34	68	1.9001	35	69	3.8003	37	70	5.7004	42
19	36	72	08	44	81	17	63	89	25	41
20	38	77	15	53	92	30	68	5.0706	45	40
21	41	81	22	62	3.1703	44	84	25	65	39
22	43	86	28	71	14	57	4.4400	42	85	38
23	45	90	35	80	26	71	16	61	5.7106	37
24	47	95	42	89	37	84	31	78	26	36
25	50	99	49	98	48	98	47	97	46	35
26	52	1.2704	55	2.5407	59	3.8111	63	5.0814	66	34
27	54	08	62	16	70	.24	78	32	86	33
28	56	13	69	25	82	88	94	50	5.7207	32
29	59	17	76	34	93	51	4.4510	68	27	31
30	61	22	82	43	3.1804	65	26	86	47	30
31	63	28	89	52	15	78	41	5.0904	67	29
32	65	31	96	61	27	92	57	22	88	28
33	68	35	1.9103	70	38	3.8205	73	40	5.7308	27
34	70	40	09	79	49	19	89	58	28	26
35	72	44	16	88	60	32	4.4604	76	48	25
36	74	48	23	97	71	45	19	94	68	24
37	77	53	30	2.5506	83	59	36	5.1012	89	23
38	79	57	36	15	94	72	51	30	5.7408	22
39	81	62	43	24	3.1905	86	67	48	29	21
40	83	66	50	33	16	99	82	66	49	20
41	85	71	56	42	27	3.8312	98	83	69	19
42	88	75	63	51	39	26	4.4714	5.1102	89	18
43	90	80	70	60	50	39	29	19	8.7509	17
44	92	84	77	69	61	53	45	38	30	16
45	94	89	83	78	72	66	61	55	50	15
46	97	93	90	86	83	80	76	73	69	14
47	99	98	97	96	95	93	92	91	90	13
48	0.6401	1.2802	1.9203	2.5604	3.2006	3.8407	4.4808	5.1209	5.7610	12
49	03	07	10	13	17	20	23	26	30	11
50	06	11	17	22	28	34	39	45	50	10
51	08	16	23	31	39	47	55	62	70	9
52	10	20	30	40	50	60	70	80	90	8
53	12	25	37	49	62	74	86	98	5.7711	7
54	15	29	44	58	73	87	4.4902	5.1316	81	6
55	17	33	50	67	84	3.8500	17	34	50	5
56	19	38	57	76	95	14	33	52	71	4
57	21	42	64	85	3.2006	27	48	70	91	3
58	23	47	70	94	17	40	64	87	5.7811	2
59	26	51	77	2.5702	28	54	79	5.1405	30	1
60	0.6428	1.2856	1.9284	2.5712	3.2140	3.8567	4.4995	5.1423	5.7851	0
	1	2	3	4	5	6	7	8	9	

LATITUDE 50 DEGREES.

K

1	2	3	4	5	6	7	8	9	10
0	0.7660	1.5821	2.2981	3.0642	3.8802	4.5962	5.3623	6.1283	6.8944
1	59	17	76	34	3.8298	52	10	69	27.59
2	57	18	70	27	84	40	5.3597	54	10.68
3	55	10	64	19	74	29	84	38	6.8883
4	53	06	59	12	65	18	71	24	77.55
5	51	02	53	04	56	07	58	09	60.55
6	49	1.5298	48	3.0597	46	4.5895	44	6.1194	43.54
7	47	95	42	89	37	84	31	78	26.53
8	46	91	37	82	28	73	19	64	10.52
9	44	87	31	74	18	62	05	49	6.8742
10	42	83	25	67	09	50	5.3492	34	75.50
11	40	80	19	59	3.8199	39	79	18	58.49
12	38	76	14	52	90	28	66	04	42.48
13	36	72	08	44	81	17	53	6.1089	25.47
14	34	68	03	37	71	05	39	74	06.46
15	32	65	2.2897	29	62	4.5794	26	58	6.8691
16	30	61	91	22	52	82	13	43	74.44
17	29	57	86	14	43	72	00	29	57.43
18	27	53	80	07	34	60	5.3387	14	40.42
19	25	50	74	3.0499	24	49	74	6.0998	23.41
20	23	46	69	92	15	37	60	83	06.40
21	21	42	63	84	05	26	47	68	6.8590
22	19	38	58	77	3.8096	15	34	54	73.38
23	17	35	52	69	87	04	21	38	56.37
24	15	31	46	62	77	4.5692	08	23	39.36
25	14	27	41	54	68	81	5.3295	08	22.35
26	12	23	35	46	58	70	81	6.0893	04.34
27	10	19	29	39	49	58	68	78	6.8487
28	08	16	23	31	39	47	55	62	70.32
29	06	12	18	24	30	35	41	47	53.31
30	04	08	12	16	21	25	29	33	87.30
31	02	04	07	09	11	13	15	18	20.29
32	00	01	01	01	02	02	02	02	03.28
33	0.7598	1.5197	2.2795	3.0394	3.7992	4.5590	5.3189	6.0787	6.8386
34	97	93	90	86	83	79	76	72	69.26
35	95	89	84	78	73	68	62	57	51.25
36	93	85	78	71	64	56	49	42	34.24
37	91	82	72	63	54	45	36	26	17.23
38	89	78	67	56	45	33	22	11	00.22
39	87	74	61	48	35	22	09	7.0696	6.8283
40	85	70	55	40	26	11	5.3096	81	66.20
41	83	66	50	33	16	4.5499	82	66	49.19
42	81	63	44	25	07	88	69	50	32.18
43	79	59	38	18	3.7897	76	56	35	15.17
44	78	55	33	10	88	65	43	20	6.8198
45	76	51	27	02	78	54	29	05	80.15
46	74	48	21	3.0295	69	48	17	6.0590	64.14
47	72	44	16	88	60	31	03	75	47.13
48	70	40	10	80	50	19	5.2989	59	29.12
49	68	36	04	72	40	08	76	44	12.11
50	66	32	2.2698	64	31	4.5397	63	29	6.8095
51	64	28	93	57	21	85	49	14	78.9
52	62	25	87	49	12	74	36	6.0498	61.8
53	60	21	81	42	02	62	23	83	44.7
54	59	17	76	34	3.7793	51	10	68	27.6
55	57	13	70	26	83	40	5.2896	53	09.5
56	55	09	64	19	74	28	83	38	6.7992
57	53	06	58	11	64	17	70	22	75.8
58	51	02	53	04	55	05	56	07	58.2
59	49	1.5098	47	3.0196	45	4.5294	43	6.0392	41.1
60	0.7547	1.5094	2.2641	3.0188	3.7786	4.5283	5.2880	6.0377	6.7924
1	2	3	4	5	6	7	8	9	

DEPARTURE 40 DEGREES.

155

1	2	3	4	5	6	7	8	9	10
0	0.6428	1.2856	2.9284	2.5712	3.2140	3.8567	4.4995	5.1423	5.7851
1	80	60	90	20	51	81	4.5011	41	71 59
2	82	65	97	29	62	94	26	58	91 58
3	85	69	1.9804	88	78	3.8608	42	77	5.7911 57
4	87	74	10	47	84	21	58	94	81 56
5	89	78	17	56	95	84	78	6.1512	51 55
6	41	82	24	65	3.2206	47	88	30	71 54
7	44	87	81	74	18	61	4.5105	48	92 53
8	46	91	87	88	29	74	20	66	5.8011 62
9	48	96	44	92	40	87	35	83	81 51
10	50	1.2900	50	2.5800	51	3.8701	51	5.1601	51 50
11	52	05	57	10	62	14	67	19	72 49
12	55	09	64	18	73	28	82	37	91 48
13	57	14	70	27	84	41	98	54	5.8111 47
14	59	18	77	36	95	54	4.5213	72	81 46
15	61	22	84	45	3.2306	67	28	90	51 45
16	64	27	91	54	18	81	45	5.1708	72 44
17	66	31	97	63	29	94	60	26	91 43
18	68	36	1.9404	72	40	3.8807	75	48	5.8211 42
19	70	40	10	80	51	21	91	61	81 41
20	72	45	17	89	62	34	4.5306	78	51 40
21	75	49	24	98	73	48	22	97	71 39
22	77	54	30	2.5907	84	61	38	5.1814	91 38
23	79	58	37	16	95	74	53	82	5.8311 37
24	81	62	44	25	3.2406	87	68	50	81 36
25	83	67	51	34	17	3.8900	84	67	51 35
26	86	71	57	42	28	14	99	85	70 34
27	88	76	63	51	39	28	4.5415	5.1902	90 33
28	90	80	70	60	51	41	81	21	5.8411 32
29	92	85	77	69	62	54	46	38	81 31
30	95	89	84	78	73	67	62	56	51 30
31	97	93	90	87	84	81	77	74	70 29
32	99	98	97	96	95	98	92	91	90 28
33	0.6501	1.8002	1.9503	2.6004	3.2506	3.9007	4.5508	5.2009	5.8510 27
34	08	07	10	13	17	20	23	26	30 26
35	06	11	17	22	28	33	39	44	50 25
36	08	15	23	31	39	46	54	62	69 24
37	10	20	30	40	50	59	69	79	89 23
38	12	24	37	49	61	78	85	98	5.8610 22
39	14	29	43	58	72	88	4.5601	5.2115	30 21
40	17	33	50	66	83	3.9100	16	33	49 20
41	19	38	56	75	94	13	32	51	69 19
42	21	42	63	84	3.2605	26	47	68	89 18
43	23	46	70	93	16	39	62	86	5.8709 17
44	25	51	76	2.6102	27	52	78	5.2208	29 16
45	28	55	83	10	38	66	94	21	48 15
46	30	60	89	19	49	79	4.5709	38	68 14
47	32	64	96	28	60	92	24	56	88 13
48	34	68	1.9608	37	71	3.9205	39	74	5.8808 12
49	36	73	09	46	82	18	55	91	28 11
50	39	77	16	54	93	32	70	6.2809	47 10
51	41	82	22	63	3.2704	45	86	26	67 9
52	43	86	29	72	15	58	4.5801	44	87 8
53	45	90	36	81	26	71	16	62	5.8907 7
54	47	95	42	90	37	84	32	79	27 6
55	50	99	49	98	48	98	47	97	47 5
56	52	1.8104	55	2.6207	59	3.9311	63	5.2414	66 4
57	54	08	62	16	70	24	78	32	86 3
58	56	12	69	25	81	37	93	50	5.9006 2
59	58	17	75	34	92	50	4.5909	67	26 1
60	0.6561	1.8121	1.9682	2.6242	3.2808	3.9364	4.5924	5.2485	5.9045 0

LATITUDE 49 DEGREES.

LATITUDE 41 DEGREES.										
	1	2	3	4	5	6	7	8	9	10
0	0.7547	1.5094	2.2641	3.0188	3.7736	4.5283	5.2830	6.0377	6.7924	7.5471
1	45	90	36	81	26	71	16	62	0759	
2	43	87	30	73	17	60	03	46	6.7890	58
3	41	83	24	66	08	48	5.2790	31	7367	
4	40	79	19	58	3.7698	37	77	16	5656	
5	38	75	13	50	88	25	63	00	3855	
6	36	71	07	42	78	14	49	6.0285	2054	
7	34	67	01	35	69	02	36	70	0853	
8	32	64	2.2595	27	59	4.5191	23	54	6.7786	53
9	30	60	90	20	50	79	09	39	6956	
10	28	56	84	12	40	68	5.2696	24	5250	
11	26	52	78	04	31	57	83	09	3549	
12	24	48	72	3.0096	21	45	69	6.0193	1748	
13	22	44	67	89	11	33	55	78	0047	
14	20	41	61	81	02	22	42	62	6.7683	46
15	18	37	55	74	3.7592	10	29	47	6645	
16	17	33	50	66	83	4.5099	16	32	4944	
17	15	29	44	58	73	88	02	17	3143	
18	13	25	38	50	63	76	5.2588	01	1342	
19	11	21	32	43	54	64	75	6.0086	6.7596	41
20	09	18	26	35	44	53	62	70	7940	
21	07	14	21	28	35	41	48	55	6239	
22	05	10	15	20	25	30	35	40	4538	
23	03	06	09	12	15	18	21	24	2737	
24	01	02	03	04	06	07	08	09	1036	
25	0.7499	1.4998	2.2498	2.9997	3.7496	4.4995	5.2494	5.9994	6.7493	35
26	97	95	92	89	87	84	81	78	7634	
27	95	91	86	81	77	72	67	62	5833	
28	93	87	80	74	67	60	54	47	4132	
29	92	83	75	66	58	49	41	32	2431	
30	90	79	69	58	48	38	27	17	0630	
31	88	75	63	50	38	26	13	01	6.7388	29
32	86	71	57	43	29	14	00	5.9886	7123	
33	84	68	51	35	19	03	5.2387	70	5427	
34	82	64	45	27	09	91	73	54	3626	
35	80	60	40	20	00	4.4879	59	39	1925	
36	78	56	34	12	3.7390	68	46	24	0224	
37	76	52	28	04	80	56	32	08	6.7234	23
38	74	48	22	2.9896	71	45	19	5.9793	6722	
39	72	44	17	89	61	33	05	78	5021	
40	70	41	11	81	52	22	5.2292	62	3320	
41	68	37	05	73	42	10	78	46	1519	
42	66	33	2.2399	66	32	4.4798	65	31	6.7198	18
43	64	29	93	58	24	86	51	15	8017	
44	63	25	88	50	13	75	38	00	6316	
45	61	21	82	42	03	64	24	5.9685	4515	
46	59	17	76	34	3.7293	52	10	69	2714	
47	57	13	70	27	84	40	5.2197	54	1013	
48	55	10	64	19	74	29	84	38	6.7093	12
49	53	06	58	11	64	17	70	22	7511	
50	51	02	53	04	55	05	56	07	5810	
51	49	1.4898	47	2.9796	45	4.4693	42	5.9591	409	
52	47	94	41	88	35	82	29	76	238	
53	45	90	35	80	26	71	16	61	067	
54	43	86	29	72	16	59	02	45	6.6988	6
55	41	82	24	65	06	47	5.2088	30	714	
56	39	78	18	57	3.7196	35	74	14	535	
57	37	75	12	49	87	24	61	5.9498	863	
58	35	71	06	41	77	12	47	82	182	
59	33	67	00	34	67	00	84	67	011	
60	0.7431	1.4863	2.2294	2.9726	3.7157	4.4588	5.2020	5.9451	6.6883	0
	1	2	3	4	5	6	7	8	9	10
. DEPARTURE 48 DEGREES.										

DEPARTURE 41 DEGREES.

157

1	1	2	3	4	5	6	7	8	9	10
0	0.6581	1.8121	1.9682	2.6242	3.2803	3.9864	4.5924	5.2485	5.9045	6.5606
1	68	26	88	51	14	77	40	5.2502	65	58
2	65	30	95	60	25	90	55	20	85	58
3	67	34	1.9702	69	36	8.9408	70	38	5.9105	57
4	69	39	08	78	47	16	86	55	25	56
5	72	43	15	86	58	80	4.6001	78	44	55
6	74	48	21	95	69	48	17	90	64	54
7	78	52	28	2.6804	80	55	81	5.2607	83	53
8	78	56	34	12	91	68	47	25	5.9208	52
9	80	61	41	21	8.2902	82	62	42	23	51
10	83	65	48	30	18	95	78	60	48	50
11	85	69	54	39	24	8.9508	98	78	62	49
12	87	74	61	48	35	21	4.6108	95	82	48
13	89	78	67	56	46	35	24	5.2718	5.9802	47
14	91	83	74	65	57	48	39	80	22	46
15	94	87	81	74	68	61	55	48	42	45
16	96	91	87	82	78	74	69	65	60	44
17	98	96	93	91	89	87	85	82	80	43
18	0.6600	1.8200	1.9800	2.6400	3.8000	3.9600	4.6200	5.2800	5.9400	42
19	02	04	07	09	11	18	15	18	20	41
20	04	09	13	18	22	26	31	35	40	40
21	07	13	20	26	33	40	46	53	59	39
22	09	18	26	35	44	53	62	70	79	38
23	11	22	33	44	55	65	76	87	98	37
24	13	26	39	52	66	79	92	5.2905	5.9518	36
25	15	31	46	61	77	92	4.6307	22	88	35
26	18	35	53	70	88	8.9705	23	40	58	34
27	20	39	59	79	99	18	38	58	77	33
28	22	44	65	87	8.8109	31	53	74	96	32
29	24	48	72	96	20	44	68	92	5.9618	31
30	26	52	79	2.6505	31	57	83	5.8090	86	30
31	28	57	85	14	42	70	99	27	56	29
32	31	61	92	22	53	84	4.6414	45	75	28
33	33	65	98	31	64	96	29	62	94	27
34	35	70	1.9905	40	75	8.9809	44	80	5.9714	26
35	37	74	11	48	86	23	60	97	84	25
36	39	79	18	57	97	36	75	5.8114	54	24
37	41	83	24	66	8.8207	48	90	31	73	23
38	44	87	31	74	18	62	4.6505	49	92	22
39	46	92	37	83	29	75	21	66	5.9812	21
40	48	96	44	92	40	88	36	84	82	20
41	50	1.8800	50	2.6600	51	8.9901	51	5.8201	51	19
42	52	05	57	09	62	14	66	18	71	18
43	55	09	64	18	73	27	82	36	91	17
44	57	13	70	26	83	40	96	53	5.9909	16
45	59	18	76	35	94	53	4.6612	70	29	15
46	61	22	83	44	8.8305	66	27	88	49	14
47	63	26	90	53	16	79	42	5.8306	69	13
48	65	31	96	61	27	92	57	22	88	12
49	68	35	2.0008	70	38	4.0005	73	40	6.0008	11
50	70	39	05	79	49	18	88	58	27	10
51	72	44	16	87	59	31	4.6708	74	46	9
52	74	48	22	96	70	44	18	92	66	8
53	76	52	29	2.6705	81	57	33	5.8410	86	7
54	78	57	35	13	92	70	48	26	6.0105	6
55	81	61	42	22	8.8408	83	64	44	25	5
56	83	65	48	31	14	96	79	62	44	4
57	85	70	54	39	24	4.0109	94	78	68	3
58	87	74	61	48	35	22	4.6809	96	83	2
59	89	78	67	56	46	35	24	5.8513	6.0202	1
60	0.6691	1.8383	2.0074	2.6765	3.8457	4.0148	4.6839	5.8530	6.0222	0
	1	2	3	4	5	6	7	8	9	

LATITUDE 48 DEGREES.

LATITUDE 42 DEGREES.									
1	2	3	4	5	6	7	8	9	10
0	0.7431	1.4863	2.2294	2.9726	3.7157	4.4588	5.2020	5.9451	6.6883
1	20	59	89	18	48	77	07	36	66
2	28	55	83	10	38	66	5.1993	21	48
3	26	51	77	02	28	54	79	05	30
4	24	47	71	2.9695	19	42	66	5.9890	13
5	22	43	65	87	09	30	52	74	6.6795
6	20	40	59	79	3.7099	20	39	58	78
7	18	36	53	71	89	07	25	42	60
8	16	32	48	64	80	4.4495	11	27	43
9	14	28	42	56	70	83	5.1897	11	25
10	12	24	36	48	60	72	84	5.9296	08
11	10	20	30	40	50	60	70	80	6.6690
12	08	16	24	32	40	48	56	64	72
13	06	12	18	24	30	37	43	49	55
14	04	08	12	16	21	25	29	33	37
15	02	04	07	09	11	13	15	18	20
16	00	00	01	01	01	01	01	02	02
17	6.7398	1.4797	2.2195	2.9593	3.6992	4.4390	5.1788	5.9186	6.6585
18	96	93	89	85	82	78	74	70	67
19	94	89	83	78	72	66	61	55	50
20	92	85	77	70	62	54	47	39	32
21	90	81	71	62	52	42	33	23	14
22	89	77	66	54	43	31	20	08	6.6497
23	87	73	60	46	33	19	06	5.9092	79
24	85	69	54	38	23	08	5.1692	77	61
25	83	65	48	30	13	4.4296	78	61	43
26	81	61	42	22	03	84	64	45	25
27	79	57	36	15	3.6894	72	51	30	08
28	77	53	20	07	84	60	37	14	6.6390
29	75	49	24	2.9499	74	48	23	5.8998	72
30	73	46	18	91	64	37	10	82	55
31	71	42	12	83	54	25	5.1596	66	37
32	69	38	06	75	44	13	82	50	19
33	67	34	01	68	35	01	68	35	02
34	65	30	2.2095	60	25	4.4189	54	19	6.6284
35	63	26	89	52	15	77	40	08	66
36	61	22	83	44	05	66	27	5.8888	49
37	59	18	77	36	3.6795	54	18	72	31
38	57	14	71	28	85	42	5.1499	56	13
39	55	10	65	30	76	31	86	41	6.6196
40	53	06	59	12	66	19	72	25	78
41	51	02	53	04	56	07	58	09	59
42	49	1.4698	47	2.9396	46	95	44	5.8793	42
43	47	94	42	89	36	83	30	78	25
44	45	90	36	81	26	71	16	62	07
45	43	86	20	72	16	59	02	42	6.6089
46	41	82	24	65	06	47	5.1388	30	71
47	39	79	18	57	2.6697	36	75	14	54
48	37	75	12	49	87	24	61	5.8698	36
49	35	71	06	41	77	12	47	82	18
50	33	67	00	33	67	00	33	66	00
51	31	63	2.1994	26	57	4.3988	20	51	6.5983
52	29	59	88	18	47	76	06	35	65
53	27	55	82	10	37	64	5.1292	19	47
54	25	51	76	02	27	52	78	08	29
55	23	47	70	2.9294	17	40	64	5.8587	11
56	22	43	65	86	08	29	51	72	6.5894
57	20	39	59	78	2.6598	17	37	56	76
58	18	35	53	70	88	05	23	40	58
59	16	31	47	62	78	4.3893	09	24	40
60	0.7314	1.4627	2.1941	2.9254	3.6568	4.3881	5.1195	5.8508	6.5822
1	2	3	4	5	6	7	8	9	

DEPARTURE 47 DEGREES.

DEPARTURE 42 DEGREES.

159

1	2	3	4	5	6	7	8	9	10	
0	0.6691	1.8883	2.0074	2.6765	3.8457	4.0148	4.6839	5.8530	6.0222	60
1	94	87	81	74	68	61	55	48	42	59
2	96	91	87	82	78	74	69	65	60	58
3	98	96	93	91	89	87	85	82	80	57
4	0.6700	1.8400	2.0100	00	3.8500	99	99	99	99	56
5	02	04	06	2.6808	11	4.0213	4.6915	5.8617	6.0309	55
6	04	09	13	17	22	26	30	34	39	54
7	06	18	19	26	32	38	45	51	58	53
8	09	17	26	34	43	52	60	69	77	52
9	11	21	32	43	54	64	75	86	96	51
10	13	26	39	52	65	77	90	5.8708	6.0416	50
11	15	30	45	60	76	91	4.7006	21	36	49
12	17	34	52	69	86	4.0808	20	38	55	48
13	19	39	58	78	97	16	36	55	75	47
14	22	43	65	86	3.8608	29	51	72	94	46
15	24	47	71	95	19	42	66	90	6.0518	45
16	26	52	77	2.6908	29	55	81	5.3806	32	44
17	28	56	84	12	40	68	96	24	52	43
18	30	60	90	20	51	81	4.7111	41	71	42
19	32	65	97	29	62	94	26	58	91	41
20	34	69	03	88	72	4.0406	41	75	6.0610	40
21	37	73	2.0210	46	83	20	56	93	29	39
22	39	77	16	55	94	32	71	5.8910	48	38
23	41	82	22	64	3.8705	45	86	27	68	37
24	43	86	29	72	15	58	4.7201	44	87	36
25	45	90	36	81	26	71	16	62	6.0707	35
26	47	95	42	89	37	84	31	78	26	34
27	50	99	49	98	48	97	47	96	46	33
28	52	1.8508	55	2.7006	58	4.0510	61	5.4018	64	32
29	54	08	61	15	69	23	77	30	84	31
30	56	12	68	24	80	35	91	47	6.0808	30
31	58	16	74	32	90	48	4.7806	64	22	29
32	60	20	81	41	3.8801	61	21	82	42	28
33	62	25	87	49	12	74	36	98	61	27
34	65	29	94	58	23	87	52	5.4116	81	26
35	67	33	2.0800	66	33	4.0600	66	33	99	25
36	69	38	06	75	44	18	82	50	6.0919	24
37	71	42	18	84	55	25	96	67	38	23
38	73	46	19	92	65	38	4.7411	84	57	22
39	75	50	26	2.7101	76	51	26	5.4202	77	21
40	77	55	32	09	87	64	41	18	96	20
41	80	59	39	18	98	77	57	36	6.1016	19
42	82	63	45	26	3.8908	90	71	53	34	18
43	84	67	51	35	19	4.0702	86	70	53	17
44	86	72	58	44	30	15	4.7501	87	78	16
45	88	76	64	52	40	28	16	5.4804	92	15
46	90	80	70	60	51	41	81	21	6.1111	14
47	92	85	77	69	62	53	46	38	81	13
48	94	89	83	78	72	66	61	55	50	12
49	97	98	90	86	83	79	76	72	69	11
50	99	97	96	95	94	92	91	90	88	10
51	0.6801	1.8602	2.0402	2.7208	3.4004	4.0805	4.7606	5.4406	6.1207	9
52	08	06	09	12	15	17	20	23	26	8
53	06	10	15	20	26	31	36	41	46	7
54	07	14	22	29	36	43	50	58	65	6
55	09	19	28	37	47	56	65	74	84	5
56	12	23	35	46	58	69	81	92	9.1804	4
57	14	27	41	54	68	82	95	5.4509	22	3
58	16	31	47	63	79	94	4.7710	26	41	2
59	18	36	54	72	90	4.0907	25	43	61	1
60	0.6820	1.8640	2.0460	2.7280	3.4100	4.0920	4.7740	5.4560	6.1880	0
1	2	3	4	5	6	7	8	9		

LATITUDE 47 DEGREES.

LATITUDE 43 DEGREES.										
160	1	2	3	4	5	6	7	8	9	10
0	0.7314	1.4627	2.1941	2.9254	3.6568	4.3881	5.1195	5.8508	6.5822	7.3135
1	12	23	35	46	58	70	81	5.8493	6.5786	7.3079
2	10	19	29	38	48	58	67	77	6.5786	7.3079
3	08	15	23	30	38	46	53	41	6857	
4	06	11	17	22	28	34	39	45	5056	
5	04	07	11	14	18	22	25	29	3255	
6	02	03	05	06	08	10	11	13	1454	
7	00	1.4599	2.1899	2.9198	3.6498	4.3798	5.1097	5.8397	6.5697	7.2997
8	0.7298	95	93	90	88	86	83	81	6.5678	7.2978
9	96	91	87	83	79	74	70	66	6151	
10	94	87	81	75	69	62	56	50	4350	
11	92	83	75	67	59	50	42	34	2549	
12	90	79	69	59	49	38	28	18	0748	
13	88	75	63	51	39	26	14	01	6.5589	7.2878
14	86	71	57	43	29	14	00	5.8286	7146	
15	84	67	51	35	19	02	5.0986	70	6245	
16	82	63	45	27	09	4.3690	72	54	3544	
17	80	59	39	19	3.6399	78	58	38	1743	
18	78	55	33	11	89	66	44	22	6.5499	7.2742
19	76	51	27	03	79	54	30	06	8141	
20	74	47	21	2.9095	69	42	16	5.8190	6340	
21	72	43	15	87	59	30	02	74	4539	
22	70	39	09	79	49	18	5.0888	58	2738	
23	68	35	03	71	39	06	74	42	0937	
24	66	31	2.1797	63	29	4.3594	60	26	6.5391	7.2636
25	64	27	91	55	19	82	46	10	7335	
26	62	23	85	47	09	70	32	5.8094	5534	
27	60	19	79	39	3.6299	58	18	78	3733	
28	58	15	73	31	89	46	04	62	1932	
29	56	11	67	23	79	34	5.0790	46	0131	
30	54	07	61	15	69	22	76	30	6.5283	7.2530
31	52	03	55	07	59	10	62	14	6529	
32	50	1.4499	49	2.8999	49	4.3498	48	5.7998	4728	
33	48	95	43	91	39	86	34	82	2927	
34	46	91	37	83	29	74	20	66	1126	
35	44	87	31	75	19	62	06	50	6.5193	7.2425
36	42	83	25	67	09	50	5.0692	34	7524	
37	40	79	19	59	3.6199	38	78	18	5723	
38	38	75	13	51	89	26	64	02	3922	
39	36	71	07	43	79	14	60	5.7886	2121	
40	34	67	01	35	69	02	36	70	0320	
41	32	63	2.1795	27	59	4.3390	22	54	6.5085	7.2319
42	30	59	89	19	49	78	08	38	6718	
43	28	55	83	11	39	66	5.0594	22	4917	
44	26	51	77	03	29	54	80	06	3116	
45	24	47	71	2.8894	18	42	65	5.7789	1215	
46	22	43	65	86	08	30	51	73	6.4994	7.2214
47	20	39	59	78	3.6098	18	37	57	7618	
48	18	35	53	70	88	06	23	41	5812	
49	16	31	47	62	78	4.3294	09	25	4011	
50	14	27	41	54	68	82	5.0495	09	2210	
51	12	23	35	46	58	70	81	5.7693	049	
52	10	19	29	38	48	57	67	76	6.4886	7.218
53	08	15	23	30	38	45	53	60	687	
54	06	11	17	22	28	33	39	44	506	
55	04	07	11	14	18	21	25	28	325	
56	02	03	05	06	08	09	11	12	144	
57	00	1.4399	2.1699	2.8798	3.5997	4.3197	5.0397	5.7596	6.4796	7.203
58	0.7197	95	92	90	87	84	82	79	772	
59	96	91	86	82	77	72	68	63	591	
60	0.7193	1.4387	2.1680	2.8774	3.5967	4.3180	5.0354	5.7547	6.4741	7.200
	1	2	3	4	5	6	7	8	9	10
DEPARTURE 46 DEGREES.										

1	1	2	3	4	5	6	7	8	9	1
0	0.6820	1.8640	2.0460	2.7280	3.4100	4.0920	4.7740	5.4560	6.1380	60
1	22	44	66	88	11	33	55	77	99	59
2	24	48	73	97	21	45	69	94	6.1418	58
3	26	53	79	2.7306	32	58	85	5.4611	38	57
4	29	57	86	14	43	71	4.7800	28	57	56
5	81	61	92	22	53	84	14	45	75	55
6	33	65	98	31	64	96	29	62	94	54
7	85	70	2.0505	40	75	4.1009	44	79	6.1514	53
8	87	74	11	48	85	22	59	96	33	52
9	89	78	17	56	96	35	74	5.4713	52	51
10	41	82	24	65	3.4206	47	88	30	71	50
11	43	87	30	73	17	60	4.7903	46	90	49
12	46	91	87	82	28	73	19	64	6.1610	48
13	48	95	43	90	38	86	33	81	28	47
14	50	99	49	99	49	98	48	98	47	46
15	52	1.8704	55	2.7407	59	4.1111	63	5.4814	66	45
16	54	08	62	16	70	23	77	31	85	44
17	56	12	68	24	81	37	93	49	6.1705	43
18	58	16	75	33	91	49	4.8007	66	24	42
19	60	21	81	41	3.4302	62	22	82	43	41
20	62	25	87	50	12	74	37	99	62	40
21	65	29	94	58	23	87	52	5.4916	81	39
22	67	33	2.0600	66	33	4.1200	66	33	99	38
23	69	38	06	75	44	13	82	50	6.1819	37
24	71	42	13	84	55	25	96	67	38	36
25	73	46	19	92	65	38	4.8111	84	57	35
26	75	50	25	2.7500	76	51	26	5.5001	76	34
27	77	54	32	09	86	63	40	18	95	33
28	79	59	38	17	97	76	55	24	6.1914	32
29	81	63	44	26	3.4407	88	70	51	33	31
30	84	67	51	34	18	4.1301	85	68	52	30
31	86	71	57	43	29	14	4.8200	86	71	29
32	88	76	63	51	39	27	15	5.5102	90	28
33	90	80	70	60	50	39	29	19	6.2009	27
34	92	84	76	68	60	52	44	36	28	26
35	94	89	83	78	72	66	61	35	50	25
36	96	92	89	85	81	77	73	70	66	24
37	98	97	95	93	92	90	88	86	85	23
38	0.6900	1.8801	2.0701	2.7602	3.4502	4.1402	4.8303	5.5203	6.2104	22
39	08	05	07	10	13	15	18	20	23	21
40	05	09	14	18	23	28	32	37	41	20
41	07	13	20	27	34	40	47	54	60	19
42	09	18	26	35	44	53	62	70	79	18
43	11	22	33	44	55	65	76	87	98	17
44	13	26	39	52	65	78	91	5.5304	6.2217	16
45	15	30	45	60	76	91	4.8406	21	36	15
46	17	34	52	69	86	4.1503	20	38	55	14
47	19	38	58	77	97	16	35	54	74	13
48	21	43	64	86	3.4607	28	50	71	93	12
49	24	47	71	94	18	41	65	88	6.2312	11
50	26	51	77	2.7702	28	54	79	5.5405	30	10
51	28	55	83	11	39	66	94	22	49	9
52	30	60	89	19	49	79	4.8509	38	68	8
53	32	64	96	28	60	91	23	55	87	7
54	34	68	2.0802	36	70	4.1604	38	72	6.2406	6
55	36	72	08	44	81	17	53	89	25	5
56	38	76	15	53	91	25	67	5.5506	44	4
57	40	80	21	61	3.4702	42	82	22	63	3
58	42	85	27	70	12	54	97	39	82	2
59	44	89	34	78	23	67	4.8612	56	6.2501	1
60	0.6947	1.8898	2.0840	2.7786	3.4733	4.1680	6.8626	5.5573	6.2519	0
	1	2	3	4	5	6	7	8	9	

	1	2	3	4	5	6	7	8	9	10
0	0 7193	1.4387	2.1580	2.8774	3.5967	4.3160	5.0354	5.7547	6.4741	60
1	91	83	74	66	57	48	40	31	23	59
2	89	79	68	58	47	36	26	15	05	58
3	87	75	62	49	37	24	11	5.7498	6.4686	57
4	85	71	56	41	27	12	97	82	68	56
5	83	67	50	33	17	00	5.0283	66	50	55
6	81	63	44	25	07	4.3088	69	50	32	54
7	79	58	38	17	3.5896	75	54	34	18	53
8	77	54	32	09	86	63	40	18	6.4695	52
9	75	50	26	01	76	51	26	02	77	51
10	73	46	20	2.8693	66	39	12	5.7386	59	50
11	71	42	13	84	56	27	5.0198	69	40	49
12	69	38	07	76	46	15	84	53	22	48
13	67	34	01	68	36	03	70	37	04	47
14	65	20	2.1495	60	25	4.2990	55	20	6.4485	46
15	63	26	89	52	15	78	41	04	67	45
16	61	22	83	44	05	66	27	5.7288	49	44
17	59	18	77	36	3.5795	54	13	72	31	43
18	57	14	71	28	85	41	5.0098	55	12	42
19	55	10	65	20	75	29	84	39	6.4394	41
20	53	06	59	12	65	17	70	23	76	40
21	51	02	52	03	54	05	56	06	57	39
22	49	1.4298	46	2.8595	44	4.2893	42	09	39	38
23	47	94	40	87	34	81	28	5.7174	21	37
24	45	89	34	79	24	68	13	58	02	36
25	43	85	28	71	14	56	4.9999	42	6.4284	35
26	41	81	22	63	04	44	85	26	66	34
27	39	77	16	54	3.5693	82	70	09	47	33
28	37	73	10	46	83	20	56	5.7093	29	32
29	35	69	04	38	73	07	42	76	11	31
30	33	65	2.1398	30	63	4.2795	28	60	6.4193	30
31	31	61	92	22	53	83	14	43	74	29
32	28	57	85	14	42	70	4.9899	27	56	28
33	26	53	79	06	32	58	85	11	37	27
34	24	49	73	2.8497	22	46	70	5.6994	19	26
35	22	45	67	89	12	34	56	78	01	25
36	20	41	61	81	02	22	42	62	6.4083	24
37	18	36	55	73	3.5591	09	27	46	64	23
38	16	32	49	65	81	4.2697	13	30	46	22
39	14	28	42	56	71	85	4.9799	13	27	21
40	12	24	36	48	61	73	85	5.6897	09	20
41	10	20	30	40	50	60	70	80	6.3990	19
42	08	16	24	32	40	48	56	64	72	18
43	06	12	18	24	30	35	41	47	53	17
44	04	08	12	16	20	23	27	31	35	16
45	02	04	06	08	10	11	13	15	17	15
46	00	00	00	2.8399	3.5499	4.2599	4.9699	5.6798	2.3898	14
47	0.7098	1.4196	2.1293	91	89	87	85	82	80	13
48	96	91	87	83	79	74	70	66	61	12
49	94	87	81	75	69	62	56	49	43	11
50	92	83	75	66	58	50	41	33	24	10
51	90	79	69	58	48	38	27	17	06	9
52	88	75	63	50	38	25	13	00	6.3788	8
53	86	71	57	42	28	13	4.9599	5.6684	70	7
54	83	67	50	34	17	00	84	67	51	6
55	81	63	44	25	07	4.2488	69	50	32	5
56	79	59	38	17	3.5397	76	55	34	14	4
57	77	54	32	09	86	63	40	18	6.3695	3
58	75	50	26	01	76	51	26	02	77	2
59	73	46	19	2.8292	66	39	12	5.6585	58	1
60	0.7071	1.4142	2.1213	2.8284	3.5356	4.2427	4.9498	5.6569	6.3640	0
	1	2	3	4	5	6	7	8	9	

DEPARTURE 44 DEGREES.

163

1	2	3	4	5	6	7	8	9	10
0 0.6947	1.3893	2.0840	2.7786	3.4733	4.1680	4.8626	5.5572	6.2519	6.9466
1 49	97	46	95	44	92	41	90	38	86
2 51	1.3902	52	2.7803	54	4.1705	56	5.5606	57	58
3 53	06	59	12	65	17	70	23	76	57
4 55	10	65	20	75	29	84	89	94	56
5 57	14	71	28	85	42	99	56	6.2613	56
6 59	18	77	36	96	55	4.8714	73	82	54
7 61	22	84	45	2.4806	67	28	90	51	53
8 63	27	90	53	17	80	43	5.5706	70	52
9 65	31	96	62	*	27	92	58	23	89
10 68	35	2.0903	70	38	4.1805	78	40	6.2708	50
11 70	39	09	78	48	18	87	57	26	49
12 72	43	15	87	59	30	4.8802	74	45	48
13 74	47	21	95	69	42	16	90	63	47
14 76	52	27	2.7903	79	55	31	5.5806	82	46
15 78	56	34	12	90	67	46	23	6.2801	45
16 80	60	40	20	2.4900	80	60	40	20	44
17 82	64	46	28	11	93	75	57	39	48
18 84	68	53	37	21	4.1905	89	74	58	42
19 86	72	59	45	31	17	4.8903	90	76	41
20 88	77	65	53	42	30	18	5.5906	95	40
21 90	81	71	62	52	42	33	28	6.2914	39
22 93	85	78	70	63	55	48	40	33	88
23 95	89	84	78	73	68	62	57	51	87
24 97	93	90	86	83	80	76	78	69	86
25 99	97	96	95	94	92	91	90	88	85
26 0.7001	1.4002	2.1002	2.8003	2.5004	4.2005	4.9006	5.6006	6.3007	84
27 03	06	09	12	15	17	20	23	26	83
28 05	10	15	20	25	29	34	39	44	82
29 07	14	21	28	35	42	49	56	63	81
30 09	18	27	36	46	55	64	73	82	80
31 11	22	34	45	56	67	78	90	6.3101	29
32 13	26	40	55	66	79	92	5.6106	19	28
33 15	31	46	61	77	92	4.9107	22	38	27
34 17	35	52	70	87	4.2104	22	39	57	26
35 20	39	59	78	98	17	37	56	76	25
36 22	43	65	86	2.5108	29	51	72	94	24
37 24	47	71	94	18	42	65	89	6.3212	23
38 26	51	77	2.8103	29	54	80	5.6206	31	22
39 28	55	83	11	39	66	94	22	49	21
40 30	60	89	19	49	79	4.9209	38	68	20
41 32	64	96	28	60	92	23	55	87	19
42 34	68	2.1102	36	70	4.2203	37	71	6.3305	18
43 36	72	08	44	80	16	52	88	24	17
44 38	76	14	52	91	29	67	5.6305	43	16
45 40	80	20	60	2.5201	41	81	21	61	15
46 42	84	27	69	11	53	95	38	80	14
47 44	89	33	77	22	66	4.9310	54	99	13
48 46	93	39	85	32	78	24	70	6.3417	12
49 48	97	45	94	42	90	39	87	86	11
50 51	1.4101	52	2.8202	53	4.2303	54	5.6404	55	10
51 53	05	58	10	63	15	68	20	78	9
52 55	09	64	18	73	28	82	37	91	8
53 57	13	70	27	84	40	97	54	6.3510	7
54 59	17	76	35	94	52	4.9411	70	28	6
55 61	22	82	43	2.5304	65	26	86	47	5
56 63	26	88	51	14	77	40	5.6502	65	4
57 65	30	95	60	25	89	54	19	84	3
58 67	34	2.1201	68	35	4.2402	69	36	6.3603	2
59 69	38	07	76	45	14	83	52	21	1
60 0.7071	1.4142	2.1213	2.8284	3.5350	4.2427	4.9496	5.6569	6.3640	0
1	2	3	4	5	6	7	8	9	

LATITUDE 45 DEGREES.

EXPLANATION OF THE TRAVERSE TABLE.

Latitude is the distance made in a north or south direction on a given meridian, by running a line at any bearing less than 90 degrees from that meridian; or it is the distance on any line parallel to a given meridian. When the given meridian is assumed at true north and south, the distance made in running on a course in a northerly direction is termed north latitude, or northing; and if ran southerly, the distance south is termed south latitude, or southing.

Departure is the distance perpendicular to the given meridian that is made by running on a given course. East departure, or easting, is when the line is run east of the meridian. West departure, or westing, is when the line is run west of the meridian.

Example. Let M N represent the meridian, such as any line (generally assumed north and south); let the point M = north, and the point N = south; let the bearing of the line N C = N. 44°, 17' E., and the distance N C = 9,74 chains = 9 chains and 74 links.

Here N B is the latitude made, and B C is the departure perpendicular to the meridian or base line N M; consequently, N B is north latitude or northing, and B C is the east departure or easting.

Or, latitude N B = cosine of the $\angle$ C N B $\times$ by the distance N C. And departure B C = sine of the $\angle$ C N B $\times$ by the distance N C.



The degrees are at the top and bottom, and the minutes in the outer columns. The distances 2, 8, etc., to 9, at top and bottom, may be used as chains, tenths of a chain, or links.

<i>Example.</i> Lat. N C for 44° 17', and distance 9 chains =	6,4431
Lat. N C for 44° 17', and distance 90 chains, remove the decimal point one place to the right =	64,431
Lat. N C for 44° 17', and distance 900 chains, remove the point two places to the right =	644,31
Lat. N C for 44° 17', and distance 90 links, or ,9 chains, remove the point one place to the left =	0,64431
Lat. N C for 44° 17', and distance 9 links, or ,09 chains, remove the point two places to the left =	/ 0,064431

Application. Given the course N 44° 17' E., and distance N C = 97,48 chains, to find the latitude N B and departure B C.

Take a piece of card paper, two inches wide, and as long as the width of the page; have it ruled, and numbered 1, 2, 8, etc., to 9, similar to the tables. Lay this across, from 17' to 23', under latitude 44°. Lay a small weight on the guide paper; then under the edge of the paper you will have the required numbers to be taken out.

Under 9 chains we have 6,4431 ∴ for ,90 chains we have	64,4310
Under 7 chains we have	5,0113
Under 4 chains we have 2,8636 ∴ for ,4 chains we have	0,2864
Under 8 chains we have 5,7272 ∴ for ,08 chains we have	0,0573

Latitude N B = 69,7860
chains.

Let the distance be 9748 links.

Under 9 we have 6,4431 ∴ 9000, remove the point 3 places =	6443,1
Under 7 we have 5,0113 ∴ 700, remove the point 2 places =	501,13
Under 4 we have 2,8636 ∴ 40, remove the point 1 place =	28,636
Under 8 we have 5,7272 ∴ 8,	= 5,723

6975,689
Latitude N C = 6975,689 links.

TABLE II.—*Expansion of Solids in Direction of their Lengths, from 32° to 212° (Change of Temperature 180°).*

Name of Substance.	Authority.	Vulgar fractions.	Dec. frac. 180° ch.	con. in 10° ch.	orex.
Platinum.	Troughton.	1 in 1008	0.0008618	0.0000551	
do.	Dulong & Petit.	1 in 1131	0.0008242	0.0000458	
do.	Borda.	1 in 1167	0.0008566	0.0000476	
do.	Hasler.	1 in 1082	0.0009242	0.0000512	
Mean of the four.		1 in 1094	0.0009142	0.0000508	
Glass, white barometer tube.	Smeaton.	1 in 1175	0.0008510	0.0000472	
“ flint.	Brunner.	1 in 1248	0.0008012	0.0000445	
“ tube, without lead (4 sorts).	Lavoisier & Laplace.	1 in 1115	0.0008969	0.0000492	
“ “ with lead.	Brunner.	1 in 1142	0.0008757	0.0000486	
Steel, not tempered.	Lavoisier & Laplace.	1 in 927	0.0010788	0.0000599	
“ tempered yellow at 149°.	do.	1 in 807	0.0012396	0.0000699	
“ rod.	Major General Roy.	1 in 847	0.0011807	0.0000656	
“ blistered.	Smeaton.	1 in 870	0.0011500	0.0000682	
“ tempered.	do.	1 in 816	0.0012583	0.0000699	
Iron wire.	Troughton.	1 in 840	0.0011899	0.0000681	
“ cast (prism).	Brunner.	1 in 812	0.0012350	0.0000686	
“ bar.	Smeaton.	1 in 785	0.0012583	0.0000699	
“ forged.	Hasler.	1 in 797	0.0012584	0.0000696	
Copper, mean of three specimens.	Lavoisier & Laplace.	1 in 819	0.0012206	0.0000678	
“ “	do.	1 in 552	0.0017122	0.0000951	
“ hammered.	Troughton.	1 in 521	0.0019188	0.0001066	
“ “	Smeaton.	1 in 598	0.0017000	0.0000944	
“ eight parts, tin 1.	Brunner.	1 in 581	0.0017211	0.0000956	
Brass, cast.	Smeaton.	1 in 550	0.0018167	0.0001009	
“ wire.	do.	1 in 538	0.0018750	0.0001042	
“ Hamburg.	do.	1 in 517	0.0019333	0.0001074	
“ English angular.	Roy.	1 in 589	0.0018566	0.0001081	
“ English round rod.	do.	1 in 528	0.0019845	0.0001053	
“ mean of three specimens.	do.	1 in 528	0.0018980	0.0001053	
Antimony.	Lavoisier & Laplace.	1 in 532	0.0018797	0.0001044	
Bismuth.	Smeaton.	1 in 923	0.0010833	0.0000602	
Lead.	do.	1 in 719	0.0013917	0.0000772	
Tin, fine.	do.	1 in 349	0.0028667	0.0001592	
“ grain.	do.	1 in 485	0.0020333	0.0001257	
Zinc.	do.	1 in 403	0.0024883	0.0001229	
Pine, white, Norway.	do.	1 in 340	0.0029417	0.0001634	
	Captain Kater.		0.0004083	0.0000227	

Example. A surveyor had adjusted his chain at a temperature of 60°, the standard chain of 66 feet or 100 links being cut in the floor of a public hall. During the time that he measured a line of 8000 links, the mean temperature had been 105° Required the *true length of the line*, the chain being of iron wire.

From col. 10° correction for 1° = 0,00000685

45°

to be added, ,00030825

1

$$1 + c = 1,00030825 \quad \text{here } c = \text{correction.}$$

8000

8002,466 links = true length.

$(1 + c) \cdot L = \text{true length, when chain or box expanded.}$

$(1 - c) \cdot L = \text{true length, when chain contracted.}$

Here $L = \text{measured length, and } c = \text{tabular correction for change of temperature.}$

The above correction 2,466 links would be subtracted if the mean temperature was 15° above zero (Fahrenheit).

Note 1. If the above line had been measured by a Norway pine pole or rod, 15 feet long (*see measuring of base lines*), the correction would only be 0,82 link, nearly eight tenths of a link in a mile.

Note 2. It appears from this table that there is no sensible or practical benefit to be derived in using a steel chain, in reference to expansion or contraction. However, steel chains are to be preferred, as they are not liable to bend like the iron wire chain.

TABLE III.—To Reduce Links to Feet.

k.	Feet	100	200	300	400	500	600	700	800	900
0	0.00	66.00	132.00	198.00	264.00	330.00	396.00	462.00	528.00	594.00
1	0.66	66.66	132.66	198.66	264.66	330.66	396.66	462.66	528.66	594.66
2	1.32	67.32	133.32	199.32	265.32	331.32	397.32	463.32	529.32	595.32
3	1.98	67.98	133.98	199.98	265.98	331.98	397.98	463.98	529.98	595.98
4	2.64	68.64	134.64	200.64	266.64	332.64	398.64	464.64	530.64	596.64
5	3.30	69.30	135.30	201.30	267.30	333.30	399.30	465.30	531.30	597.30
6	3.96	69.96	135.96	201.96	267.96	333.96	399.96	465.96	531.96	597.96
7	4.62	70.62	136.62	202.62	268.62	334.62	400.62	466.62	532.62	598.62
8	5.28	71.28	137.28	203.28	269.28	335.28	401.28	467.28	533.28	599.28
9	5.94	71.94	137.94	203.94	269.94	335.94	401.94	467.94	533.94	599.94
10	6.60	72.60	138.60	204.60	270.60	336.60	402.60	468.60	534.60	600.60
11	7.26	73.26	139.26	205.26	271.26	337.26	403.26	469.26	535.26	601.26
12	7.92	73.92	139.92	205.92	271.92	337.92	403.92	469.92	535.92	601.92
13	8.58	74.58	140.58	206.58	272.58	338.58	404.58	470.58	536.58	602.58
14	9.24	75.24	141.24	207.24	273.24	339.24	405.24	471.24	537.24	603.24
15	9.90	75.90	141.90	207.90	273.90	339.90	405.90	471.90	537.90	603.90
16	10.56	76.56	142.56	208.56	274.56	340.56	406.56	472.56	538.56	604.56
17	11.22	77.22	143.22	209.22	275.22	341.22	407.22	473.22	539.22	605.22
18	11.88	77.88	143.88	209.88	275.88	341.88	407.88	473.88	539.88	605.88
19	12.54	78.54	144.54	210.54	276.54	342.54	408.54	474.54	540.54	606.54
20	13.20	79.20	145.20	211.20	277.20	343.20	409.20	475.20	541.20	607.20
21	13.86	79.86	145.86	211.86	277.86	343.86	409.86	475.86	541.86	607.86
22	14.52	80.52	146.52	212.52	278.52	344.52	410.52	476.52	542.52	608.52
23	15.18	81.18	147.18	213.18	279.18	345.18	411.18	477.18	543.18	609.18
24	15.84	81.84	147.84	213.84	279.84	345.84	411.84	477.84	543.84	609.84
25	16.50	82.50	148.50	214.50	280.50	346.50	412.50	478.50	544.50	610.50
26	17.16	83.16	149.16	215.16	281.16	347.16	413.16	479.16	545.16	611.16
27	17.82	83.82	149.82	215.82	281.82	347.82	413.82	479.82	545.82	611.82
28	18.48	84.48	150.48	216.48	282.48	348.48	414.48	480.48	546.48	612.48
29	19.14	85.14	151.14	217.14	283.14	349.14	415.14	481.14	547.14	613.14
30	19.80	85.80	151.80	217.80	283.80	349.80	415.80	481.80	547.80	613.80
31	20.46	86.46	152.46	218.46	284.46	350.46	416.46	482.46	548.46	614.46
32	21.12	87.12	153.12	219.12	285.12	351.12	417.12	483.12	549.12	615.12
33	21.78	87.78	153.78	219.78	285.78	351.78	417.78	483.78	549.78	615.78
34	22.44	88.44	154.44	220.44	286.44	352.44	418.44	484.44	550.44	616.44
35	23.10	89.10	155.10	221.10	287.10	353.10	419.10	485.10	551.10	617.10
36	23.76	89.76	155.76	221.76	287.76	353.76	419.76	485.76	551.76	617.76
37	24.42	90.42	156.42	222.42	288.42	354.42	420.42	486.42	552.42	618.42
38	25.08	91.08	157.08	223.08	289.08	355.08	421.08	487.08	553.08	619.08
39	25.74	91.74	157.74	223.74	289.74	355.74	421.74	487.74	553.74	619.74
40	26.40	92.40	158.40	224.40	290.40	356.40	422.40	488.40	554.40	620.40
41	27.06	93.06	159.06	225.06	291.06	357.06	423.06	489.06	555.06	621.06
42	27.72	93.72	159.72	225.72	291.72	357.72	423.72	489.72	555.72	621.72
43	28.38	94.38	160.38	226.38	292.38	358.38	424.38	490.38	556.38	622.38
44	29.04	95.04	161.04	227.04	293.04	359.04	425.04	491.04	557.04	623.04
45	29.70	95.70	161.70	227.70	293.70	359.70	425.70	491.70	557.70	623.70
46	30.36	96.36	162.36	228.36	294.36	360.36	426.36	492.36	558.36	624.36
47	31.02	97.02	163.02	229.12	295.02	361.02	427.02	493.02	559.02	625.02
48	31.68	97.68	163.68	229.78	295.68	361.68	427.68	493.68	559.68	625.68
49	32.34	98.34	164.34	230.34	296.34	362.34	428.34	494.34	560.34	626.34
50	33.00	99.00	165.00	231.00	297.00	363.00	429.00	495.00	561.00	627.00
51	33.66	99.66	165.66	231.66	297.66	363.66	429.66	495.66	561.66	627.66
52	34.32	100.32	166.32	232.32	298.32	364.32	430.32	496.32	562.32	628.32
53	34.98	100.98	166.98	232.98	298.98	364.98	430.98	496.98	562.98	628.98
54	35.64	101.64	167.64	233.64	299.64	365.64	431.64	497.64	563.64	629.64
55	36.30	102.30	168.30	234.30	300.30	366.30	432.30	498.30	564.30	630.30
56	36.96	102.96	168.96	234.96	300.96	366.96	432.96	498.96	564.96	630.96
57	37.62	103.62	169.62	235.62	301.62	367.62	433.62	499.62	565.62	631.62
58	38.28	104.28	170.28	236.28	302.28	368.28	434.28	500.28	566.28	632.28
59	38.94	104.94	170.94	236.94	302.94	368.94	434.94	500.94	566.94	632.94
60	39.60	105.60	171.60	237.60	303.60	369.60	435.60	501.60	567.60	633.60

TABLE III.—To Reduce Links to Feet.

167

lk.	Feet.	100	200	300	400	500	600	700	800	900
61	40.26	106.26	172.26	238.26	304.26	370.26	436.26	502.26	568.26	634.26
62	40.92	106.92	172.92	238.92	304.92	370.92	436.92	502.92	568.92	634.92
63	41.58	107.58	173.58	239.58	305.58	371.58	437.58	503.58	569.58	635.58
64	42.24	108.24	174.24	240.24	306.24	372.24	438.24	504.24	570.24	636.24
65	42.90	108.90	174.90	240.90	306.90	372.90	438.90	504.90	570.90	636.90
66	43.56	109.56	175.56	241.56	307.56	373.56	439.56	505.56	571.56	637.56
67	44.22	110.22	176.22	242.22	308.22	374.22	440.22	506.22	572.22	638.22
68	44.88	110.88	176.88	242.88	308.88	374.88	440.88	506.88	572.88	638.88
69	45.54	111.54	177.54	243.54	309.54	375.54	441.54	507.54	573.54	639.54
70	46.20	112.20	178.20	244.20	310.20	376.20	442.20	508.20	574.20	640.20
71	46.86	112.86	178.86	244.86	310.86	376.86	442.86	508.86	574.86	640.86
72	47.52	113.52	179.52	245.52	311.52	377.52	443.52	509.52	575.52	641.52
73	48.18	114.18	180.18	246.18	312.18	378.18	444.18	510.18	576.18	642.18
74	48.84	114.84	180.84	246.84	312.84	378.84	444.84	510.84	576.84	642.80
75	49.50	115.50	181.50	247.50	313.50	379.50	445.50	511.50	577.50	643.50
76	50.16	116.16	182.16	248.16	314.16	380.16	446.16	512.16	578.16	644.16
77	50.82	116.82	183.82	248.82	314.82	380.82	446.82	512.82	578.88	644.82
78	51.48	117.48	183.48	249.48	315.48	381.48	447.48	513.48	579.48	645.48
79	52.14	118.14	188.14	250.14	316.14	382.14	448.14	514.14	580.14	646.14
80	52.80	118.80	184.80	250.80	316.80	382.80	448.80	514.80	580.80	646.80
81	53.46	119.46	185.46	251.46	317.46	383.46	449.46	515.46	581.46	647.46
82	54.12	120.12	186.12	252.12	318.12	384.12	450.12	516.12	582.12	648.12
83	54.78	120.78	186.78	252.78	318.78	384.78	450.78	516.78	582.78	648.78
84	55.44	121.44	187.44	253.44	319.44	385.44	451.44	517.44	583.44	649.44
85	56.10	122.10	188.10	254.10	320.10	386.10	452.10	518.10	584.10	650.10
86	56.76	123.76	188.76	254.76	320.76	386.76	452.76	518.76	584.76	650.76
87	57.42	123.42	189.42	265.42	321.42	387.42	453.42	519.42	585.42	651.42
88	58.08	124.08	190.08	256.08	322.08	388.08	454.08	520.08	586.08	652.08
89	58.74	124.74	190.74	256.74	322.74	388.74	454.74	520.74	586.74	653.74
90	59.40	125.40	191.40	257.40	323.40	389.40	455.40	521.40	587.40	653.40
91	60.06	126.06	192.06	258.06	324.06	390.06	456.06	522.06	588.06	654.06
92	60.72	126.72	192.72	258.72	324.72	390.72	456.72	522.72	588.72	654.72
93	61.38	127.38	193.38	259.38	325.38	391.38	457.38	523.38	589.38	655.38
94	62.04	128.04	194.04	260.04	326.04	392.04	458.04	524.04	590.04	656.04
95	62.70	128.70	164.70	260.70	326.70	392.70	458.70	524.70	590.70	656.70
96	63.36	129.36	195.36	261.36	327.36	393.36	459.36	525.36	591.36	657.36
97	64.02	130.02	196.02	262.02	328.02	394.02	460.02	526.02	592.02	658.02
98	64.68	130.68	196.68	262.68	328.68	394.68	460.68	526.68	592.68	658.68
99	65.34	131.34	197.34	263.34	329.34	395.34	461.34	527.34	593.34	659.34

Lks.	1000	2000	3000	4000	5000	6000	7000	8000	9000
000	660	1320	1980	2640	3300	3960	4620	5280	5940
100	726	1386	2046	2706	3366	4026	4686	5346	6006
200	792	1452	2112	2772	3432	4092	4752	5412	6072
300	858	1518	2178	2838	3498	4158	4818	5478	6138
400	924	1584	2244	2904	3564	4224	4884	5544	6204
500	990	1650	2310	2970	3630	4290	4950	5610	6270
600	1056	1716	2376	3036	3696	4356	5016	5676	6336
700	1122	1782	2442	3102	3762	4422	5082	5742	6402
800	1188	1848	2508	3168	3828	4488	5148	5808	6468
900	1254	1914	2574	3234	3894	4554	5214	5874	6534

Example. Reduce 9664 links to feet.

From the bottom table, under 9000 at top, and opposite 600 in the left hand column, we find 6336

Opposite 64, in upper table, and under 6— 42,24

6378,24 feet.

TABLE IV.—To Reduce Feet to Links.

Feet	Links	100	200	300	400	500	600	700	800	900
0	0.00	151.52	303.03	454.55	606.06	757.58	909.09	1060.60	1212.12	1363.64
1	1.515	153.03	304.55	5.07	7.58	9.10	910.61	2.12	3.64	5.16
2	3.03	154.55	6.06	7.58	9.09	760.61	2.12	3.63	5.15	6.67
3	4.55	6.07	7.58	9.10	610.61	2.13	3.64	5.15	6.67	8.19
4	6.06	7.58	9.09	460.61	2.12	3.64	5.15	6.66	8.18	9.70
5	7.58	9.10	310.61	2.13	4.64	5.16	6.67	8.18	9.70	1371.22
6	9.09	160.61	2.12	3.64	5.15	6.67	8.18	9.69	1221.21	2.73
7	10.61	2.13	3.64	5.16	6.67	8.19	9.70	1071.21	2.73	4.25
8	12.12	3.64	5.15	6.67	8.18	9.70	921.21	2.72	4.24	6.76
9	13.64	5.16	6.67	8.19	9.70	771.22	2.73	4.24	5.76	7.28
10	15.15	6.67	8.18	9.70	621.21	2.73	4.24	5.75	7.27	8.79
11	16.67	8.19	9.70	471.22	2.73	4.25	5.76	7.27	8.79	1380.31
12	18.18	9.70	321.21	2.73	4.24	5.76	7.27	8.78	1230.30	1.82
13	19.70	171.22	2.73	4.25	5.76	7.28	8.79	1080.30	1.82	3.34
14	21.21	2.73	4.24	5.76	7.27	8.79	980.30	1.81	3.33	4.85
15	22.73	4.25	5.76	7.28	8.79	780.31	1.82	3.33	4.85	6.37
16	24.24	5.76	7.27	8.79	630.30	1.82	3.33	4.84	6.36	7.88
17	25.76	7.28	8.79	480.31	1.82	3.34	4.85	6.36	7.88	9.40
18	27.27	8.79	330.30	1.82	3.33	4.85	6.36	7.87	9.39	1390.91
19	28.79	180.31	1.82	3.34	4.85	6.37	7.88	9.39	1240.91	2.43
20	30.30	1.82	3.33	4.85	6.36	7.88	9.39	1090.90	2.42	3.94
21	31.82	3.34	4.85	6.37	7.85	9.40	940.91	2.42	4.09	5.46
22	33.33	4.85	6.36	7.88	8.39	790.91	2.42	3.93	5.45	6.97
23	34.85	6.37	7.88	9.40	640.91	2.43	3.94	5.95	6.97	8.49
24	36.36	7.88	9.39	490.91	2.42	3.94	5.45	6.96	8.48	1400.00
25	37.88	9.40	340.91	2.43	3.94	5.46	6.97	8.45	1250.00	1.52
26	39.39	190.91	2.42	3.94	5.45	6.97	8.48	9.99	1.51	3.08
27	40.91	2.43	3.94	5.56	6.97	8.49	950.00	1101.51	3.03	4.55
28	42.43	3.95	5.46	6.98	8.49	800.01	1.52	3.03	4.54	6.07
29	43.94	4.46	6.97	8.49	650.00	1.52	3.03	4.54	6.06	7.58
30	45.46	6.98	8.49	500.01	1.52	3.04	4.55	6.06	7.58	9.10
31	46.97	8.49	350.00	1.52	3.03	4.55	6.06	7.57	9.09	1410.61
32	48.48	200.00	1.51	3.03	4.54	6.06	7.57	9.08	1260.60	2.12
33	50.00	1.52	3.03	4.55	6.06	7.58	9.09	1110.60	2.12	3.64
34	51.52	3.04	4.55	6.07	7.58	9.10	960.61	2.12	3.64	5.16
35	53.03	4.55	6.06	7.58	9.09	810.61	2.12	3.63	5.15	6.67
36	54.54	6.06	7.58	9.09	660.60	2.12	3.63	5.14	6.66	8.18
37	56.06	7.58	9.09	510.61	2.12	3.64	5.15	6.66	8.18	1420.70
38	57.57	9.09	360.60	2.12	3.63	5.15	6.66	8.17	9.69	1.21
39	59.09	210.61	2.12	3.64	5.15	6.67	8.18	9.69	1271.22	2.73
40	60.60	2.12	3.63	5.15	6.66	8.18	9.69	1121.20	2.72	4.24
41	62.12	3.64	5.15	6.67	8.18	9.70	971.21	2.72	4.24	5.76
42	63.63	5.15	6.66	8.18	9.69	821.21	2.72	4.23	5.75	7.27
43	65.15	6.17	8.18	9.70	521.21	2.73	4.24	5.75	7.27	8.79
44	66.66	8.18	9.69	521.21	2.72	4.24	5.75	7.26	8.78	1430.30
45	68.18	9.70	371.21	2.73	4.24	5.76	7.27	8.78	1280.30	1.82
46	69.69	221.21	2.72	4.24	5.75	7.27	8.78	1130.29	1.81	3.33
47	71.21	2.73	4.24	5.76	7.27	8.79	980.30	1.81	3.33	4.85
48	72.73	4.25	5.76	7.28	8.79	830.31	1.82	3.33	4.85	6.37
49	74.24	5.76	7.27	8.79	680.30	1.82	3.33	4.84	6.36	7.88
50	75.77	7.29	8.80	530.32	1.83	3.35	4.86	6.37	7.89	9.41
51	77.27	8.79	380.30	1.82	3.33	4.85	6.36	7.87	9.39	1440.91
52	78.79	230.31	1.82	3.34	4.85	6.37	7.88	9.38	1290.91	2.43
53	80.30	1.82	3.33	4.85	6.36	7.88	9.39	1140.90	2.42	3.94
54	81.81	3.33	4.84	6.36	7.87	9.39	990.90	2.41	3.93	5.45
55	83.33	4.85	6.36	7.88	9.39	840.91	2.42	3.93	5.45	6.97
56	84.85	6.37	7.88	9.40	690.91	2.43	3.94	5.45	6.97	8.49
57	86.36	7.88	9.39	540.91	2.43	3.94	5.45	6.96	8.48	1450.00
58	87.88	9.40	390.91	2.43	4.94	5.46	6.97	8.48	1300.00	1.52
59	89.29	240.91	2.42	3.94	5.45	6.97	8.48	9.96	1.51	3.03
60	90.91	2.43	3.94	5.46	6.97	8.49	1000.00	1151.51	3.03	4.55

TABLE IV.—To Reduce Feet to Links.

16

ft.	Links	100	200	300	400	500	600	700	800	900
61	92.42	243.94	394.45	546.97	698.48	850.00	1001.51	1153.02	1304.54	1456.06
62	98.94	5.46	6.97	8.49	700.00	1.52	3.03	4.54	6.06	7.57
63	96.06	6.97	8.48	9.99	1.51	3.03	4.54	6.06	7.57	9.08
64	96.57	248.99	400.00	551.52	703.03	854.55	1006.06	1157.57	1309.09	1460.60
65	98.08	250.00	401.51	553.02	704.53	856.04	1007.55	1159.06	1310.57	1462.08
66	100.00	251.52	403.03	554.54	706.04	857.55	1009.06	1160.57	1312.08	1463.59
67	101.52	3.04	4.55	6.07	7.58	9.09	1010.61	2.12	3.63	5.14
68	103.03	4.55	6.06	7.58	9.09	1012.12	2.12	3.63	5.14	6.65
69	104.55	6.07	7.58	9.10	1013.63	2.12	3.64	5.15	6.67	8.18
70	106.06	7.58	9.09	1015.14	2.12	3.64	1015.15	6.66	8.17	9.68
71	107.58	9.10	1016.65	2.18	3.64	5.16	6.67	8.18	9.70	1471.2
72	109.09	10.61	2.12	3.64	5.16	6.67	8.18	9.69	1121.21	2.7
73	110.61	2.12	3.64	5.16	6.67	8.19	9.70	1171.21	2.73	4.2
74	112.12	3.64	5.16	6.67	8.18	9.70	1023.21	2.72	4.24	5.7
75	113.64	5.16	6.67	8.19	9.70	1024.72	2.73	4.24	5.76	7.2
76	115.15	6.67	8.18	9.70	1026.23	2.73	4.24	5.75	7.27	8.7
77	116.67	8.19	9.70	1027.74	2.73	4.25	5.76	7.27	8.79	1480.8
78	118.18	9.70	1029.25	2.73	4.24	5.76	7.27	8.75	1029.25	1.8
79	119.70	10.21	2.73	4.25	5.76	7.28	8.79	1030.76	1.82	3.3
80	121.21	2.73	4.24	5.76	7.27	8.79	1032.27	1.81	3.33	4.8
81	122.73	4.25	5.76	7.28	8.79	1033.78	1.82	3.33	4.85	6.3
82	124.24	5.76	7.27	8.79	1035.29	1.82	3.33	4.85	6.33	7.8
83	125.76	7.28	8.79	1036.80	1.82	3.34	4.85	6.36	7.88	9.4
84	127.27	8.79	1038.31	1.82	3.33	4.85	6.36	7.87	9.39	1490.9
85	128.79	10.30	1.82	3.34	4.85	6.37	7.88	9.39	1040.41	2.4
86	130.30	11.82	3.33	4.85	6.36	7.88	9.39	1190.90	2.42	3.9
87	131.82	13.34	4.85	6.37	7.88	9.40	1040.91	2.42	4.00	5.5
88	133.33	14.85	6.36	7.88	9.39	1042.42	2.42	3.93	5.45	6.9
89	134.85	16.37	7.88	9.40	1043.93	2.43	3.93	5.45	6.97	8.4
90	136.36	17.88	9.39	1045.44	2.42	3.94	5.45	6.96	8.48	1500.0
91	137.88	19.40	10.90	2.42	3.94	5.46	6.97	8.48	1250.00	1.5
92	139.39	20.91	2.42	3.93	5.45	6.97	8.48	9.99	1.51	3.0
93	140.91	1.43	3.94	5.45	6.97	8.49	1050.00	1201.51	3.03	4.5
94	142.42	2.96	5.45	6.98	8.48	9.00	1.51	3.03	4.54	6.0
95	143.94	4.46	6.97	8.48	750.00	1.52	3.03	4.54	6.06	7.5
96	145.46	6.98	8.49	9.00	1.52	3.03	4.55	6.06	7.58	9.1
97	146.97	8.49	9.00	1.52	3.03	4.55	6.06	7.57	9.00	1510.6
98	148.48	10.00	1.51	3.03	4.54	6.06	7.58	9.08	1260.60	2.1
99	150.00	11.52	3.03	4.55	6.06	7.58	9.09	1210.60	2.12	3.6

Ft.	1000	2000	3000	4000	5000	6000	7000	8000	9000
0	1515.15	3030.30	4545.45	6060.61	7575.78	9090.91	10606.06	12121.21	13636.38
100	1666.67	3333.33	4999.99	6666.66	8333.33	10000.00	11666.66	13333.33	15000.00
200	1818.18	3636.36	5454.54	7272.72	8909.09	10581.81	12254.54	13927.27	15606.06
300	1969.70	3939.39	5909.09	7636.36	9363.63	11045.45	12727.27	14409.09	16111.11
400	2121.21	4242.42	6363.63	8181.81	9818.18	11500.00	13181.81	14863.63	16616.16
500	2272.73	4545.45	6818.18	8636.36	10272.72	11954.54	13636.36	15318.18	17121.21
600	2424.24	4848.48	7272.72	9090.90	10727.27	12409.09	14090.90	15772.72	17626.26
700	2575.76	5151.51	7727.27	9545.45	11181.81	12863.63	14545.45	16227.27	18131.31
800	2727.27	5454.55	8181.81	10000.00	11636.36	13318.18	15000.00	16681.81	18636.36
900	2878.79	5757.58	8636.36	10454.54	12090.90	13772.72	15454.54	17136.36	19141.41

inches.	Feet.	Links.	inches.	Feet.	Links.	inches.	Feet.	Links.
1	0.083	0.126	5	0.416	0.631	9	0.750	1.126
2	0.176	0.253	6	0.500	0.757	10	0.833	1.262
3	0.250	0.379	7	0.583	0.883	11	0.947	1.388
4	0.333	0.505	8	0.667	1.010	12	1.000	1.515

Example. Reduce 9874 feet to links.

From the bottom table we find 9800 feet =

14848.48 links

From the upper table, 74 links =

112.12

14960.60 links

Deg.	Arc.	Deg.	Arc.	Deg.	Arc.	M	Arc.	S	Arc.
1	0.017453	61	1.064651	121	1.111848	1	291	1	5
2	0.034907	2	1.082104	2	1.129302	2	582	2	10
3	0.052360	3	1.099557	3	1.146750	3	873	3	15
4	0.069813	4	1.117011	4	1.164208	4	1164	4	19
5	0.087267	5	1.134464	5	1.181662	5	1454	5	24
6	0.104720	6	1.151917	6	1.199115	6	1745	6	29
7	0.122173	7	1.169371	7	1.216568	7	2036	7	34
8	0.139626	8	1.186824	8	1.234021	8	2327	8	39
9	0.157080	9	1.204277	9	1.251475	9	2618	9	44
10	0.174533	10	1.221731	10	1.268928	10	2909	10	49
11	0.191986	11	1.239184	11	1.286381	11	3201	11	52
12	0.209439	12	1.256637	12	1.303834	12	3491	12	58
13	0.226892	13	1.274090	13	1.321287	13	3782	13	63
14	0.244345	14	1.291543	14	1.338740	14	4072	14	68
15	0.261799	15	1.308997	15	1.356194	15	4363	15	73
16	0.279252	16	1.326450	16	1.373647	16	4654	16	78
17	0.296705	17	1.343903	17	1.391100	17	4945	17	83
18	0.314158	18	1.361356	18	1.408553	18	5235	18	87
19	0.331611	19	1.378809	19	1.426006	19	5526	19	92
20	0.349066	20	1.396263	20	1.443461	20	5818	20	97
21	0.366519	21	1.413716	21	1.460914	21	6109	21	102
22	0.383972	22	1.431169	22	1.478367	22	6400	22	107
23	0.401425	23	1.448622	23	1.495820	23	6690	23	112
24	0.418878	24	1.466075	24	1.513273	24	6981	24	116
25	0.436332	25	1.483529	25	1.530727	25	7272	25	121
26	0.453785	26	1.500982	26	1.548180	26	7563	26	126
27	0.471238	27	1.518435	27	1.565633	27	7854	27	131
28	0.488691	28	1.535888	28	1.583086	28	8145	28	136
29	0.506144	29	1.553341	29	1.600539	29	8436	29	141
30	0.523599	30	1.570793	30	1.617994	30	8727	30	145
31	0.541052	31	1.588249	31	1.635447	31	9018	31	150
32	0.558505	32	1.605702	32	1.652900	32	9308	32	155
33	0.575958	33	1.623155	33	1.670353	33	9599	33	160
34	0.593411	34	1.640608	34	1.687806	34	9890	34	165
35	0.610865	35	1.658062	35	1.705260	35	10181	35	170
36	0.628318	36	1.675515	36	1.722713	36	10471	36	175
37	0.645771	37	1.692968	37	1.740165	37	10762	37	179
38	0.663224	38	1.710421	38	1.757618	38	11053	38	184
39	0.680677	39	1.727874	39	1.775071	39	11344	39	189
40	0.698132	40	1.745329	40	1.792527	40	11636	40	194
41	0.715585	41	1.762782	41	1.809980	41	11926	41	199
42	0.733038	42	1.780235	42	1.827433	42	12217	42	204
43	0.750491	43	1.797688	43	1.844886	43	12508	43	208
44	0.767944	44	1.815141	44	1.862339	44	12799	44	213
45	0.785398	45	1.832595	45	1.879793	45	13090	45	218
46	0.802851	46	1.850048	46	1.897246	46	13381	46	223
47	0.820304	47	1.867501	47	1.914699	47	13672	47	228
48	0.837757	48	1.884954	48	1.932152	48	13963	48	233
49	0.855210	49	1.902407	49	1.949605	49	14254	49	238
50	0.872665	50	1.919862	50	1.967060	50	14544	50	242
51	0.890118	51	1.937315	51	1.984513	51	14835	51	247
52	0.907571	52	1.954768	52	2.001966	52	15126	52	252
53	0.925024	53	1.972221	53	2.019419	53	15417	53	257
54	0.942477	54	1.989674	54	2.036872	54	15708	54	262
55	0.959931	55	2.007128	55	2.054326	55	15999	55	267
56	0.977384	56	2.024581	56	2.071779	56	16290	56	272
57	0.994837	57	2.042034	57	2.089232	57	16581	57	277
58	1.012290	58	2.059487	58	2.106685	58	16872	58	281
59	1.029743	59	2.076940	59	2.124138	59	17162	59	286
60	1.047198	60	2.094395	60	2.141593	60	17453	60	291

Rule 1. Multiply the tabular number corresponding to the given angle by the radius, the product will be the length of the arc.

Rule 2. Multiply the length of the arc found by rule 1 by half the radius, the product will be the area of the sector of which the angle is given.

Here M = minutes, S = seconds.

TABLE VI.—Lengths of Circular Arcs obtained by having the Chord or Base, and Height or Versed Sine given.

$\frac{h}{b}$	Length.	$\frac{h}{b}$	Length.	$\frac{h}{b}$	Length.	$\frac{h}{b}$	Length.	$\frac{h}{b}$	Length.	$\frac{h}{b}$	Length.
.100	1.0265	.160	1.0669	.220	1.1245	.280	1.1974	.340	1.2843	.400	1.3832
1	70	1	78	1	56	1	89	1	58	1	50
2	75	2	86	2	66	2	1.2001	2	74	2	67
3	81	3	94	3	77	3	15	3	90	3	85
4	86	4	1.0703	4	89	4	28	4	1.2905	4	1.3902
.105	91	.165	11	.225	1.1300	.285	42	.345	21	.405	20
6	97	6	19	6	11	6	56	6	37	6	37
7	1.0803	7	28	7	22	7	70	7	52	7	55
8	08	8	37	8	33	8	83	8	68	8	72
9	14	9	45	9	44	9	97	9	84	9	90
.110	20	.170	54	.230	56	.290	1.2110	.350	1.3000	.410	1.4008
1	85	1	62	1	67	1	24	1	16	1	25
2	81	2	71	2	79	2	38	2	32	2	48
3	87	3	80	3	90	3	52	3	47	3	61
4	43	4	89	4	1.1402	4	66	4	63	4	79
.115	49	.175	98	.235	14	.295	79	.355	79	.415	97
6	55	6	1.0807	6	25	6	98	6	95	6	1.4115
7	61	7	16	7	36	7	1.2206	7	1.8112	7	32
8	67	8	25	8	48	8	20	8	28	8	50
9	73	9	34	9	60	9	35	9	44	9	68
.120	80	.180	43	.240	71	.300	50	.360	60	.420	86
1	86	1	52	1	83	1	64	1	76	1	1.4204
2	92	2	61	2	95	2	78	2	92	2	22
3	99	3	70	3	1.1507	3	92	3	1.8209	3	40
4	1.0405	4	80	4	19	4	1.2306	4	25	4	58
.125	12	.185	89	.245	81	.305	21	.365	41	.425	76
6	18	6	98	6	43	6	35	6	58	6	95
7	25	7	1.0908	7	55	7	49	7	74	7	1.4813
8	31	8	17	8	67	8	64	8	91	8	81
9	38	9	27	9	79	9	78	9	1.3807	9	49
.130	45	.190	86	.250	91	.310	98	.370	23	.430	67
1	52	1	46	1	1.1603	1	1.2407	1	40	1	86
2	58	2	56	2	16	2	22	2	56	2	1.4404
3	65	3	65	3	28	3	36	3	78	3	22
4	72	4	75	4	40	4	51	4	90	4	41
.135	79	.195	85	.255	53	.315	65	.375	1.3406	.435	59
6	86	6	95	6	65	6	80	6	23	6	77
7	93	7	1.1005	7	77	7	95	7	40	7	96
8	1.0600	8	15	8	90	8	1.2510	8	56	8	1.4514
9	08	9	25	9	1.1702	9	24	9	73	9	83
.140	15	.200	35	.260	15	.320	39	.380	90	.440	51
1	22	1	45	1	28	1	54	1	1.3507	1	70
2	29	2	55	2	40	2	69	2	24	2	88
3	37	3	65	3	53	3	84	3	41	3	1.4607
4	44	4	75	4	66	4	99	4	58	4	26
.145	52	.205	85	.265	78	.325	1.2614	.385	74	.445	44
6	59	6	96	6	91	6	29	6	91	6	63
7	67	7	1.1106	7	1.1804	7	41	7	1.3608	7	82
8	74	8	17	8	16	8	59	8	25	8	1.4700
9	82	9	27	9	29	9	74	9	43	9	19
.150	90	.210	37	.270	48	.330	89	.390	60	.450	88
1	97	1	48	1	56	1	1.2704	1	77	1	57
2	1.0605	2	58	2	69	2	20	2	94	2	75
3	13	3	69	3	82	3	35	3	1.3711	3	94
4	21	4	80	4	97	4	50	4	28	4	1.4813
.155	29	.215	90	.275	1.1908	.335	66	.395	46	.455	32
6	37	6	1.1201	6	21	6	81	6	63	6	51
7	45	7	12	7	34	7	96	7	80	7	70
8	53	8	23	8	48	8	1.2812	8	97	8	89
9	61	9	33	9	61	9	27	9	15	9	1.4908
.160	1.6069	.220	1.1245	.280	1.1974	.340	1.2843	.400	1.3832	.460	1.4927

TABLE VI.—Lengths of Circular Arcs obtained by having the Chord or Base, and Height or Versed Sine given.

$\frac{h}{b}$	Length.	$\frac{h}{b}$	Length.	$\frac{h}{b}$	Length.	$\frac{h}{b}$	Length.	$\frac{h}{b}$	Length.	$\frac{h}{b}$	Length.
.460	1.4927	.467	1.5061	.474	1.5196	.481	1.5332	.488	1.5470	.495	1.5608
1	46	8	80	5	1.5215	2	52	9	89	1	28
2	65	9	99	6	35	3	71	.490	1.5509	2	48
3	84	.470	1.5119	7	54	4	91	1	29	3	68
4	1.5008	1	38	8	74	.485	1.5411	2	49	4	88
.465	22	2	57	9	93	6	80	3	69	.500	1.5708
6	42	3	76	.480	1.5318	7	50	4	85		
7	1.5061	4	1.5196	1	1.5332	8	1.5470	.595	1.5608		

Example. Given the chord = 12,16 feet, and the height 3,48 feet, to find the length of the arc. Here $h = 3,48$, and $b = 12,16$,

$$\text{and } \frac{h}{b} = \frac{3,48}{12,16} = \text{tabular height} = ,2862 \text{ nearly.}$$

Tabular arc corresponding to 286 = 1,2056 = 1,2056

Tabular arc corresponding to 287 = 1,2070

difference, 14 multiplied by ,2, = 0002,8

1,2058,8

12,16

Length of the curve = 14,6635 feet nearly.

Rule. To the tabular arc corresponding to the first three figures, add the product of the fourth decimal, if any, by the difference of the tabular heights, of the one less and the other greater than the given tabular number. The sum will be the required tabular length to the nearest ten thousandth part, which sum multiplied by the given chord, will give the required length.

Example 2. Let chord $b = 40,20$ feet, and height $h = 5,16$ feet.

$$\text{Here } \frac{h}{b} = \frac{5,16}{40,2} = ,1277 = \text{tabular height.}$$

,127 = 1,0425

= 1,0425

,128 = 1,0481

difference = ,0006, multiplied by 7 = 4,2

1,0429,2

Base or chord

= 40,2

The required length of the curve = 41,92 feet.

TABLE VII.—Areas of Segments of a Circle whose Diameter is Unity.

Tab. h'ght	Area seg.	Tab. h'ght	Area seg.	Tab. h'ght	Area seg.	Tab. h'ght	Area seg.	Tab. h'ght	Area seg.
.001	.000042	.063	.020681	.125	.056668	.187	.101553	.249	.152680
2	119	4	1168	6	7326	8	2834	.250	8546
3	219	5	1659	7	7991	9	3116	1	4412
4	337	6	2154	8	8658	.190	3900	2	5280
5	470	7	2652	9	9327	1	4685	3	6149
6	618	8	3154	.180	9999	2	5472	4	7019
7	779	9	3659	1	.060672	3	6261	5	7890
8	951	.070	4168	2	1348	4	7051	6	8762
9	.001185	1	4680	3	2026	5	7842	7	9636
.010	1329	2	5195	4	2707	6	8686	8	.160510
1	1533	3	5714	5	3389	7	9480	9	1386
2	1746	4	6286	6	4074	8	.110226	.260	2263
3	1968	5	6761	7	4760	9	1024	1	3140
4	2199	6	7289	8	5449	.200	1823	2	4019
5	2438	7	7821	9	6140	1	2624	2	4899
6	2685	8	8356	.140	6833	2	3426	4	5780
7	2940	9	8894	1	7528	3	4230	5	6663
8	3202	.080	9435	2	8225	4	5035	6	7546
9	3471	1	9979	3	8924	5	5842	7	8480
.020	3748	2	.080528	4	9625	6	6650	8	9315
1	4031	3	1076	5	.070828	7	7460	9	.170202
2	4322	4	1629	6	1083	8	8271	.270	1089
3	4618	5	2180	7	1741	9	9083	1	1978
4	4921	6	2745	8	2450	.210	9897	2	2867
5	5230	7	3307	9	3161	1	.120712	3	3758
6	5546	8	3872	.150	3874	2	1529	4	4649
7	5867	9	4441	1	4589	3	2347	5	5542
8	6194	.090	5011	2	5306	4	3167	6	6435
9	6527	1	5585	3	6026	5	3988	7	7330
.030	6865	2	6162	4	6747	6	4810	8	8225
1	7209	3	6741	5	7469	7	5634	9	9122
2	7558	4	7323	6	8194	8	6459	.280	.180019
3	7913	5	7909	7	8921	9	7285	1	9918
4	8273	6	8496	8	9649	.220	8113	2	1817
5	8438	7	9087	9	.080380	1	8942	3	2718
6	9008	8	9680	.160	1112	2	9773	4	3619
7	9383	9	.040276	1	1846	3	.130605	5	4521
8	9763	.100	0875	2	2582	4	1438	6	5425
9	.010148	1	1476	3	3320	5	2272	7	6329
.040	0637	2	2080	4	4059	6	3108	8	7234
1	0981	3	2687	5	4801	7	3945	9	8140
2	1330	4	3296	6	5544	8	4784	.290	9047
3	1734	5	3908	7	6289	9	5624	1	9955
4	2142	6	5522	8	7036	.230	6465	2	.190864
5	2554	7	5139	9	7785	1	7307	3	1775
6	2971	8	5759	.170	8535	2	8150	4	2684
7	3392	9	6381	1	9287	3	8995	5	3596
8	3818	.110	7005	2	.090041	4	9841	6	4509
9	4247	1	7632	3	0797	5	.140688	7	5422
.050	4631	2	8262	4	1554	6	1537	8	6337
1	5119	3	8894	5	2313	7	2387	9	7252
2	5561	4	9523	6	3074	8	3238	.300	8168
3	6007	5	.050165	7	3836	9	4091	1	9085
4	6457	6	0804	8	4601	.240	4944	2	.200008
5	6911	7	1446	9	5366	1	5799	3	9922
6	7369	8	2090	.180	6134	2	6655	4	1841
7	7831	9	2736	1	6903	3	7512	5	2761
8	8296	.120	3385	2	7674	4	8371	6	3683
9	8766	1	4036	3	8447	5	9230	7	4605
.060	9239	2	4689	4	9221	6	.150091	8	5527
1	9716	3	5345	5	9997	7	0953	9	6451
2	.020196	4	6003	6	.100774	8	1861	.310	7376

TABLE VII.—Areas of Segments of a Circle whose Diameter is Unity.

Tab. h'ght	Area seg.	Tab. h'ght	Area seg.	Tab. h'ght	Area seg.	Tab. h'ght	Area seg.	To find the Tabular Versed Sine.
.311	.208301	.373	.267078	.435	.327882	.497	.389699	Rule. Divide the height of the given segment by the diameter of the circle of which it is a segment. The quotient will be the required tabular height.
2	9227	4	8045	6	8874	8	.390699	
3	.210154	5	9013	7	9866	9	.391699	And because the areas of circles are to one another as the squares of their diameters, multiply the tabular areas in this table by the square of the diameter of the circle of which a segment is given. The product will be the area of the required segment.
4	1082	6	9982	8	.330858	.500	.392699	
5	2011	7	.270951	9	1850			<i>Example.</i> (See fig. 17.) Let the chord A B = 42, versed sine D C = 7.
6	2940	8	1920	.440	2843			
7	3871	9	2890	1	3836			By Euclid III, prop. 3 and 35, the diameter cuts the chord of the arc at right angles, consequently making the rectangle contained by the versed sine, and the remaining part of the diameter, equal to the square of half the chord.
8	4802	.380	3861	2	4829			
9	5733	1	4832	3	5822			∴ 21 ² divided by 7 = V D = 63; therefore the diameter V C = 63 + 7 = 70, and 7 divided by 70 = .100 = tabular versed sine, whose corresponding area = .040875, which multiplied by the square of the diameter = 4900, gives the required area = 200,2875.
.320	6666	2	5803	4	6816			
1	7599	3	6775	5	7810			<i>Example 2.</i> Let the tabular versed sine = .3456, which is not to be found in the table. Tab. versed sine 346, area segment = .240218 Tab. versed sine 347, area segment = .241169 difference, 951 As 10 : 951 :: 6 : 570,6, nearly 571, ∴ .240218 more 571 240789 = the required area of segment.
2	8533	4	7748	6	8804			
3	9468	5	8721	7	9798			
4	.220404	6	9694	8	.340793			
5	1344	7	.280668	9	1787			
6	2277	8	1642	.450	2782			
7	3215	9	2617	1	3777			
8	4154	.390	3592	2	4772			
9	5093	1	4568	3	5768			
.330	6033	2	5544	4	6764			
1	6974	3	6521	5	7759			
2	7915	4	7498	6	8755			
3	8858	5	8476	7	9752			
4	9801	6	9453	8	.350748			
5	.230745	7	.290432	9	1745			
6	1689	8	1411	.460	2742			
7	2634	9	2390	1	3789			
8	3580	.400	3369	2	4786			
9	4526	1	4349	3	5782			
.340	5473	2	5330	4	6786			
1	6421	3	6311	5	7727			
2	7369	4	7292	6	8729			
3	8318	5	8273	7	9723			
4	9268	6	9255	8	.360721			
5	.240218	7	.300238	9	1719			
6	1169	8	1220	.470	2717			
7	2121	9	2203	1	3715			
8	3074	.410	3187	2	4413			
9	4026	1	4171	3	5712			
.350	4980	2	5155	4	6710			
1	5934	3	6140	5	7709			
2	6889	4	7125	6	8708			
3	7845	5	8116	7	9707			
4	8801	6	9095	8	.370706			
5	9757	7	.310081	9	1705			
6	.250715	8	1069	.480	2764			
7	1673	9	2054	1	3703			
8	2631	.420	3041	2	4702			
9	3590	1	4029	3	5702			
.360	4550	2	5016	4	6702			
1	5510	3	6004	5	7701			
2	6471	4	6992	6	8701			
3	7433	5	7981	7	9700			
4	8395	6	8970	8	.380700			
5	9357	7	9959	9	1699			
6	.260320	8	.320948	.490	2699			
7	1284	9	1938	1	3699			
8	2248	.430	2928	2	4699			
9	3213	1	3918	3	5699			
.370	4178	2	4909	4	6699			
1	5144	3	5900	5	7699			
2	6111	4	6892	6	8699			

Rule. Take out the areas corresponding to the nearest tabular versed sine,—one greater and the other less than the given tabular versed sine; take the difference of the area segments; multiply this difference by the fourth decimal figure of the given tabular versed sine; cut off one figure to the right, and add the remainder to the lesser area segment. The sum will be the required area segment. (See the last example.)

Note. When the tabular versed sine is greater than ,500, the segment is greater than a semicircle; in which case subtract it from 1, find the area seg. of the difference, which take from ,785398. Multiply this difference by the square of the diameter. The product will be the required area.

Examp^l. Let tabular versed sine = ,867, and let 60 = diameter of the circle.

From	1,000	Area circle = ,785398
take tabular versed sine	<u>0,867</u>	
	difference, ,133	Area segment = ,062026
Correct area of segment		= ,723372
Square of 60 (the diameter)		= 3600
Required area of the segment		= 2604,039200

TABLE VIII.—To Reduce Square Feet to Acres, and Vice Versa.

Ac.	Sq. feet.	Acres.	Sq. feet.	Acres.	Sq. feet.	Acres.	Sq. feet.	Acres.	Sq. feet.
1	43560	11	479160	21	914760	81	1850360	41	1785960
2	87120	12	522720	22	958320	82	1893920	42	1829520
3	130680	13	566280	23	1001880	83	1437480	43	1873080
4	174240	14	609840	24	1045440	84	1481040	44	1916640
5	217800	15	653400	25	1089000	85	1524600	45	1960200
6	261360	16	696960	26	1132560	86	1568160	46	2003760
7	304920	17	740520	27	1176120	87	1611720	47	2047320
8	348480	18	784080	28	1219680	88	1655280	48	2090880
9	392040	19	827640	29	1263240	89	1698840	49	2134440
10	435600	20	871200	30	1306800	40	1742400	50	2178000
0.1	4356	.01	435.6	0.001	48.56	0.0001	4.86	0.00001	0.44
.2	8712	.02	871.2	.002	87.12	.0002	8.71	.00002	0.87
.3	13068	.03	1306.8	.003	130.68	.0003	13.07	.00003	1.31
.4	17424	.04	1742.4	.004	174.24	.0004	17.42	.00004	1.74
.5	21780	.05	2178.0	.005	217.80	.0005	21.78	.00005	2.18
.6	26136	.06	2613.6	.006	261.36	.0006	26.14	.00006	2.61
.7	30492	.07	3049.2	.007	304.92	.0007	30.49	.00007	3.05
.8	34848	.08	3484.8	.008	348.48	.0008	34.85	.00008	3.49
0.9	39204	0.09	3920.4	0.009	392.04	0.0009	39.20	0.00009	3.92

Example. Reduce 1283446 square feet to acres.

From the first part, $\frac{1283240}{21206} = 29$

From the second part, $\frac{17424}{2782} = 4$
 $\frac{26136}{168.4} = .06$
 $\frac{180.68}{37.72} = .008$
 $\frac{84.85}{2.87} = .0006$
 $\frac{2.61}{0.26} = .00006$ nearly; $\therefore 29,46885 = \text{Answer}.$

This example, being one of the most difficult that can occur, is sufficient to show the application.

BLE VIIIa.—*Properties of Polygons whose Sides are = Unity.*

Name of polygon	Area of polygon.	Angle at the centre.	Angle made by two of its sides.	Radius of the inscribed circle.	Side of a polygon inscribed in a circle whose diameter = 1
		D I N	D I N		
Trigon.	0.4330127	120,00,00	60,00,00	0.2886751	1.732051
Tetragon.	1.0000000	90,00,00	90,00,00	0.5000000	1.414214
Pentagon.	1.7204774	72,00,00	108,00,00	0.6881910	1.175571
Hexagon.	2.5980762	60,00,00	120,00,00	0.8660254	1.000000
Heptagon.	8.6389124	51,25,42 ¹ ₂	128,34,17 ¹ ₂	0.0882617	0.867768
Octagon.	4.8284271	45,00,00	135,00,00	1.2071068	0.765367
Nonagon.	6.1818242	40,00,00	140,00,00	1.3787387	0.684040
Decagon.	7.6942088	36,00,00	144,00,00	1.5388418	0.618084
Undecagon.	9.8656404	32,00,16 ¹ ₂	147,16,21 ¹ ₂	1.7028437	0.568366
Dodecagon.	11.1961524	30,00,00	150,00,00	1.8660254	0.517638

TABLE IX.—*Properties of the Five Regular Bodies.*

Name of polygon	Area of regular polygon whose side is = 1.	Solidity of regular polygon whose side is = 1.	Side of a poly. ins. in sphere whose diameter = 1.	Side of poly. circ. inscrib'd sphere whose diam. = 1.	Side of a poly. = a sphere whose diameter = 1.
Tetradron.	1.732051	0.117851	0.117851	0.816497	2.44948
Hexadron.	6.000000	1.000000	1.000000	0.577350	1.00000
Octadron.	3.464102	0.471405	0.471405	0.707107	1.22474
Dodecadron.	20.645729	7.668119	7.668119	0.525731	0.66158
Icosadron.	8.660254	2.181695	2.181695	0.356822	0.44903

TABLE X.—*To Reduce Square Links to Acres, Roods and Perches.*

Sq. links	Per.	Sq. links	Per.	Sq. links	Perches	Sq. links	Perches	Sq. links
100000	29	18125	15	9375	1.0	625	0.05	31.25
75000	28	17500	14	8750	0.1	62.5	0.06	37.50
50000	27	16875	13	8125	0.2	125.0	0.07	43.75
25000	26	16250	12	7500	0.3	187.5	0.08	50.00
24375	25	15625	11	6875	0.4	250.0	0.09	56.25
23750	24	15000	10	6250	0.5	312.5	0.001	0.63
23125	23	14375	9	5625	0.6	375.0	0.002	1.25
22500	22	13750	8	5000	0.7	437.5	0.003	1.88
21875	21	13125	7	4375	0.8	500.0	0.004	2.50
21250	20	12500	6	3750	0.9	562.5	0.005	3.13
20625	19	11875	5	3125	0.01	6.25	0.006	3.75
20000	18	11250	4	2500	0.02	12.50	0.007	4.37
19375	17	10625	3	1875	0.03	18.75	0.008	5.00
18750	16	10000	2	1250	0.04	25.00	0.009	5.63

Example 1. Reduce 47682854 links to roods and perches.

47,68285	A. R.
Always places to right.	47,50000 = 47, 2
ches	18285
	= 18125
	160
ch	= 125
rch	= 35,
	81,25
	8,75
rch	= 8,75

Example 2. Reduce 1753 square links to perches.

This being less than 25000, shows that there are no roods in the answer.

	1753 square links.
2 perches	= 1250
	503
,8 perch	= 500
	8
005	= 3,15
	Answer, 2,805 perches.

TABLE XI.—Showing the Reduction on Each Chain of 100 Links to Reduce Hypothenusal to Base or Horizontal Measurements.

Angle of inclinat'n.	Red. in lks.	Angle of inclinat'n.	Red. in lks.	Angle of inclinat'n.	Red. in lks.	Angle of inclinat'n.	Red. in lks.	
° / ' / "		° / ' / "		° / ' / "		° / ' / "		
2 33 40	0.10	19 26 19	5.70	27 30 05	11.80	33 47 54	16.90	Example 1. The angle of elevation being 23d. 4m. 26s., and the distance measured along the slope = 15 chains, to find the base. For angle 23d. 4m. 26s., the correction is 8 links, which multiplied by 15 chains gives 1.20 chains, which taken from 15, gives the base = 13.80.
3 27 29		20 19 36 34	80	27 37 30		40 33 53 04	17.00	
4 26 20		80 19 46 47	90	27 44 50		50 34 00 14	10	
5 07 35		40 19 56 55	6.00	27 52 20		60 34 06 26	20	
6 43 55		50 20 06 56		10 27 54 40		70 34 12 30	80	
6 16 45		60 20 16 54		20 28 06 56		80 34 18 36	40	
6 47 00		70 20 26 46		30 28 14 12		90 34 24 41	50	
7 15 07		80 20 36 35		40 28 21 27	12.00	34 30 46	60	
7 30 33		90 20 46 19		50 28 28 41		10 34 36 50	70	
8 06 34	1.00	20 55 58	60	28 35 52		20 34 42 53	80	
8 30 23		10 21 05 33	70	28 43 02		30 34 48 54	90	Example 2. Given the hypothenuse = 12 chains, and the angle of elevation = 25d. 12m., to find the base. Tab. angle greater than the given angle = 25d. 18m. 40s., its reduction = 9.60 lks. Tab. angle less than the given angle = 25d. 19m. 40s., its reduction = 9.50 lks. 8m. As 8m. : 0.10 links :: 1m. 20s. : = 0.0167, which added to 9.60, and multiplied by 12, gives the required correction 1m. 20s.
8 53 07		20 21 15 04	80	28 50 11		40 34 54 55	18.00	
9 14 55		30 21 24 32	90	28 57 18		50 35 00 55	10	
9 35 55		40 21 33 54	7.00	29 04 23		60 35 06 54	20	
9 56 11		50 21 43 14		10 29 11 27		70 35 12 52	30	
10 15 47		60 21 52 30		20 29 18 29		80 35 18 49	40	
10 34 48		70 22 01 41		30 29 25 30		90 35 24 45	50	
10 53 16		80 22 10 50		40 29 32 29	13.00	35 30 41	60	
11 11 12		90 22 19 54		50 29 39 27		10 35 36 36	70	
11 28 42	2.00	22 28 55	60	29 46 20		20 35 42 30	80	
11 45 46		10 22 37 53	70	29 53 18		30 35 48 22	90	Example 3. The angle of elevation being 23d. 4m. 26s., and the distance measured along the slope = 15 chains, to find the base. For angle 23d. 4m. 26s., the correction is 8 links, which multiplied by 15 chains gives 1.20 chains, which taken from 15, gives the base = 13.80.
12 02 26		20 22 46 47	80	30 30 10		40 35 54 15	19.00	
12 18 44		80 22 55 38	90	30 07 02		50 36 00 06	10	
12 34 41		40 53 04 26	8.00	30 13 52		60 36 05 56	20	
12 50 20		50 23 13 12		10 30 20 42		70 46 11 46	30	
13 05 38		60 23 21 52		20 30 27 29		80 36 17 36	40	
13 20 00		70 23 30 31		30 30 34 15		90 36 23 23	50	
13 35 37		80 23 39 07		40 30 41 00	14.00	36 29 10	60	
13 49 56		90 23 47 40		50 30 47 44		10 36 34 57	70	
14 03 12	3.00	23 55 59	60	30 54 26		20 36 40 43	80	
14 18 13		10 24 04 36	70	31 01 07		30 36 46 27	90	Example 4. The angle of elevation being 23d. 4m. 26s., and the distance measured along the slope = 15 chains, to find the base. For angle 23d. 4m. 26s., the correction is 8 links, which multiplied by 15 chains gives 1.20 chains, which taken from 15, gives the base = 13.80.
14 32 02		20 24 13 00	80	31 07 47		40 36 52 12	20.00	
14 45 37		80 24 21 22	90	31 14 25		50 36 57 55	10	
14 59 01		40 24 29 40	9.00	31 21 02		60 37 03 37	20	
15 12 14		50 24 37 10		10 31 27 38		70 37 09 20	30	
15 25 14		60 24 46 10		20 31 34 12		80 37 15 01	40	
15 38 05		70 24 54 20		30 31 40 46		90 37 20 41	50	
15 50 45		80 25 02 30		40 31 47 18	15.00	37 26 21	60	
16 03 05		90 25 10 40		50 31 53 49		10 37 32 00	70	
16 15 06	4.00	25 18 40	60	32 00 19		20 37 37 88	80	
16 27 48		10 25 26 40	70	32 06 47		30 37 43 15	60	Example 5. The angle of elevation being 23d. 4m. 26s., and the distance measured along the slope = 15 chains, to find the base. For angle 23d. 4m. 26s., the correction is 8 links, which multiplied by 15 chains gives 1.20 chains, which taken from 15, gives the base = 13.80.
16 39 52		20 25 34 40	80	32 13 15		40 37 48 52	21.00	
16 51 48		80 25 42 40	90	32 19 41		50 37 54 28	10	
17 03 35		40 25 50 30	10.00	32 26 06		60 38 00 04	20	
17 15 14		50 25 58 20		10 32 32 36		70 38 05 38	30	
17 26 45		60 26 06 10		20 32 38 53		80 38 11 12	40	
17 35 10		70 26 14 00		30 32 45 15		90 38 16 46	50	
17 49 27		80 26 21 50		40 32 51 36	16.00	38 22 18	60	
18 00 38		90 26 29 30		50 32 57 55		10 38 27 50	70	
18 11 42	5.00	26 37 10	60	33 04 14		20 38 33 21	80	
18 22 39		10 26 44 50	70	33 10 31		30 38 38 52	90	Example 6. The angle of elevation being 23d. 4m. 26s., and the distance measured along the slope = 15 chains, to find the base. For angle 23d. 4m. 26s., the correction is 8 links, which multiplied by 15 chains gives 1.20 chains, which taken from 15, gives the base = 13.80.
18 33 30		20 26 52 30	80	33 16 47		40 38 44 22	22.00	
18 44 18		80 27 00 00	90	33 23 03		50 38 49 51	10	
18 54 55		40 27 07 40	11.00	33 29 17		60 38 55 20	20	
19 05 27		50 27 15 10		10 33 35 32		70 39 00 48	30	
19 15 56		60 27 22 40		20 33 41 43		80 39 06 15	40	

TABLE XII.—To Reduce Sidereal Time to Mean Solar Time.

Minutes.			Seconds.		
sid. t.	Mean time.	sid. t.	Mean t.	sid. t.	Mean t.
m.	m. s.	s.	s.	m.	m. s.
1	0 59.84	1	0.997	1	0 59.84
2	1 59.67	2	1.995	2	1 59.67
3	2 59.51	3	2.992	3	2 59.51
4	3 59.35	4	3.989	4	3 59.35
5	4 59.18	5	4.986	5	4 59.18
6	5 59.02	6	5.984	6	5 59.02
7	6 58.85	7	6.98	7	6 58.85
8	7 58.69	8	7.98	8	7 58.69
9	8 58.53	9	8.98	9	8 58.53
10	9 58.36	10	9.97	10	9 58.36
11	10 58.20	11	10.97	11	10 58.20
12	11 58.03	12	11.97	12	11 58.03
13	12 57.87	13	12.97	13	12 57.87
14	13 57.71	14	13.96	14	13 57.71
15	14 57.54	15	14.96	15	14 57.54
16	15 57.38	16	15.96	16	15 57.38
17	16 57.22	17	16.95	17	16 57.22
18	17 57.05	18	17.95	18	17 57.05
19	18 56.89	19	18.95	19	18 56.89
20	19 56.72	20	19.95	20	19 56.72
21	20 56.56	21	20.94	21	20 56.56
22	21 56.40	22	21.94	22	21 56.40
23	22 56.23	23	22.94	23	22 56.23
24	23 56.07	24	23.93	24	23 56.07
25	24 55.90	25	24.93	25	24 55.90
26	25 55.74	26	25.93	26	25 55.74
27	26 55.58	27	26.93	27	26 55.58
28	27 55.41	28	27.92	28	27 55.41
29	28 55.25	29	28.92	29	28 55.25
30	29 55.09	30	29.92	30	29 55.09
31	30 54.92	31	30.92	31	30 54.92
32	31 54.76	32	31.91	32	31 54.76
33	32 54.59	33	32.91	33	32 54.59
34	33 54.43	34	33.91	34	33 54.43
35	34 54.26	35	34.90	35	34 54.26
36	35 54.10	36	35.90	36	35 54.10
37	36 53.94	37	36.90	37	36 53.94
38	37 53.78	38	37.90	38	37 53.78
39	38 53.61	39	38.89	39	38 53.61
40	39 53.44	40	39.89	40	39 53.44
41	40 53.28	41	40.89	41	40 53.28
42	41 53.12	42	41.89	42	41 53.12
43	42 52.96	43	42.88	43	42 52.96
44	43 52.79	44	43.88	44	43 52.79
45	44 52.63	45	44.88	45	44 52.63
46	45 52.46	46	45.87	46	45 52.46
47	46 52.30	47	46.87	47	46 52.30
48	47 52.14	48	47.87	48	47 52.14
49	48 51.97	49	48.87	49	48 51.97
50	49 51.81	50	49.86	50	49 51.81
51	50 51.65	51	50.86	51	50 51.65
52	51 51.48	52	51.86	52	51 51.48
53	52 51.32	53	52.86	53	52 51.32
54	53 51.15	54	53.85	54	53 51.15
55	54 50.99	55	54.85	55	54 50.99
56	55 50.83	56	55.85	56	55 50.83
57	56 50.66	57	56.84	57	56 50.66
58	57 50.41	58	57.84	58	57 50.41
59	58 50.33	59	58.84	59	58 50.33

TABLE XIII.—To Reduce Mean Solar Time to Sidereal Time.

hours mean time.	Hours of sidereal time.	Min. mean time.	equivalents in sidereal time.	Sec. mean time.
h. m. s.	h. m. s.	m. s.	m. s.	s.
1	1 0 9.86	1	1 0.16	1
2	2 0 19.71	2	2 0.33	2
3	3 0 29.57	3	3 0.49	3
4	4 0 39.43	4	4 0.66	4
5	5 0 49.28	5	5 0.82	5
6	6 0 59.14	6	6 0.99	6
7	7 1 9.00	7	7 1.15	7
8	8 1 18.85	8	8 1.31	8
9	9 1 28.71	9	9 1.48	9
10	10 1 38.56	10	10 1.64	10
11	11 1 48.42	11	11 1.81	11
12	12 1 58.28	12	12 1.97	12
13	13 2 8.13	13	13 2.14	13
14	14 2 17.99	14	14 2.30	14
15	15 2 27.85	15	15 2.46	15
16	16 2 37.70	16	16 2.63	16
17	17 2 47.56	17	17 2.79	17
18	18 2 57.42	18	18 2.96	18
19	19 3 7.27	19	19 3.12	19
20	20 3 17.13	20	20 3.29	20
21	21 3 26.99	21	21 3.45	21
22	22 3 36.84	22	22 3.61	22
23	23 3 46.70	23	23 3.78	23
24	24 3 56.56	24	24 3.94	24
25		25	25 4.11	25
26		26	26 4.27	26
27		27	27 4.44	27
28		28	28 4.60	28
29		29	29 4.76	29
30		30	30 4.93	30
31		31	31 5.09	31
32		32	32 5.26	32
33		33	33 5.42	33
34		34	34 5.59	34
35		35	35 5.75	35
36		36	36 5.91	36
37		37	37 6.08	37
38		38	38 6.24	38
39		39	39 6.41	39
40		40	40 6.57	40
41		41	41 6.74	41
42		42	42 6.90	42
43		43	43 7.06	43
44		44	44 7.23	44
45		45	45 7.39	45
46		46	46 7.56	46
47		47	47 7.72	47
48		48	48 7.89	48
49		49	49 8.05	49
50		50	50 8.21	50
51		51	51 8.38	51
52		52	52 8.54	52
53		53	53 8.71	53
54		54	54 8.87	54
55		55	55 9.04	55
56		56	56 9.20	56
57		57	57 9.36	57
58		58	58 9.53	58
59		59	59 9.69	59

TABLE XII.

Continued.

hours of sid. time.	Hours of mean time.
h. m. s.	h. m. s.
1	0 59 50.17
2	1 59 40.34
3	2 59 30.51
4	3 59 20.68
5	4 59 10.85
6	5 59 1.02
7	6 58 51.19
8	7 58 41.36
9	8 58 31.53
10	9 58 21.70
11	10 58 11.88
12	11 58 2.05
13	12 57 52.22
14	13 57 42.39
15	14 57 32.56
16	15 57 22.73
17	16 57 12.90
18	17 57 3.07
19	18 56 53.24
20	19 56 43.41
21	20 56 33.58
22	21 56 23.75
23	22 56 13.92
24	23 56 4.09

The fractions of seconds agree to two places of decimals for mean and sidereal.

**TABLE XIV.—To Reduce
Sidereal Time to Longitude
or Degrees.**

Hours.	Degrees.	Time. Min. Sec.	Arc. Deg. Min. Sec.
1	15	1	0 15
2	30	2	0 30
3	45	3	0 45
4	60	4	1 00
5	75	5	1 15
6	90	6	1 30
7	105	7	1 45
8	120	8	2 00
9	135	9	2 15
10	150	10	2 30
11	165	11	2 45
12	180	12	3 00
13	195	13	3 15
14	210	14	3 30
15	225	15	3 45
16	240	16	4 00
17	255	17	4 15
18	270	18	4 30
19	285	19	4 45
20	300	20	5 00
21	315	21	5 15
22	330	22	5 30
23	345	23	5 45
24	360	24	6 00
		25	6 15
		26	6 30
		27	6 45
		28	7 00
		29	7 15
		30	7 30
		31	7 45
		32	8 00
		33	8 15
		34	8 30
		35	8 45
		36	9 00
		37	9 15
		38	9 30
		39	9 45
		40	10 00
		41	10 15
		42	10 30
		43	10 45
		44	11 00
		45	11 15
		46	11 30
		47	11 45
		48	12 00
		49	12 15
		50	12 30
		51	12 45
		52	13 00
		53	13 15
		54	13 30
		55	13 45
		56	14 00
		57	14 15
		58	14 30
		59	14 45
		60	15 00

**TABLE XV.
To Reduce Longitude or Degrees
to Sid. Time.**

Arc. Deg. Min. Sec.	Time. Hr. Min. Sec. Th.
1	0 4
2	0 8
3	0 12
4	0 16
5	0 20
6	0 24
7	0 28
8	0 32
9	0 36
10	0 40
11	0 44
12	0 48
13	0 52
14	0 56
15	1 00
16	1 04
17	1 08
18	1 12
19	1 16
20	1 20
21	1 24
22	1 28
23	1 32
24	1 36
25	1 40
26	1 44
27	1 48
28	1 52
29	1 56
30	2 00
31	2 04
32	2 08
33	2 12
34	2 16
35	2 20
36	2 24
37	2 28
38	2 32
39	2 36
40	2 40
41	2 44
42	2 48
43	2 52
44	2 56
45	3 00
46	3 04
47	3 08
48	3 12
49	3 16
50	3 20
51	3 24
52	3 28
53	3 32
54	3 36
55	2 40
56	3 44
57	3 48
58	3 52
59	3 56
60	4 00

**TABLE XVI.
Showing the Dip or Depression
of the Horizon, and the Dis-
tance at Sea in Miles, Corres-
ponding to given Heights.**

Height in feet.	Dip in arc.	Dist. seen at sea in miles.
1	0 58	1.32
2	1 22	1.87
3	1 40	2.29
4	1 55	2.65
5	2 09	2.96
6	2 22	3.24
7	2 33	3.50
8	2 44	3.74
9	2 54	3.97
10	3 03	4.18
11	3 12	4.39
12	3 21	4.58
13	3 29	4.77
14	3 37	4.95
15	3 45	5.12
16	3 53	5.29
17	4 01	5.45
18	4 08	5.61
19	4 15	5.77
20	4 22	5.92
21	4 28	6.06
22	4 34	6.21
23	4 40	6.34
24	4 46	6.48
25	4 52	6.61
26	4 58	6.75
28	5 10	7.00
30	5 21	7.25
32	5 31	7.48
34	5 40	7.71
36	5 50	7.94
38	6 00	8.16
40	6 10	8.37
42	6 19	8.57
44	6 28	8.78
46	6 37	8.97
48	6 45	9.17
50	6 53	9.35
55	7 11	9.81
60	7 29	10.25
65	7 47	10.67
70	8 05	11.07
75	8 23	11.46
80	8 44	11.83
85	8 57	12.20
90	9 14	12.55
95	9 30	12.89
100	9 46	13.23
105	10 01	13.56
110	10 16	13.88
115	10 30	14.19
120	10 43	14.49
125	10 56	14.79
130	11 09	15.08
135	11 22	15.37
140	11 35	15.65
145	11 47	15.93
150	11 59	16.20
155	12 11	16.46
160	12 23	16.73

TABLE XVII.—*Correction of the Apparent Altitude for Refraction*
(Subtractive).

Ap. altitude	Mean refract.	Cor. in 1 unit	Ap. altitude	Mean ref.	cor. 1 un.	Ap. altitude	Mean ref.	cor. 1 un.	Ap. altitude	Mean ref.	cor. 1 un.
0	/	/	0	/	/	0	/	/	0	/	/
4 00	11 47	19.9	12 00	4 28	6.9	20 00	2 39	4.1	40 00	1 10	1.8
10	26	19.2	10	25	6.8	20	36	4.0	41 00	7	1.7
20	6	18.5	20	21	6.8	40	34	3.9	42 00	5	1.7
30	10 46	18.0	30	18	6.7	21 00	31	3.9	43 00	3	1.6
40	28	17.5	40	14	6.6	20	29	3.8	44 00	0	1.5
50	11	16.9	50	11	6.4	40	26	3.7	45 00	0 58	1.5
5 00	9 54	16.4	13 00	8	6.4	22 00	24	3.7	46 00	56	1.4
10	38	16.0	10	5	6.4	20	21	3.7	47 00	54	1.4
20	23	15.6	20	2	6.2	40	19	3.5	48 00	53	1.4
30	9	15.1	30	3 59	6.2	23 00	17	3.5	49 00	51	1.3
40	8 55	14.7	40	56	6.0	20	15	3.5	50 00	49	1.2
50	42	14.3	50	53	6.0	40	13	3.5	51 00	47	1.2
6 00	30	13.9	14 00	50	6.0	24 00	11	3.3	52 00	46	1.1
10	18	13.5	10	47	5.9	20	09	3.3	53 00	44	1.1
20	7	13.3	20	45	5.8	40	07	3.3	54 00	42	1.0
30	7 56	12.9	30	42	5.7	25 00	05	3.3	55 00	41	1.0
40	45	12.6	40	40	5.6	20	03	2.2	56 00	39	1.0
50	35	12.3	50	37	5.6	40	01	3.1	57 00	38	0.9
7 00	25	12.0	15 00	35	5.5	26 00	1 59	3.1	58 00	36	.9
10	16	11.8	10	32	5.4	20	58	3.0	59 00	35	.9
20	7	11.5	20	30	5.4	40	56	2.9	60 00	34	.8
30	6 59	11.3	30	28	5.3	27 00	54	2.9	61 00	32	.8
40	50	11.0	40	25	5.2	20	53	2.9	62 00	31	.8
50	42	10.8	50	23	5.2	40	51	2.9	63 00	30	.8
8 00	35	10.6	16 00	21	5.2	28 00	49	2.9	64 00	28	.7
10	27	10.4	10	19	5.2	20	48	2.8	65 00	27	.6
20	20	10.2	20	17	5.1	40	46	2.7	66 00	26	.6
30	13	10.0	30	15	5.0	29 00	45	2.7	67 00	25	.6
40	7	9.8	40	13	5.0	30	44	2.7	68 00	24	.6
50	0	9.6	50	11	4.9	40	42	2.7	69 00	22	.5
9 00	5 44	9.4	17 00	9	4.9	30 00	41	2.7	70 00	21	.5
10	48	9.2	10	7	4.8	20	39	2.6	71 00	20	.5
20	42	9.1	20	5	4.8	40	38	2.6	72 00	19	.4
30	36	8.9	30	3	4.7	31 00	37	2.5	73 00	18	.4
40	31	8.7	50	1	4.7	20	35	2.5	74 00	17	.4
50	25	8.4	50	0	4.7	40	34	2.4	75 00	16	.4
10 00	20	8.5	18 00	2 58	4.6	32 00	33	2.3	76 00	15	.4
10	15	8.3	10	56	4.6	20	32	2.3	77 00	13	.3
20	10	8.1	20	55	4.5	40	31	2.2	78 00	12	.3
30	6	8.0	30	53	4.5	33 00	30	2.2	79 00	11	.3
40	1	8.0	40	51	4.4	20	28	2.2	80 00	10	.3
50	4 56	7.8	50	50	4.3	40	27	2.1	81 00	9	.2
11 00	52	7.7	19 00	48	4.3	34 00	26	2.1	82 00	8	.2
10	48	7.6	10	47	4.3	35 00	23	2.1	83 00	7	.1
20	44	7.5	20	45	4.2	36 00	20	2.1	84 00	6	.1
30	40	7.4	30	44	4.2	37 00	17	2.0	86 00	4	.0
40	36	7.3	40	42	4.2	38 00	15	1.9	88 00	2	.0
50	32	7.1	50	41	4.1	39 00	12	1.8	90 00	0	.0

The first column contains the apparent altitude of the object.

The second column contains the mean refraction; that is, the refraction corresponding to the state of the atmosphere, when the height of the barometer is 30 inches and that of the thermometer 50 degrees Fahrenheit. Here 30 — one-tenth of the thermometer = 30 — 5 = 25 = given standard tabular height for mean refraction.

The third column contains the correction for every unit that the tabular height is greater or less than the standard height (25). *Example.* Let the apparent altitude be 20°; the barometer 29.5 inches; the thermometer 15° above zero. Here we have 29.5 — one-tenth of 15 = 29.5 — 1.5 = 28, which is 3 units greater than 25.

Mean refraction for apparent altitude 20° = 2' 39".

Correction for 1 unit = 4". This multiplied by 3 = plus 12.3.

Required refraction = 2' 51.3".

When the tabular height is greater than 25, add the correction to the mean refraction. The sum will be the required refraction. When the tabular height is less than 25, subtract the correction from the mean refraction. The difference will be the required refraction.

TABLE XVIII.
Sun's Parallax in Altitude.

Sun's alt. in degrees.	Parallax in seconds.
0	9
10	9
20	8
30	8
40	7
50	6
55	5
60	4
65	4
70	3
75	2
80	2
85	1
90	0

Parallax is to be added to apparent altitude, or it may be added after the corrections, for dip and refraction are taken from the ap. alt., or it may be taken from the sum of the dip and ref., and the difference taken from ap. alt.

TABLE XIX.—Parallax in Altitude of the Planets.
PLANETS' HORIZONTAL PARALLAX. (From Naut. Alm.)

Ap. Alt.	1"	3"	5"	7"	9"	11"	13"	15"	17"	19"	21"	23"	25"	27"	29"	31"
0	10	13	15	17	19	11	13	15	17	19	21	23	25	27	29	30
15	13	15	17	19	11	13	15	17	19	21	23	25	26	28	29	
20	13	15	17	19	10	12	14	16	18	20	22	24	25	27	28	
25	13	15	17	19	10	12	14	15	17	19	21	23	24	26	27	
30	13	14	16	18	10	11	13	15	16	18	20	22	23	25	26	
33	12	14	16	18	9	11	13	14	16	18	19	21	23	24	25	
36	12	14	16	17	9	11	12	14	15	17	19	20	22	23	25	
39	12	14	15	17	9	10	12	13	15	16	18	19	21	23	24	
42	12	14	15	17	8	10	11	13	14	16	17	19	20	22	23	
45	12	14	15	16	8	9	11	12	13	15	16	18	19	21	22	
48	12	13	15	16	7	9	10	11	13	14	15	17	18	19	20	
51	12	13	14	16	7	8	9	11	12	13	14	16	17	18	19	
54	12	13	14	15	6	8	9	10	11	12	14	15	16	17	18	
57	12	13	14	15	6	7	8	9	10	11	12	14	15	16	17	
60	12	13	14	15	6	7	8	9	10	11	12	13	14	15	15	
62	12	13	14	15	5	6	7	8	10	10	11	12	13	14	14	
64	12	13	14	15	5	6	7	8	9	10	10	11	12	13	13	
66	12	13	14	14	4	5	6	7	8	9	9	10	11	12	12	
68	12	13	13	14	4	5	5	6	7	8	9	9	10	11	11	
70	12	12	13	14	4	5	5	6	7	7	8	9	9	10	10	
72	12	12	12	13	3	4	5	5	6	6	7	8	8	9	9	
74	12	12	12	13	3	3	4	4	5	5	6	6	7	7	7	
76	12	12	12	13	3	3	4	4	4	4	5	5	6	6	6	
78	12	12	12	13	2	3	3	3	3	3	4	4	4	5	5	
81	12	12	12	13	2	2	2	2	2	2	2	3	3	3	3	
84	12	12	12	13	1	1	1	2	2	2	2	2	3	3	3	
87	12	12	12	13	1	1	1	1	1	1	1	1	1	1	2	
90	12	12	12	13	0	0	0	0	0	0	0	0	0	0	0	

TABLE XX.—Reduction of the Time of the Moon's Passage over the Meridian at Greenwich to that over any other Meridian.

DAILY VARIATION OF THE MOON'S MERIDIAN PASSAGE.

Long. in deg.	40'	42'	44'	46'	48'	50'	52'	54'	56'	58'	60'	62'	64'	66'
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	2	2	2	2	3	3	3	3	3	3	3	3	4	4
30	3	3	4	4	4	4	4	4	4	5	5	5	5	5
40	4	4	5	5	5	5	6	6	6	6	6	7	7	7
50	5	6	6	6	6	6	7	7	7	8	8	8	9	9
60	6	7	7	8	8	8	9	9	9	10	10	10	11	11
70	8	8	9	9	9	10	10	10	11	11	12	12	12	13
80	9	9	10	10	11	11	12	12	12	13	13	14	14	15
90	10	10	11	11	12	12	13	13	14	14	15	15	16	16
100	11	12	12	13	13	14	14	15	16	16	17	17	18	18
110	12	13	13	14	15	15	16	16	17	18	18	19	20	20
120	13	14	15	15	16	17	17	18	19	19	20	21	21	22
130	14	15	16	17	17	18	19	19	20	21	22	22	23	24
140	16	16	17	18	19	19	20	21	22	23	23	24	25	26
150	17	17	18	19	20	21	22	22	23	24	25	26	27	27
160	18	19	20	20	21	22	23	24	25	26	27	28	28	29
170	19	20	21	22	23	24	25	25	26	27	28	29	30	31
180	20	21	22	23	24	25	26	27	28	29	30	31	32	33

Note. In this table, longitude means the moon's longitude, or time in arc, from her meridian passage. The corrections are in minutes of a degree.

TABLE XXI.—*Best Time for Obtaining Apparent Time.*

Declination of the same name with the Latitude.

2°	4°	6°	8°	10°	12°	14°	16°	18°	20°	22°	24°
m. h. m.	m. h. m.	m. h. m.	m. h. m.	m. h. m.	m. h. m.	m. h. m.	m. h. m.	m. h. m.	m. h. m.	m. h. m.	m. h. m.
0	6 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0
0	5 2	5 22	5 81	5 87	5 41	5 44	5 46	5 48	5 49	5 50	5 51
0	4 0	4 42	5 2	5 14	5 22	5 28	5 32	5 35	5 38	5 40	5 42
13	2 46	4 0	4 32	4 51	5 3	5 11	5 18	5 23	5 27	5 30	5 33
0	0 0	3 13	4 1	4 27	4 43	4 55	5 4	5 10	5 16	5 20	5 24
26	2 28	2 15	3 26	4 1	4 23	4 38	4 49	4 58	5 4	5 10	5 15
42	3 13	0 0	2 46	3 34	4 1	4 20	4 34	4 45	4 53	5 0	5 5
54	3 41	2 5	1 56	3 3	3 39	4 2	4 19	4 31	4 41	4 49	4 56
2	4 1	2 46	0 0	2 29	3 14	3 43	4 3	4 17	4 29	4 39	4 46
9	4 15	3 14	1 50	1 44	2 47	3 22	3 46	4 3	4 17	4 28	4 37
14	4 27	3 34	2 29	0 0	2 16	3 0	3 28	3 49	4 4	4 16	4 27
19	4 36	3 49	2 55	1 40	1 35	2 35	3 9	3 33	3 51	4 5	4 16
22	4 43	4 1	3 14	2 16	0 0	2 6	2 49	3 17	3 37	3 53	4 6
25	4 49	4 12	8 80	2 41	1 32	1 29	2 26	2 59	3 23	3 41	3 55
28	4 55	4 20	3 43	3 0	2 6	0 0	1 58	2 40	3 7	3 28	3 44
30	4 59	4 28	3 53	3 15	2 30	1 26	1 23	2 18	2 50	3 13	3 32
32	5 4	4 34	4 8	3 28	2 49	1 58	0 0	1 52	2 32	2 59	3 20
34	5 7	4 40	4 11	3 39	3 4	2 21	1 21	1 19	2 11	2 43	3 7
35	5 10	4 45	4 17	3 49	3 17	2 40	1 52	0 0	1 47	2 26	2 53
37	5 13	4 49	4 24	3 57	3 28	2 54	2 14	1 17	1 16	2 6	2 37
38	5 16	4 53	4 29	4 4	3 37	3 7	2 32	1 47	0 0	1 43	2 21
39	5 18	4 56	4 34	4 11	3 46	3 18	2 47	2 9	1 14	1 13	2 2
40	5 20	5 0	4 39	4 16	3 53	3 28	2 59	2 26	1 43	0	1 39
41	5 22	5 3	4 43	4 22	4 0	3 36	3 10	2 40	2 41	1 11	1 10
42	5 24	5 5	4 46	4 27	4 6	3 44	3 20	2 53	2 21	1 39	0
43	5 26	5 8	4 50	4 31	4 12	3 51	3 28	3 2	3 52	0	1 9
44	5 27	5 10	4 53	4 35	4 17	3 57	3 36	3 13	2 47	2 16	1 36
44	5 28	5 12	4 56	4 39	4 21	4 3	3 43	3 22	3 0	2 30	1 56
45	5 30	5 14	4 59	4 43	4 26	4 8	3 49	3 29	3 8	2 42	2 13
46	5 31	5 16	5 1	4 46	4 30	4 13	3 55	3 36	3 16	2 53	2 26
46	5 32	5 18	5 4	4 49	4 34	4 18	4 1	3 43	3 24	3 2	2 38
47	5 33	5 20	5 6	4 52	4 37	4 22	4 6	3 49	3 31	3 11	2 49
47	5 34	5 21	5 8	4 54	4 40	4 26	4 11	3 55	3 38	3 19	2 58
48	5 35	5 23	5 10	4 57	4 44	4 30	4 15	4 0	3 44	3 26	3 7
48	5 36	5 24	5 12	4 59	4 47	4 33	4 19	4 5	3 49	3 33	3 15
49	5 37	5 25	5 14	5 2	4 49	4 37	4 23	4 9	3 55	3 39	3 22
49	5 38	5 27	5 15	5 4	4 52	4 40	4 27	4 14	4 0	3 45	3 29
49	5 39	5 28	5 17	5 6	4 54	4 43	4 31	4 18	4 4	3 50	3 35
50	5 39	5 29	5 19	5 8	4 57	4 46	4 34	4 22	4 9	3 55	3 41
50	5 40	5 30	5 20	5 10	4 59	4 48	4 37	4 25	4 13	4 0	3 47
50	5 41	5 31	5 21	5 11	5 1	4 51	4 40	4 29	4 17	4 5	2 52
51	5 42	5 32	5 23	5 13	5 3	4 54	4 43	4 32	4 21	4 9	2 57
51	5 42	5 33	5 24	5 15	5 5	4 56	4 46	4 35	4 25	4 13	1
51	5 43	5 34	5 25	5 16	5 7	4 58	4 48	4 38	4 28	4 17	4 6
52	5 43	5 35	5 27	5 18	5 9	5 0	4 51	4 41	4 31	4 21	10
52	5 44	5 36	5 28	5 19	5 11	5 2	4 53	4 44	4 35	4 25	14
52	5 45	5 37	5 29	5 21	5 13	5 4	4 56	4 47	4 38	4 28	18
53	5 45	5 38	5 30	5 22	5 14	5 6	4 58	4 49	4 41	4 31	22
53	5 46	5 38	5 31	5 23	5 16	5 8	5 0	4 52	4 43	4 35	25
53	5 46	5 39	5 32	5 25	5 17	5 10	5 2	4 54	4 46	4 38	29
53	5 47	5 40	5 33	5 26	5 19	5 12	5 4	4 57	4 49	4 41	32
54	5 47	5 41	5 35	5 28	5 22	5 15	5 8	5 1	4 54	4 46	39
54	5 48	5 42	5 37	5 31	5 24	5 18	5 12	5 5	4 59	4 52	45
55	5 49	5 44	5 38	5 33	5 27	5 21	5 15	5 9	5 3	4 57	50
55	5 50	5 45	5 40	5 35	5 29	5 24	5 19	5 13	5 7	5 2	55
55	5 51	5 46	5 41	5 37	5 32	5 27	5 22	5 17	5 11	5 6	0
56	5 51	5 47	5 43	5 38	5 34	5 30	5 25	5 20	5 15	5 10	5
56	5 52	5 48	5 44	5 40	5 36	5 32	5 28	5 24	5 19	5 15	10
56	5 53	5 49	5 46	5 42	5 38	5 35	5 31	5 27	5 23	5 19	14
57	5 54	5 50	5 47	5 44	5 40	5 37	5 33	5 30	5 26	5 22	19
57	5 54	5 51	5 48	5 45	5 42	5 39	5 36	5 33	5 30	5 26	23

TABLE XXII.—*Best Altitudes for Obtaining Apparent Time.*

Lat.	Declination of the same name with the Latitude.											
	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°	22°	24°
0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
1	30 0	14 29	9 37	7 12	5 46	4 49	4 8	3 38	8 14	2 55	2 40	2 28
2	90 0	30 1	19 30	14 31	11 36	9 40	8 18	7 16	6 29	5 51	5 21	4 55
3	41 49	48 37	30 3	22 5	17 32	14 35	12 30	10 57	9 45	8 48	8 2	7 24
4	30 1	90 0	41 52	30 5	23 41	19 36	16 45	14 40	13 8	11 46	10 44	9 53
5	23 36	58 10	56 30	38 46	30 8	24 47	21 7	18 26	16 23	14 46	13 27	12 22
6	19 30	41 52	90 0	48 41	37 1	30 11	25 36	22 17	19 46	17 48	16 12	14 53
7	16 38	34 55	59 4	61 7	44 34	35 53	30 15	26 14	23 14	20 52	18 59	17 26
8	14 31	30 5	48 41	60 0	53 16	42 1	35 7	30 20	26 46	24 1	21 49	20 1
9	12 53	26 29	41 56	92 50	64 16	48 48	40 17	34 35	30 25	27 13	24 41	22 37
10	11 36	23 41	37 1	53 16	90 0	56 38	45 52	39 8	34 11	30 31	27 37	25 16
11	10 32	21 27	33 13	46 50	65 31	66 36	52 4	43 48	38 8	33 55	30 37	27 59
12	9 40	19 36	30 11	42 1	56 38	90 0	59 15	48 58	42 17	37 26	33 48	30 45
13	8 56	18 4	27 41	38 13	50 32	67 33	68 25	54 42	46 43	41 8	36 54	33 35
14	8 18	16 45	25 36	35 7	45 52	59 15	90 0	61 22	51 32	45 1	40 14	36 30
15	7 45	15 38	23 49	32 32	42 8	53 27	69 11	69 53	56 93	49 11	43 42	39 31
16	7 16	14 40	22 17	30 20	39 8	48 58	61 22	90 0	63 7	53 42	47 22	42 40
17	6 51	13 48	20 57	28 26	36 26	45 20	55 50	70 81	71 7	58 44	51 18	45 57
18	6 29	12 3	19 46	26 46	34 11	42 17	51 32	68 7	90 0	64 37	55 35	49 27
19	6 9	12 22	18 44	25 18	32 14	39 41	48 0	57 51	71 39	72 9	60 21	53 10
20	5 51	11 46	17 48	24 1	30 31	37 26	45 1	53 42	64 37	90 0	65 55	57 14
21	5 35	11 13	16 58	22 51	28 59	35 28	42 28	50 17	59 34	72 38	73 4	61 47
22	5 21	10 44	16 12	21 49	27 37	33 43	40 14	47 22	55 35	65 55	90 0	67 5
23	5 7	10 17	15 31	20 52	26 23	32 9	38 15	44 52	52 16	61 5	73 29	53 53
24	4 55	9 53	14 53	20 1	25 16	30 45	36 30	42 40	49 27	57 14	67 5	90 0
25	4 44	9 30	14 19	19 14	24 16	29 28	34 55	40 43	46 59	54 1	62 25	74 16
26	4 34	9 9	13 48	18 31	23 20	28 19	33 30	38 58	44 49	51 17	58 43	68 5
27	4 25	8 50	13 19	17 51	22 29	27 15	32 12	37 23	42 54	48 53	55 36	63 37
28	4 16	8 33	12 52	17 15	21 42	26 17	31 8	35 57	41 10	46 46	52 56	60 3
29	4 8	8 16	12 27	16 41	20 59	25 24	29 56	34 39	39 36	44 52	50 36	57 2
30	4 0	8 1	12 4	16 10	20 19	24 34	28 56	33 27	38 10	43 10	48 31	54 26
31	3 53	7 47	11 43	15 41	19 42	23 49	28 1	32 21	36 52	41 37	46 40	52 10
32	3 47	7 34	11 23	15 14	19 8	23 6	27 10	31 21	35 40	40 12	44 59	50 9
33	3 40	7 22	11 4	14 48	18 36	22 26	26 22	30 24	34 34	38 54	43 27	48 18
34	3 35	7 10	10 46	14 25	18 5	21 50	25 38	29 32	33 33	37 42	42 4	46 39
35	3 29	6 59	10 30	14 8	17 37	21 15	24 57	28 43	32 36	36 36	40 47	45 11
36	3 24	6 49	10 16	13 42	17 11	20 43	24 18	27 58	31 43	35 35	39 36	43 46
37	3 19	6 39	10 0	13 22	16 46	20 13	23 42	27 16	30 54	34 38	38 30	42 31
38	3 15	6 31	9 47	13 4	16 23	19 44	23 8	26 36	30 8	33 45	37 29	41 20
39	3 11	6 22	9 34	12 47	16 1	19 18	22 36	25 59	29 25	32 55	36 32	40 16
40	3 7	6 14	9 22	12 30	15 40	18 52	22 7	25 24	28 44	32 9	35 39	39 15
41	3 3	6 6	9 10	12 15	15 21	18 29	21 38	24 51	28 6	31 25	34 49	38 18
42	2 59	5 59	8 59	12 0	15 2	18 6	21 12	24 20	27 30	30 44	34 3	37 25
43	2 56	5 52	8 49	11 47	14 45	17 45	20 47	23 50	26 57	30 6	33 19	36 36
44	2 53	5 46	8 39	11 33	14 29	17 25	20 23	23 23	26 25	29 30	32 38	35 49
45	2 50	5 40	8 30	11 21	14 13	17 6	20 2	22 57	25 55	28 56	31 59	35 6
46	2 47	5 34	8 21	11 9	13 58	16 48	19 39	22 32	25 26	28 23	31 25	34 25
47	2 44	5 28	8 13	10 58	13 44	16 31	19 19	22 8	25 0	27 53	30 49	33 47
48	2 42	5 23	8 5	10 48	13 31	16 15	19 0	21 46	24 34	27 24	30 16	33 12
49	2 39	5 18	7 58	10 38	13 18	15 59	18 42	21 25	24 10	26 57	29 46	32 35
50	2 37	5 13	7 51	10 28	13 6	15 45	18 25	21 5	22 47	26 31	29 17	32 4
51	2 32	5 5	7 37	10 10	12 44	15 18	17 53	20 28	23 5	25 43	28 23	31 7
52	2 28	4 57	7 25	9 54	12 24	14 54	17 24	19 55	22 27	25 1	27 35	30 10
53	2 25	4 50	7 15	9 40	12 5	14 31	16 58	19 25	21 53	24 22	26 52	29 22
54	2 22	4 43	7 5	9 27	11 49	14 11	16 35	18 58	21 22	23 47	26 13	28 39
55	2 19	4 37	6 56	9 15	11 34	13 53	16 13	18 34	20 54	23 16	25 38	28 0
56	2 16	4 31	6 47	8 3	11 19	13 37	15 54	18 12	20 29	22 48	25 6	27 26
57	2 14	4 27	6 41	8 54	11 8	13 23	15 37	17 52	20 8	22 22	24 38	26 54
58	2 11	4 23	6 34	8 46	10 57	13 9	15 21	17 34	19 46	21 58	24 13	26 26
59	2 9	4 19	6 28	8 38	10 48	12 57	15 7	17 18	19 28	21 39	23 50	26 1
60	2 8	4 15	6 23	8 31	10 39	12 47	14 55	17 8	19 12	21 21	23 30	25 39

TABLE XXIII.—Azimuths or Bearings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.)

star	Sigma in Octantis.			Alpha in Ursaminoris (Polaris).							
	40'	45'	50'	55'	1°	1° 5'	1° 10'	1° 15'	1° 20'		
lat	° ' "	° ' "	° ' "	° ' "	° ' "	° ' "	° ' "	° ' "	° ' "	° ' "	° ' "
1	10 00 45	00 0	50 00 0	55 00 1	0 1 1	5 00 1	10 01 1	15 01 1	20 01 1		
2	02 02	02	02	02	2	02	03	03	03		
3	03 03	03	04	04	5	05	05	06	06		
4	06 06	06	07	08	9	09	10	11	12		
5	09 10	10	11	13	14	15	16	14	18		
6	13 14	16	16	18	20	22	23	25	26		
7	17 20	23	23	25	27	29	32	34	36		
8	24 26	29	29	33	35	38	41	45	47		
9	30 33	37	37	41	45	49	52	56	21 00		
10	37 41	46	46	51	56	6 01	05	16 09	14		
11	45 50	56	56	56 02	1 7	13	14	24	30		
12	54 16 00	51 07	51 07	14	20	27	34	41	47		
13	11 03 11	19	19	27	35	43	51	58	22 06		
14	14 23	32	32	41	50	59	12 08	17 17	27		
15	25 35	46	46	56	2 7	7 18	28	39	49		
16	37 49	52 01	57 13	25	37	49	18 02	23 13			
17	50 47 03	17	31	46	58	13 12	26	39			
18	42 03 18	34	50	3 05	8 21	36	52	24 07			
19	18 35	53	58 10	27	45	14 02	19 19	37			
20	34 53	53 12	32	51	9 10	30	49	25 08			
21	51 48 12	33	55	4 16	38	59	20 20	42			
22	59 22	45	59 07	29	52	15 14	37	59			
23	18 41	54 07	32	57	21	46	21 11	35			
24	27 53	19	45	5 11	37	16 03	29	55			
25	37 49 04	32	58	26	53	20	47	27 14			
26	47 15	44 1	0 12	41	11 09	37	22 06	34			
27	57 27	57	26	56	26	56	26	55			
28	44 08 39	55 10	41	6 12	43	14	45	28 16			
29	19 51	24	56	29	12 01	17 33	23 06	38			
30	30 50 04	38	1 12	45	19	53	27	29 01			
31	42 17	52	27	7 03	38	18 13	48	24			
32	54 30	56 07	44	20	57	34	24 11	47			
33	45 06 41	22	2 00	39	13 17	55	34	30 11			
34	18 58	38	17	57	37	19 17	57	36			
35	31 51 12	54	35	8 16	58	39	25 21	31 02			
36	44 27	57 10	53	36	14 19	20 02	45	28			
37	57 42	27	3 12	56	41	26	26 10	55			
38	16 11 57	44	31	9 17	15 03	50	36	32 23			
39	25 52 13	58 02	60	38	26	21 15	27 03	51			
40	40 30	20	4 10	10 00	50	40	30	33 20			
41	55 46	38	30	22	16 14	22 06	58	50			
42	10 53 04	59 58	50	45	37	33	28 26	34 20			
43	26 21	17	5 13	11 09	17 04	23 00	56	51			
44	41 39	37	35	32	31	28	29 26	35 23			
45	57 57	58	57	57	57	57	57	56			
46	15 54 17 1	0 19	6 20	12 22	18 25	24 26	30 28	36 30			
47	32 42	40	44	48	53	57	31 01	37 04			
48	50 56	1 02	7 09	13 15	19 21	25 27	33	40			
49	19 08 46	25	33	42	51	59	32 08	38 16			
50	26 55 37	48	59	14 10	20 21	26 32	42	55			
51	46 59	2 12	8 25	38	52	27 05	33 18	39 31			
52	50 56 20	36	50	15 08	21 23	29	55	40 11			
53	25 43	3 02	9 20	38	56	28 14	34 32	51			
54	46 57 06	27	48	16 09	22 29	50	35 11	41 32			
55	51 07 30	53	10 17	40	23 03	29 27	50	42 14			
56	28 54	4 20	46	17 12	39	30 04	36 31	57			
57	50 58 29	48	11 17	46	24 15	43	37 12	43 41			
58	13 54	5 16	48	18 20	51	31 23	54	44 27			
59	36 59 10	45 1	12 19 1	18 55 1	25 29 1	32 04 1	38 38 1	45 13			

TABLE XXIII.—Azimuths or Bearings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.)

Polaris.		δ Ursa Minoris.		ϵ Ursa Minoris.			
1° 25'	1° 30'	3° 20'	3° 23'	7° 45'	7° 50'	7° 55'	8° 00'
o / / /	o / / /	o / / /	o / / /	o / / /	o / / /	o / / /	o / / /
1 25 01	1 30 00	3 20 02	3 23 55	7 45 04	7 50 04	7 55 05	8 0 0
03	03	08	24 01	45 17	50 17	55 18	0 1
07	08	17	24 10	45 89	50 89	55 39	0 4
12	13	29	24 23	46 08	51 10	56 10	1 1
20	21	46	24 40	46 47	51 49	56 50	1 5
28	30	21 06	25 00	47 35	52 36	57 38	1 8
38	40	21 30	25 25	48 31	53 33	58 35	3 3
50	53	21 58	25 53	49 36	54 39	59 42	4 4
26 03	31 07	22 30	26 26	50 50	55 54	8 0 58	6 0
19	23	23 06	27 02	52 13	57 18	2 28	7 2
36	41	23 45	27 42	53 46	58 51	3 57	9 0
54	82 01	24 29	28 27	55 27	8 0 34	5 41	10 4
27 14	22	25 16	29 15	57 19	2 27	7 84	12 5
36	45	26 08	30 08	59 20	4 29	9 88	14 4
28 00	33 10	27 04	31 05	8 1 31	6 41	11 52	17 0
26	38	28 04	32 06	8 52	9 04	14 01	19 2
53	34 07	29 09	33 15	6 23	11 87	16 51	22 0
29 29	38	30 18	34 23	9 05	14 21	19 87	24 5
54	35 11	31 32	35 39	11 58	17 16	22 34	27 5
30 27	47	32 51	36 59	15 03	20 22	25 42	31 0
81 03	36 24	34 15	38 25	18 19	23 40	29 02	34 2
22	44	34 59	39 09	20 01	25 24	30 47	36 1
41	37 04	35 44	39 55	21 46	27 11	32 34	37 5
32 00	25	36 30	40 42	23 35	29 00	34 25	39 5
20	46	37 18	41 31	25 26	30 53	36 20	41 4
42	38 09	38 07	42 21	27 21	32 49	38 16	43 4
33 03	31	38 57	43 12	29 19	34 48	40 17	45 4
25	54	39 49	44 05	31 20	36 51	42 21	47 5
47	39 19	40 42	44 59	33 25	38 57	44 28	50 0
34 11	48	41 37	45 55	35 33	41 06	46 39	52 1
34	40 08	42 33	46 52	37 44	43 19	48 53	54 2
59	34	43 31	47 51	39 59	45 35	51 11	56 4
35 24	41 01	44 30	48 52	42 17	47 55	53 33	59 1
50	28	45 31	49 54	44 33	50 19	55 58	9 1 3
36 16	56	46 33	50 57	47 06	52 47	58 28	4 0
44	42 27	47 37	52 02	49 36	55 26	9 1 01	6 4
37 12	55	48 42	53 09	52 10	57 54	3 88	9 2
40	43 25	49 50	54 18	54 47	9 0 33	6 19	12 0
38 09	55	50 59	55 28	57 29	4 32	9 04	14 5
39	44 27	52 10	56 41	9 0 15	6 05	11 54	17 4
39 10	45 00	53 22	57 54	8 05	8 57	14 48	20 3
42	38	54 37	59 10	6 00	11 53	17 46	23 3
40 15	46 08	55 53	4 0 29	8 59	14 54	20 49	26 4
47	48	57 12	1 48	12 02	17 59	23 56	29 5
41 21	47 19	58 32	3 10	15 10	21 09	27 09	33 0
56	56	59 54	4 34	18 23	24 24	30 26	36 2
42 32	48 29	4 1 18	5 59	21 41	27 44	33 48	39 2
43 09	49 13	2 45	7 28	25 04	31 09	37 15	43 2
43	52	4 13	8 58	28 31	34 39	40 47	46 5
44 25	50 33	5 44	10 30	32 04	38 15	44 25	50 3
45 07	51 15	7 17	12 05	35 42	41 55	48 08	54 2
45	51 58	8 53	13 45	39 26	45 42	51 57	58 1
46 26	52 42	10 31	15 22	43 17	49 34	55 51	10 1 5
47 09	53 27	12 11	17 05	47 11	53 31	59 52	6 1
52	54 13	13 53	18 50	51 13	57 35	10 3 58	10 2
48 37	55 00	15 33	20 37	55 20	10 1 46	8 11	14 3
49 23	55 49	17 27	22 27	59 33	6 02	12 30	18 5
50 10	56 39	19 18	24 20	10 3 53	10 25	16 56	23 2
58	57 30	21 11	26 16	8 20	14 54	21 29	28 0
51 47	58 22	23 03	28 14	12 54	19 31	26 08	32 4

TABLE XXIII.—Azimuths or Bearings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.)

star	Sigma in Octant.				Alpha in Ursa Minoris (Polaris).																		
	40'	45'	50'	55'	1°	1° 5'	1° 10'	1° 15'	1° 20'														
lat.	0	10	20	30	40	50	60	70	80														
41	0	53 00	0	59 38	1	6 15	1	12 53	1	19 30	1	26 08	1	32 45	1	39 23	1	46 01					
1	1	53 24	1	0 43	6 46	18 26	20 07	26 48	33 28	40 09	46 50												
42	2	53 50	2	0 23	7 17	14 01	20 44	27 28	34 12	40 56	47 40												
1	1	54 15	1	1 02	7 49	14 36	21 23	28 10	34 57	41 44	48 31												
43	3	54 42	3	1 31	8 22	15 11	22 03	28 53	35 44	42 34	49 24												
1	1	55 09	2	2 02	8 56	15 50	22 43	29 37	36 31	43 24	50 18												
44	4	55 37	2	2 34	9 31	16 28	23 25	30 22	37 19	44 16	51 18												
1	1	56 05	3	3 05	10 06	17 07	24 08	31 09	38 09	45 11	52 10												
45	5	56 34	3	3 39	10 43	17 47	24 52	31 56	39 00	46 04	53 09												
1	1	57 04	4	4 12	11 20	18 28	25 36	32 44	39 53	47 01	54 09												
46	6	57 35	4	4 47	11 59	19 11	26 23	33 35	40 47	47 55	55 10												
1	1	58 07	5	5 23	12 38	19 53	27 10	34 26	41 42	48 58	56 14												
47	7	58 39	5	5 59	13 19	20 39	27 59	35 19	42 39	49 59	57 19												
1	1	59 12	6	6 37	14 01	21 25	28 49	36 13	43 28	51 01	58 26												
48	8	59 47	7	7 15	14 54	22 12	29 40	37 09	44 37	52 06	59 34												
1	1	0 22	7	7 55	15 28	23 06	30 33	38 06	45 39	53 12	0 45												
49	9	0 58	8	8 36	16 13	23 50	31 28	39 05	46 42	54 20	1 57												
1	1	1 36	9	9 17	17 00	24 42	32 24	40 06	47 28	55 30	3 12												
50	10	2 14	10	10 01	17 47	25 34	33 21	41 08	48 55	56 41	4 28												
1	1	2 53	1	10 45	18 37	26 28	34 20	42 12	50 04	57 56	5 47												
51	11	3 34	11	11 31	19 27	27 24	35 21	43 09	51 15	59 11	7 09												
1	1	4 16	12	12 17	20 19	28 21	36 23	44 26	52 28	0 30	8 32												
52	12	4 58	13	06	21 13	29 20	37 28	45 35	53 43	1 50	9 58												
1	1	5 43	13	55	22 08	30 21	38 34	46 47	54 00	3 11	11 26												
53	1	6 28	14	47	23 28	31 24	39 42	48 01	56 20	4 38	12 57												
1	1	7 15	15	39	24 05	32 28	40 53	49 17	57 42	6 06	14 31												
54	2	8 03	16	34	25 04	33 35	42 05	50 35	59 06	7 37	16 08												
1	1	8 53	17	30	26 06	34 36	43 20	51 57	0 34	9 10	17 47												
55	3	9 44	18	28	27 11	35 54	44 37	53 20	2 03	10 47	19 30												
1	1	10 37	19	27	28 20	37 20	45 56	55 02	3 53	12 45	21 16												
56	4	11 32	20	29	29 25	38 22	47 19	56 15	5 12	14 08	23 06												
1	1	12 28	21	32	30 36	39 39	48 43	57 57	6 51	15 54	24 58												
57	5	13 37	22	38	32 01	41 14	50 20	59 38	8 50	18 03	26 15												
1	1	14 27	23	46	33 04	42 23	51 41	0 59	10 18	19 37	28 55												
58	6	15 49	24	56	34 22	43 48	53 14	2 41	12 07	21 34	31 00												
1	1	16 34	26	08	35 42	45 16	54 51	4 26	14 00	23 34	33 09												
59	7	17 40	27	23	37 05	46 47	56 31	6 13	15 56	25 39	35 22												
1	1	18 49	28	40	38 30	48 23	58 18	8 05	17 57	27 48	37 40												
60	8	20 00	30	00	40 02	50 01	0 11	10 02	20 02	30 02	40 03												
1	1	21 14	31	23	41 33	51 42	1 52	12 01	22 11	32 21	42 31												
61	9	22 31	32	50	43 09	54 26	3 52	14 06	24 25	34 44	45 04												
1	1	23 50	34	19	44 48	55 17	5 46	16 15	26 44	37 18	47 43												
62	10	25 12	35	52	46 31	57 10	7 49	18 29	29 18	39 48	50 27												
1	1	26 38	37	28	48 18	59 07	9 58	20 48	31 38	42 28	53 19												
63	11	28 07	39	08	50 09	2	1 10	12 11	23 12	34 14	45 15	56 17											
1	1	29 39	40	51	52 04	3	1 17	14 80	25 42	36 56	48 08	59 22											
64	12	31 15	42	40	54 05	5	2 29	16 54	28 19	39 44	51 09	3 23											
1	1	32 55	44	33	56 09	7	4 47	19 24	31 11	42 39	54 16	5 54											
65	1	34 39	46	29	58 20	10	10	22 00	33 51	45 41	57 32	9 22											
1	1	36 28	48	31	0 35	12	13	24 43	36 47	48 51	0 56	13 00											
66	2	38 21	50	39	2 57	15	15	27 33	39 52	52 10	4 28	16 46											
1	1	40 19	52	52	5 25	17	58	30 30	43 04	55 36	8 10	20 45											
67	3	42 23	55	11	8 00	20	48	33 36	46 25	59 13	12 02	24 51											
1	1	44 32	57	37	10 41	23	46	36 50	49 55	3 00	16 04	29 09											
68	4	46 48	2	0 09	13 30	26	52	40 13	53 35	6 57	20 18	33 41											
1	1	49 09	2	4 48	16 27	30	06	43 57	57 25	11 04	24 25	38 25											
69	5	51 38	5	8 55	19 33	33	31	47 29	3	1 27	15 25	29 24	43 22										
1	1	54 14	8	32	22 48	37	06	51 25	5	4 41	19 59	34 17	48 35										
70	1	56 58	11	36	26 14	2	40	51	2	55	30	3	10	08	3	24	47	3	39	25	3	54	04

TABLE XXIII.—Azimuths or Bearings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.

Polaris.		δ Ursa Minoris.		ε Ursa Minoris.			
1° 25'	1° 30'	3° 20'	3° 23'	7° 45'	7° 50'	7° 55'	8° 00'
0 / / /	0 / / /	0 / / /	0 / / /	0 / / /	0 / / /	0 / / /	0 / / /
1 52 38	1 59 16	4 25 07	4 30 16	10 17 08	10 24 06	10 30 55	10 37 36
53 30	2 0 11	27 09	32 21	22 22	29 06	35 49	42 33
54 23	1 07	29 00	34 29	27 17	34 04	40 51	47 38
55 18	2 05	31 15	36 14	32 20	38 26	46 01	52 51
56 14	3 04	33 24	38 55	36 38	44 25	51 18	58 12
57 12	4 05	35 52	41 13	42 50	51 18	56 44	11 8 42
58 11	5 08	38 45	44 10	48 18	55 18	11 2 19	9 20
59 11	6 12	40 30	46 01	53 54	11 0 58	8 03	15 07
2 0 13	7 18	43 00	48 30	59 40	68 20	13 57	21 04
1 17	8 25	45 31	51 04	11 5 43	12 36	19 58	27 10
2 23	9 35	48 05	53 41	11 37	18 53	26 10	38 26
3 30	10 46	50 43	56 23	17 51	25 12	32 32	39 53
4 39	11 59	53 27	59 09	24 15	31 40	39 05	46 30
5 50	13 14	56 14	5 2 00	30 50	38 19	45 48	53 18
7 00	14 31	59 06	4 55	37 36	45 09	52 44	12 0 17
8 18	15 50	5 2 03	7 55	44 28	52 11	59 50	7 28
9 35	17 12	5 05	11 01	51 42	59 24	12 7 07	14 51
10 52	18 36	8 12	14 11	59 03	12 6 51	14 29	22 27
12 16	20 02	11 24	17 27	12 6 36	14 30	22 22	30 16
13 39	21 13	14 42	20 49	14 23	22 21	30 20	38 19
15 05	23 02	18 05	24 16	22 24	30 27	38 31	48 33
16 34	24 36	21 34	27 49	30 39	38 48	46 57	55 07
18 05	26 13	25 09	31 29	39 09	47 33	55 38	13 03 54
19 39	27 52	28 51	35 18	47 53	56 03	13 4 26	12 56
21 16	29 35	32 40	39 08	56 55	13 5 21	13 46	22 16
22 56	31 20	36 35	43 08	13 6 10	14 45	23 18	31 52
24 38	33 09	40 38	47 15	15 47	24 27	33 07	41 47
26 24	35 01	44 48	51 30	25 41	34 26	43 12	52 00
28 14	36 57	49 06	55 53	35 52	44 45	53 39	14 2 33
30 06	38 56	53 32	6 0 24	46 24	55 25	14 4 24	13 20
32 02	40 59	58 06	5 05	57 16	14 6 24	15 33	24 41
34 03	43 06	6 2 50	9 54	14 8 31	17 46	27 02	36 18
36 07	45 41	7 44	14 52	20 09	29 34	38 54	48 18
38 14	47 33	12 45	19 59	32 07	41 39	51 12	15 0 44
40 27	49 53	17 58	25 20	44 33	54 13	15 3 54	13 34
42 44	52 15	23 21	30 49	57 24	15 7 13	17 03	26 52
45 05	54 48	28 56	36 30	15 10 43	20 44	30 39	40 38
47 31	57 23	34 42	42 24	24 30	34 38	44 46	54 48
50 03	3 0 04	40 41	48 29	38 50	49 05	59 22	16 9 41
52 40	2 50	46 52	54 48	53 37	16 4 05	16 14 33	25 02
55 23	5 42	53 18	7 1 21	16 6 43	19 37	27 57	40 57
58 12	8 14	59 57	8 10	29 04	38 10	46 38	55 04
3 1 08	11 47	7 6 52	15 11	43 34	55 00	17 3 37	17 14 39
4 09	15 00	14 03	22 30	58 49	17 10 03	21 17	32 31
7 18	18 20	21 30	30 10	17 16 47	28 13	39 39	51 06
10 34	21 48	30 18	38 02	35 28	47 40	58 47	18 13 04
13 59	25 24	37 19	46 15	54 57	18 6 50	18 18 43	30 38
17 32	29 10	45 48	54 51	18 15 16	27 23	39 31	51 40
21 13	33 04	54 29	8 3 45	36 27	48 50	19 49 13	19 13 37
25 04	37 09	8 3 36	13 13	58 36	19 11 14	23 53	36 56
29 05	41 24	12 08	22 46	19 21 45	34 40	47 36	20 0 33
33 17	45 51	23 05	32 54	45 59	59 04	20 12 25	25 40
37 40	50 29	33 28	43 31	20 11 17	20 24 53	38 28	51 56
42 15	55 20	44 22	54 37	37 59	51 51	21 5 41	21 19 34
47 03	4 0 25	55 46	9 6 14	21 5 56	21 20 08	34 28	48 33
52 05	5 45	9 7 42	18 25	35 20	49 53	22 4 27	22 19 02
57 08	11 20	20 15	31 13	22 6 15	22 21 11	36 09	51 07
4 2 54	17 12	33 25	44 39	38 05	54 11	23 9 34	23 24 57
8 43	23 02	47 17	58 48	23 13 16	23 29 02	44 50	24 0 40

TABLE XXIII.—*Azimuths or Bearings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.*

Star.	β Chamaeleontis.			β Hydri and ζ Ursa Minoris.			
P.D.	11° 30'	11° 35'	11° 40'	11° 45'	11° 50'	11° 55'	12° 00'
Lat.	° / ' / "	° / ' / "	° / ' / "	° / ' / "	° / ' / "	° / ' / "	° / ' / "
1	11 30 06	11 35 06	11 40 06	11 45 07	11 50 07	11 55 07	12 0 07
2	30 26	35 26	40 26	45 26	50 26	55 27	0 27
3	30 58	35 58	40 59	45 59	50 59	56 00	1 00
4	31 42	36 43	41 44	46 45	51 46	56 46	1 47
5	32 40	37 42	42 43	47 45	52 43	57 47	2 48
6	33 51	38 53	43 55	48 56	53 58	59 00	4 02
7	35 15	40 18	45 19	50 22	55 25	12 0 27	5 29
8	36 53	41 56	46 59	52 02	57 05	2 08	7 11
9	38 43	43 27	48 51	53 55	58 59	4 03	9 07
10	40 48	45 53	50 57	56 02	12 1 07	6 12	11 17
11	43 06	48 12	53 18	58 23	3 29	8 35	13 40
12	45 88	50 45	55 52	12 0 59	6 06	11 13	16 20
13	48 24	53 33	58 41	3 49	8 51	14 06	19 14
14	51 86	56 35	12 1 45	6 55	12 04	17 14	22 23
15	54 41	59 53	5 04	10 15	15 26	20 37	25 48
16	58 13	12 3 25	8 88	13 51	19 03	24 16	28 28
17	12 1 56	7 14	12 28	17 42	22 57	28 11	31 40
18	5 81	11 18	16 34	21 51	27 07	32 23	36 39
19	10 21	15 27	21 08	26 15	31 33	36 52	42 10
20	14 57	20 17	25 87	30 57	36 18	41 38	47 09
21	19 50	25 12	30 35	35 57	41 20	46 42	52 04
22	22 23	27 48	33 10	38 34	43 58	49 21	55 01
23	25 01	30 26	35 51	41 16	46 40	52 05	57 30
24	27 44	33 10	38 36	44 02	49 27	54 55	13 0 19
25	30 30	35 58	41 25	46 52	52 20	57 47	3 14
26	33 23	38 51	44 20	49 48	55 17	13 0 45	6 14
27	36 20	41 49	47 19	52 49	58 19	3 49	9 18
28	39 21	44 52	50 23	55 55	13 1 26	6 57	12 28
29	42 29	48 00	53 33	57 16	4 27	10 11	15 43
30	45 40	51 14	56 48	13 8 32	7 54	13 30	19 04
31	48 57	54 32	13 0 08	5 21	11 19	16 54	23 30
32	52 19	57 56	3 33	9 10	14 47	20 24	26 02
33	55 47	13 1 26	7 04	12 43	18 22	24 00	29 39
34	59 19	5 01	10 41	16 21	21 54	27 41	33 22
35	13 2 37	8 32	14 23	20 05	25 47	31 29	37 11
36	6 44	12 28	18 11	23 55	29 38	35 22	41 06
37	10 12	16 12	22 05	27 51	33 24	39 21	45 07
38	14 31	20 18	26 05	32 06	37 40	43 27	49 14
39	18 34	24 23	30 12	36 01	41 50	47 39	53 28
40	22 43	28 34	34 25	40 15	46 06	51 57	57 48
41	26 58	32 57	38 44	44 37	50 29	56 22	14 2 15
42	31 21	37 25	43 10	49 05	54 48	14 5 54	6 49
43	35 45	41 46	47 42	53 39	59 36	5 33	11 30
44	40 13	46 23	52 22	58 21	14 4 20	10 19	16 18
45	45 07	50 56	57 09	14 3 11	9 11	15 12	21 13
46	49 57	56 00	14 2 03	8 06	14 09	20 13	26 16
47	54 54	14 0 59	6 29	13 14	19 15	25 21	31 37
48	59 58	6 06	12 14	18 20	24 29	30 37	36 45
49	14 5 11	11 21	17 31	23 41	29 29	36 02	42 17
50	10 32	16 44	22 56	29 09	35 21	41 34	47 47
51	15 59	22 13	28 29	34 44	41 00	47 15	53 30
52	21 36	28 06	34 01	40 29	46 34	52 04	59 22
53	27 21	33 42	40 02	46 22	52 30	59 03	15 5 24
54	33 15	39 39	46 01	52 24	58 41	15 5 11	11 34
55	39 19	45 48	52 10	58 22	15 5 02	11 28	17 54
56	47 36	52 00	58 28	15 4 45	11 26	17 55	24 24
57	53 59	58 25	15 4 56	11 28	18 00	24 31	31 03
58	58 25	15 5 00	11 34	18 09	24 43	31 18	37 53
59	15 5 07	11 45	18 22	25 00	31 24	38 16	44 54
60	11 59	18 40	25 32	32 02	38 43	45 24	52 05

TABLE XXIII.—Azimuths or Bearings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.)

Star.	β Chamaeleontis.			β Hydri and ζ Ursæ Minoris.			
P.D.	11° 30'	11° 35'	11° 40'	11° 45'	11° 50'	11° 55'	12° 00'
Lat.	° / ' / "	° / ' / "	° / ' / "	° / ' / "	° / ' / "	° / ' / "	° / ' / "
41	15 19 08	15 26 00	15 32 30	15 39 50	15 45 59	15 52 43	15 59 27
1/2	26 17	33 17	39 57	46 39	53 26	16 0 14	16 7 02
42	33 42	40 33	45 10	52 00	16 1 05	7 57	14 48
1/2	41 20	48 14	55 08	16 2 03	8 57	15 50	22 46
43	49 10	56 07	16 3 05	10 02	17 01	23 59	30 58
1/2	57 11	16 4 12	11 14	18 16	25 18	32 20	39 22
44	16 5 06	12 13	19 57	26 42	33 49	40 54	48 00
1/2	18 54	21 03	28 12	35 22	42 32	49 42	56 02
45	22 36	29 49	37 08	44 16	51 16	58 44	17 5 58
1/2	31 32	38 49	46 07	53 56	17 0 43	17 8 01	15 19
46	40 13	48 05	55 27	17 2 49	10 11	17 34	24 56
1/2	50 09	17 57 00	17 5 02	12 28	19 57	27 22	34 49
47	59 51	7 22	14 53	22 24	29 55	37 27	44 58
1/2	17 9 49	17 25	25 00	32 37	40 12	47 49	55 25
48	20 04	27 44	35 25	43 06	50 47	58 28	18 5 34
1/2	30 37	38 23	46 08	53 54	18 1 40	18 9 26	17 12
49	41 25	49 19	57 09	18 5 01	12 52	20 43	28 18
1/2	52 38	18 0 35	18 8 30	16 27	24 23	32 20	40 16
50	18 4 08	12 10	20 11	28 12	36 14	44 16	52 19
1/2	15 58	24 05	32 12	40 20	48 27	56 35	19 4 43
51	28 10	36 23	44 36	53 05	19 1 02	19 9 16	17 30
1/2	40 44	49 02	57 21	19 5 41	14 00	22 20	30 22
52	53 40	19 2 05	19 10 30	18 56	27 21	35 47	44 13
1/2	19 7 01	15 19	24 05	32 36	41 08	49 40	58 13
53	20 47	29 25	38 03	46 42	55 20	20 4 00	20 12 39
1/2	34 59	43 40	52 28	20 1 14	20 10 00	19 00	27 32
54	49 38	58 35	20 7 02	16 15	25 07	34 01	42 54
1/2	20 5 52	20 13 44	22 45	31 44	40 47	49 46	58 46
55	20 22	29 30	38 29	47 45	56 53	21 6 00	21 15 10
1/2	36 31	45 47	55 02	21 3 21	21 13 39	22 52	32 07
56	53 14	21 2 37	21 12 00	21 24	30 48	40 13	49 38
1/2	21 10 29	20 01	29 33	39 05	48 38	58 11	22 7 45
57	31 28	41 10	50 53	22 0 35	22 10 18	22 16 47	26 30
1/2	46 50	56 40	22 6 30	16 20	26 24	36 13	45 55
58	22 6 00	22 15 59	25 58	35 58	46 00	56 00	23 6 02
1/2	25 51	36 00	46 10	56 19	23 6 30	28 16 41	26 58
59	46 26	56 47	23 7 05	23 17 26	27 47	38 09	48 31
1/2	23 7 47	23 18 17	28 48	39 19	49 50	24 0 24	24 10 58
60	29 27	40 39	51 20	24 2 03	24 13 21	23 31	34 15
1/2	52 59	24 3 52	24 14 46	25 41	36 37	47 33	58 27
61	24 16 56	28 02	39 07	50 15	25 1 42	25 12 32	25 23 41
1/2	41 29	53 09	25 4 28	25 15 49	27 09	38 32	49 54
62	25 7 46	25 19 18	30 52	42 25	54 00	26 5 36	26 17 12
1/2	34 47	46 34	58 21	26 10 17	26 21 58	33 48	45 40
63	26 2 58	26 15 00	26 27 02	39 05	51 09	27 8 15	27 15 20
1/2	32 22	44 40	56 55	27 9 17	27 21 37	34 00	46 21
64	27 3 06	27 15 39	27 28 14	40 51	53 27	28 6 06	28 18 46
1/2	35 04	48 05	28 0 57	28 13 51	28 26 46	39 42	43 57
65	28 8 52	28 22 02	35 12	48 24	29 1 39	29 14 54	29 28 08
1/2	43 27	57 36	29 11 06	29 24 39	38 12	51 48	30 5 22
66	29 21 05	29 34 56	48 47	30 2 40	30 16 36	30 30 33	44 31
1/2	59 56	30 14 08	30 28 22	42 38	56 56	31 11 15	31 25 37
67	30 40 48	55 25	31 10 02	31 24 41	31 39 23	54 07	32 8 53
1/2	31 23 42	31 38 53	53 56	32 9 01	32 24 08	32 39 18	54 31
68	32 9 17	32 24 15	32 40 16	55 49	33 11 24	33 27 02	33 42 42
1/2	57 17	33 13 15	33 29 14	33 45 17	34 1 22	34 17 30	34 33 38
69	33 48 07	34 4 36	34 21 07	34 37 45	54 19	35 11 00	35 27 43
1/2	34 42 02	59 16	35 16 10	35 33 19	35 50 32	36 7 48	36 25 08
70	35 39 21	35 57 00	36 15 43	36 33 30	36 50 32	37 8 16	37 26 14

TABLE XXIII.—Azimuths or Bearings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.)

Star.	γ Cephei.						
P. D.	12° 05'	12° 40'	12° 45'	12° 50'	22° 55'	13° 00'	13° 05'
Lat.	° / ' / "	° / ' / "	° / ' / "	° / ' / "	° / ' / "	° / ' / "	° / ' / "
1	12 5 07	12 40 07	12 45 07	12 50 07	12 55 07	13 0 07	13 5 7
2	5 27	40 28	45 29	50 29	55 29	0 29	5 29
8	6 01	41 36	46 04	51 05	56 05	1 05	6 06
4	6 48	41 53	46 57	51 55	56 56	1 56	6 57
5	7 49	42 57	47 58	53 00	58 01	3 02	8 03
6	9 03	44 16	49 17	54 19	59 21	4 21	9 24
7	10 32	45 48	50 51	55 53	13 0 55	5 58	11 00
8	12 14	47 36	52 39	57 42	2 45	7 48	12 51
9	14 11	49 38	54 42	59 46	4 50	9 54	14 58
10	16 21	51 55	57 00	13 2 05	7 10	12 15	17 20
11	18 48	54 28	59 34	4 40	9 46	14 52	19 58
12	21 27	57 16	13 2 23	7 31	12 28	17 52	22 52
13	24 24	13 0 15	5 29	10 37	15 45	20 33	26 02
14	27 33	3 40	8 50	14 07	19 09	24 19	29 29
15	30 59	7 17	12 10	17 39	22 49	28 02	33 18
16	34 41	11 10	16 23	21 35	26 48	32 01	37 14
17	38 40	15 21	20 35	25 50	31 04	36 19	41 03
18	42 55	19 49	25 05	30 22	35 38	40 54	46 10
19	47 28	24 35	29 54	35 12	40 30	45 48	51 07
20	52 18	29 40	35 01	40 21	45 41	51 02	56 22
21	57 27	35 04	40 25	45 49	51 12	56 35	14 1 67
13	0 08	37 54	43 17	48 41	54 06	59 29	4 53
22	2 54	40 48	46 13	51 38	57 03	14 2 28	7 53
14	5 45	43 48	49 14	54 40	14 0 06	5 33	10 59
23	8 41	46 53	52 20	57 48	3 15	8 42	14 10
14	11 51	50 02	55 31	14 1 00	6 29	11 58	17 20
24	14 48	53 18	58 48	4 18	9 49	15 18	20 46
14	18 00	56 39	14 2 10	7 42	13 13	18 45	24 19
25	21 15	14 0 06	5 38	11 11	16 44	22 17	27 56
14	24 38	3 37	9 12	14 46	20 21	25 55	31 30
26	28 06	7 15	12 51	18 27	24 03	29 39	35 15
15	31 50	10 59	16 37	22 14	27 52	33 29	39 07
27	35 07	14 49	20 28	26 07	31 46	37 00	43 04
15	39 02	18 45	24 25	30 07	35 47	41 58	37 03
28	42 53	22 48	28 30	34 12	39 54	45 36	51 19
15	46 49	26 56	32 40	38 24	44 08	49 52	55 36
29	50 52	31 11	36 57	42 43	48 28	54 04	15 0 00
15	55 01	35 32	41 21	47 08	52 43	58 43	4 31
30	59 17	40 01	45 51	51 40	57 36	15 3 19	9 09
14	3 39	44 37	50 28	56 15	15 2 11	8 02	13 54
31	8 08	49 19	55 13	15 1 15	6 59	12 53	18 46
15	12 44	54 09	15 0 05	6 00	11 55	17 51	23 46
32	17 27	59 06	5 04	11 01	16 29	22 56	28 53
15	22 17	15 4 12	10 11	16 10	22 10	28 10	34 09
33	27 14	9 24	15 26	21 27	27 29	33 31	39 33
15	32 20	14 44	20 49	26 53	32 57	39 01	45 05
34	37 32	20 14	26 20	32 26	38 32	44 39	50 45
15	42 53	25 51	32 00	38 07	44 00	50 45	56 34
35	48 22	31 37	37 47	43 58	50 10	56 34	2 32
15	54 00	37 31	43 45	49 58	56 12	16 2 32	8 39
36	59 46	43 35	49 51	56 07	16 2 23	8 39	14 55
15	5 40	49 48	56 06	16 2 25	8 44	14 55	21 21
37	11 44	56 10	16 2 40	8 53	15 14	21 21	27 57
16	17 47	16 2 48	9 06	15 31	21 54	27 57	34 43
38	24 21	9 25	15 52	22 18	28 46	34 43	41 39
16	30 52	16 18	22 47	29 17	35 47	41 39	48 47
39	37 35	23 21	29 53	36 26	42 59	48 47	56 06
16	44 14	30 35	37 11	43 47	50 23	56 06	3 34
40	51 32	38 00	44 39	51 18	57 57	17 3 34	11 16
16	58 46	45 35	52 20	59 02	17 5 24	11 16	19 09

TABLE XXIII.—Azimuths or Bearings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.)

Star.	γ Cephi.													
P.D.	12° 05'		12° 40'		12° 45'		12° 50'		12° 55'		13° 00'		13° 05'	
Lat.	°	'	°	'	°	'	°	'	°	'	°	'	°	'
41	16	6 12	16	53 27	17	0 12	17	6 57	17	13 43	17	20 29	17	27 14
1/2		13 50	17	1 28		8 17		15 06		22 23		28 44		35 33
42		21 39		9 42		16 34		23 27		30 19		37 12		44 05
1/2		29 41		18 09		25 05		32 01		38 57		45 53		52 50
43		37 56		26 50		33 49		40 49		47 49		54 49	18	1 49
1/2		46 24		35 44		42 48		49 51		56 55	18	3 59		11 03
44		55 06		44 58		52 01		59 14	18	6 16		13 23		20 31
1/2	17	4 02		54 17	18	1 29	18	8 40		15 52		23 03		30 15
45		13 13	18	3 56		11 12		18 28		25 43		33 00		40 16
1/2		22 38		13 52		21 11		28 31		35 51		43 11		50 32
46		32 19		24 31		31 28		38 52		46 16		53 41	19	1 06
1/2		42 16		34 33		42 00		49 13		56 59	19	4 23		11 57
47		52 50		45 19		52 52	19	0 25	19	7 59		15 28		23 07
1/2	18	3 01		56 23	19	4 01		11 39		19 18		26 55		34 36
48		13 51	19	7 47		15 30		23 13		30 57		38 40		46 25
1/2		24 59		19 30		27 18		34 50		42 55		50 44		58 33
49		36 26		31 34		39 27		47 21		55 15	20	8 09	20	11 04
1/2		48 13		43 59		51 58		59 57	20	7 57		15 57		23 56
50	19	0 38		56 46	20	4 57	20	12 55		21 01		29 03		37 11
1/2		12 21	20	9 56		18 07		26 17		34 28		42 39		50 51
51		25 44		23 30		31 47		40 03		48 20		56 37	21	4 55
1/2		39 00		37 29		45 52		54 14	21	2 38	21	11 01		19 25
52		52 40		51 54	21	0 22	21	8 52		17 29		25 52		34 22
1/2	20	6 47	21	6 46		15 22		23 58		32 24		41 10		49 47
53		21 00		22 06		30 48		39 31		48 14		56 58	22	5 42
1/2		36 19		37 55		46 45		55 35	22	4 25	22	13 01		22 07
54		51 48		54 16	22	3 13	22	12 10		21 08		30 06		39 04
1/2	21	7 48	22	11 08		20 13		29 18		38 23		47 29		56 35
55		24 18		28 35		37 47		47 00		56 13	23	5 27	23	14 41
1/2		41 24		46 36		55 57	23	5 18	23	14 29		24 02		33 24
56		59 03	23	5 14	23	14 43		24 13		33 17		43 04		51 45
1/2	22	17 19		24 32		34 10		43 48		53 28	24	8 07	24	12 48
57		36 13		44 30		54 17	24	4 05	24	13 54		23 43		33 32
1/2		55 48	24	5 11	24	15 08		25 06		35 03		45 02		55 02
58	23	16 04		26 37		32 05		46 51		57 00	25	7 08	25	17 18
1/2		37 06		48 50		59 07	25	9 26	25	19 45		30 04		40 24
59		58 54	25	11 54	25	22 22		31 43		43 21		53 52	26	4 23
1/2	24	21 32		35 50		46 30		57 10	26	7 52	26	13 34		29 17
60		45 02	26	0 43	26	11 34	26	22 27		33 20		44 15		55 10
1/2	25	9 27		26 34		37 38		48 44		59 50	27	10 57	27	22 05
61		34 52		53 30	27	4 46	27	16 14	27	27 24		38 45		50 05
1/2	26	1 18	27	21 29		33 01		44 32		56 06	28	7 40	28	19 15
62		28 50		50 41	28	2 26	28	14 12		28 26 01		37 50		49 46
1/2		57 32		28 21 07		33 08		45 10		57 13	29	9 17	29	21 23
63	27	27 29		52 54	29	5 11	29	17 29	29	29 49		42 13		54 31
1/2		54 33		29 26 07		38 41		51 16	30	3 52	30	16 30	30	24 30
64	28	27 09	30	0 51	30	13 43		30 26 36		39 30		51 26 31		5 24
1/2	29	4 39		37 14		50 24	31	3 36	31	16 56	31	30 57		43 23
65		41 29	31	15 20	31	28 51		42 24		55 59	32	9 34	32	23 13
1/2	30	18 41		55 22		32 9 14	32	23 08	32	37 59		51 18	33	5 03
66		58 31	32	37 25		53 28	33	5 58	33	20 17	33	34 39		49 33
1/2	31	40 00	33	21 41	33	36 20		51 03	34	5 47	34	20 34	34	35 23
67	32	23 40	34	8 20	34	23 26		34 38 43		53 46	35	9 00	35	24 16
1/2	33	9 46		57 36	35	13 10	35	28 47		35 44 28	36	0 10	36	15 56
68		58 25	35	49 42	36	5 47	36	21 55		36 38 07		54 21	37	10 38
1/2	34	49 55	36	44 55	37	1 33	37	18 14		37 35 00	37	51 48	38	8 40
69	35	44 30	37	43 33	38	0 50	38	18 05		38 35 27	38	52 53	39	10 23
1/2	36	42 31	38	45 57	39	3 51	39	29 49		39 39 52	39	57 59	40	16 10
70		37 44 17	39	59 11	40	11 11		40 29 53		40 48 40	41	7 23	41	24 29

TABLE XXIII.—Azimuths or Bearings of Certain Stars when at their greatest Elongations from the Meridian. (The numbers at top denote polar dist.)

β (Kochab) Ursa Minoris.													
1875		1895		1915		1935		1955		1975		1995	
15° 20'	15° 25'	15° 30'	15° 35'	15° 40'	15° 45'	15° 50'	15° 55'	16° 00'	16° 05'	16° 10'	16° 15'	16° 20'	16° 25'
15 20 09	15 25 19	15 30 09	15 35 10	15 40 09	15 45 23	15 50 09							
20 35	25 35	30 35	35 36	40 36	45 36	50 36							
21 18	26 18	31 19	36 20	41 20	46 20	51 19							
22 18	27 19	32 20	37 20	42 21	47 22	52 23							
23 36	28 38	33 29	38 40	43 41	48 43	53 44							
25 12	30 14	35 15	40 17	45 19	50 21	55 23							
27 05	32 07	37 10	42 12	47 15	52 17	57 19							
29 16	34 19	39 22	44 26	49 29	54 32	59 35							
31 45	36 49	41 53	46 57	52 01	57 06	62 10							
34 33	39 38	44 43	49 48	54 53	59 58	65 03							
37 39	42 45	47 52	52 57	58 04	63 10	68 16							
41 05	46 12	51 19	56 26	61 34	66 41	71 48							
44 49	49 58	55 06	60 15	65 23	70 32	75 40							
48 54	54 04	59 13	64 23	69 33	74 43	79 53							
53 18	58 29	63 41	68 52	74 04	79 15	84 27							
58 08	63 16	68 29	73 42	78 55	84 08	89 21							
16 3 09	8 24	13 38	18 53	24 07	29 23	34 38							
8 87	13 53	19 10	24 27	29 43	35 00	40 17							
14 26	19 45	25 13	30 22	35 41	41 00	46 19							
20 39	26 00	31 21	36 41	42 17	47 23	52 44							
27 15	32 38	38 01	43 24	48 47	54 11	59 34							
30 42	36 06	41 31	46 55	52 20	57 47	63 08							
34 15	39 41	45 06	50 36	55 58	61 23	66 49							
37 55	43 22	48 47	54 15	59 48	65 09	70 36							
41 41	47 09	52 37	58 08	63 34	69 02	74 33							
45 33	51 03	56 32	62 02	67 31	73 01	78 30							
49 32	55 03	60 34	66 05	71 36	77 07	82 38							
53 38	59 10	64 43	70 15	75 47	81 20	86 23							
57 50	63 24	68 58	74 32	80 06	85 40	91 14							
17 2 10	7 29	13 21	18 57	24 32	30 07	35 42							
6 37	12 14	17 50	23 27	29 04	34 41	40 17							
11 11	16 49	22 27	28 06	33 44	39 23	45 02							
15 52	21 32	27 12	32 52	38 32	44 12	49 53							
20 41	26 22	32 04	37 46	43 28	49 10	54 52							
25 37	31 21	37 04	42 48	48 31	54 15	59 58							
30 42	36 27	42 12	47 57	53 43	59 28	65 13							
35 54	41 41	47 28	53 15	59 03	64 50	70 37							
41 15	47 04	52 52	58 41	64 31	70 20	76 09							
46 44	52 34	58 25	64 16	70 07	75 58	81 49							
52 21	58 14	64 07	70 00	75 52	81 46	87 39							
58 07	64 02	70 57	76 54	82 47	88 42	94 37							
18 4 03	10 00	16 02	22 12	28 51	35 09	41 46							
10 07	16 07	22 05	28 05	34 04	40 03	46 03							
16 21	22 22	28 24	34 25	40 27	46 28	52 30							
22 45	28 48	34 52	40 55	47 00	53 03	59 07							
29 18	35 24	41 30	47 36	53 42	59 48	65 55							
36 01	42 10	48 18	54 26	60 35	66 43	72 52							
42 55	49 06	55 17	61 29	67 39	73 50	79 57							
49 18	56 13	62 26	68 41	74 53	81 07	87 20							
57 15	63 11	69 27	75 43	81 59	88 15	94 31							
19 4 42	11 00	17 19	23 37	29 56	36 15	42 34							
12 20	18 41	25 02	31 24	37 45	44 07	50 28							
20 10	26 34	32 58	39 22	45 46	52 11	58 35							
28 12	34 39	41 06	47 33	54 00	60 27	66 54							
36 26	42 56	49 20	55 56	62 33	69 10	75 47							
44 54	51 27	58 00	64 34	71 06	77 41	84 17							
53 35	60 11	66 46	73 17	79 59	86 36	93 12							
0 2 29	9 08	15 47	22 27	29 06	35 46	42 26							
11 37	18 20	25 08	31 45	38 28	45 11	51 24							

TABLE XXIII.—Azimuths or Bearings of Certain Stars when at their Greatest Elongations from the Meridian. (The numbers at top denote polar dist.)

Star.	(Kochab) β Ursa Minoris.																					
Year.	1875		1895		1915		1935		1955		1975		1995									
P.D.	15° 20'		15° 25'		15° 30'		15° 35''		15° 40'		15° 45'		5° 5'									
Lat.	°	'	°	'	°	'	°	'	°	'	°	'	°	'								
41'	20	30	38	20	37	27	20	46	17	20	51	6	20	54	54	21	4	46	21	14	36	
$\frac{1}{2}$	20	41	47	24	54	47	21	1	10	21	8	4	14	57	21	51			21	51		
42	50	4	57	36	21	4	33	11	30	18	29	25	25	32	22			32	22			
$\frac{1}{2}$	21	1	5	21	8	6	15	6	22	8	29	8	36	9	43	10			43	10		
43	11	48	18	52	25	57	33	1	40	1	47	11	54	10					54	10		
$\frac{1}{2}$	22	49	29	56	37	4	44	15	51	22	58	31	22	24	41			22	24	41		
44	34	6	41	18	48	31	55	44	22	2	57	22	10	10	19	24			19	24		
$\frac{1}{2}$	45	42	54	59	22	16	22	7	38	14	50	22	8	29	26			22	8	29		
45	57	38	22	5	6	21	21	19	42	27	4	34	26	41	48			34	26	41		
$\frac{1}{2}$	22	9	54	17	19	24	45	31	14	39	38	47	5	54	32			47	5	54		
46	22	30	30	2	37	31	45	3	52	34	23	0	5	23	7	37			23	7		
$\frac{1}{2}$	35	29	43	4	50	39	58	15	23	5	2	10	4	21	4			10	4	21		
47	48	49	56	3	23	4	10	23	11	51	19	32	22	14	34	55			19	32		
$\frac{1}{2}$	23	2	23	23	10	18	18	4	25	50	30	37	41	23	48	10			30	37		
48	16	40	24	31	32	22	40	14	48	5	55	57	24	3	50			48	5	55		
$\frac{1}{2}$	31	13	39	10	47	6	55	3	24	3	00	24	10	58	18	58			24	3		
49	46	12	54	14	24	2	16	24	10	19	18	22	26	23	34	29			18	22		
$\frac{1}{2}$	24	1	38	24	9	46	17	54	26	2	34	11	42	20	50	30			26	2		
50	17	31	25	45	34		42	14	50	29	58	45	25	7	0			50	29	58		
$\frac{1}{2}$	33	54	42	14	50	35	58	56	25	7	17	25	15	39	24	1			25	7		
51	50	48	59	15	25	7	42	25	15	9	24	37	33	6	25	41	34			24	37	
$\frac{1}{2}$	25	8	12	25	16	47	25	21	33	56	42	30	42	30	59	40			42	30		
52	26	3	34	53	43	34	51	15	26	0	46	26	9	40	25	18	22			26	0	
$\frac{1}{2}$	44	46	53	33	26	2	22	26	11	10	20	00	28	51	37	41			20	00		
53	26	3	55	26	12	50	21	47	30	43	39	40	48	43	57	35			39	40		
$\frac{1}{2}$	23	44	32	46	41	50	50	54	27	0	2	27	9	4	27	18	10			27	0	
54	44	10	53	22	27	2	34	27	11	45	21	0	30	14	39	36			21	0		
$\frac{1}{2}$	27	5	21	27	14	40	24		33	16	42	43	52	4	28	1	37			33	16	
55	27	13	36	41	46	16	55	40	28	5	10	14	42	24	13			28	5	10		
$\frac{1}{2}$	49	52	59	20	28	9	8	28	18	50	28	27	38	7	47	48			28	18		
56	28	13	18	28	23	6	32	52	42	43	52	48	29	2	23	29	12	13				
$\frac{1}{2}$	37	36	47	39	57	28	29	7	31	29	17	31	27	32	37	32			29	17		
57	29	2	48	29	12	56	29	23	5	33	4	43	14	53	36	30	3	48				
$\frac{1}{2}$	28	56	39	13	49	35	59	55	30	10	23	30	20	51	31	2			30	10		
58	56	4	30	6	34	30	17	57	30	27	38	38	11	48	40	59	48					
$\frac{1}{2}$	30	24	15	34	58	45	41	56	25	31	7	10	31	17	57	31	28	44				
59	58	17	31	4	28	31	15	21	31	26	21	37	21	48	22	59	18					
$\frac{1}{2}$	31	24	1	35	10	46	19	57	30	32	8	43	32	15	8	32	31	4				
60	55	45	32	7	7	32	18	31	32	29	55	41	23	52	49	33	4	13				
$\frac{1}{2}$	32	28	43	40	25	52	4	33	3	43	33	15	23	33	27	9	38	48				
61	33	3	17	33	15	10	33	24	4	39	50	55	34	2	54	34	14	53				
$\frac{1}{2}$	39	17	51	2	34	3	37	34	15	48	34	28	2	40	17	52	33					
62	34	16	53	34	20	20	41	49	54	19	35	6	50	35	19	22	35	31	56			
$\frac{1}{2}$	56	14	35	9	35	21	46	35	34	35	47	25	36	0	17	36	13	28				
63	35	37	27	50	38	36	3	39	36	16	47	36	30	2	43	10	56	26				
$\frac{1}{2}$	36	20	40	36	34	6	47	34	37	1	43	37	14	37	28	11	37	41	48			
64	37	6	4	37	19	52	37	33	42	47	38	38	1	30	38	15	29	38	29	29		
$\frac{1}{2}$	53	47	38	8	38	22	15	38	36	32	38	50	52	39	5	15	39	19	40			
65	38	44	2	38	58	40	39	13	23	39	28	12	39	42	54	39	57	44	40	12	48	
$\frac{1}{2}$	39	43	38	40	7	21	40	7	21	40	22	33	40	37	51	40	53	10	41	8	33	
66	40	34	29	40	48	45	41	4	26	41	20	10	41	36	0	41	51	50	42	7	46	
$\frac{1}{2}$	41	32	29	41	48	40	42	4	55	42	21	14	42	37	36	42	54	2	43	10	32	
67	42	35	30	42	52	18	43	9	9	43	26	4	43	43	8	44	0	11	44	17	21	
$\frac{1}{2}$	43	42	34	44	0	3	44	17	35	44	35	13	44	53	5	45	10	43	45	28	36	
68	44	54	8	45	12	19	45	38	43	45	49	3	46	7	33	46	33	40	46	44	50	
$\frac{1}{2}$	46	10	46	29	34	46	40	34	47	8	13	47	27	37	47	7	48	6	42			
69	47	33	5	47	53	5	48	13	13	48	23	28	48	53	50	49	14	20	49	35	7	
$\frac{1}{2}$	49	1	57	49	23	49	44	14	50	5	34	50	27	7	50	48	47	50	13	35		
70	50	38	17	51	23	35	51	23	5	51	45	43	52	8	41	52	31	37	52	54	50	

TABLE XXIV.

Showing the Azimuths of Polaris when on the same vertical plane with γ (Gamma) in Cassiopea at its under transit.

All the Azimuths or bearings are North-west.

The column headings are the years or dates.

	1870	1880	1890	1900	1910	1920	1930	1940
2	8 6	8 46	9 24	10 02	10 38	11 12	11 41	12 10
4	7	47	26	04	40	14	43	12
6	9	49	28	06	42	16	45	15
8	12	52	41	09	45	19	49	18
10	15	55	34	13	49	23	53	23
12	19	59	38	18	54	28	58	28
14	23	9 04	44	23	11 00	35	12 05	35
16	28	10	50	30	07	42	12	42
18	34	16	57	37	14	50	20	51
20	8 41	23	10 04	45	23	59	30	13 01
22	48	32	13	54	33	12 9	41	12
24	57	41	23	11 05	44	21	53	25
26	9 06	51	34	16	56	33	13 05	38
28	17	10 02	46	29	12 09	48	21	54
30	28	15	59	43	25	13 04	37	14 14
32	41	28	11 14	59	41	21	55	29
34	55	44	30	12 16	59	40	14 15	51
36	10 11	11 11	48	35	13 19	14 01	37	13
38	28	18	12 8	56	42	24	15 02	15 38
40	47	39	30	13 19	14 06	50	28	16 06
41	57	49	41	22	20	15 04	43	21
42	11 7	12 01	54	45	33	18	58	37
43	19	14	13 07	59	48	34	16 14	54
44	32	27	20	14 14	15 03	50	31	17 12
45	43	40	35	29	20	16 08	49	30
46	56	54	50	45	37	26	17 08	49
47	12 11	13 09	14 06	15 03	55	44	28	18 10
48	25	25	23	21	16 14	17 05	48	32
49	41	42	41	40	35	26	18 10	53
50	57	14 0	15 00	16 00	56	48	34	19 19
51	13 15	19	21	22	17 19	18 12	59	45
52	33	39	42	44	43	37	19 25	20 12
53	53	15 0	16 05	17 09	18 08	19 04	53	41
54	14 14	23	29	34	36	33	20 22	21 12
55	36	47	54	18 01	19 05	20 03	54	45
56	15 0	16 12	17 22	31	35	35	21 28	22 19
57	25	40	51	19 02	20 08	21 10	22 05	57
58	53	17 09	18 22	35	43	46	42	23 36
59	16 22	40	56	20 11	21 21	22 26	23 23	24 19
60	53	18 14	19 32	49	22 01	23 08	24 07	25 05
61	17 26	50	20 11	21 20	45	23 54	54	25 54
62	18 03	19 29	53	22 14	23 32	24 43	25 45	26 47
63	42	20 11	21 38	23 03	24 22	25 36	26 41	27 44
64	19 24	58	22 27	55	25 17	26 34	27 40	28 46
65	20 11	21 47	23 20	24 52	26 17	27 36	28 45	29 54
66	21 02	22 42	24 18	25 53	27 22	28 45	29 56	31 07
67	57	23 41	25 21	27 01	28 33	29 56	31 14	32 28
68	22 57	24 47	26 32	28 15	29 52	31 22	32 39	33 50
69	24 05	25 59	27 49	29 37	31 18	32 52	34 13	35 33
70	25 20	27 19	29 15	31 09	32 54	34 23	35 32	37 22

TABLE XXV.

*Showing the Azimuths of Polaris, when vertical with Alioth in Ursa
Majoris at its under transit.*

All the Azimuths or bearings are North-east.

(The top column is years, beginning Jan. 1.)

North Lat.	1870	1880	1890	1900	1910	1920	1930	1940
°	' "	' "	' "	' "	' "	' "	' "	' "
2	8 01	8 41	9 30	10 03	10 30	11 15	11 47	12 18
4	02	43	32	04	35	17	49	19
6	03	45	34	05	40	19	52	20
8	05	47	37	07	45	22	55	24
10	07	50	39	11	50	26	58	28
12	11	54	42	15	54	31	12 03	33
14	15	58	48	20	59	36	09	40
16	19	9 04	53	26	11 06	43	16	46
18	25	10	58	34	13	49	23	54
20	31	16	10 06	41	21	57	32	13 03
22	38	23	14	49	30	12 07	42	14
24	45	31	23	59	40	19	54	26
26	53	40	34	11 09	52	31	13 06	39
28	9 02	50	45	20	12 05	45	21	54
30	13	10 02	59	33	18	13 00	36	14 11
32	24	16	11 12	50	34	13 16	54	28
34	38	30	28	12 06	51	35	14 12	49
36	53	45	44	24	13 11	55	34	15 10
38	10 08	11 03	12 01	44	32	14 18	57	34
40	25	22	23	13 06	55	41	15 22	16 01
41	34	32	34	18	14 07	56	36	16
42	44	42	46	30	21	15 10	51	31
43	53	53	58	43	35	23	16 07	55
44	11 04	12 06	13 11	56	50	39	22	17 04
45	15	19	24	14 11	15 05	56	40	21
46	29	33	38	26	21	16 11	57	40
47	42	51	52	42	37	31	17 16	18 00
48	55	13 08	14 07	59	55	50	36	20
49	12 09	16	25	15 17	16 15	17 12	58	42
50	24	32	43	35	35	38	18 20	19 05
51	40	49	15 04	56	57	18 01	43	30
52	56	14 11	25	16 16	17 19	26	19 08	56
53	13 14	29	48	40	43	50	34	20 24
54	34	48	16 11	17 04	18 08	19 10	20 02	53
55	52	15 10	34	28	35	40	33	21 23
56	14 14	33	58	56	19 04	20 11	21 04	54
57	38	58	17 26	18 25	37	45	38	22 28
58	15 01	16 25	54	56	20 07	21 17	22 14	32
59	27	54	18 26	19 28	42	52	35	23 10
60	55	17 24	58	20 03	21 19	22 33	23 21	50
61	16 24	57	19 57	41	59	23 16	24 10	24 33
62	56	18 32	20 16	21 21	22 42	59	25 03	25 18
63	17 30	19 10	58	22 05	23 28	24 51	25 56	26 02
64	18 06	51	21 41	22 52	24 19	25 46	26 55	27 00
65	18 47	20 35	22 33	23 43	25 13	26 48	27 51	28 02
66	19 30	21 23	23 27	24 39	26 13	27 46	28 51	30 10
67	20 17	22 15	24 20	25 39	27 17	28 58	30 15	31 21
68	21 10	23 13	25 14	26 45	28 27	30 19	31 28	32 46
69	22 04	24 16	26 31	27 58	29 44	31 35	32 46	34 14
70	23 05	25 26	27 49	29 18	31 09	33 00	34 22	35 55

TABLE XXVI.

Mean places of Gamma, (Casiopeæ) and Epsilon (Alioth), Ursa Majoris, at Greenwich.

Mean noon for the first day of January of each year, from 1870 to 1950.

	Gamma in Cassiopeæ.		Alioth in Ursa Majoris.	
	Right Ascen'n.	N. Polar Dist.	Right Ascen'n.	N. Polar Dist.
	α β γ	δ ϵ ζ	η θ ι	κ λ μ
1870	0 48 52.6	29 59 15.6	12 48 18.2	33 20 3.8
1	56.1	58 56.0	20.9	20 23.5
2	59.7	58 36.4	23.5	20 43.2
3	49 3.3	58 16.8	26.2	21 2.9
4	0 49 6.8	58 57.2	28.9	21 22.5
5	10.4	57 37.9	31.5	21 42.2
6	14.0	57 18.0	34.2	22 1.8
7	17.5	57 58.3	36.8	22 21.6
8	21.1	56 38.7	39.5	22 41.2
9	24.7	56 4.1	42.2	23 0.9
1880	28.2	55 59.5	12 48 44.8	33 23 20.5
1	0 49 31.8	55 39.9	47.5	23 40.2
2	35.4	55 20.3	50.1	23 59.9
3	38.9	55 0.7	52.8	24 19.5
4	42.5	54 41.1	55.5	24 39.2
5	46.1	54 21.4	48 58.12	24 58.9
6	49.7	54 1.9	49 0.8	25 18.5
7	53.2	53 42.3	3.4	25 38.2
8	0 49 56.7	53 22.7	6.1	25 57.8
9	0 50 0.3	53 3.9	8.7	29 17.5
1890	4.0	52 43.5	12 49 11.4	33 26 37.2
1	7.5	52 23.9	14.1	29 56.8
2	11.1	52 4.3	16.7	27 16.5
3	14.7	51 44.7	19.4	27 36.2
4	18.3	51 25.1	22.0	27 55.8
5	21.8	51 5.5	24.7	28 15.4
6	25.4	50 45.9	27.3	28 25.1
7	29.0	50 26.3	30.0	28 54.8
8	32.6	58 6.7	32.6	29 14.4
9	36.2	49 47.1	35.3	26 34.1
1900	0 58 39.7	49 27.6	12 49 37.9	29 53.7
1	43.3	49 8.0	40.6	30 13.5
2	46.9	48 48.4	43.2	30 33.0
3	50.5	48 28.8	45.9	30 52.7
4	54.1	48 9.2	48.5	31 12.3
5	57.7	47 49.6	51.2	30 32.0
6	0 51 01.2	47 30.0	53.9	31 51.6
7	04.8	47 10.4	56.5	32 11.3
8	08.4	46 50.8	59.2	32 30.9
9	12.0	46 31.7	01.9	32 50.6
1910	0 51 15.6	29 43 11.6	12 50 04.5	33 33 10.2
1911	0 51 19.2	45 12.4	12 50 07.2	29.8
12	51 22.8	44 52.8	9.8	49.5
13	26.4	44 33.2	12.5	34 09.1
14	30.0	29 44 53.2	15.12	28.7
15	33.6	44 33.6	17.8	48.0
16	37.2	44 14.0	20.5	35 08.0
17	40.8	43 54.4	23.12	27.9
18	44.4	43 34.8	25.8	47.3
19	48.0	43 24.2	28.1	36 06.0
1920	0 51 51.5	42 55.8	12 50 30.7	36 26.6
30	52 27.6	39 40.2	57.4	39 42.9
40	53 03.7	36 24.7	51 23.8	42 59.1
50	53 32.8	29 23 9.6	50.2	34 43 15.6

NOTE.— γ in Cassiopeæ is No. 253, and ϵ (Alioth) in Ursa Majoris is 4335, in the British Association's Catalogue of Stars, published in 1850. These stars, so essential to the Surveyor, have not been given in the British or American Nautical Almanacs. We have sent a proof of this sheet, with seven other pages, to Mr. Coffin, Superintendent of the American Ephemerides, on March 26, 1875, requesting of him to give them a place in his future years.

TABLE XXVII.

Showing the Azimuth or bearing of Alpha in the foot of the Southern Cross (Crucis), when on the same vertical plane with Beta in Hydri, or in the tail of the Serpent.

Bearings are all South-east when Alpha Crucis is at its under transit, and for the 1st of January of the years given at top.

Lat.	1850	1900	1950	2000	2050	2100	2150
	' "	' "	' "	' "	2 15	' "	' "
12	1 12	1 14	1 19	1 43	15	2 58	3 53
13	12	14	19	43	16	58	54
14	12	14	20	43	16	58	56
15	12	15	21	44	17	59	57
16	13	15	22	44	17	3 01	58
17	13	16	23	45	17	03	59
18	14	16	24	46	18	3 06	4 00
19	14	17	24	47	19	07	01
20	14	17	25	48	20	3 09	02
21	1 15	18	1 25	1 49	2 21	3 10	4 05
22	15	18	26	1 50	22	10	06
23	16	19	27	50	23	11	07
24	16	19	28	51	24	12	09
25	17	20	28	52	25	13	11
26	17	20	29	52	26	14	13
27	18	21	29	53	27	15	15
28	19	22	30	1 54	29	17	18
29	20	22	31	55	30	19	20
30	21	23	32	56	32	3 21	23
31	1 22	1 24	1 34	1 57	2 34	3 23	4 26
32	22	25	35	58	36	3 25	29
33	23	26	36	59	37	27	32
34	24	27	37	2 01	39	30	35
35	25	28	38	02	41	32	38
36	26	29	39	03	43	35	42
37	28	30	40	05	45	38	45
38	29	31	42	07	48	41	49
39	30	32	43	09	50	45	53
40	31	34	44	11	53	48	58
41	1 32	1 35	1 46	2 13	2 55	3 51	5 02
42	34	37	48	15	57	58	07
43	36	39	50	17	59	59	11
44	37	41	51	20	3 03	4 03	16
45	39	43	53	22	06	07	22
46	41	44	55	25	10	12	28
47	43	46	57	28	13	17	34
48	45	48	2 00	2 31	17	22	40
49	47	50	02	33	21	28	47
50	49	52	05	36	26	34	54
51	1 51	1 54	2 07	38	3 30	4 37	6 02
52	54	57	10	40	34	43	12
53	57	2 00	13	45	39	49	17
54	59	02	16	51	44	55	27
55	2 02	06	19	53	50	5 02	39
56	05	09	23	3 01	56	10	51
57	08	13	27	05	4 01	18	7 01
58	12	16	32	09	09	27	11
59	16	19	36	14	16	36	23
60	20	24	40	20	24	44	36
61	24	29	45	26	32	6 04	7 50
62	29	34	51	33	41	6 12	8 01
63	34	38	56	40	50	20	23
64	40	43	3 02	48	5 00	27	41
65	2 45	2 47	09	3 56	5 10	6 35	9 01

TABLE XXVIII.

ALTITUDES AND GREATEST AZIMUTHS FOR 1st Jan., 1907, (SIDERIAL TIME.)
 FOR CHICAGO—LAT. $41^{\circ} 50' 30''$ LONG. $87^{\circ} 34' 7''$ W.; AND BUENOS AYRES—LAT. $34^{\circ} 36' 40''$ S., LONG. $58^{\circ} 24' 3''$ W.

British Associated Catalogue of Stars.	NAME OF STAR AND CONSTELLATION.	Magn.	Pol. dist.	R. A.		Up-Meridian Passage.		Time to Gr. Az.		Time of G. E. Az.		Time of G. W. Az.		Greatest Azimuth.	Alt. at G. Az.
360	α (Polaris) Ursa Minoris.....	2	1 24	h. m. s.	1 10 17	h. m. s.	6 26 38	h. m. s.	5 55 0	h. m. s.	0 31 38	h. m. s.	12 21 38	1 52 46	11 51 25
6281	α Ursa Minoris.....	4.5	3 23 44	18 15 15	23 31 36	5 47 51	5 47 51	5 32 9	17 43 45	10 43 45	5 19 20	4 33 36	4 44 55	34	55 34
5780	ϵ ".....	4.5	7 44 55	16 50 42	22 16 03	5 16 59	5 16 59	5 3 21	16 43 54	3 48 12	10 24 9	10 24 9	10 24 9	42 18 58	42 18 58
5285	ζ ".....	4.5	11 47 52	15 48 52	21 5 13	5 16 59	5 16 59	5 3 21	15 48 14	2 22 12	15 55 44	42 57 31	42 57 31	43 45 22	43 45 22
4936	β " (Kochab).....	2	15 18 5	14 51 7	20 7 28	5 12 5	5 12 5	5 3 21	15 4 7	1 10 49	20 44 46	43 43 47	43 43 47	43 43 47	43 43 47
8238	γ Cephei.....	3.4	13 6 35	23 33 55	4 50 16	3 30 56	3 30 56	2 18 24	23 38 11	10 02 21	48 58 0	53 45 9	53 45 9	53 45 9	53 45 9
169	α Cassiopeiæ.....	Var.	34 11 33	0 32 59	5 49 20	3 5 13	3 5 13	10 34 54	2 18 24	17 15 20	55 14 9	57 29 27	57 29 27	57 29 27	57 29 27
3242	ϕ Ursa Majoris.....	.3	37 43 7	9 23 56	14 40 7	4 9 17	4 9 17	12 2 34	2 34 54	20 21 08	38 21 11	48 47 14	48 47 14	48 47 14	48 47 14
3777	α " (Dubhe).....	2	27 31 55	10 55 30	16 11 51	3 21 12	3 21 12	13 41 58	20 24 22	51 19 39	55 5 27	55 5 27	55 5 27	55 5 27	55 5 27
4017	γ ".....	2.3	35 33 57	11 46 49	17 3 10	3 6 8	3 6 8	19 37 39	1 49 55	54 58 59	57 20 48	57 20 48	57 20 48	57 20 48	57 20 48
5937	β Draconis.....	3.2	37 35 57	17 27 26	22 43 47	2 58 57	2 58 57	20 10 55	2 2 8	49 56 28	2 58 27	48	48	48	48
6091	γ ".....	2.3	38 29 40	17 53 31	23 09 52	4 6 41	4 6 41	22 25 04	6 38 26	39 1 42	58 27 48	58 27 48	58 27 48	58 27 48	58 27 48
7416	α Cephei.....	3.2	27 58 39	21 15 24	2 31 45	5 58 0	5 58 0	17 19 51	11 02 00	14 39 21	35 24 52	35 24 52	35 24 52	35 24 52	35 24 52
5959	δ Octantis.....	6	0 43 17	18 0 55	23 17 51	5 26 21	5 26 21	0 9 18	22 55 27	13 55 20	35 24 52	35 24 52	35 24 52	35 24 52	35 24 52
88	β Hydræ.....	3	11 59 45	0 18 43	5 35 39	5 27 55	5 27 55	16 53 46	2 49 20	26 4 50	37 32 27	37 32 27	37 32 27	37 32 27	37 32 27
4131	β Chameleontis.....	5	11 25 34	10 10 36	17 27 32	4 57 47	4 57 47	2 32 12	11 07 10	40 10 43	42 5 56	42 5 56	42 5 56	42 5 56	42 5 56
5578	α Trianguli Australis.....	2	21 13 18	16 34 37	21 51 33	4 17 29	4 17 29	7 44 52	15 31 0	47 29 59	45 37 27	45 37 27	45 37 27	45 37 27	45 37 27
507	α Eridani (Achernær).....	1	32 5 13	1 32 45	6 49 41	4 33 53	4 33 53	10 9 38	18 51 18	39 5 57	41 39 14	41 39 14	41 39 14	41 39 14	41 39 14
2096	α Argus (Anapus).....	1	37 22 33	6 21 30	11 37 56	4 20 50	4 20 50	13 0 41	22 11 37	34 17 37	39 52 45	39 52 45	39 52 45	39 52 45	39 52 45
3186	ϵ ".....	2	31 16 58	9 13 02	14 30 28	4 35 28	4 35 28	13 0 41	22 11 37	34 17 37	39 52 45	39 52 45	39 52 45	39 52 45	39 52 45
4187	α Crucis.....	1	27 38 22	12 19 13	17 36 9	4 24 59	4 24 59	14 46 25	5 46 05	41 11 29	42 31 59	42 31 59	42 31 59	42 31 59	42 31 59
4659	β Centauri.....	1	30 16 13	13 54 28	19 11 24	4 14 12	4 14 12	21 17 41	5 46 05	41 11 29	42 31 59	42 31 59	42 31 59	42 31 59	42 31 59
7004	α Pavonis.....	2	32 50 11	20 14 57	1 31 53	4 14 12	4 14 12	21 17 41	5 46 05	41 11 29	42 31 59	42 31 59	42 31 59	42 31 59	42 31 59

Southern Hemisphere.

Northern Hemisphere.

TABLE XXVIII.
TABLE OF EQUAL ALTITUDES.

Int'val h. m.	Log. A.	Log. B.	Int'val h. m.	Log. A.	Log. B.	Int'val h. m.	Log. A.	Log. B.
2. 0	7.7297	7.7146	4. 2	7.7451	7.6815	6. 0	7.7703	7.6198
2	98	43	4	54	07	2	08	84
4	7300	39	6	58	800	4	13	70
6	02	36	8	61	792	6	19	56
8	04	32	10	64	84	8	24	42
10	05	28	12	68	76	10	29	27
12	07	25	14	72	68	12	35	13
14	09	21	16	75	59	14	40	.6098
16	11	17	18	79	51	16	45	82
18	13	13				18	51	68
20	15	09	20	82	43	20	56	53
22	17	05	22	86	34	22	62	38
24	19	7.7101	24	90	26	24	67	23
26	21	7.7097	26	94	17	26	73	07
28	23	92	28	97	08	28	79	.5991
30	25	88	30	.7501	.6700	30	84	75
32	27	83	32	05	.6691	32	90	59
34	29	79	34	.09	82	34	96	43
36	31	75	36	13	73	36	.7801	27
38	33	70	38	7.7517	7.6663	38	07	10
40	36	65	40	7.7521	7.6654	40	13	.5894
42	38	61	42	25	45	42	19	77
44	40	56	44	29	35	44	25	60
46	42	51	46	33	26	46	31	43
48	45	46	48	37	16	48	36	25
50	47	41	50	41	.6606	50	42	08
52	49	36	52	45	.6597	52	48	.5790
54	52	31	54	49	87	54	54	72
56	54	26	56	53	77	56	60	54
58	57	21	58	57	.6567	58	67	36
3.00	59	15	5.00	62	56	7.00	73	17
2	62	10	2	66	46	2	79	.5699
4	64	7.7005	4	70	36	4	85	80
6	67	7.6999	6	75	25	6	91	61
8	69	93	8	79	14	8	98	41
10	72	88	10	83	.6504	10	.7904	22
12	74	82	12	88	.6493	12	10	02
14	77	76	14	92	82	14	16	.5582
16	80	70	16	97	71	16	23	62
18	7.7383	7.6964	18	.7601	60	18	29	42
20	7.7386	7.6958	20	06	48	20	36	22
22	88	52	22	10	37	22	42	01
24	91	46	24	15	25	24	49	.5480
26	94	40	26	20	14	26	55	59
28	97	34	28	24	.6402	28	62	37
30	.7400	27	30	29	.6390	30	69	16
32	03	21	32	34	78	32	75	.5394
34	06	14	34	38	66	34	82	72
36	09	08	36	43	54	36	89	50
38	12	.6901	38	48	42	38	95	27
40	15	.6894	40	53	30	40	.8002	04
42	18	88	42	58	17	42	09	.5281
44	21	81	44	63	.6304	44	16	58
46	24	74	46	68	.6291	46	23	34
48	28	67	48	73	78	48	30	11
50	31	59	50	78	65	50	37	.5186
52	34	52	52	83	52	52	44	62
54	37	45	54	88	39	54	51	37
56	41	38	56	93	25	56	58	12
58	44	30	58	7.7698	.6212	58	65	.5087
4.00	47	23	6.00	7.7703	.6199	8.00	7.8072	72

TABLE XXVIIIb.

To Convert Metres into Stat. Miles.		Lat.	Statute Miles.
Metres.	Miles.		
		0	69.07
		1	69.06
		2	69.08
10	.006	3	68.07
20	.012	4	68.90
30	.019	5	68.81
40	.025	6	68.62
50	.031	7	68.48
60	.037	8	68.31
70	.044	9	69.15
80	.050	10	67.95
90	.056	11	67.73
100	.062	12	67.48
200	.124	13	67.21
300	.186	14	66.95
400	.249	15	66.65
500	.311	16	69.31
600	.373		
700	.435		
800	.497		
900	.559	51	43.43
1000	.621	52	42.48
2000	1.243	53	41.53
3000	1.864	54	40.56
4000	2.485	55	39.58
5000	3.107	56	38.58
6000	3.728	57	37.58
7000	4.319	58	36.57
8000	4.971	59	35.54
9000	5.592	60	34.50
10000	6.213	61	33.45
11000	12.427	62	32.40
12000	18.640	63	31.33
13000	24.854	64	30.24
14000	31.067	65	29.15
15000	37.281	66	28.06
16000	43.494	67	26.96

TABLE XXVIIIc.

Showing the length of a Degree of Lat.
and Long. in Metres and Miles.

In Lat.	Length of a Degree of Lat. in Metres.	Length of a Degree of Lon. in Metres.	Length of a D. of Lon. in Stat. Miles.
17	11.658.4	106.473.4	66.157
18	11.669.5	105.892.6	66.796
19	11.681.1	106.279.7	65.415
20	693.3	104.634.8	65.015
21	706.0	103.958.7	64.594
22	719.2	103.250.0	64.154
23	732.9	102.510.0	63.695
24	747.1	101.739.7	63.216
25	761.7	100.938.2	62.718
26	776.7	100.105.9	62.200
27	792.2	90.243.2	61.664
28	808.3	98.350.2	61.109
29	824.4	97.427.4	60.536
30	841.9	96.474.8	59.944
31	858.0	95.492.9	59.334
32	875.2	94.481.9	58.706
33	110.892.8	93.442.1	58.060
34	910.7	92.373.8	57.396
35	928.8	91.277.3	56.715
36	947.2	90.152.9	56.016
37	995.8	89.001.0	55.300
38	984.6	87.821.6	54.568
39	111.003.5	86.616.0	53.819
40	022.6	85.383.9	53.053
41	041.8	84.125.1	52.271
42	061.1	82.840.8	51.473
43	080.5	81.531.1	50.659
44	100.0	80.196.5	49.830
45	119.4	78.837.3	48.986
46	138.9	77.453.9	48.126
47	158.4	79.046.8	47.251
48	177.8	74.612.3	46.362
49	197.2	73.162.9	45.460
50	216.4	71.687.0	44.543

That part of Table C, from Lat. 17° to 50°, is calculated according to Bessel's formula, as given in the United States Coast Survey of 1853, page 100.

Tables C and D are from the same volume, pp. 103 and 106, excepting that showing the length of a degree of Longitude, from Latitudes 0 to 17, and 50 to 90 degrees, which is taken from Keith on the Globe, p. 193.

Those having occasion to project a map on an extensive scale, will find, in the above volume, tables from pp. 107 to 163 which have been calculated for the United States Coast Survey, under the superintendence of the late A. D. BACHE.

To find the length of a degree of Longitude in any degree of Lat. :
Rad. is to the length of a degree on the Equator—as the Cosine of the given Latitude is to the length of a degree of Longitude in that Lat.

Length of a degree on the Equator is 365144 feet. Radius of the Equator = 20921180. Polar Semiaxis = 20853180.

NOTE.—There has been nothing printed to fill from page 200 to 249.

TABLE XXIX.—To Reduce French Litres to Cubic Feet and Imperial Gallons.

1 Litre = 0.0353166 Cubic Feet, or 0.2200967 Imperial Gallons.

Litre.	Cubic Feet	English or Imper Gallons.	Litre.	Cubic Feet	English or Imperial Gallons.
1	0.0354	0.2201	60	.1190	18.2058
2	.0706	.4401	61	.1543	.4258
3	.1059	.6608	62	.1896	.6460
4	.1418	.8804	63	2.2249	18.8661
5	.1766	1.1005	64	.2603	14.0862
6	.2119	.3206	65	.2956	.3063
7	.2472	.5407	66	.3309	.5264
8	.2825	.7608	67	.3662	.7465
9	.3178	.9809	68	.4015	.9666
10	.3532	2.2010	69	.4368	15.1867
11	.3885	.4211	70	.4722	.4068
12	.4238	.6412	71	.5075	.6268
13	.4591	.8613	72	.5428	.8470
14	.4944	8.0814	73	.5781	16.0671
15	.5297	.3014	74	.6134	.2872
16	.5651	.5215	75	.6487	.5073
17	.6004	.7416	76	.6841	.7273
18	.6357	.9617	77	.7194	.9474
19	.6710	4.1818	78	.7547	17.1675
20	.7063	.4019	79	.7900	.3876
21	.7416	.6220	80	.8253	.6077
22	.7770	.8421	81	.8606	.8278
23	.8123	5.0622	82	.8960	18.0479
24	.8476	.2825	83	.9313	.2680
25	.8829	.5024	84	.9666	.4881
26	.9182	.7225	85	3.0019	.7082
27	.9535	.9426	86	.0372	.9283
28	.9889	6.1627	87	.0725	19.1484
29	1.0242	.3828	88	.1079	.3685
30	.0595	.6029	89	.1932	.5886
31	.0948	.8230	90	.1785	.8087
32	.1301	7.0431	91	.2138	20.0288
33	.1654	.2631	92	.2491	.2489
34	.2008	.4832	93	.1844	.4690
35	.2361	.7034	94	.8198	.6891
36	.2714	.9235	95	.8551	.9092
37	.3067	8.1436	96	.8904	21.1293
38	.3420	.3637	97	.4257	.3494
39	.3773	.5838	98	.4610	.5695
40	.4127	.8039	99	.4963	.7896
41	.4480	9.0240	100	8.5317	22.0097
42	.4833	.2441	200	7.0633	44.0193
43	.5186	.4642	300	10.5950	166.0290
44	.5539	.6843	400	14.1266	188.0387
45	.5892	.9044	500	17.6583	110.0484
46	.6246	10.1244	600	21.1900	132.0580
47	.6599	.3445	700	24.7216	154.0677
48	.6952	.5646	800	28.2533	176.0774
49	.7305	.7848	900	31.7849	198.0871
50	.7658	11.0048	1000	35.3166	220.0967
51	.8011	.2249	2000	70.6332	440.019
52	.8365	.4450	3000	105.950	660.029
53	.8718	.6651	4000	141.266	880.039
54	.9071	.8852	5000	176.583	1100.48
55	.9424	12.1053	6000	211.901	1820.58
56	.9777	.8254	7000	247.216	1540.68
57	2.0130	.5455	8000	282.533	1760.77
58	.0484	.7656	9000	317.849	1980.87
59	.0837	.9857	10000	353.166	2200.97

TABLE XXX.—Foreign Weights and Measures.

French, new system.	English inches.
Millimetre equals	0.039371
Centimetre	0.393708
Decimetre	3.937079
Metre	39.37079
Decametre	393.7079
Hectometre	3937.079
Kilometre	39370.79
Myriametre	393707.9
Foot (Pied de Roi)	= 12.7925
Spanish foot	= 11.034 inches
French "	= 12.7925 "
Swedish "	= 11.690 "
Austrian "	= 12.448 "
Lisbon "	= 12.96 "
Toise, or 6 ft. Fr.	= 76.735 in.
Sq. metre	= 1550.85 sq. in.
Sq. metre	= 10.7638 sq. feet

	Measure.	Sq. yds.
England	Acre	4840
Amsterdam	Moyen	9722
Hamburgh	Moyen	11545
Ireland	Acre	7840
Naples	Moggia	3996
Portugal	Geira	6970
Prussia	Morgen	3053
Rome	Pizza	3158
Russia	Desiatina	1806.8
Spain	Fanegada	6500
Sweden	Tunneland	5900
Scotland	Acre	6150

SURFACE.

French, old system.	English.
Square inch	= 1.384 inches
Arpent (Paris)	= 900 sq. toises
" (woodland)	= 100 sq. royal perches
New system.	
Are	= 100 sq. metres
Are	= 1076.38 sq. feet
Centare	= 1 sq. metre
Decare	= 10 ares
Hectare	= 100 ares

CAPACITY.

Litre taken as a standard.	
Myrialitre	= 10000 litres
Kilolitre	= 1000 "
Hectolitre	= 100 "
Decalitre	= 10 "
Litre	= 1 "
Decilitre	= 0.1 "
Centilitre	= 0.01 "
Millilitre	= 0.001 "
Litre	= cubic centimetre
Troy grains.	
Milligramme	= .0154
Centigramme	= .1544
Decigramme	= 1.5444
Gramme	= 15.4440
Decagramme	= 154.4403
Hectogramme	= 1544.4023
Kilogramme	= 15444.0234
Milligramme	= 154440.2344

For a valuable collection of tables of weights and measures, see Oliver Byrne's Dictionary of Mechanics and Engineering.

TABLE XXXI.—Discharge of Water through New Pipes.
 Compiled from Henry Darcy's French Tables of 1857.

Velocity per sec.		10 Centimetres.		12 Centimetres.		14 Centimetres.		16 Centimetres.	
Diam. Metre.	Area of section.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.
0.01	0.0001	0.8602	0.008	0.5187	0.009	0.7060	0.011	0.9221	0.013
2	03	1154	081	1662	088	2262	44	2954	060
3	07	626	071	0901	085	1226	99	1601	113
4	13	415	126	598	161	0811	176	1063	201
5	20	806	196	441	286	600	275	0784	314
6	28	241	283	847	839	472	898	617	452
7	38	198	885	285	462	887	589	506	616
8	50	168	508	241	608	828	704	428	804
9	64	145	686	208	768	283	891	370	1.018
10	79	127	785	183	948	249	1.100	326	257
11	95	114	950	164	1.140	223	880	291	521
12	118	102	1.181	148	857	201	583	262	810
13	133	093	827	134	598	183	858	239	2.124
14	164	86	539	123	847	168	2.155	219	463
15	177	79	767	114	2.121	155	474	203	827
16	201	73	2.011	106	418	144	815	188	3.217
17	227	69	270	99	724	134	8.178	176	632
18	254	64	545	93	8.054	126	563	165	4.072
19	284	61	885	87	402	119	969	155	536
20	314	57	8.142	82	770	112	4.398	146	5.027
21	346	54	464	78	4.156	106	849	139	542
22	380	51	801	74	562	101	5.322	132	6.082
23	415	49	4.155	71	980	096	817	125	648
24	452	47	524	67	5.429	92	6.333	120	7.285
25	491	45	909	64	6.091	88	872	114	854
26	531	43	5.309	62	871	84	7.438	110	8.495
27	573	41	726	59	871	81	8.016	106	9.161
28	616	40	6.158	57	7.889	77	621	101	862
29	661	38	606	55	926	75	9.247	097	10.568
30	.0707	.1037	7.069	.0153	8.482	.0072	9.896	.0094	11.310
31	755	35	548	51	9.057	69	10.567	91	12.076
32	804	34	8.048	49	651	67	11.259	88	868
33	865	33	558	48	10.264	65	974	85	13.685
34	908	32	9.079	46	895	63	12.711	82	14.527
35	962	31	621	45	11.545	61	13.470	80	15.394
36	1018	30	10.179	43	12.215	59	14.250	77	16.286
37	1075	29	752	42	903	57	15.053	75	17.203
38	1134	28	11.341	41	13.609	56	078	73	18.146
39	1195	28	946	40	14.835	54	16.724	71	19.113
40	1257	27	12.566	39	15.080	53	17.593	69	20.106
41	1320	26	13.203	38	843	51	18.484	67	21.124
42	1385	26	855	37	16.625	50	19.396	66	22.167
43	1452	25	14.522	36	17.427	49	20.331	64	23.235
44	1521	24	15.205	35	18.246	48	21.287	62	24.328
45	1590	24	904	34	19.085	47	22.266	60	25.446
46	1662	23	16.619	34	943	46	23.267	60	26.591
47	1735	23	17.350	33	20.819	45	24.289	58	27.759
48	1810	22	18.096	32	21.715	44	25.334	57	28.953
49	1886	22	858	31	22.629	43	26.401	56	30.172
50	1964	21	19.685	31	23.562	42	27.489	55	31.416
55	2376	19	23.758	28	28.510	38	33.262	49	38.013
60	2827	18	28.274	25	33.929	35	39.584	45	45.239
65	3318	16	33.183	23	39.820	32	46.456	42	53.093
70	3848	15	38.485	22	46.182	29	53.878	38	61.575
75	4418	14	44.179	20	53.015	27	61.850	36	70.686
80	5027	13	50.266	19	60.319	26	70.372	33	80.425
85	5675	12	56.745	18	68.994	24	79.443	31	90.792
90	6362	12	63.617	17	76.841	23	89.064	30	101.788
95	7088	11	70.882	16	85.059	21	99.235	28	113.412
1.00	.7854	.1010	78.540	.0115	94.248	.0020	109.956	.0016	125.664

TABLE XXXI. — Discharge of Water through New Pipes.
 Compiled from Henry Darcy's French Tables of 1857.

18 Centimetres.		20 Centimetres.		22 Centimetres.		24 Centimetres.		26 Centimetres.	
Hgt. in 100 met.	Disch'ge in litres	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'r. in litres	Hgt. in 100 met.	Disch'r. in litres	Hgt. in 100 met.	Disch'r. in litres
1.1670	0.014	1.4408	0.016	1.7484	0.017	2.0748	0.019	2.4350	0.020
9.8739	057	4616	068	0.5685	069	0.6647	75	0.7801	082
2027	127	2502	141	8028	155	8603	170	4229	184
1345	226	2661	251	2010	277	2892	302	2807	327
992	358	1225	398	1488	432	1764	471	2071	511
780	509	0.0964	566	1168	622	1888	679	1628	735
640	698	791	770	957	847	1189	924	1386	1.001
542	906	669	1.005	809	1.106	963	1.206	1130	307
469	1.145	579	272	700	400	888	527	0978	654
412	414	509	571	616	728	783	885	860	2.042
868	711	454	901	550	091	654	281	768	471
882	2.086	410	2.262	496	488	590	714	698	941
802	889	873	655	452	920	587	8.186	631	3.451
277	771	848	3.079	414	387	498	695	579	4.002
256	8.181	816	584	383	888	456	4.241	535	595
288	619	294	4.021	356	4.423	428	825	497	5.228
222	4.086	274	540	332	999	395	5.448	464	901
208	580	257	5.089	311	5.588	370	6.107	435	6.616
196	5.104	242	671	293	6.238	349	805	409	7.372
185	655	229	6.283	377	912	329	7.540	386	8.168
175	6.235	216	927	262	7.620	312	8.313	366	9.005
167	842	206	7.603	249	8.363	1296	9.123	348	883
159	7.479	196	8.310	237	9.140	82	971	331	10.802
151	8.143	187	9.048	226	953	69	10.857	316	11.762
145	836	179	818	216	10.799	57	11.781	302	12.768
139	9.557	171	10.619	207	11.680	47	12.742	290	13.804
133	10.306	164	11.451	199	12.596	37	13.741	278	14.886
128	11.084	158	12.315	191	13.547	28	14.778	267	16.010
123	889	152	13.210	184	14.531	19	15.858	257	17.174
.0119	12.723	.0147	14.137	.0178	15.551	.1211	16.965	.0248	18.378
115	13.586	142	15.095	171	16.605	04	18.114	239	19.624
111	14.476	137	16.085	166	17.693	197	19.302	231	20.910
107	15.395	132	17.106	160	18.817	91	20.527	224	22.238
104	16.343	128	18.158	155	19.974	85	21.790	217	23.606
101	17.318	124	19.242	150	21.167	79	23.091	210	25.015
98	18.322	121	20.358	146	22.393	74	24.429	204	26.465
95	19.354	117	21.504	142	23.655	69	25.805	198	27.956
92	20.414	114	22.682	138	24.951	64	27.219	93	29.487
90	21.508	111	23.892	134	26.281	60	28.670	87	31.059
87	22.620	108	25.133	131	27.646	55	30.159	82	32.673
85	23.765	105	26.405	127	29.046	51	31.686	78	34.327
83	24.938	102	27.709	124	30.480	48	33.251	73	36.022
81	26.140	100	29.044	121	31.940	44	34.853	69	37.753
79	27.370	098	30.411	118	33.452	40	36.498	65	39.534
77	28.628	95	31.809	115	34.989	187	38.170	161	41.351
75	29.914	93	33.238	113	36.562	34	39.886	57	43.210
74	31.229	91	34.699	110	38.169	31	41.636	54	45.109
72	32.572	89	36.191	108	39.810	28	43.429	50	47.049
71	33.944	87	37.715	105	41.487	25	45.258	47	49.030
69	35.343	85	38.270	103	43.197	23	47.124	44	51.051
63	42.765	77	47.517	093	52.268	11	57.020	20	61.772
57	50.894	70	56.549	85	62.204	101	67.869	19	73.513
53	59.780	65	66.866	78	73.003	093	79.646	10	86.276
49	69.272	60	76.969	73	84.666	86	92.363	102	100.06
45	79.522	56	88.353	68	97.193	81	106.03	095	114.87
42	90.478	52	100.581	63	110.58	75	120.64	88	130.69
40	102.141	49	113.490	59	124.84	71	136.19	83	147.54
38	114.511	46	127.285	56	139.96	67	152.68	78	165.41
36	127.588	44	141.765	53	155.94	63	170.12	74	184.30
34	141.372	.0042	157.080	.0050	172.79	.0060	188.50	.0070	204.20

TABLE XXXI—Discharge of Water through New Pipes.
Compiled from Henry Darcy's French Tables of 1857.

Velocity per sec.		30 Centimetres.		34 Centimetres.		38 Centimetres.		42 Centimetres.	
Diam. Metre	Area of sect'n	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.
0.01	0.0001	3.2418	0.024	4.1639	0.027	5.2013	0.030	6.3539	0.033
2	03	1.0886	094	1.8340	107	1.6664	1.119	2.0857	132
8	07	0.5630	212	0.7231	240	0.9033	269	1.1035	297
4	13	3737	877	4800	427	5996	478	0.7325	528
5	20	2757	589	3541	668	4423	746	5404	825
6	28	2168	848	2785	961	3479	1.074	4249	188
7	38	1779	1.155	2285	1.308	2854	462	3487	1.616
8	50	1505	508	1933	709	2414	910	2949	2.111
9	64	1302	909	1672	2.163	2088	2.417	2651	672
10	79	1146	2.356	1471	670	1838	985	2245	3.299
11	95	1022	851	1313	231	1640	3.611	2003	991
12	113	0922	3.393	1185	845	1480	4.298	1808	4.750
13	133	840	982	1079	4.513	1348	5.044	1646	5.575
14	154	771	618	0980	5.234	1237	850	1511	6.465
15	177	712	5.301	914	6.008	1142	6.715	1395	7.422
16	201	661	6.032	849	8.36	1061	7.640	1296	8.445
17	227	617	809	793	7.717	0991	8.625	1210	9.533
18	254	579	7.634	744	8.652	929	9.670	1135	10.688
19	284	45	8.506	700	9.640	874	10.774	1058	11.908
20	314	15	9.425	661	10.681	826	11.938	1009	13.195
21	346	487	10.391	626	11.776	782	13.162	0955	14.547
22	380	63	11.404	595	12.925	43	14.445	907	15.966
23	415	41	12.464	66	14.126	07	15.788	864	17.450
24	452	21	13.572	40	15.381	675	17.191	825	19.000
25	491	02	14.726	17	16.690	45	18.653	788	20.617
26	531	385	15.928	495	18.052	18	20.175	755	22.299
27	573	70	17.177	75	19.467	594	21.757	725	24.047
28	616	56	18.473	57	20.936	71	23.398	697	25.862
29	661	42	19.816	40	22.458	49	25.100	71	27.742
30	0707	0330	21.206	0624	24.033	0530	26.861	0647	29.688
31	755	19	22.643	09	25.662	511	28.681	25	31.700
32	804	08	24.127	396	27.344	494	30.561	04	33.778
33	855	298	25.659	83	29.080	78	32.501	84	35.923
34	908	89	27.238	71	30.869	63	34.501	66	38.133
35	962	80	28.863	59	32.712	49	36.560	48	40.409
36	1018	71	30.536	49	34.608	36	38.679	32	42.751
37	1075	64	32.256	39	36.557	23	40.858	17	45.159
38	1134	56	34.024	29	38.560	11	43.096	02	47.683
39	1195	49	35.838	20	40.616	00	45.395	489	50.173
40	1257	43	37.699	12	42.726	389	47.752	76	52.779
41	1320	36	39.608	04	44.889	79	50.170	63	55.451
42	1385	30	41.564	206	47.105	70	52.647	52	58.189
43	1452	25	43.566	89	49.375	61	55.184	41	60.993
44	1521	19	45.616	82	51.698	52	57.780	30	63.862
45	1590	14	47.713	75	54.075	44	60.436	20	66.798
46	1662	09	49.857	69	56.505	36	63.153	10	69.800
47	1735	05	52.049	63	58.988	28	65.928	01	72.868
48	1810	00	54.286	57	61.525	21	68.763	392	76.002
49	1886	0196	56.573	52	64.106	14	71.659	84	79.002
50	1964	92	68.905	46	66.759	08	74.613	76	82.467
55	2376	74	71.275	23	80.778	279	90.282	40	99.785
60	2827	59	84.823	04	96.133	54	107.44	11	118.75
65	3318	46	99.550	187	112.82	34	126.10	286	139.37
70	3848	35	115.45	74	130.85	17	146.24	65	161.64
75	4418	26	132.54	62	150.21	02	167.88	47	185.55
80	5027	18	150.80	51	170.90	189	191.01	31	211.12
85	5675	11	170.24	42	192.93	77	215.63	17	238.33
90	6362	04	190.85	34	216.30	67	241.75	04	267.19
95	7088	99	212.65	27	241.00	58	269.35	193	297.71
100	7854	0094	235.62	0120	267.04	0150	298.45	0183	329.87

TABLE XXXI. — Discharge of Water through New Pipes.
 Compiled from Henry Darcy's French Tables of 1857.

46 Centimetres.		50 Centimetres.		54 Centimetres.		58 Centimetres.		62 Centimetres.	
Hgt. in 100 met.	Disch'r. in litres	Hgt. in 100 met.	Disch'r. in litres	Hgt. in 100 met.	Disch'r. in litres	Hgt. in 100 met.	Disch'r. in litres	Hgt. in 100 met.	Disch'r. in litres
7.6218	0.086	9.0050	0.039	10.5080	0.042	12.117	0.046	18.846	0.049
2.4419	145	2.8850	157	3.8651	170	3.8820	182	4.4360	195
1.3287	325	1.5639	353	1.8241	382	2.1044	410	2.4046	438
0.8787	578	0.881	628	2.109	679	1.8969	729	1.5962	779
6482	903	0.7658	982	0.8932	1.060	0.805	1.139	1715	1.217
5097	301	6022	414	7024	527	0.8104	640	0.9260	753
4188	770	4942	924	5764	2.078	6650	2.282	7599	2.886
8538	2.812	4180	2.518	4875	714	5628	915	6427	8.116
3060	926	3616	8.181	4217	8.435	4865	8.689	5529	944
2693	613	3182	927	3712	4.241	4282	4.555	4893	4.869
2403	372	2839	4.752	3812	5.131	3821	5.511	4366	5.892
2168	5.202	562	5.655	2988	6.107	3448	6.559	8939	7.012
1970	6.106	833	6.637	2721	7.167	8139	7.698	8587	8.229
1812	7.081	141	7.697	2497	8.318	2881	8.928	8292	9.544
1674	8.129	1978	8.836	2307	9.543	2661	10.249	3041	10.956
1555	9.249	837	10.053	2143	10.857	2472	11.661	2825	12.465
1452	10.441	715	11.349	2000	12.257	2308	13.165	2637	14.073
1861	11.686	608	12.723	1876	13.741	164	14.759	2472	15.778
1281	13.042	514	14.176	765	15.310	037	16.445	2327	17.579
1210	14.451	429	15.708	667	16.965	1923	18.221	2198	19.477
1146	15.933	354	17.318	579	18.703	822	20.089	2082	21.474
1088	17.486	286	19.007	500	20.527	730	22.047	1978	23.568
1036	19.112	225	20.774	428	22.435	648	24.097	883	25.759
0999	20.810	169	22.620	365	24.429	572	26.239	797	28.048
946	22.580	118	24.544	304	26.507	503	28.471	718	30.434
906	24.423	071	26.547	249	28.670	441	30.793	646	32.917
870	26.338	028	28.628	199	30.918	383	33.208	580	35.498
826	28.325	0988	30.788	152	33.251	329	35.718	519	38.177
805	30.384	951	33.026	109	35.668	280	38.310	462	40.952
.0776	32.516	.0917	35.843	.1070	38.170	.1234	40.997	.1410	43.825
749	34.719	885	37.738	82	40.757	191	43.777	361	46.795
724	36.995	855	40.212	998	43.429	151	46.646	315	49.863
700	39.344	28	42.765	65	46.186	114	49.607	273	53.029
678	41.764	02	45.396	35	49.027	079	52.659	233	56.291
58	44.257	777	48.106	06	51.954	46	55.803	195	59.651
38	46.822	54	50.894	0880	54.965	15	59.087	160	63.108
20	49.460	32	53.761	54	58.061	0986	62.362	126	66.663
03	52.169	12	56.706	30	61.242	58	65.779	1095	70.315
586	54.951	693	59.730	08	64.508	32	69.286	65	74.065
71	57.805	74	62.832	786	67.858	07	72.885	37	77.911
56	60.732	57	66.013	66	71.294	884	76.575	10	81.856
42	63.731	40	69.273	47	74.814	62	80.356	0984	85.897
29	66.802	25	72.611	28	78.419	40	84.228	960	90.037
16	69.944	10	76.027	11	82.109	20	88.191	937	94.273
04	73.160	595	79.522	694	85.883	801	92.245	915	98.607
492	70.648	82	83.096	78	89.743	783	96.391	894	103.04
81	79.808	69	86.748	63	93.687	65	100.63	874	107.57
71	83.240	56	90.478	49	97.716	48	104.95	855	112.19
61	86.745	44	94.288	35	101.88	32	109.37	837	116.92
51	90.321	33	98.175	22	106.03	17	113.88	819	121.74
08	109.29	482	118.79	563	128.30	649	137.80	742	147.80
373	130.06	41	141.37	14	152.68	93	163.99	677	175.30
43	152.64	05	165.92	473	179.19	545	192.46	628	205.74
18	177.03	375	192.42	88	207.82	05	223.21	577	238.60
296	203.22	50	220.89	08	238.57	470	256.24	537	273.91
77	231.22	27	251.33	381	271.43	40	291.54	508	311.65
60	261.03	07	283.73	58	306.42	13	329.12	472	351.82
45	292.64	290	318.09	38	343.53	90	368.98	445	394.43
32	326.06	274	354.41	820	382.76	69	411.12	421	439.47
.0220	361.28	.0260	382.70	.0303	424.12	.0350	455.53	.0400	486.95

TABLE XXXI. — Discharge of Water through New Pipes.
 Compiled from Henry Darcy's French Tables of 1857.

Veloc'y per sec		66 Centim.-tres.		70 Centimetres.		74 Centimetres.		78 Centimetres.	
Diam. Metre	Area of section	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.
0.01	0.0001	15.690	0.052	17.650	0.055	19.725	0.059	21.915	0.061
2	08	5.0268	207	5.6547	220	6.3193	232	7.0209	245
3	07	2.7249	467	3.0652	495	3.4255	523	3.8050	551
4	18	1.8088	829	2.0847	880	2.2789	930	2.5264	980
5	20	1.3343	1.296	1.5010	1.374	1.6774	1.453	1.8637	1.532
6	26	1.0493	1.866	1.1804	979	8191	2.092	4556	2.205
7	38	0.8611	2.540	0.9686	2.694	0825	848	2027	3.002
8	50	7288	3.317	8192	3.619	0.9155	3.719	0172	921
9	64	6300	4.199	7086	4.453	7919	4.707	0.8799	4.962
10	79	5544	5.183	6237	5.498	6970	5.811	7744	6.126
11	95	4947	6.272	5565	6.652	6219	7.032	6910	7.412
12	118	4464	7.464	5021	7.917	5611	8.369	6234	8.821
13	138	4065	8.760	4572	9.291	5110	9.822	5677	10.353
14	154	3780	10.159	4196	10.776	4689	11.391	5210	12.007
15	177	3446	11.663	3876	12.370	4332	13.077	4813	13.783
16	201	3201	13.270	601	14.074	4024	14.079	471	15.663
17	227	2988	14.981	862	15.889	3757	16.796	174	17.704
18	254	802	16.795	152	17.813	522	18.831	3913	19.849
19	284	637	18.713	2966	19.347	815	20.981	683	22.115
20	314	490	20.784	801	21.991	131	23.247	478	24.504
21	346	359	22.859	654	24.245	2966	25.031	295	27.016
22	380	241	25.089	521	26.609	817	28.129	180	29.650
23	415	184	27.421	400	29.083	682	30.745	2980	32.407
24	452	036	29.857	290	31.667	560	33.477	844	35.286
25	491	1947	32.397	190	34.361	448	36.325	720	38.288
26	531	866	35.041	098	37.165	345	39.289	606	41.412
27	573	791	37.789	014	40.079	251	42.369	501	44.659
28	616	721	40.639	1986	43.103	164	45.565	404	48.029
29	661	657	43.694	864	46.236	083	48.876	315	51.521
30	0707	1598	46.653	1797	49.480	2008	52.307	2231	55.135
31	755	542	49.859	735	52.834	1989	55.853	164	58.871
32	804	490	53.080	677	56.297	874	59.514	082	62.731
33	855	442	56.449	622	59.871	813	63.292	014	66.713
34	908	397	59.923	571	63.555	756	67.186	1951	70.817
35	962	354	63.499	523	67.348	702	71.196	891	75.045
36	1018	314	67.179	478	71.251	652	75.323	835	79.394
37	1075	276	70.964	436	75.265	604	79.565	782	83.866
38	1134	240	74.861	395	79.388	559	83.925	733	88.461
39	1195	207	78.843	357	83.622	517	88.399	685	93.178
40	1257	175	82.938	321	87.965	477	92.991	641	98.017
41	1320	144	87.137	287	92.418	439	97.699	598	102.98
42	1385	116	91.439	255	96.982	402	102.52	558	108.07
43	1452	088	95.845	224	101.66	368	107.46	520	113.27
44	1521	062	100.36	195	106.44	336	112.52	483	118.60
45	1590	037	104.97	167	111.33	304	117.69	449	124.05
46	1662	014	109.69	140	116.33	274	122.98	416	129.63
47	1735	0991	114.51	115	121.45	246	128.29	384	135.32
48	1810	969	119.43	090	126.67	218	133.91	354	141.15
49	1886	948	124.46	067	132.00	192	139.55	325	147.09
50	1964	928	129.59	044	137.45	167	145.30	297	153.15
55	2376	840	156.81	0945	166.31	056	175.81	174	185.32
60	2827	767	186.61	863	197.92	0965	209.23	072	220.54
65	3318	706	219.01	794	232.28	888	245.56	0986	258.83
70	3848	654	254.00	736	269.39	822	284.79	913	300.18
75	4418	609	291.58	685	309.25	766	326.92	851	344.60
80	5027	570	331.75	641	351.86	716	371.97	796	392.07
85	5675	535	374.52	602	397.22	673	419.91	748	442.61
90	6362	505	419.88	568	445.22	634	470.77	705	496.22
95	7088	477	467.82	537	496.18	600	524.53	667	552.88
1.00	7854	0452	518.36	0510	549.78	0569	581.20	0633	612.61

TABLE XXXI. — *Discharge of Water through New Pipes.*
Compiled from Henry Darcy's French Tables in 1867.

82 Centimetres.		86 Centimetres.		90 Centimetres.		94 Centimetres.		98 Centimetres.	
Hgt. in 100 met.	Disch'g in litres.	Hgt. in 100 met.	Disch'g in litres.	Hgt. in 100 met.	Disch'g in litres.	Hgt. in 100 met.	Disch'g in litres.	Hgt. in 100 met.	Disch'g in litres.
24.220	0.064	26.640	0.068	29.176	0.071	31.827	0.074	34.594	0.078
7.7594	258	8.5350	270	9.8474	288	10.197	295	11.088	307
4.2062	580	4.6266	608	5.0670	636	5.5274	664	6.0078	698
2.7921	1.080	3.0712	1.081	3.3685	1.181	3.6691	1.181	3.9881	1.232
2.0597	610	2.2655	689	2.4812	767	2.7066	846	2.9418	924
1.6197	2.819	1.7816	2.482	1.9512	2.545	2.1285	2.658	2.3185	2.771
8292	3.156	4620	8.810	6012	8.464	1.7467	8.618	1.8985	8.771
1242	4.121	2365	4.828	3542	4.524	4.778	4.725	6057	4.926
0.9724	5.217	0696	5.471	1714	5.726	2778	5.980	3889	6.284
8558	6.440	0.9414	6.754	0810	7.068	1246	7.383	2224	7.697
7687	7.798	8400	8.178	0.9199	8.553	085	8.933	0907	9.313
6890	9.274	7579	9.726	8800	10.179	9054	10.631	0.9841	11.088
6274	10.884	6902	11.415	7558	11.946	8245	12.477	8962	13.007
5758	12.628	6338	13.229	6936	13.854	7567	14.470	8224	15.085
5319	14.491	5859	15.197	6407	15.904	6990	16.611	7597	17.818
4941	16.487	485	17.291	5952	18.096	498	18.896	7058	19.704
618	18.612	078	19.520	557	20.428	462	21.386	6589	22.244
825	20.866	4757	21.884	210	22.902	688	23.880	6177	24.938
071	23.249	477	24.383	4904	25.518	649	26.651	5814	27.785
3844	25.761	228	27.017	631	28.274	052	29.531	491	30.787
641	28.401	005	29.787	387	31.178	4785	32.557	201	33.948
459	31.171	3804	32.691	167	34.212	545	35.782	4940	37.258
298	34.069	628	35.731	3967	37.393	828	39.055	704	40.717
143	37.096	457	38.205	786	40.715	130	42.625	489	44.384
006	40.251	806	42.215	621	44.177	8950	46.142	293	48.105
2880	43.536	168	45.660	469	47.584	8784	49.907	113	52.081
764	46.949	040	49.239	330	51.530	682	53.820	3948	56.110
657	50.491	2928	52.955	201	55.418	492	57.681	795	60.343
558	54.163	814	56.805	082	59.447	802	62.089	654	64.781
.2466	57.962	.2713	60.789	.2971	63.627	.3241	66.445	.3522	69.272
881	61.891	618	64.910	868	67.929	128	70.948	402	73.967
301	65.948	581	69.165	771	72.882	023	75.599	286	78.816
226	70.135	448	73.555	681	76.977	2925	80.896	179	83.819
156	74.449	871	78.081	597	81.713	883	85.345	079	88.978
090	78.893	299	82.741	518	86.590	747	90.439	2982	94.287
028	83.466	281	87.537	443	91.609	655	95.680	897	99.752
1970	88.167	167	92.468	378	96.769	589	101.07	814	105.87
915	92.997	106	97.584	307	102.07	516	106.61	735	111.14
863	97.957	049	102.74	244	107.51	448	112.29	661	117.07
813	103.10	1995	108.07	184	113.10	383	118.13	590	123.15
767	108.26	948	113.54	128	118.82	321	124.10	523	129.39
722	113.61	804	119.15	074	124.69	263	130.28	460	135.77
680	119.08	848	124.89	024	130.70	207	136.51	399	142.82
640	124.68	803	130.77	1975	136.85	154	142.98	342	149.01
601	130.42	761	136.78	929	143.14	104	149.50	287	155.86
564	136.38	721	142.92	885	149.57	056	156.22	235	162.87
529	142.27	682	149.21	842	156.15	010	163.09	185	170.03
496	148.38	646	155.62	802	162.86	1966	170.10	137	177.34
464	154.63	610	162.17	764	169.72	924	177.26	091	184.80
433	161.01	572	168.86	727	176.72	883	184.57	047	192.42
297	194.32	427	204.32	563	213.33	705	223.33	1853	232.33
185	231.85	203	243.16	427	254.47	557	265.78	692	277.09
090	272.10	199	285.38	313	298.65	433	311.92	557	325.20
010	315.57	110	330.97	216	346.37	327	361.76	442	377.15
0940	362.27	084	379.94	132	397.61	235	415.28	343	432.95
879	412.18	967	432.28	059	452.59	156	472.50	256	492.60
826	465.31	909	488.01	995	510.71	086	533.40	180	556.10
779	521.66	857	547.11	939	572.56	024	598.00	113	623.45
737	581.24	811	609.59	888	637.94	969	666.29	053	694.65
.0699	644.03	.0769	675.44	.0842	706.86	.0919	738.28	.0999	769.69

TABLE XXXI. -- Discharge of Water through New Pipes.
 Compiled from Henry Darcy's French Tables of 1857.

Velocity per sec.		102 Centimetres.		100 Centimetres.		110 Centimetres.		114 Centimetres.	
Diam.	Area of section.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.
0.01	0.0001	37.475	0.080	40.472	0.083	43.584	0.086	46.812	0.090
2	03	12.006	320	12.966	333	18.964	346	14.997	358
3	07	6.5083	721	7.0287	749	7.5692	778	8.1297	806
4	13	4.3208	1.282	4.6658	1.332	5.0245	1.382	5.3966	1.433
5	20	3.1870	2.003	3.4418	2.081	3.7065	2.160	3.9809	2.238
6	28	2.5062	2.884	2.7066	2.997	2.9148	3.110	3.306	3.223
7	38	0.666	8.925	2.2211	4.079	3.919	4.233	2.5690	4.387
8	50	1.7394	5.128	1.8786	5.327	0.230	5.529	1.728	5.731
9	64	5046	6.489	6249	6.743	1.7499	6.998	1.8795	7.251
10	79	3242	8.011	4391	8.325	5401	8.639	6541	8.953
11	95	1816	9.693	2761	10.072	3742	10.454	4760	10.832
12	113	0661	11.536	1514	11.988	2399	12.441	3317	12.892
13	133	9708	13.538	0485	14.069	1291	14.601	2127	15.131
14	154	8909	15.702	0.9621	16.318	1362	16.933	1129	17.548
15	177	8230	18.025	8888	18.731	0.9571	19.439	0280	20.145
16	201	7645	20.508	8257	21.312	8892	22.117	9550	22.920
17	227	7137	23.152	7708	24.060	8301	24.968	8916	25.876
18	254	6692	25.956	7227	26.973	7783	27.992	8359	29.009
19	284	6298	28.920	6802	30.053	7325	31.188	7868	32.321
20	314	5948	32.044	6424	33.300	6918	34.558	7430	35.814
21	346	534	35.329	6085	36.715	553	38.100	7088	39.485
22	380	352	38.773	5780	40.294	224	41.815	6685	43.334
23	415	096	42.379	503	44.041	5927	45.702	6365	47.365
24	452	4863	46.144	252	47.953	656	49.763	6075	51.571
25	491	651	50.070	023	52.032	409	53.996	5809	55.960
26	531	456	54.155	4812	56.278	182	58.402	5566	60.526
27	573	277	58.401	619	60.691	4974	62.981	5342	65.271
28	616	111	62.807	440	65.270	781	67.733	5135	70.196
29	661	8958	67.374	275	70.015	608	72.657	4944	75.299
30	.0707	.3816	72.100	.4121	74.928	.4438	77.755	.4766	80.582
31	755	683	76.987	3978	80.206	284	83.025	4601	86.014
32	804	560	82.033	844	85.250	140	88.467	4447	91.682
33	855	444	87.241	720	90.661	006	94.083	4302	97.505
34	908	336	92.608	603	96.238	880	99.071	4167	103.50
35	962	234	98.136	493	101.98	761	105.83	4040	109.68
36	1018	138	103.82	389	107.89	650	111.97	3920	116.04
37	1075	048	109.67	292	113.97	545	118.27	3807	122.57
38	1134	2963	115.68	200	120.22	446	124.75	3701	129.29
39	1195	882	121.25	113	126.63	352	131.41	3600	136.18
40	1257	806	128.18	030	133.20	263	138.23	3505	143.26
41	1320	733	134.67	2952	139.95	179	145.28	3414	150.51
42	1385	665	141.32	878	146.86	098	152.40	3328	157.94
43	1452	596	148.13	807	153.93	023	159.74	3247	165.55
44	1521	537	155.09	740	161.18	2950	167.26	3169	173.34
45	1590	477	162.23	676	168.59	881	174.95	3095	181.31
46	1662	421	169.52	614	176.16	815	182.81	3024	189.46
47	1735	367	176.97	556	183.90	752	190.85	2956	197.78
48	1810	315	184.58	500	191.81	692	199.05	2891	206.29
49	1886	265	192.35	446	199.89	634	207.43	2830	214.98
50	1964	217	200.28	395	208.13	579	215.99	2770	223.84
55	2376	007	242.34	168	251.84	334	261.34	2507	270.84
60	2827	1833	288.40	1980	299.71	182	311.02	2289	322.33
65	3318	687	338.47	822	315.74	1962	365.01	2107	378.29
70	3848	562	392.54	687	407.94	817	423.33	1951	438.72
75	4418	455	450.62	571	468.29	692	485.97	1817	503.64
80	5027	361	513.71	470	532.82	583	552.92	1700	573.03
85	5675	278	578.80	381	601.50	487	624.20	1597	646.89
90	6362	205	648.90	302	674.34	402	699.79	1506	725.24
95	7088	140	723.00	232	751.35	326	779.71	1424	808.06
1.00	.7854	.1082	801.11	.1168	882.52	.1258	863.94	.1351	895.36

TABLE XXXI. — Discharge of Water through New Pipes.
 Compiled from Henry Darcy's French Tables of 1857.

118 Centimetres.		122 Centimetres.		126 Centimetres.		130 Centimetres.		184 Centimetres.	
Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.
50.154	0.098	58.612	0.096	57.185	0.099	60.874	0.102	64.678	0.106
16.068	0.871	17.176	0.888	18.821	0.896	19.508	0.408	20.721	0.421
8.7102	0.834	9.8107	0.862	9.9318	0.891	10.872	0.919	11.282	0.947
5.7819	1.483	6.1806	1.588	6.5925	1.588	7.0177	1.684	7.4562	1.684
4.2652	2.817	4.5598	2.895	4.8681	2.474	5.1768	2.558	5.5008	2.681
3.8542	3.836	3.5854	3.449	3.8244	3.563	4.0710	3.676	4.8256	3.789
2.7524	4.541	2.9482	4.695	3.1888	4.849	3.3407	5.008	3.5594	5.157
2.8279	5.981	4.884	6.182	2.6543	6.388	2.8255	6.535	3.0020	6.735
2.0187	7.505	2.1525	7.761	2.2960	8.015	4440	8.270	2.5968	8.525
1.7722	9.267	1.8944	9.582	2.0207	9.895	1510	10.210	2.2854	10.528
5814	11.212	6904	11.594	8081	11.974	1.9198	12.854	9898	12.784
4268	13.844	5252	18.797	6268	14.250	7318	14.708	1.8400	15.154
2998	15.661	3889	16.192	4815	16.728	5770	17.255	6755	17.785
1924	18.164	2746	18.781	3595	19.896	4472	20.012	5876	20.626
1.1014	20.851	1.1774	21.558	1.2558	22.265	3868	22.978	1.4204	23.679
0.282	23.724	0.988	24.529	1.666	25.332	2419	26.188	8195	26.942
0.9552	26.784	0.211	27.691	0.891	28.600	1594	29.507	2318	30.416
8956	30.027	0.9574	31.045	0.212	32.068	0.870	33.081	1549	34.099
8429	33.457	9011	34.590	0.9611	35.725	0.231	36.859	0.870	37.998
7960	37.070	8509	38.827	9076	39.582	0.9662	40.841	0.265	42.096
7540	40.871	8060	42.256	8598	43.641	9152	45.027	0.9724	46.411
7162	44.854	7656	46.376	8166	47.896	8698	49.417	9286	50.938
6820	49.025	7290	50.688	7778	52.849	8278	54.012	8795	55.678
6509	53.888	6957	55.192	7421	57.001	7899	58.811	8398	60.619
6224	57.924	6653	59.887	7097	61.851	7554	63.814	8027	65.776
5963	62.648	6375	64.773	6799	66.896	7238	69.021	7690	71.144
5724	67.561	6118	69.852	6526	72.141	6947	74.432	7381	76.728
5502	72.658	5881	75.122	6278	77.586	6678	80.048	7095	82.510
5297	77.941	5662	80.588	6040	83.225	6429	85.868	6881	88.609
0.5107	88.408	0.5459	86.237	0.5823	89.064	0.6198	91.892	6585	94.720
4980	89.064	5269	92.082	5621	95.100	5983	98.120	6357	101.14
4764	94.900	5098	98.117	5432	101.38	5782	104.55	6144	107.77
4609	100.98	4927	104.85	5256	107.77	5595	111.19	5944	114.61
4464	107.18	4772	110.77	5090	114.40	5419	118.08	5757	121.66
4328	113.58	4627	117.88	4935	121.28	5253	125.08	5581	128.92
4200	120.11	4490	124.18	4789	128.25	5098	132.32	5416	136.89
4079	126.87	4360	131.18	4651	135.48	4951	139.78	5260	144.08
3965	133.88	4238	138.86	4521	142.90	4813	147.44	5118	151.97
3857	140.96	4123	145.74	4398	150.52	4682	155.80	4974	160.08
3755	148.28	4014	153.31	4281	158.84	4558	163.86	4842	168.39
3658	155.79	3910	161.07	4171	166.85	4440	171.63	4717	176.92
3566	163.48	3812	169.08	4066	174.57	4328	180.11	4599	185.66
3478	171.36	3718	177.17	3966	182.98	4222	188.79	4486	194.59
3395	179.42	3629	185.50	3871	191.59	4121	197.67	4378	203.75
0.3316	187.67	0.3544	194.03	0.3780	200.40	4024	206.76	0.4276	218.12
3240	196.11	3463	202.75	3694	209.40	3982	216.05	4177	222.70
3167	204.72	3386	211.66	3611	218.60	3844	225.54	4084	232.48
3098	213.58	3311	220.77	3532	228.01	3760	235.24	3995	242.48
3032	222.52	3241	230.06	3457	237.60	3679	245.14	3900	252.69
2968	231.69	3173	239.55	3384	247.40	3602	255.26	3827	263.11
2896	240.85	2871	249.85	3068	259.85	3260	268.86	3464	278.86
2458	288.64	2622	344.95	2797	356.26	2978	367.57	3164	378.88
2288	391.56	2418	404.84	2574	418.11	2740	431.88	2911	444.66
2091	454.12	2235	469.51	2384	484.91	2537	500.80	2696	515.69
1947	521.81	2081	538.98	2210	556.65	2368	574.82	2510	591.99
1821	593.18	1947	618.24	2077	638.35	2210	658.45	2349	678.56
1711	669.59	1829	692.29	1951	714.99	2077	737.69	2206	760.88
1618	750.69	1725	776.18	1839	801.59	1958	827.08	2080	852.47
1526	836.41	1631	864.76	1740	898.12	1852	921.47	1968	949.81
0.1447	926.77	0.1548	958.19	0.1651	989.60	0.1757	1021.00	0.1867	1062.44

TABLE XXXI.—Discharge of Water through New Pipes.
Compiled from Henry Darcy's French Tables of 1857.

Veloc'y per sec		138 Centimetres.		142 Centimetres.		146 Centimetres.		150 Centimetres.	
Diam.	Area of section.	Hgt. in 100 met.	Disch'r. in litres	Hgt. in 100 met.	Disch'r. in litres	Hgt. in 100 met.	Disch'r. in litres	Hgt. in 100 met.	Disch'r. in litres
0.01	0.0001	68.696	0.108	72.631	0.112	76.780	0.116	81.045	0.118
2	03	21.977	0.434	23.269	446	24.599	0.459	25.965	0.471
3	07	11.913	0.975	12.613	1.004	13.334	1.032	14.075	1.060
4	13	7.9080	1.784	8.8781	1.784	8.8515	1.835	9.8431	1.886
5	20	5.8336	2.710	6.1766	2.788	6.5295	2.867	6.8922	2.945
6	28	4.5873	3.902	4.8678	4.015	5.1848	4.128	5.4200	4.241
7	38	3.7645	5.811	3.9859	5.405	4.2186	5.619	4.4477	5.772
8	50	3.1839	6.937	3.8712	7.138	3.6638	7.839	3.7617	7.540
9	64	2.7541	8.777	2.9161	9.083	3.0827	9.287	3.2539	9.543
10	79	2.4239	10.839	2.5665	11.153	2.7131	11.467	2.8638	11.781
11	95	2.1628	13.114	2.2900	13.495	2.4809	13.874	2.5553	14.255
12	113	1.9519	15.608	2.0662	16.606	2.1843	16.512	3.056	16.965
13	133	7771	18.315	1.8816	18.849	1.9891	19.879	0996	19.910
14	154	6808	21.242	7267	21.859	1.8254	22.474	1.9266	23.091
15	177	5064	24.387	5950	25.093	1.6862	25.799	1.7798	26.507
16	201	3995	27.746	4818	28.501	5664	29.354	5584	30.159
17	227	3065	31.324	3833	32.231	4623	33.140	5436	34.047
18	254	2249	35.115	2976	36.135	3711	37.151	4472	38.170
19	284	1529	39.125	2207	40.261	2904	41.395	3621	42.529
20	314	0887	43.354	1528	44.610	2186	45.866	2863	47.124
21	345	0813	47.797	0920	49.184	1544	50.569	2185	51.964
22	380	0.9796	52.458	0872	53.979	0965	55.498	1574	57.020
23	415	9328	57.885	0.9876	58.998	1.0440	60.659	1020	62.322
24	452	8902	62.429	9425	64.240	0.9964	66.049	0617	67.859
25	491	8513	67.740	9014	69.705	9528	71.668	1.0058	73.631
26	531	8156	73.268	8636	75.892	9129	77.516	0.9636	79.640
27	573	7828	79.011	8289	81.803	8762	83.598	9249	85.884
28	616	7525	87.974	7968	87.433	8423	89.900	8891	92.363
29	661	7245	91.151	7671	93.794	8109	96.435	8560	99.078
30	707	6985	97.546	0.7395	100.87	0.7818	103.20	0.8252	106.03
31	755	6742	104.16	7139	107.18	7546	110.20	7966	113.22
32	804	6516	110.99	6899	114.20	7293	117.42	7698	120.64
33	855	6304	118.08	6675	121.45	7056	124.87	7448	128.30
34	908	6106	125.29	6465	128.93	6834	132.56	7214	136.19
35	962	5920	132.77	6268	136.62	6626	140.47	6994	144.32
36	1018	5744	140.47	6082	144.54	6430	148.61	6787	152.69
37	1075	5579	148.38	5907	152.68	6245	156.98	6592	161.28
38	1134	5423	156.21	5742	161.04	6070	165.58	6407	170.12
39	1195	5276	164.85	5586	169.63	5905	174.41	6233	179.19
40	1257	5136	173.42	5438	178.44	5748	183.47	6068	188.50
41	1320	5003	182.20	5297	187.48	5600	192.76	5911	198.04
42	1385	4877	191.19	5164	196.78	5459	202.28	5762	207.82
43	1452	4757	200.40	5037	206.21	5325	212.02	5621	217.83
44	1521	4643	209.88	4916	215.92	5197	222.00	5486	228.08
45	1590	4535	219.48	0.4801	225.84	0.5076	232.20	0.5358	238.57
46	1662	4431	229.34	4692	235.99	4959	242.84	5235	249.29
47	1735	4332	239.42	4587	246.36	4849	253.30	5118	260.24
48	1810	4237	249.72	4486	256.96	4743	264.20	5006	271.45
49	1886	4146	260.23	4390	267.78	4641	275.32	4899	282.86
50	1964	4059	270.96	4298	278.82	4544	286.67	4796	294.53
55	2376	3674	327.86	3890	337.37	4112	346.87	4341	356.88
60	2827	3355	390.19	3553	401.50	3756	412.81	3964	424.12
65	3318	3088	457.93	3269	471.20	3456	484.48	3648	497.75
70	3848	2859	531.09	3027	546.48	3200	561.88	3378	577.27
75	4418	2662	609.67	2819	627.84	2980	645.01	3146	562.68
80	5027	2491	698.67	2637	713.77	2788	733.88	2943	753.98
85	5675	2340	788.08	2478	805.78	2619	828.48	2765	851.18
90	6362	2207	877.92	2336	903.37	2470	928.81	2607	954.26
95	7088	2087	978.17	2210	1006.53	2336	1034.9	2466	1063.24
1.00	0.7854	0.1980	1088.9	0.2097	1115.27	0.2217	1146.7	0.2340	1178.12

TABLE XXXI.—*Discharge of Water through New Pipes.*
Compiled from Henry Darcy's French Tables in 1857.

154 Centimetres.		158 Centimetres.		162 Centimetres.		166 Centimetres.		170 Centimetres.	
Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.
85.425	0.121	89.920	0.124	94.581	0.127	99.257	0.130	104.10	0.134
27.368	0.494	28.808	0.496	30.286	0.509	31.800	0.522	33.351	0.534
14.886	1.089	15.616	1.117	16.417	1.145	17.288	1.173	18.078	1.202
9.8441	1.985	10.866	1.985	10.898	2.035	11.443	2.086	12.001	2.136
7.2647	3.024	7.6470	3.102	8.0891	3.181	8.441	3.259	8.8527	3.338
5.7129	4.364	6.0186	4.467	6.3219	4.580	6.6880	4.694	6.9617	4.807
4.6880	5.927	4.9847	6.081	5.1878	6.235	5.4471	6.388	5.7128	6.542
3.9650	7.741	4.1787	7.943	4.3877	8.144	4.6070	8.343	4.8817	8.545
3.4298	9.797	3.6102	10.051	3.7953	10.306	3.9851	10.561	4.1795	10.815
3.0186	12.095	3.1774	12.409	3.3393	12.723	3.5073	13.037	3.6784	13.852
2.6984	14.034	2.8552	15.014	2.9806	15.395	1296	15.776	3.2822	16.156
4802	17.416	5581	17.868	6888	18.322	2.8237	18.774	2.9614	19.227
2180	20.439	3295	20.971	4489	21.602	5714	22.033	6968	22.665
0809	23.706	1377	24.322	2473	24.938	8597	25.554	4748	26.170
1.8760	27.218	1.9747	27.919	2.0760	28.627	2.1798	29.335	2.2861	30.042
7428	30.962	1.8345	31.768	1.9285	32.572	2.0250	33.376	1237	34.181
6270	34.954	7126	35.862	8004	36.771	1.8904	37.678	1.9826	38.587
5254	39.187	6057	40.207	6880	41.224	7724	42.241	8589	43.260
4357	43.668	5118	44.797	5888	45.932	6682	47.065	7495	48.200
3558	48.280	4272	49.636	5004	50.894	5754	52.150	6522	53.407
2843	53.389	3519	54.725	4212	56.111	4923	57.495	5651	58.881
2199	58.540	2841	60.060	3489	61.681	4174	63.102	4866	64.623
1616	63.983	2227	65.645	2854	67.307	3497	68.969	4155	70.681
1086	69.667	1669	71.477	2267	73.287	2881	75.097	3509	76.906
0601	75.594	1159	77.558	1781	79.522	2318	81.484	2919	83.449
0157	81.764	0692	83.886	1240	86.011	1802	88.134	2377	90.258
0.9749	88.178	1.0262	90.463	0788	92.754	1827	95.043	1880	97.335
9371	94.826	9865	97.290	0870	99.753	0889	102.21	1420	104.68
9022	101.72	9497	104.36	9984	107.00	0483	109.65	0994	112.29
8698	108.86	0.9156	111.68	0.9625	114.51	1.0106	117.34	1.0599	120.17
8396	116.23	8838	119.25	9291	122.27	0.9756	125.29	0281	128.31
8114	123.35	8541	127.07	8979	130.29	9428	133.50	9888	136.72
7851	131.72	8264	135.14	8688	138.56	9122	141.98	9567	145.40
7604	139.32	8004	143.45	8414	147.08	8835	150.71	9266	154.35
7373	148.17	7760	152.01	8158	155.86	8566	159.71	8983	163.56
7154	156.76	7530	160.82	7916	164.90	8312	168.97	8717	173.04
6948	165.68	7318	169.88	7688	174.18	8073	178.48	8466	182.79
6758	174.65	7109	179.19	7473	183.73	7847	188.26	8230	192.80
6570	183.97	6915	188.75	7270	193.52	7633	198.30	8006	203.08
6396	193.52	6732	198.55	7077	203.58	7431	208.60	7794	213.68
6231	203.32	6558	208.60	6895	213.88	7239	219.16	7592	224.44
6074	213.36	6393	218.90	6721	224.44	8057	229.99	7401	235.53
5925	223.64	6236	229.45	6556	235.26	6884	241.07	7220	246.88
5783	234.16	6087	240.24	6399	246.32	6719	252.41	7047	258.49
0.5647	244.93	0.5944	251.29	0.6249	257.65	0.6562	264.01	0.6882	270.37
5518	255.93	5808	262.58	6106	269.23	6411	275.88	6724	282.53
5394	267.13	5678	274.12	5969	281.06	6268	289.00	6574	294.94
5276	278.67	5554	286.00	5839	293.15	6131	300.39	6430	307.63
5163	290.41	5435	297.95	5714	305.49	5996	313.03	6292	320.58
5055	302.88	5321	310.23	5594	318.09	5874	325.94	6160	333.80
4575	365.88	4316	375.38	5063	384.99	5316	394.39	5575	408.89
4179	435.43	4398	446.74	4624	458.05	4855	469.36	5092	480.67
3845	511.02	4047	524.30	4255	537.57	4468	550.84	4685	564.11
3561	592.66	3748	608.06	3940	623.45	4137	638.84	4390	654.24
3316	680.25	3490	698.03	3669	715.70	3852	733.37	4040	751.04
3102	774.09	3265	794.20	3433	814.30	3604	834.41	3780	854.52
2914	873.88	3068	896.57	3225	919.27	3386	941.97	3551	964.67
2748	979.71	2892	1005.0	3041	1031.0	3198	1056.0	3348	1081.0
2599	1091.6	2736	1120.0	2877	1148.0	3020	1177.0	3168	1205.0
0.2466	1209.5	0.2596	1241.0	0.2729	1272.0	0.2866	1304.0	0.3006	1335.0

TABLE XXXI.—Discharge of Water through New Pipes.
Compiled from Henry Darcy's French Tables of 1857.

Velocity per sec.		174 Centimetres.		178 Centimetres.		182 Centimetres.		186 Centimetres.	
Diam.	Area	Hgt. in	Disch'ge	Hgt. in	Disch'ge	Hgt. in	Disch'ge	Hgt. in	Disch'ge
Metre	of sect'n.	100 met.	in litres.	100 met.	in litres.	100 met.	in litres.	100 met.	in litres.
0.01	0.0001	109.05	0.137	114.13	0.140	119.31	0.143	124.62	0.146
2	03	84.939	0.547	36.563	0.159	88.225	0.572	39.924	0.584
3	07	18.939	1.230	19.820	1.258	20.721	1.287	21.646	1.315
4	13	12.572	2.187	18.157	2.237	18.755	2.287	14.367	2.337
5	20	9.2741	3.416	9.7054	3.495	10.147	3.574	10.597	3.662
6	28	7.2932	4.920	7.6324	5.033	7.9792	5.146	8.3338	5.259
7	38	5.9848	6.696	6.2631	6.850	6.5478	7.004	6.8387	7.158
8	50	5.0618	8.747	5.2972	8.947	5.5379	9.149	5.7840	9.349
9	64	4.3784	11.069	4.5821	11.323	4.7903	11.578	5.0032	11.833
10	79	3.8535	13.665	4.0327	13.979	4.2160	14.294	4.4034	14.607
11	95	3.4385	16.536	3.5984	16.916	3.7619	17.296	3.9291	17.676
12	113	3.1024	19.678	3.2467	20.130	3.3943	20.584	3.5451	21.035
13	133	2.8252	23.095	2.9566	23.625	3.0909	24.157	3.2283	24.687
14	154	5926	26.786	2.7132	27.400	2.8365	28.017	2.9626	28.632
15	177	2.3949	30.747	2.5063	31.455	2.6201	32.162	2.7366	32.867
16	201	2.248	34.984	2.3283	35.788	2.411	36.591	2.5423	37.396
17	227	2.0770	39.494	2.1736	40.402	2.2724	41.311	2.3734	42.218
18	254	1.9474	44.277	2.0379	45.215	2.1806	46.814	2.2537	47.251
19	284	8328	49.333	1.9181	50.467	2.0053	51.602	2.0944	52.737
20	314	7309	54.662	1.8114	55.920	1.8937	57.177	1.9779	58.434
21	346	6396	60.267	1.7158	61.651	1.7938	63.038	1.8785	64.423
22	380	5573	66.142	1.6298	67.662	1.7088	69.184	1.7796	70.704
23	415	4829	72.298	1.5519	73.963	1.6224	75.617	1.6645	77.277
24	452	4152	78.715	1.4810	80.525	1.5483	82.355	1.6171	84.143
25	491	3534	85.412	1.4168	87.376	1.4807	89.340	1.5465	91.302
26	531	2967	92.382	1.3570	94.504	1.4186	96.630	1.4817	98.752
27	573	2445	99.623	1.3024	101.92	1.3616	104.21	1.4221	106.50
28	616	1964	107.14	1.2520	109.60	1.3089	112.07	1.3671	114.53
29	661	1518	114.93	1.2058	117.57	1.2601	120.22	1.3161	122.86
30	707	1.1104	122.99	1.1620	125.82	1.2148	128.65	1.2688	131.48
31	755	0718	131.33	1.1217	134.35	1.1727	137.37	1.2248	140.39
32	804	0359	139.94	1.0841	143.15	1.1333	146.37	1.1837	149.69
33	855	0023	149.82	1.0489	152.24	1.0965	155.67	1.1452	159.09
34	908	0.9707	157.98	1.0159	161.61	1.0620	165.24	1.0922	168.87
35	962	9411	167.41	0.9849	171.28	1.0296	175.11	1.0754	178.95
36	1018	9132	177.11	0.9557	181.18	0.9991	185.25	1.0435	189.33
37	1075	8870	187.09	0.9282	191.39	0.9704	195.69	1.0135	199.99
38	1134	8622	197.34	0.9022	201.87	0.9432	206.41	0.9852	210.95
39	1195	8387	207.86	0.8777	212.64	0.9176	217.42	0.9584	222.19
40	1257	8165	218.66	0.8544	223.68	0.8933	228.71	0.9330	233.74
41	1320	7954	229.73	0.8324	235.01	0.8702	240.29	0.9089	245.57
42	1385	7754	241.07	0.8114	246.61	0.8483	252.15	0.8860	257.69
43	1452	7563	252.68	0.7915	258.49	0.8275	264.30	0.8642	270.11
44	1521	7382	264.57	0.7725	270.66	0.8076	276.74	0.8435	282.82
45	1590	0.7209	276.74	0.7544	283.10	0.7887	289.46	0.8238	295.82
46	1662	7044	289.17	0.7372	295.82	0.7707	302.47	0.8040	309.11
47	1735	6887	301.88	0.7207	308.82	0.7534	315.76	0.7869	322.70
48	1810	6736	314.86	0.7049	322.10	0.7370	329.34	0.7697	336.58
49	1886	6592	328.12	0.6898	335.66	0.7212	343.21	0.7532	350.75
50	1964	6453	341.65	0.6754	349.50	0.7060	357.36	0.7374	365.21
55	2376	5841	413.39	0.6113	422.90	0.6390	432.40	0.6674	441.90
60	2827	5334	491.97	0.5582	508.29	0.5836	514.59	0.6096	525.90
65	3318	4909	577.39	0.5137	590.66	0.5370	603.94	0.5609	617.21
70	3848	4546	669.63	0.4757	685.03	0.4973	700.42	0.5194	716.81
75	4418	4233	768.71	0.4429	786.38	0.4631	804.05	0.4837	821.73
80	5027	3960	874.62	0.4144	894.73	0.4333	914.33	0.4525	934.94
85	5675	3720	987.36	0.3898	1010.0	0.4070	1033.0	0.4251	1055.0
90	6362	3508	1107.0	0.3671	1132.0	0.3838	1158.0	0.4008	1203.0
95	7088	3318	1233.0	0.3472	1262.0	0.3631	1290.0	0.3792	1318.0
100	0.7854	0.3148	1367.0	0.3295	1398.0	0.3445	1429.0	0.3598	1461.0

TABLE XXXI. -- Discharge of Water through New Pipes.
 Compiled from Henry Darcy's French Tables of 1857.

190 Centimetres.		194 Centimetres.		200 Centimetres.		210 Centimetres.		220 Centimetres.	
Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'r. in litres.	Hgt. in 100 met.	Disch'r. in litres.	Hgt. in 100 met.	Disch'ge in litres.
180.08	0.149	185.56	0.152	144.08	0.157	158.85	0.165	174.84	0.178
41.659	0.597	48.482	0.609	46.160	0.628	50.891	0.660	55.854	0.691
22.582	1.348	23.548	1.372	25.022	1.414	27.587	1.484	30.277	1.555
34.981	2.388	15.628	2.438	16.610	2.518	18.818	2.639	20.098	2.765
11.058	8.731	11.529	8.809	12.253	8.927	18.509	4.123	14.826	4.320
8.696	5.372	9.066	5.485	9.636	5.655	10.623	5.988	11.659	6.220
7.186	7.312	7.440	7.466	7.907	7.697	8.717	8.082	9.567	8.446
6.036	9.551	6.292	9.751	6.687	10.053	7.878	10.556	8.092	11.058
5.221	12.087	5.448	12.841	5.785	12.723	6.878	13.861	6.999	13.996
4.595	14.928	4.790	15.235	5.091	15.708	5.618	16.498	6.160	17.279
4.100	18.056	4.274	18.486	548	19.007	5.009	19.957	5.497	20.907
3.699	21.489	3.857	21.940	099	22.620	4.519	23.751	4.960	24.881
369	25.219	512	25.749	8.788	26.547	4.115	27.874	516	29.201
091	29.248	229	29.864	425	30.788	3.776	32.827	145	33.866
2.856	33.576	2.9771	34.283	8.164	35.348	488	37.110	8.829	38.877
658	38.208	766	39.004	2.939	40.212	241	42.228	557	44.284
477	48.126	582	44.084	744	45.896	025	47.666	820	49.936
322	48.349	421	49.867	573	50.894	2.837	53.430	118	55.883
185	53.871	278	55.005	422	56.706	670	59.541	2.930	62.376
064	59.690	151	60.946	287	62.832	521	65.974	767	69.116
1.955	65.809	038	67.195	166	69.272	888	72.786	621	76.200
857	72.225	1.986	73.746	058	76.027	268	79.828	490	88.629
768	78.941	843	80.601	1.959	83.095	160	87.250	371	91.405
687	85.954	759	87.763	870	90.478	061	95.002	262	99.526
614	98.266	682	95.280	788	98.175	1.971	108.08	164	107.99
546	100.88	612	103.00	718	106.16	889	111.50	078	116.81
484	108.79	547	111.08	644	114.51	818	120.24	1.990	125.96
427	116.99	487	119.46	581	123.15	748	129.81	918	135.47
378	125.50	432	128.14	522	132.10	678	138.71	847	145.82
1.324	134.80	1.8808	137.18	1.467	141.87	1.617	148.44	1.775	155.51
278	148.41	382	146.42	416	150.95	561	158.50	714	166.05
235	152.81	288	156.02	361	160.85	509	168.89	656	176.94
195	162.51	246	165.98	324	171.06	460	179.61	602	188.17
158	172.51	207	176.14	282	181.68	414	190.66	552	199.74
122	182.80	170	186.65	243	192.42	371	202.04	505	211.67
089	198.40	135	197.47	207	203.58	330	218.75	460	223.93
058	204.29	108	208.59	172	215.04	292	225.80	418	236.55
028	215.48	072	220.02	139	226.82	256	238.17	378	249.51
000	226.97	048	231.75	108	238.92	222	250.87	341	262.81
0.974	238.76	015	248.79	079	281.38	189	268.89	305	276.46
948	250.85	0.989	256.18	051	264.05	159	277.26	272	290.46
925	268.24	964	268.78	024	277.09	129	290.95	240	304.80
902	275.92	940	281.78	0.999	290.44	102	304.96	209	319.49
880	288.90	918	294.98	975	304.11	075	319.31	180	334.52
0.860	302.18	0.896	308.54	0.958	318.09	1.050	333.99	1.153	349.89
940	315.76	876	322.41	921	332.38	026	349.00	126	365.62
821	329.64	856	336.58	910	346.99	008	364.84	101	381.69
803	343.82	837	351.05	890	361.91	0.981	380.01	077	398.10
786	358.29	819	365.84	871	377.15	961	396.01	054	414.87
769	373.07	802	380.92	853	392.70	940	412.84	032	431.97
698	451.41	726	460.91	772	475.17	851	498.92	0.984	522.68
636	537.21	663	548.52	705	565.29	777	593.76	853	622.04
585	630.48	610	643.76	749	663.66	715	696.85	785	730.03
542	781.21	565	746.60	607	759.69	662	808.18	727	846.66
505	839.40	526	857.07	559	883.58	617	927.76	677	971.93
472	955.05	492	975.15	528	1005.0	577	1056.0	638	1106.0
444	1078.0	462	1101.0	492	1155.0	542	1192.0	595	1248.0
418	1209.0	436	1284.0	463	1272.0	511	1336.0	561	1400.0
396	1347.0	412	1375.0	438	1478.0	488	1489.0	531	1559.0
0.375	1492.0	0.8914	1524.0	0.416	1571.0	0.459	1649.0	0.503	1728.0

TABLE XXXI—Discharge of Water through New Pipes.
 Compiled from Henry Darcy's French Tables of 1857.

Velocity per sec.		230 Centimetres.		240 Centimetres.		250 Centimetres.		260 Centimetres.	
Diam. Metre	Area of section.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.
0.01	0.0001	190.55	0.181	207.48	0.188	225.12	0.196	248.50	0.204
2	08	61.047	0.723	66.470	0.754	72.125	0.785	78.010	0.817
3	07	33.092	1.626	36.032	1.697	39.097	1.768	42.287	1.838
4	13	21.967	2.890	23.918	3.016	25.953	3.142	28.071	3.267
5	20	16.204	4.516	17.644	4.712	19.146	4.909	20.707	5.105
6	28	12.743	6.503	13.875	6.786	15.056	7.069	16.284	7.351
7	38	10.457	8.851	11.386	9.236	12.355	9.621	13.863	10.006
8	50	8.844	11.561	9.630	12.064	10.449	12.566	11.302	13.069
9	64	7.650	14.632	8.330	15.268	9.039	15.904	9.776	16.541
10	79	6.733	18.064	7.331	18.850	7.955	19.635	8.604	20.420
11	95	6.008	21.858	6.542	22.808	7.098	23.758	7.677	24.709
12	113	5.421	26.012	5.902	27.143	6.405	28.274	6.927	29.405
13	133	4.986	30.529	5.375	31.856	5.832	33.183	6.308	34.515
14	154	4.580	35.406	4.932	36.945	5.352	38.485	5.789	40.024
15	177	4.185	40.644	4.556	42.412	4.944	44.179	5.347	45.946
16	201	3.887	46.244	4.233	48.255	4.622	50.266	4.968	52.276
17	227	3.629	52.206	3.952	54.475	4.348	56.745	4.638	59.015
18	254	3.403	58.428	3.705	61.073	4.102	63.617	4.348	66.162
19	284	3.203	65.212	3.487	68.047	3.874	70.882	4.092	73.718
20	314	3.024	72.257	3.293	75.398	3.678	78.540	3.865	81.682
21	346	2.865	79.663	3.119	83.127	3.505	86.590	3.661	90.064
22	380	2.721	87.431	2.963	91.232	3.355	95.033	3.477	98.835
23	415	2.591	95.560	2.821	99.714	3.225	103.871	3.311	108.022
24	452	2.478	104.065	2.692	108.577	3.112	113.101	3.160	117.621
25	491	2.385	112.901	2.575	117.811	2.994	122.722	3.022	127.631
26	531	2.306	122.111	2.467	127.422	2.897	132.731	2.895	138.041
27	573	2.239	131.691	2.368	137.411	2.819	143.141	2.779	148.871
28	616	2.182	141.621	2.276	147.778	2.750	153.941	2.671	160.101
29	661	2.133	151.921	2.191	158.581	2.688	165.131	2.572	171.741
30	707	2.090	162.581	2.113	169.651	2.632	176.721	2.479	183.781
31	755	2.052	173.601	2.039	181.151	2.581	188.691	2.393	196.241
32	804	2.018	184.981	1.971	193.021	2.535	201.061	2.313	209.111
33	855	1.988	196.721	1.907	205.271	2.493	213.881	2.238	222.381
34	908	1.960	208.821	1.847	217.901	2.455	226.981	2.167	236.061
35	962	1.934	221.291	1.790	230.911	2.420	240.531	2.101	250.151
36	1018	1.910	234.111	1.737	244.291	2.388	254.471	2.039	264.651
37	1075	1.887	247.301	1.687	258.051	2.359	268.801	1.980	279.561
38	1134	1.865	260.851	1.640	272.191	2.333	283.541	1.925	294.871
39	1195	1.844	274.761	1.596	286.701	2.310	298.651	1.873	310.591
40	1257	1.823	289.031	1.553	301.591	2.288	314.161	1.823	326.781
41	1320	1.803	303.661	1.513	316.861	2.268	330.071	1.776	343.271
42	1385	1.783	318.651	1.475	332.511	2.249	346.361	1.731	360.221
43	1452	1.764	334.011	1.439	348.531	2.231	363.061	1.689	377.581
44	1521	1.745	349.721	1.404	364.931	2.214	380.131	1.648	395.841
45	1590	1.726	365.801	1.372	381.701	2.198	397.611	1.610	413.611
46	1662	1.708	382.241	1.340	398.861	2.183	415.481	1.573	432.101
47	1735	1.690	399.041	1.310	416.891	2.169	433.741	1.538	451.091
48	1810	1.673	416.201	1.282	434.291	2.156	452.391	1.504	470.491
49	1886	1.656	433.721	1.254	452.581	2.143	471.441	1.472	490.801
50	1964	1.639	451.611	1.228	471.241	2.131	490.881	1.441	510.611
55	2376	1.544	546.441	1.111	570.201	2.068	593.961	1.304	617.721
60	2827	1.431	650.811	1.015	678.591	1.917	706.861	1.191	735.131
65	3318	1.308	763.211	0.934	796.401	1.788	829.581	1.096	862.761
70	3848	1.179	885.151	0.865	923.631	1.683	962.121	1.015	1001.011
75	4418	1.044	1016.011	0.805	1060.011	1.594	1104.011	0.945	1149.011
80	5027	0.902	1156.011	0.753	1206.011	1.518	1257.011	0.884	1307.011
85	5675	0.755	1305.011	0.708	1362.011	1.454	1419.011	0.831	1475.011
90	6362	0.608	1463.011	0.667	1527.011	1.399	1590.011	0.783	1654.011
95	7088	0.461	1630.011	0.631	1701.011	1.352	1772.011	0.741	1843.011
1.00	0.7854	0.350	1806.011	0.599	1885.011	1.312	1964.011	0.703	2042.011

TABLE XXXI. — Discharge of Water through New Pipes.
Compiled from Henry Darcy's French Tables of 1857.

270 Centimetres.		280 Centimetres.		290 Centimetres.		300 Centimetres.	
Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.	Hgt. in 100 met.	Disch'ge in litres.
262.59	0.212	282.40	0.220	302.98	0.228	324.18	0.236
81.127	0.848	90.474	0.880	97.061	0.911	108.86	0.942
45.608	1.909	49.048	1.979	52.609	2.050	56.800	2.121
30.272	3.898	32.556	3.519	34.923	3.645	37.478	3.770
22.381	5.801	24.015	5.498	25.762	5.694	27.569	5.891
17.561	7.684	18.886	7.917	20.259	8.200	21.680	8.482
14.410	10.891	15.498	10.776	16.824	11.161	17.791	11.545
12.188	18.572	13.107	14.074	14.060	14.577	15.047	15.080
10.548	17.177	11.388	17.818	12.162	18.449	13.016	15.459
9.279	21.206	9.979	21.991	10.704	22.777	11.455	23.562
8.279	25.659	8.904	26.609	9.551	27.560	10.221	28.510
7.470	30.586	8.084	31.667	8.618	32.798	9.222	33.929
6.808	35.888	7.816	37.165	7.848	38.492	8.398	39.820
6.248	41.568	6.714	43.108	7.202	44.642	7.707	46.182
5.767	47.713	6.202	49.480	6.653	51.247	7.119	53.015
5.279	54.287	5.761	56.297	6.180	58.308	6.614	60.819
4.889	61.285	5.378	63.555	5.770	65.824	6.174	68.094
4.489	68.707	5.043	71.252	5.409	73.796	5.789	76.841
4.18	76.558	4.746	79.888	5.091	82.228	5.448	85.059
3.88	84.828	4.482	87.965	4.808	91.106	5.145	94.248
3.58	93.518	4.246	96.981	4.554	100.45	4.874	108.91
3.28	102.64	4.038	106.44	4.326	110.24	4.629	114.04
2.98	112.18	3.840	116.38	4.119	120.49	4.408	124.64
2.68	122.15	3.655	126.67	3.931	131.19	4.207	135.72
2.38	132.54	3.505	137.45	3.759	142.35	4.023	147.26
2.08	143.85	3.358	148.66	3.602	153.97	3.855	159.28
1.78	154.59	3.228	160.32	3.457	166.04	3.700	171.77
1.48	166.25	3.098	172.41	3.323	178.57	3.556	184.78
1.18	178.84	2.983	184.95	3.199	191.55	3.424	198.16
0.88	190.85	2.875	197.92	3.084	204.99	3.301	212.06
0.58	203.79	2.776	211.84	2.977	218.88	3.186	226.43
0.28	217.15	2.683	225.19	2.878	233.23	3.079	241.28
0.08	230.98	2.595	239.48	2.784	248.04	2.979	256.59
0.01	245.14	2.514	254.22	2.696	263.30	2.886	272.87
0.00	259.77	2.437	269.39	2.614	279.01	2.798	288.64
0.00	274.88	2.365	285.01	2.537	295.19	2.715	305.86
0.00	290.81	2.297	301.06	2.464	311.81	2.637	322.56
0.00	306.21	2.233	317.55	2.395	328.89	2.563	340.24
0.00	322.54	2.172	334.49	2.330	346.43	2.493	358.38
0.00	339.29	2.114	351.86	2.268	364.43	2.427	376.99
0.00	356.74	2.060	369.67	2.209	382.88	2.364	396.08
0.00	374.07	2.008	387.93	2.154	401.78	2.305	415.64
0.00	392.10	1.959	406.62	2.101	421.14	2.248	435.66
0.00	410.54	1.912	425.75	2.051	440.95	2.194	456.16
0.00	429.42	1.867	445.32	2.003	461.28	2.143	477.13
0.00	448.72	1.824	465.34	1.957	481.95	2.094	498.57
0.00	468.44	1.783	485.79	1.913	503.14	2.047	520.49
0.00	488.58	1.744	506.68	1.871	524.77	2.002	542.87
0.00	509.15	1.707	528.01	1.831	546.87	1.959	565.73
0.00	530.15	1.671	549.78	1.793	569.42	1.918	589.05
0.00	551.47	1.638	571.99	1.758	593.99	1.878	612.75
0.00	573.41	1.607	594.68	1.725	619.96	1.838	638.23
0.00	595.95	1.577	617.87	1.694	646.31	1.799	665.60
0.00	619.00	1.548	641.56	1.665	674.06	1.761	693.95
0.00	642.57	1.520	665.73	1.637	702.21	1.724	722.75
0.00	666.64	1.493	690.38	1.610	730.76	1.688	752.09
0.00	691.21	1.467	715.51	1.584	759.71	1.653	781.95
0.00	716.28	1.442	741.12	1.559	789.06	1.619	812.31
0.00	741.84	1.418	767.21	1.535	818.81	1.586	843.17
0.00	767.89	1.394	793.78	1.512	848.96	1.554	874.53
0.00	794.44	1.371	820.83	1.490	879.51	1.522	906.39
0.00	821.49	1.348	848.38	1.468	910.46	1.491	938.75
0.00	849.04	1.326	876.43	1.447	941.81	1.461	971.61
0.00	877.09	1.304	904.98	1.426	973.56	1.431	1004.97
0.00	905.64	1.282	934.03	1.406	1005.71	1.402	1038.83
0.00	934.69	1.261	963.58	1.386	1038.26	1.373	1073.19
0.00	964.24	1.240	993.63	1.367	1071.21	1.345	1108.05
0.00	994.29	1.220	1024.18	1.348	1104.66	1.317	1143.41
0.00	1024.84	1.200	1055.23	1.330	1138.61	1.290	1179.27
0.00	1055.89	1.180	1086.78	1.312	1173.06	1.263	1215.63
0.00	1087.44	1.160	1118.83	1.295	1208.01	1.237	1252.49
0.00	1119.49	1.140	1151.38	1.278	1243.46	1.211	1289.85
0.00	1152.04	1.120	1184.43	1.262	1279.41	1.186	1327.21
0.00	1185.09	1.100	1217.98	1.246	1315.86	1.161	1365.07
0.00	1218.64	1.080	1252.03	1.231	1352.81	1.137	1403.43
0.00	1252.69	1.060	1286.58	1.216	1390.26	1.113	1442.29
0.00	1287.24	1.040	1321.63	1.201	1428.21	1.090	1481.65
0.00	1322.29	1.020	1357.18	1.186	1466.66	1.067	1521.51
0.00	1357.84	1.000	1393.23	1.172	1505.61	1.045	1561.87
0.00	1393.89	0.980	1429.78	1.158	1545.06	1.023	1602.73
0.00	1430.44	0.960	1466.83	1.145	1585.01	1.002	1644.09
0.00	1467.49	0.940	1504.38	1.132	1625.46	0.981	1685.95
0.00	1505.04	0.920	1542.43	1.119	1666.41	0.961	1728.31
0.00	1543.09	0.900	1580.98	1.107	1707.86	0.941	1771.17
0.00	1581.64	0.880	1619.53	1.095	1749.81	0.921	1814.53
0.00	1620.69	0.860	1658.58	1.083	1792.26	0.901	1858.39
0.00	1660.24	0.840	1698.13	1.072	1835.21	0.881	1902.75
0.00	1700.29	0.820	1738.18	1.061	1878.66	0.861	1947.61
0.00	1740.84	0.800	1778.73	1.050	1922.61	0.841	1992.97
0.00	1781.89	0.780	1819.78	1.040	1967.06	0.821	2038.83
0.00	1823.44	0.760	1861.33	1.030	2012.01	0.801	2085.19
0.00	1865.49	0.740	1903.38	1.020	2057.46	0.781	2132.05
0.00	1908.04	0.720	1945.93	1.010	2103.41	0.761	2179.41
0.00	1951.09	0.700	1988.98	1.000	2149.86	0.741	2227.27
0.00	1994.64	0.680	2032.53	0.990	2196.81	0.721	2275.63
0.00	2038.69	0.660	2076.58	0.980	2244.26	0.701	2324.49
0.00	2083.24	0.640	2121.13	0.970	2292.21	0.681	2373.85
0.00	2128.29	0.620	2166.18	0.960	2340.66	0.661	2423.71
0.00	2173.84	0.600	2211.73	0.950	2389.61	0.641	2474.07
0.00	2219.89	0.580	2257.78	0.940	2439.06	0.621	2524.93
0.00	2266.44	0.560	2304.33	0.930	2489.01	0.601	2576.29
0.00	2313.49	0.540	2351.38	0.920	2539.46	0.581	2628.15
0.00	2361.04	0.520	2398.93	0.910	2590.41	0.561	2680.51
0.00	2409.09	0.500	2446.98	0.900	2641.86	0.541	2733.37
0.00	2457.64	0.480	2495.03	0.890	2693.81	0.521	2786.73
0.00	2506.69	0.460	2543.58	0.880	2746.26	0.501	2840.59
0.00	2556.24	0.440	2592.63	0.870	2799.21	0.481	2894.95
0.00	2606.29	0.420	2642.18	0.860	2852.66	0.461	2949.81
0.00	2656.84	0.400	2692.23	0.850	2906.61	0.441	3005.17
0.00	2707.89	0.380	2742.78	0.840	2961.06	0.421	3061.03
0.00	2759.44	0.360	2793.83	0.830	3016.01	0.401	3117.39
0.00	2811.49	0.340	2845.38	0.820	3071.46	0.381	3174.25
0.00	2864.04	0.320	2897.43	0.810	3127.41	0.361	3231.61
0.00	2917.09	0.300	2950.48	0.800	3183.86	0.341	3289.47
0.00	2970.64	0.280	3004.03	0.790	3240.81	0.321	3347.83
0.00	3024.69	0.260	3058.08	0.780	3298.26	0.301	3406.69
0.00	3079.24	0.240	3112.63	0.770	3356.21	0.281	3466.05
0.00	3134.29	0.220	3167.68	0.760	3414.66	0.261	3525.91
0.00	3189.84	0.200	3223.23	0.750	3473.61	0.241	3586.27
0.00	3245.89	0.180	3279.28	0.740	3533.06	0.221	3647.13
0.00	3302.44	0.160	3335.83	0.730	3593.01	0.201	3708.49
0.00	3359.49	0.140	3392.88	0.720	3653.46	0.181	3770.35
0.00	3417.04	0.120	3450.43	0.710	3714.41	0.161	3832.71
0.00	3475.09	0.100	3508.48	0.700	3775.86	0.141	3895.57
0.00	3533.64	0.080	3567.03	0.690	3837.81	0.121	3958.93
0.00	3592.69	0.060	3626.08	0.680	3899.76	0.101	4022.79
0.00	3652.24	0.040	3685.63	0.670	3962.71	0.081	4087.15
0.00	3712.29	0.020	3745.68	0.660	4026.66	0.061	4152.01
0.00	3772.84	0.000	3806.23	0.650	4091.61	0.041	4217.37
0.00	3833.89	0.000	3867.28	0.640	4157.56	0.021	4283.23
0.00	3895.44	0.000	3928.83	0.630	4224.51	0.000	4349.59
0.00	3957.49	0.000	3990.88	0.620	4292.46	0.000	4416.45
0.00	4020.04	0.000	4053.43	0.610	4361.41	0.000	4483.81
0.00	4083.09	0.000	4116.48				

M. Darcy compared the various formulas on the discharge of water through pipes, and found that the theoretical and practical discharges differed considerably. The experiments made by him on the discharge through 68 pipes of cast and wrought iron, lead and bitumen, with velocities from 10 centimetres to 3 metres per second, have shown, that former formulas were not correct. His being employed by the municipal government of Paris to make the necessary experiments, no pains or expenses were spared. From these careful experiments, he has been able to determine a formula which reconciles theory with practice. M. Darcy's great work has been published with the approval of the French Academy of Sciences, and ought to have a place in every engineer's library. It is a quarto of 268 pages, with a folio atlas.

M. Darcy's formula is as follows:

$$R i = \left(0.000507 + \frac{0.00000647}{R} \right) V^2$$

$$\text{Or } R i = \left(0.00051 + \frac{0.0000065}{R} \right) V^2$$

$$i = \left(\frac{0.00051}{R} + \frac{0.0000065}{R^2} \right) V^2 = \text{charge per metre.}$$

$$100 i = \left(\frac{0.0507}{R} + \frac{0.000647}{R^2} \right) V^2 = \text{charge per 100 metres.}$$

Let $a = 0.00051$,

$b = 0.0000065$; then we have, by quadratics,

$$R = \frac{a V^2}{2 i} + \left(\frac{b V^2}{i} + \frac{a V^4}{4 i^2} \right)^{\frac{1}{2}} = \text{radius of conduit or pipe.}$$

Example. Given V = velocity in metres per second = 0.30 met.

R = radius of pipe or conduit = 0.20 metres.

To find the head, height or charge in 100 metres.

$$100 \text{ m. } \left(\frac{0.0507}{.2} + \frac{0.000647}{.04} \right) 0.09 = 0.02427075.$$

$Q = S V$, that is the quantity discharged, is found by multiplying the sectional area by the velocity. The product will be cubic metres, which, if divided into cubic decimetres, will give the discharge in litres. Or, having the product $S V$, remove the decimal point three places to the right, for litres.

Example. $S = .7854$ = area of pipe in metres.

$V = 1$. = velocity per second in metres.

.001 = 1 cubic decimetre = 1 litre.

.001 divided into .7854 = 785.4 litres.

From the above, we have:

$$V = \left(\frac{R i}{0.000507 + \frac{0.00000647}{R}} \right)^{\frac{1}{2}} \text{ in metres, in terms of its radius.}$$

$$\text{Or } V = \left(\frac{D i}{.001014 + \frac{.00001294}{D}} \right)^{\frac{1}{2}} \text{ in metres, in terms of its diameter.}$$

$$V = \left(\frac{D i}{0.000809 + \frac{0.00002588}{D}} \right)^{\frac{1}{2}} \text{ in English feet.}$$

